

DARK ENERGY SURVEY VIA THE TECHNIQUE OF COUNTS OF GALAXY CLUSTERS

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## ABSTRACT

Dark Energy Survey via the technique of Counts of Galaxy Clusters, Salem Cherenet (University of Illinois at Urbana-Champaign, Urbana, IL 61801), Juan Estrada, and Jiangang Hao (Fermi National Laboratory, Batavia, IL 60510).

Recently it was discovered that the universe is expanding at an accelerated rate. This acceleration can be explained with the concept of Dark Energy. Dark Energy Survey is an imaging survey to make precise measurement of dark energy. The first part of this paper describes in detail the simulation done to estimate the equation of state parameter,  $w$  of the dark energy and its error by comparing it with a simulation bias data. All of the calculations were done using a mathematica based tool created specifically for this project. Comparing the model bias with the simulation bias data given  $\sigma_D = 0.03$  gave  $w = -0.716488 \pm 0.098$ . The second part of this paper focuses on the method used to build a GUI that is used to analyze images from CCD. The GUI is mostly stable with some bugs still to be fixed.

## INTRODUCTION

The Dark Energy Survey “is an optical imaging survey project led by Fermilab to make precision measurements of dark energy using four independent techniques: Counts of Galaxy Clusters, Weak Lensing Shear Tomography, Supernovae redshift, and Baryon Acoustic Oscillations.” [1]

There are several projects the Dark Energy Survey is involved at Fermilab. As a summer intern in the Summer Internships in Science and Technology (SIST) 2010 program I was involved in two separate projects in this group.

My first project, which is discussed in detail in part I of this paper, focuses on calculating the equation of state parameter for the dark energy,  $w$ . This was done by comparing model data with simulation done by Julia Campa who is doing her PHD thesis on the same area.

My second project focuses on creating a GUI that enables analyzing pictures taken by CCDs much easier and user friendly.

## **PART I**

### **INTRODUCTION**

#### **1.1 BACKGROUND**

The universe is a mysterious place. In studying cosmology we discover a lot of new facts, most of which not yet explained, that are weird but interesting at the same time. One of these weird facts is that the observable matter we see in the universe is only about 5% of the critical density. The rest is made of Dark matter, which is theorized to only act gravitationally, and Dark energy, which is responsible for the accelerating expansion of the universe. The prefix “Dark” in both dark matter and dark energy comes from the fact that we cannot “see” these phenomenon (at least until now), and we can only measure them by doing simulations or studying other phenomenon that are related to dark matter or dark energy.

Here is where we need to introduce the term “bias”, which is the ratio of the power spectrum of galaxy cluster to dark matter. A power spectrum is in simple terms the Fourier transform of the correlation function which is, as Grin defined it, “[a function that] determines the probability of finding two objects at separation  $r$  *in excess* of the probability one would expect for independent distribution.” [2]

That previous paragraph was loaded with new terms. For a more mathematical and detailed explanation of the correlation function and the power spectrum please refer to Daniel Grin’s paper titled [Galaxy Bias in the Halo Model](#). However, for this particular paper a different

method for obtaining the power spectrum was used; therefore I will not be discussing that. Nonetheless, I still want to elaborate a little bit more about why we need to calculate bias and what it has to do with dark energy or cosmology in general.

As stated above the bias or technically the bias square is proportional to the ratio of the power spectrums of the galaxy cluster and the dark matter. In practice it is almost impossible and very complicated to simulate the cluster of galaxy distributions accurately and find the power spectrum. Therefore, if we can get the power spectrum for a dark matter, which can be simulated accurately, then using that, we can calculate the bias which would enable us to estimate the power spectrum of the galaxy cluster. This is very important because galaxy clusters are the largest concentrations of masses in the universe and knowing how they are correlated helps building a more refined framework for different cosmological models that predict the behavior of dark energy.

For this paper we found that using the power spectrum at a different redshift than zero is very close to that of using the power spectrum at zero redshift and multiplying it by the square of the growth factor at the given redshift. Therefore, we chose to use the Growth factor just for the sake of minimizing computational memory and time.

In the past there were several different simulations and models to predict the value of  $w$ , and most of the models predict that  $w$  is approximately negative one. The significance of this is that at this  $w$  value the dark energy density parameter is constant, which implies it could be modeled as a vacuum energy. The problem arises when trying to compute this energy because the prediction from high energy physics assuming it acts like a vacuum energy are way too big when compared to what the dark energy should be. If the dark energy doesn't act like a vacuum energy, then there is no known way of computing it. In both cases we are stuck, so the question

is to create a new way of analysis in high energy physics or a totally new type of physics? And the answer lies on the value of what  $w$  is.

## 1.2 OBJECTIVE

In this project we first calculated bias values for different mass and redshift values in order to calculate the probability as a function of the dark energy equation of state,  $w$ , (which is defined as the ratio of pressure to density) so that we could determine the maximum likelihood of  $w$ , which eventually helps us to compute the error in  $w$  given an error in the measured bias data or vice versa. This paper will discuss in detail the method used in determining all the different parameters and compares the results with other previously done projects/researches.

## 1.3 EQUATIONS and CONSTANTS

The following equations were used in this paper.

$$H^2 = \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G^* \rho}{3} \quad (1)$$

$$H_0 = 2.133 * 10^{-33} eV/\hbar \quad (2)$$

$$H^2 = H_0^2 (\Omega_m (1+z)^3 + \Omega_{DE} (1+z)^{3(1+w)}) \quad (3)$$

$$\delta_{sc} = \frac{3}{5} \left(\frac{3\pi}{2}\right)^{\frac{2}{3}} * (1+z) \quad (4)$$

$$R(m) = \left(\frac{3m}{4\pi\rho}\right)^{\frac{1}{3}} \quad (5)$$

$$\sigma(m)^2 = \int_0^\infty \frac{dk}{k} \frac{k^3 P^{lin}(k)}{2\pi^2} |W(kR)|^2 \quad (6)$$

$$W(x) = \frac{3}{x^3} (\text{Sin}(x) - x\text{Cos}(x)) \quad (7)$$

$$\nu \equiv \frac{\delta_{sc}^2}{D(z)^2 * \sigma(m)^2} \quad (8)$$

$$E_1 = \frac{\frac{2*p}{\delta_{sc}}}{1+(q*\nu)^p} \quad (9)$$

$$\epsilon_1 = \frac{q*v-1}{\delta_{sc}} \quad (10)$$

$$b_1 = 1 + \epsilon_1 + E_1 \quad (11)$$

$$P(b_0, w) = \frac{1}{(2\pi\sigma_b)^{\frac{Nm}{2}}} e^{-\frac{\sum_{i=0}^{Nm} (b_{data_i} - b_0 * b_{model_i})^2}{2 * \sigma_b^2}} \quad (12)$$

$$P(w) = \int P(b_0, w) db_0 \quad (13)$$

$$b(\text{mass threshold}) = \frac{\int b_i * \frac{\rho m}{m} * f(v) dv}{\int \frac{\rho m}{m} * f(v) dv} \quad (14)$$

$$f(v) = A(p) (1 + (qv)^{-p}) \left(\frac{qv}{2\pi}\right)^{1/2} e^{-\frac{qv}{2}} \quad (15)$$

$$A(p) = \left[1 + \frac{2^{-p} \Gamma(\frac{1}{2} - p)}{\sqrt{\pi}}\right]^{-1} \quad (16)$$

Where,  $G = 6.67 * 10^{-11} \text{m}^3 \text{kg}^{-1} \text{s}^{-2}$

$$\hbar = 1.055 * 10^{-34} \text{Js}$$

$$\text{eV} = 1.602 * 10^{-19} \text{J}$$

$\Omega_{DE}$  – Dark Energy density parameter

$\Omega_m$  – mass density parameter

Z – red shift,  $w = p/\rho$ ,  $\delta_{sc}(z)$  is the critical density required for spherical collapse, m-mass, (in Ms)

Ms-mass of sun =  $1.989 * 10^{30} \text{kg}$ ,  $\rho$ -density in (Ms/Mpc), Mpc - Mega parsec =  $3.086 * 10^{24} \text{cm}$

$P^{lin}(k)$  – power spectrum  $\left(\frac{\text{Mpc}^3}{h^3}\right)$ , k (h/Mpc), h, p and q are dimensionless constants, b1-

bias, L- likelihood,  $\sigma_b$  – the error in Bias

$\sigma_w$  – error in w, D(z) = Growth factor, Nm-Sample number

b0 – a constant that is used to scale the bias

## 2. METHOD

The first step we took in this project was to calculate the matter power spectrum. As described in the introduction section we go about doing this in a slightly different and much easier way. This was achieved by using an online program called icosmos, which takes different cosmological parameters as an input and outputs a table of data for the matter power spectrum and other values such as the growth factor and the Hubble constant. Both the power spectrum and the growth factor data were found setting the cosmological parameters as follows:  $\Omega_{DE} = 0.70$ ,  $\Omega_m = 0.3$ ,  $w_o = -1$ ,  $w_a = 0$ ,  $z = 0$ ,  $\Omega_b = 0.045$ ,  $h = 0.7$ ,  $n_s = 0.95$ ,  $\sigma_8 = 0.9$ . Once the data table was obtained we imported it to mathematica and generated an interpolated function that describes the power spectrum as a function of  $k$ ,  $P^{lin}(k)$ , and the growth factor as a function of  $z$ ,  $D(Z)$  (NB. take the linear data from the table for the power spectrum).

The next step was to calculate the critical density by combining equation 1 and equation 3 then converting it to the “right” unit (i.e.  $M_s \cdot h / \text{Mpc}^3$ ). Next we used equation 6 to evaluate the value of  $\sigma(m)^2$  by substituting equation 5 for  $R(m)$  and setting  $p = 0.3$  and  $q = 0.75$ .

Figure 1 shows the GUI created in mathematica in order to calculate a bias,  $b$  as a function of mass,  $p$ ,  $q$ ,  $w$ , and redshift. This GUI exports different bias values at several masses and redshifts for a given  $w$ . Table 1 is the collection of the output values from this tool.

Once bias values were obtained, it has to be compared with a simulation or real measured bias data in order to find  $w$  and  $\sigma_w$ . In order to achieve this I developed a mathematica based tool that does just that.

Four main steps were taken in calculating a final result for  $w$ . The first step was to use equation 12 while  $b_0$  was kept constant at 1. Now, plotting the probability versus  $w$  and analyzing its maximum gives the  $w$  value. In step 1  $b_0$  was kept at 1; however, we do not know

if it is actually 1 or not. Therefore, in step 2  $w$  was kept constant and we solved for  $b_0$ . Since there were twenty one different models that correspond to different  $w$  values, twenty one different results were found for  $b_0$ . Finally the maximum of the  $b_0$  values was taken to be the actual  $b_0$ . Step 1 and step 2 are less accurate since only one parameter was variable at a given time. So in step 3 both  $b_0$  and  $w$  were left as variables. Then by using equation 13 a probability as a function of only  $w$  was obtained. The last step was to find the maximum value of this function and the error in  $w$  by analyzing the plot of  $P(w)$ .

### 3. RESULTS

Table 1 shows the bias values calculated in the analysis. The table was arranged in this particular order so that it would be easier to transfer the files to evaluate the probability. For each values of given “ $w$ ” there are different bias values that are dependent on redshift and mass combination we choose. “ $w$ ” values less than -1 were ignored since it does not really make any sense to evaluate that.

In the Ideal case where I set my model at  $w = -0.975$  as my measured, the maximum likelihood of  $w$  using equation 13 was at  $w = -0.975182$  and  $\sigma_w = 0.0715$ . Then same calculations were done for Campa's simulation data and the results are below.

For step 1 (i.e. when the probability was only dependent of  $w$  and  $b_0 = 1$ ) the maximum likelihood was at  $w = -1$  and  $\sigma_w = 0.002853$ . In Step 2 max value for  $b_0$  was 0.61224. on the third step where both  $b_0$  and  $w$  were kept as variables, plotting the probability and analyzing it resulted a maximum likelihood at  $b_0 = 0.61$  and  $w = -0.71$ . In order to get the probability just as a function of “ $w$ ” we have to take the integral of  $P(b_0, w)$  with respect to  $b_0$  (eqn 13). This was done in two different methods.

The first method was to just sum (Riemann sum) the values of  $P(b_0, w)$  over  $b_0$  then find the maximum and compute  $\sigma_w$ . with this method I found max likelihood of  $w$  at  $w = -0.725$  and  $\sigma_w = 0.0265$ . The second method was to take an interpolation of  $P(b_0, w)$  then take the integral over  $b_0$ . This method resulted a maximum likelihood of  $w$  at  $w = -0.716488$  and  $\sigma_w = 0.098$ .

Figure 4 shown in the Appendix A show the bias values listed in table 1 for  $w = -1$  graphically. The simulated bias values from Campa are displayed in Figures 2 and 3. Figures 6 and 7 depict equation 12 and 13 respectively.

#### **4. DISCUSSION and CONCLUSION**

Looking at Table 1 and both the measured and model bias plots we observe a repeating trend that the bias increases both with increase in mass and redshift values. Also taking a closer look at table reveals that for a redshift of zero, the bias is invariant with  $w$  whereas it varies with mass.

Figures 8 in Appendix A shows the estimation of the density parameter assuming  $w = -1$ . By combining the measurements and simulations done by the Union SNe Ia set, the Cosmic Microwave Background, and the Baryonic Acoustic Oscillations the density parameter for the dark energy was about 0.7 and that for mass/matter was about 0.3. The calculation carried out in this paper tried to do the reverse by setting the dark energy parameter at 0.7 and that of mass/matter at 0.3 and solving for  $w$ . The result obtained was to some degrees in agreement with Figure 8. Of course since we actually do not know what the value of  $w$  is we only expect to get a

less error in  $w$  than other experiments. In Figure 9 we see, again a combination of several simulations, trying to estimate  $w$  and  $\sigma w$ . In both the ideal case and when compared to simulated bias data our result appears to be smaller than that of shown in Figure 9 in Appendix A. Therefore, the results from this paper have results that can compete and contribute in the research of measuring  $w$ .

## **5. RECOMMENDATION**

- When calculating the bias, the first mass value of a mass threshold was used. Therefore, instead using a constant mass for the bias, using a mass threshold would result in a better final result.
- A tool for computing a bias for mass threshold is already made but it can only output one bias value at a time; so creating a GUI that would do multiple iterations would be something to consider.

## **PART II**

### **INTRODUCTION**

A charge-coupled device (CCD) is “a device for the movement of electrical charge, usually from within the device to an area where the charge can be manipulated, for example conversion into a digital value. This is achieved by "shifting" the signals between stages within the device one at a time.”[3]

As shown in Figure 1 in the Appendix B section, the camera consists of several CCDs. Each CCD contains two thousand columns and four thousand rows of pixels. So when a person takes a picture the photons hit the CCDs exciting electrons. Then by changing the voltage between the pixels, the electrons can be transferred from pixel to pixel finally reaching the readout point as shown in Figure 2 of the Appendix B section. Each Image is read out as combinations of Matrices.

The use of CCD cameras is useful in many astronomical research fields including the Dark Energy Survey. Therefore, getting accurate and good data would be really important in order to get better results, which is why we are interested in developing different tools to analyze the pictures taken by the CCDs.

## **OBJECTIVE**

Currently, the Dark Energy Survey group uses several codes to analyze the pictures taken by CCDs. Anyone who want to perform different types of analysis would have to type all the codes in the Linux terminal manually, which is time consuming and confusing for someone who is not familiar with programming and using the terminal. Therefore, the main goal of this project is to convert those codes in to a GUI, using python and TKinter, where users click buttons instead of typing codes and get the same result.

## **DISCUSSION and CONCLUSION**

The GUI I created can perform several things. Figure 3 shown in the Appendix B section shows the main window of the GUI. In this window users can choose an image file, a CCD number (each image contains several images) the min/max row/col and then get a plot of row versus median of columns (left side) or column versus median of rows (right side) that results a plot like Figure 4.

The “Bias Median” option gives the user the option of choosing a directory of several bias images and taking a median of those images. A bias image is an image taken while the shutter of the camera is covered. We want to take a picture with the shutter closed because we want to know the initial noise in the camera. Eventually, we will subtract this bias median image from the images we take while the shutter is open.

This subtraction is done using the “Image Subtraction” option under Tools. When the user clicks this button a prompt would appear asking the user if he/she has a bias median file. If the user does not have a median file, clicking “No” would open up a new window where the user inputs different information and the program would automatically create a median file and

subtract that from the image the user chose. On the other hand, if the user chooses “Yes” then a new window will appear prompting him/her to choose a median file, the image that is going to be edited and a name for the final file and the program would do the subtraction with a click of a button.

On the main window there is a button that says “Correct IMG\_Subtraction”. When taking an Image some portions of the columns (usually columns 1025-1125) on each CCD are called over scans. Over scans tell us how the image is changing. For instance, Figure 4 shows an over scan plot where the x-axis represents the rows and the y-axis represents the median of the bias. If the plot changes with rows as shown then we have to subtract that from both the image and the bias. However, if the plot is constant then we do not have to subtract the over scan.

Almost all the functions work properly and give the desired results; therefore, this project was a success.

## **RECOMENDATION**

As of now the “Correct IMG\_Subtraction” function is not working properly. Therefore fixing that problem would be a start, and adding a status bar would be a plus too.

Finally, there are still a lot of functions that are still in code format. Therefore, converting those and incorporating them into this GUI would be something one should consider in the future.

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Jiangang Hao, a post doc at Fermilab

Mr. Hao helped me by providing me links to learn python and Tkinter. He was very helpful in explaining and giving me some tutorials on how to work with python programming and was essential for the completion of my second project.

Julia Campa, a PHD student from Spain

My project on the Dark Energy Survey was mostly theoretical. Ms. Campa did the simulation that I used as my data for the bias comparison and computation of the probability.

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The first part of this paper was written mainly based on the previous work done by Cooray and Grin.

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## Appendix A

**Table 1:** Bias values for variable redshift and mass values with constant  $w$ .

mass	W = -1						
4.00E+13	0.71316378	0.72013609	0.7234765	0.72591217	0.72706295	0.726275	0.717377
6.00E+13	0.73789786	0.74075068	0.7429214	0.74575087	0.74911622	0.751304	0.740083
8.00E+13	0.76976078	0.76547206	0.76481348	0.76534222	0.76806838	0.771333	0.767886
1.00E+14	0.80972966	0.79387208	0.78972025	0.78699863	0.78730509	0.789605	0.800649
z	0	0.5	0.7	0.9	1.1	1.3	0.3
mass	W = -0.995						
4.00E+13	0.71316378	0.72004284	0.72337674	0.72580541	0.72694897	0.726162	0.717296
6.00E+13	0.73789786	0.74058117	0.74276501	0.74561098	0.74898377	0.751171	0.739928
8.00E+13	0.76976078	0.76520752	0.7645727	0.76514133	0.76789986	0.771182	0.767641
1.00E+14	0.80972966	0.79349616	0.78938333	0.78671942	0.78708143	0.789422	0.80029
z	0	0.5	0.7	0.9	1.1	1.3	0.3
mass	W = -0.985						
4.00E+13	0.71316378	0.71985563	0.72317617	0.72558988	0.72671733	0.725932	0.717136
6.00E+13	0.73789786	0.74024241	0.74245138	0.74532923	0.74871508	0.7509	0.739621
8.00E+13	0.76976078	0.76467865	0.76409008	0.76473795	0.7675595	0.770876	0.767153
1.00E+14	0.80972966	0.79274609	0.78870778	0.78615806	0.78663028	0.789051	0.799574
z	0	0.5	0.7	0.9	1.1	1.3	0.3
mass	W = -0.975						
4.00E+13	0.71316378	0.71966823	0.72297422	0.72537169	0.72648317	0.725698	0.716976
6.00E+013	0.737898	0.739904	0.742137	0.745045	0.748442	0.750625	0.739314
8.00E+013	0.769761	0.764150	0.763606	0.764331	0.767215	0.770564	0.766665
1.00E+014	0.809730	0.791996	0.788030	0.785593	0.786174	0.788674	0.79886
z	0	0.5	0.7	0.9	1.1	1.3	0.3
mass	W = -0.95						
4.00E+013	0.713164	0.719199	0.722463	0.724814	0.725881	0.725096	0.716577
6.00E+013	0.737898	0.739059	0.741346	0.744323	0.747742	0.749911	0.73855
8.00E+013	0.769761	0.762830	0.762391	0.763303	0.766334	0.769759	0.765453
1.00E+014	0.809730	0.790127	0.786328	0.784163	0.785011	0.787704	0.797087
z	0	0.5	0.7	0.9	1.1	1.3	0.3

mass	W = -0.925						
4.00E+013	0.713164	0.718729	0.721944	0.724240	0.725254	0.724465	0.716182
6.00E+013	0.737898	0.738216	0.740549	0.743584	0.747016	0.749162	0.737791
8.00E+013	0.769761	0.761514	0.761168	0.762257	0.765426	0.768918	0.764252
1.00E+014	0.809730	0.788262	0.784617	0.782712	0.783817	0.786694	0.795329
z	0	0.5	0.7	0.9	1.1	1.3	0.3
mass	W = -0.90						
4.00E+013	0.713164	0.718257	0.721416	0.723648	0.724604	0.723808	0.716182
6.00E+013	0.737898	0.737376	0.739746	0.742829	0.746263	0.748378	0.737791
8.00E+013	0.769761	0.760201	0.759940	0.761196	0.764491	0.768039	0.764252
1.00E+014	0.809730	0.786405	0.782896	0.781239	0.782612	0.785645	0.795329
z	0	0.5	0.7	0.9	1.1	1.3	0.3
mass	W = -0.875						
4.00E+013	0.713164	0.717785	0.720879	0.723039	0.723930	0.723124	0.715399
6.00E+013	0.737898	0.736539	0.738939	0.742059	0.745482	0.747555	0.736291
8.00E+013	0.769761	0.758893	0.758707	0.760118	0.763528	0.767121	0.761875
1.00E+014	0.809730	0.784554	0.781168	0.779747	0.781338	0.784555	0.791864
z	0	0.5	0.7	0.9	1.1	1.3	0.3
mass	W = -0.85						
4.00E+013	0.713164	0.717312	0.720334	0.722412	0.723231	0.722412	0.715011
6.00E+013	0.737898	0.735705	0.738127	0.741271	0.744673	0.746694	0.735555
8.00E+013	0.769761	0.757590	0.757470	0.759026	0.762538	0.766164	0.760702
1.00E+014	0.809730	0.782712	0.779435	0.778238	0.780054	0.783424	0.790156
z	0	0.5	0.7	0.9	1.1	1.3	0.3
mass	W = -0.825						
4.00E+013	0.713164	0.716839	0.719780	0.721768	0.722508	0.721672	0.714635
6.00E+013	0.737898	0.734876	0.737311	0.740469	0.743836	0.745793	0.734814
8.00E+013	0.769761	0.756293	0.756230	0.757920	0.761520	0.765167	0.759537
1.00E+014	0.809730	0.780878	0.777697	0.776712	0.778743	0.782252	0.788465
z	0	0.5	0.7	0.9	1.1	1.3	0.3
mass	W = -0.80						

4.00E+013	0.713164	0.716364	0.719217	0.721106	0.721760	0.720906	0.714246
6.00E+013	0.737898	0.734051	0.736492	0.739651	0.742970	0.744851	0.734085
8.00E+013	0.769761	0.755003	0.754990	0.756800	0.760475	0.764127	0.758383
1.00E+014	0.809730	0.779056	0.775957	0.775171	0.777402	0.781037	0.786791
z	0	0.5	0.7	0.9	1.1	1.3	0.3
mass	W = -0.775						
4.00E+013	0.713164	0.715889	0.718646	0.720426	0.720988	0.720113	0.713867
6.00E+013	0.737898	0.733232	0.735669	0.738816	0.742075	0.743868	0.733362
8.00E+013	0.769761	0.753721	0.753749	0.755675	0.759403	0.763045	0.757239
1.00E+014	0.809730	0.777244	0.774217	0.773618	0.776035	0.779781	0.785134
z	0	0.5	0.7	0.9	1.1	1.3	0.3
mass	W = -0.75						
4.00E+013	0.713164	0.715414	0.718067	0.719728	0.720193	0.719292	0.713492
6.00E+013	0.737898	0.732417	0.734843	0.737966	0.741150	0.742842	0.732645
8.00E+013	0.769761	0.752446	0.752509	0.754525	0.758303	0.76192	0.756106
1.00E+014	0.809730	0.775445	0.772477	0.772054	0.774643	0.778482	0.783493
z	0	0.5	0.7	0.9	1.1	1.3	0.3
mass	W = -0.725						
4.00E+013	0.713164	0.714938	0.717479	0.719013	0.719374	0.718446	0.713119
6.00E+013	0.737898	0.731608	0.734014	0.737101	0.740194	0.741774	0.731935
8.00E+013	0.769761	0.751180	0.751271	0.753371	0.757176	0.76075	0.754982
1.00E+014	0.809730	0.773658	0.770740	0.770481	0.773226	0.777141	0.78187
z	0	0.5	0.7	0.9	1.1	1.3	0.3
mass	W = -0.70						
4.00E+013	0.713164	0.714461	0.716882	0.718281	0.718533	0.717576	0.712749
6.00E+013	0.737898	0.730806	0.733183	0.736219	0.739208	0.740661	0.731232
8.00E+013	0.769761	0.749924	0.750036	0.752207	0.756022	0.759535	0.75387
1.00E+014	0.809730	0.771885	0.769008	0.768901	0.771785	0.775757	0.780264
z	0	0.5	0.7	0.9	1.1	1.3	0.3
mass	W = -0.675						
4.00E+013	0.713164	0.713984	0.716277	0.717532	0.717669	0.716682	0.712383
6.00E+013	0.737898	0.730009	0.732350	0.735323	0.738191	0.739506	0.730535

8.00E+013	0.769761	0.748679	0.748806	0.751034	0.754841	0.758274	0.752768
1.00E+014	0.809730	0.770128	0.767282	0.767315	0.770321	0.77433	0.778675
z	0	0.5	0.7	0.9	1.1	1.3	0.3
mass	W = -0.65						
4.00E+013	0.713164	0.713507	0.715664	0.716766	0.716785	0.715765	0.712019
6.00E+013	0.737898	0.729220	0.731515	0.734410	0.737142	0.738305	0.729845
8.00E+013	0.769761	0.747444	0.747580	0.749852	0.753633	0.756966	0.751677
1.00E+014	0.809730	0.768385	0.765563	0.765726	0.768836	0.772859	0.777104
z	0	0.5	0.7	0.9	1.1	1.3	0.3
mass	W = -0.625						
4.00E+013	0.713164	0.713029	0.715042	0.715983	0.715882	0.714828	0.711658
6.00E+013	0.737898	0.728437	0.730679	0.733482	0.736062	0.737061	0.729163
8.00E+013	0.769761	0.746221	0.746361	0.748662	0.752398	0.75561	0.750597
1.00E+014	0.809730	0.766659	0.763854	0.764135	0.767329	0.771345	0.77555
z	0	0.5	0.7	0.9	1.1	1.3	0.3
mass	W = -0.60						
4.00E+013	0.713164	0.712551	0.714413	0.715186	0.714960	0.713872	0.711301
6.00E+013	0.737898	0.727661	0.729841	0.732539	0.734951	0.735774	0.728487
8.00E+013	0.769761	0.745010	0.745149	0.747464	0.751136	0.754205	0.749528
1.00E+014	0.809730	0.764950	0.762157	0.762544	0.765803	0.769788	0.774014
z	0	0.5	0.7	0.9	1.1	1.3	0.3
mass	W = -0.575						
4.00E+013	0.713164	0.712072	0.713775	0.714374	0.714022	0.712899	0.710946
6.00E+013	0.737898	0.726893	0.729001	0.731580	0.733807	0.734444	0.727818
8.00E+013	0.769761	0.743812	0.743945	0.746260	0.749846	0.752752	0.748471
1.00E+014	0.809730	0.763259	0.760472	0.760955	0.764258	0.768186	0.772495
z	0	0.5	0.7	0.9	1.1	1.3	0.3
mass	W = -0.55						
4.00E+013	0.713164	0.711593	0.713129	0.713548	0.713070	0.711912	0.710594
6.00E+013	0.737898	0.726133	0.728161	0.730606	0.732633	0.733074	0.727157
8.00E+013	0.769761	0.742628	0.742749	0.745049	0.748530	0.751248	0.747425
1.00E+014	0.809730	0.761587	0.758802	0.759369	0.762694	0.76654	0.770994
z	0	0.5	0.7	0.9	1.1	1.3	0.3
mass	W = -0.525						

4.00E+013	0.713164	0.711113	0.712475	0.712709	0.712105	0.710913	0.710245
6.00E+013	0.737898	0.725379	0.727319	0.729616	0.731427	0.731664	0.726504
8.00E+013	0.769761	0.741458	0.741562	0.743832	0.747186	0.749695	0.74639
1.00E+014	0.809730	0.759933	0.757147	0.757786	0.761112	0.76485	0.76951
z	0	0.5	0.7	0.9	1.1	1.3	0.3
mass	W = -0.50						
4.00E+013	0.713164	0.710632	0.711814	0.711858	0.711129	0.709905	0.709899
6.00E+013	0.737898	0.724634	0.726476	0.728611	0.730191	0.730217	0.725857
8.00E+013	0.769761	0.740302	0.740385	0.742608	0.745815	0.748092	0.745367
1.00E+014	0.809730	0.758301	0.755511	0.756210	0.759513	0.763115	0.768044
z	0	0.5	0.7	0.9	1.1	1.3	0.3

**Table 3:** Simulated Bias data values for Julia Campa

Mass threshold	Bias Square at z =											
	0.3	$\sigma_b$	0.5	$\sigma_b$	0.7	$\sigma_b$	0.9	$\sigma_b$	1.1	$\sigma_b$	1.3	$\sigma_b$
4.00E+013 1E+015	1	0	1.01	0.04	1.12	0.04	1.23	0.02	1.41	0.07	1.48	0.1
6.00E+013 1E+015	1.08	0.05	1.13	0.07	1.25	0.06	1.35	0.07	1.6	0.1	1.65	0.1
8.00E+013 1E+015	1.28	0.07	1.3	0.07	1.16	0.1	1.64	0.1	1.94	0.1	2.05	0.2
1.00E+014 1E+015	1.5	0.1	1.47	0.1	1.71	0.1	1.84	0.1	2.51	0.3	2.52	0.4

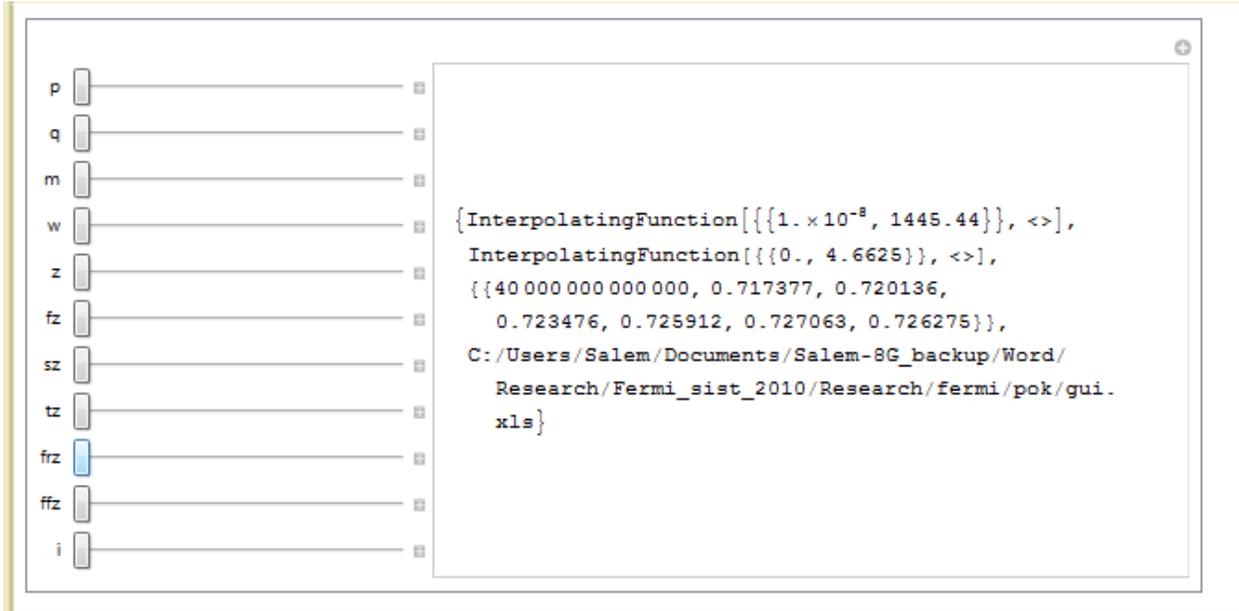


Figure 1: GUI for calculating Bias as a function of p, q, mass, w and redshift

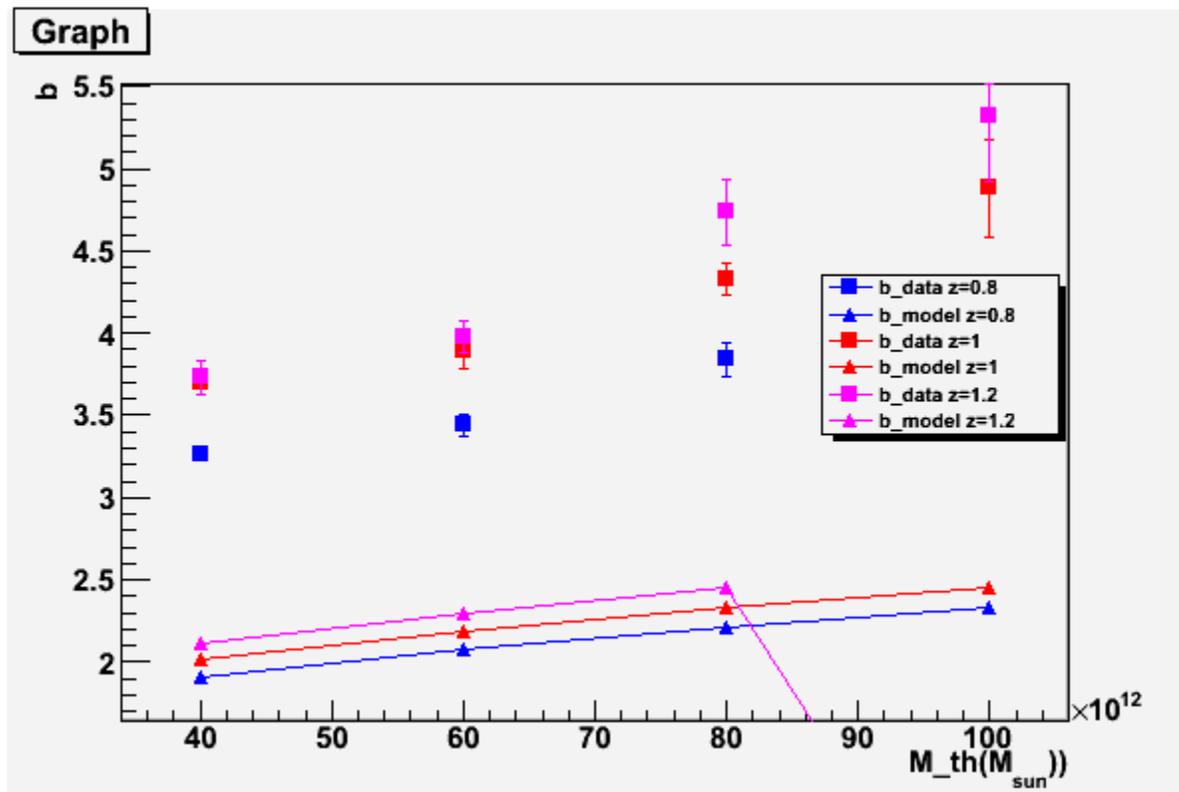


Figure 2: Measured Bias Value from Julia Campa

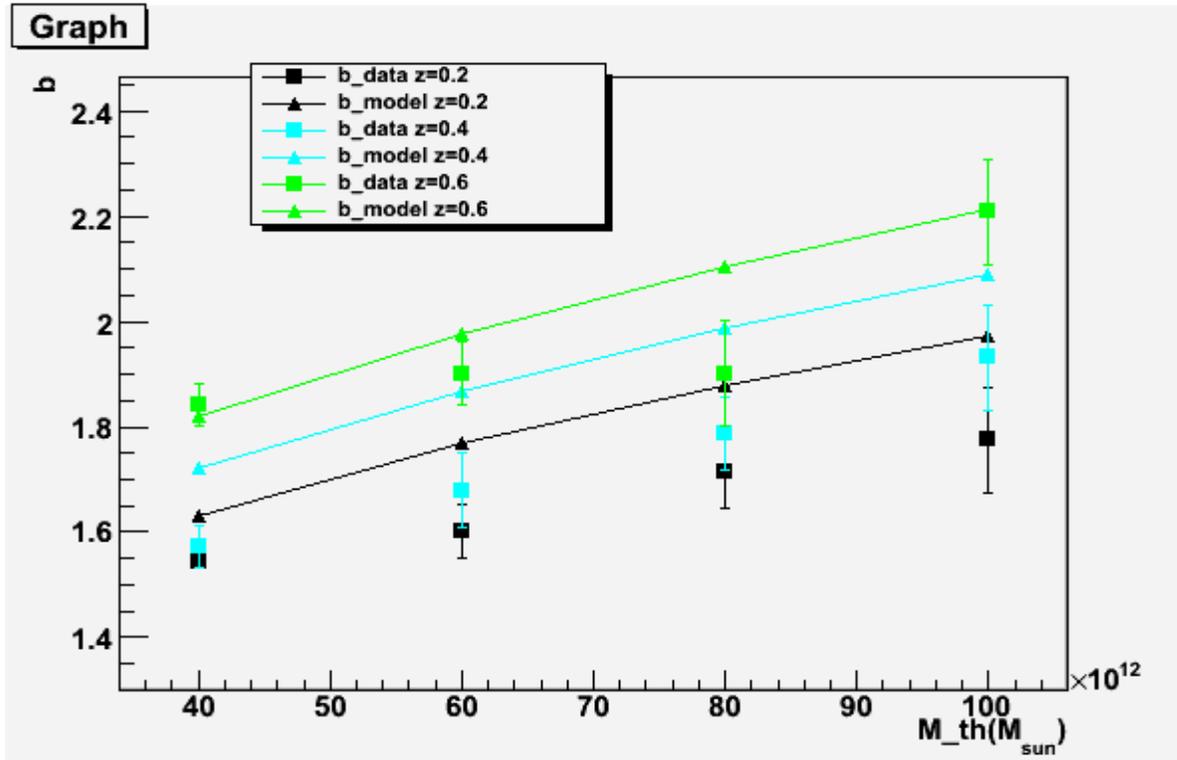


Figure 3: Measured Bias Value from Julia Campa

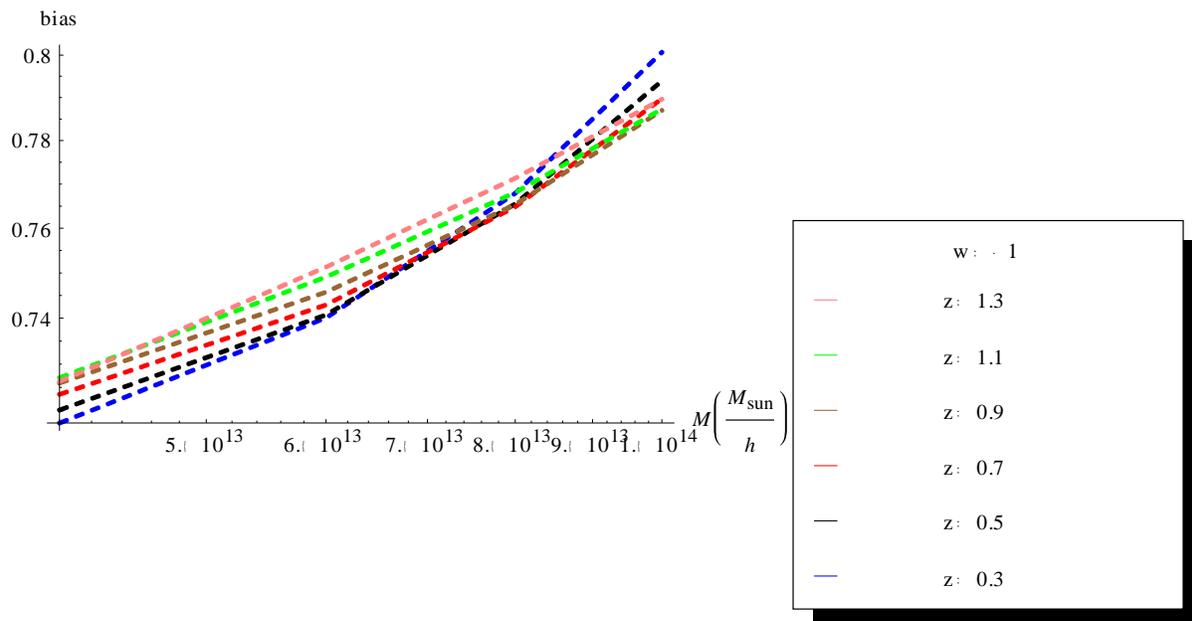


Figure 4: Bias values as a function of mass for different redshift values and  $w = -1$

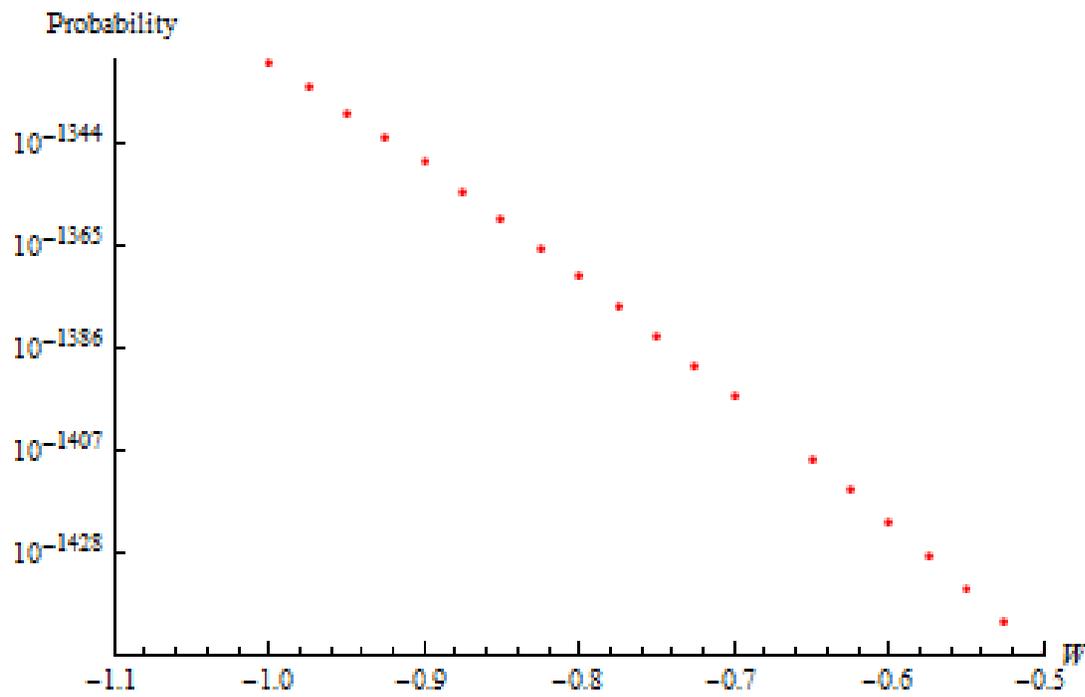


Figure 5:  $P(b_0, w)$  at  $b_0 = 1$

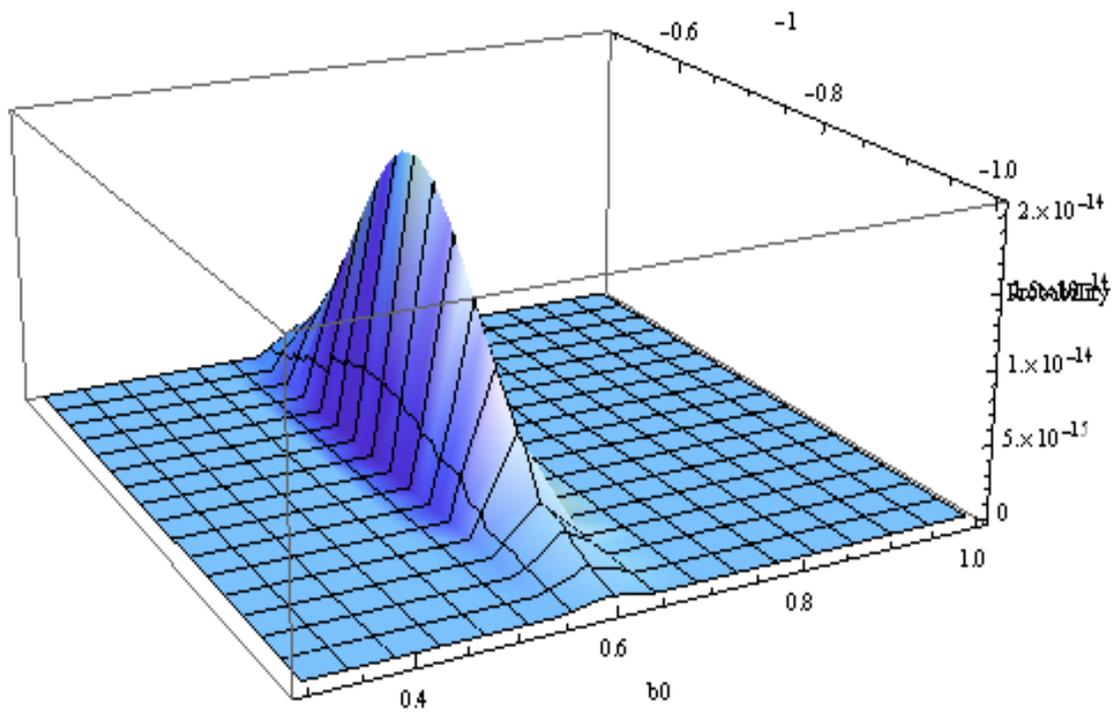


Figure 6:  $P(b_0, w)$  where both  $b_0$  and  $w$  are variables.

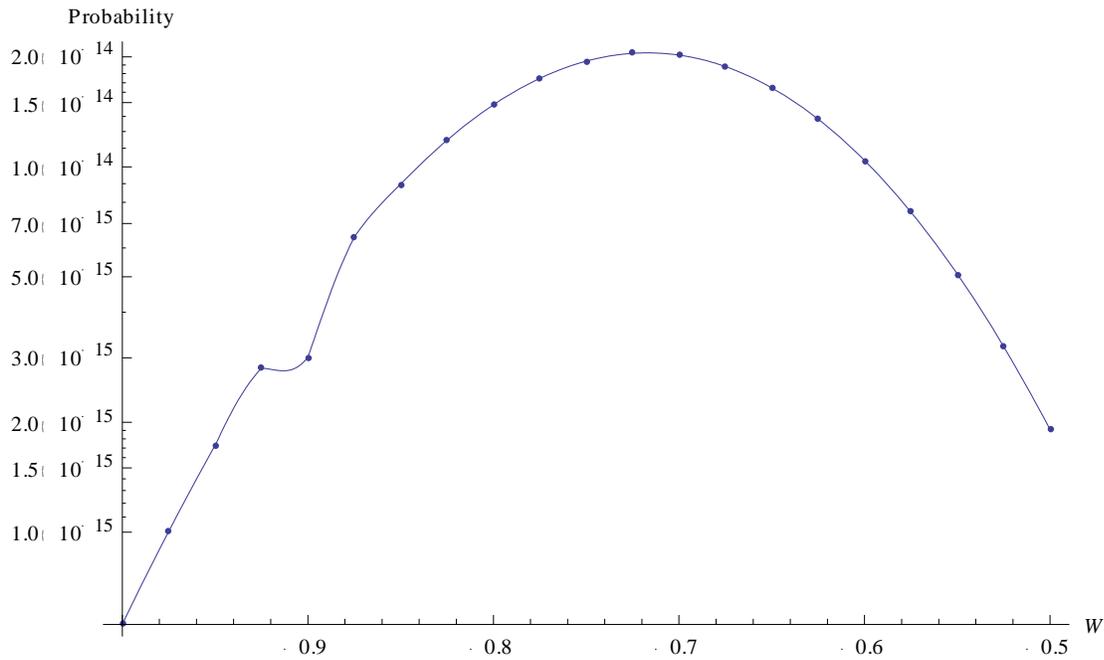


Figure 7:  $P(w) = \int P(b_0, w) db_0$  with iteration function.

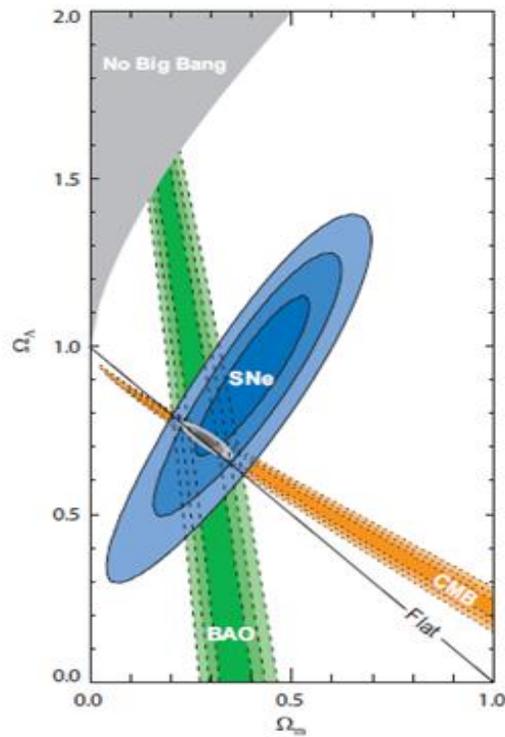


Figure 8 : Confidence level contours of 68.3%, 95.4% and 99.7% in the  $\Omega_\Lambda$ - $\Omega_m$  plane from the Cosmic Microwave Background, Baryonic Acoustic Oscillations and the Union SNe Ia set, as well as their combination (assuming  $w = -1$ ). [Courtesy of Kowalski *et al.* [22]]

[4]

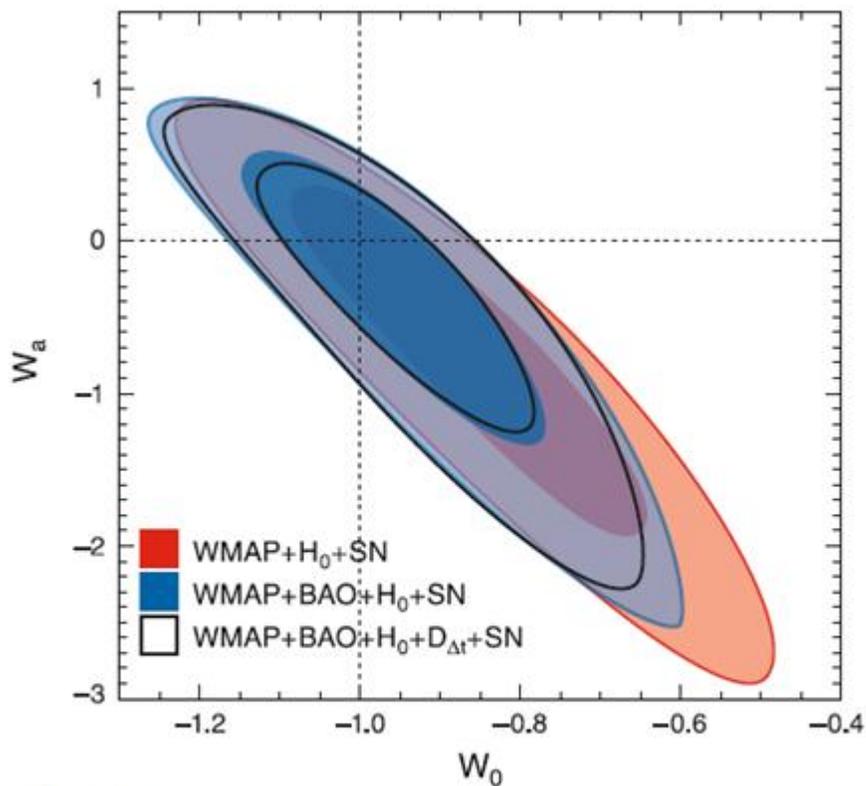


Figure 9.— Joint two-dimensional marginalized constraint on the linear evolution model of dark energy equation of state,  $w(a) = w_0 + w_a(1 - a)$ . The contours show the 68% and 95% CL from WMAP+ $H_0$ +SN (red), WMAP+BAO+ $H_0$ +SN (blue), and WMAP+BAO+ $H_0$ + $D_{\Delta t}$ +SN (black), for a flat universe.

[5]

## Appendix B

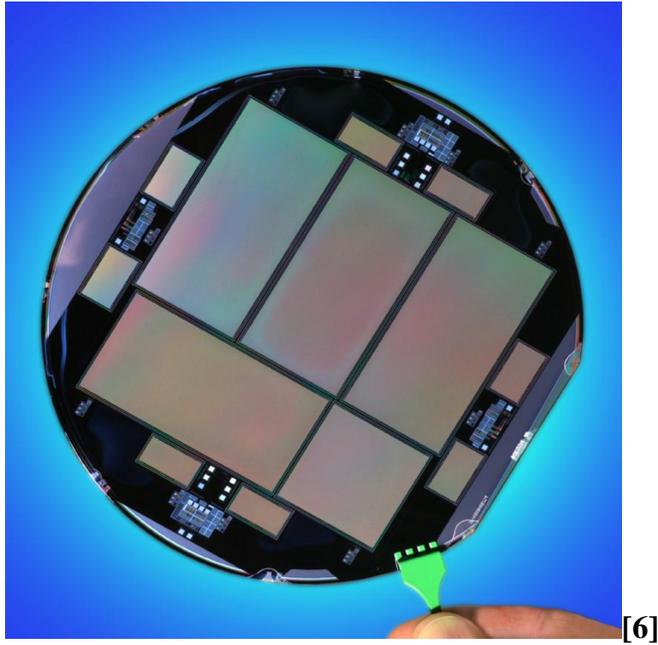


Figure 1: A CCD Camera

7 pixels

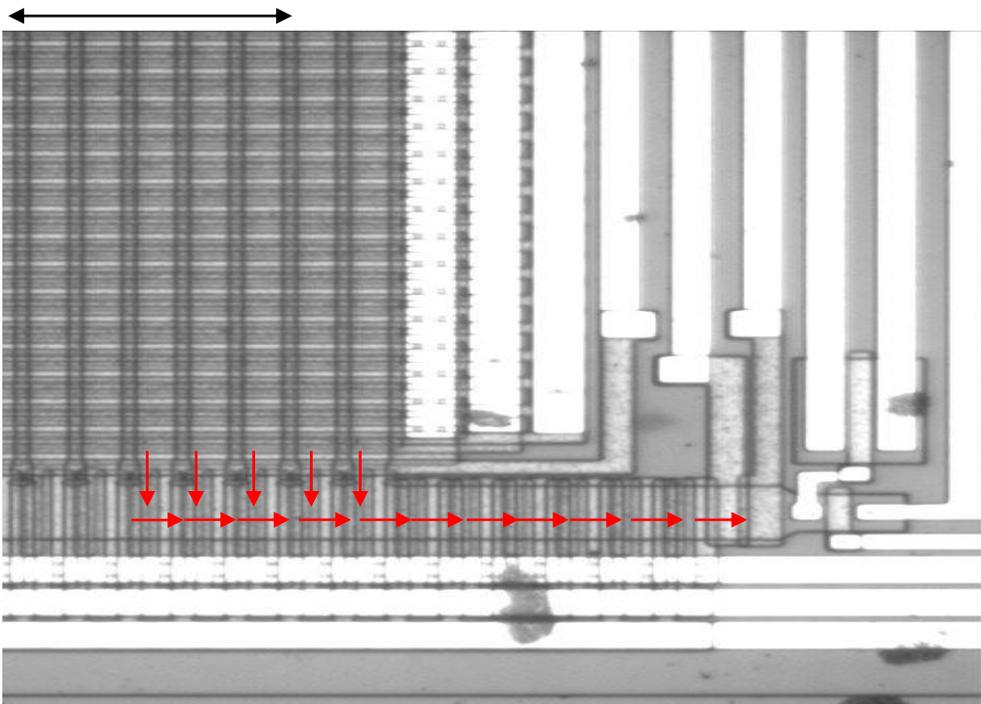


Figure 2: the red arrows showing the flow of electrons from pixel to pixel

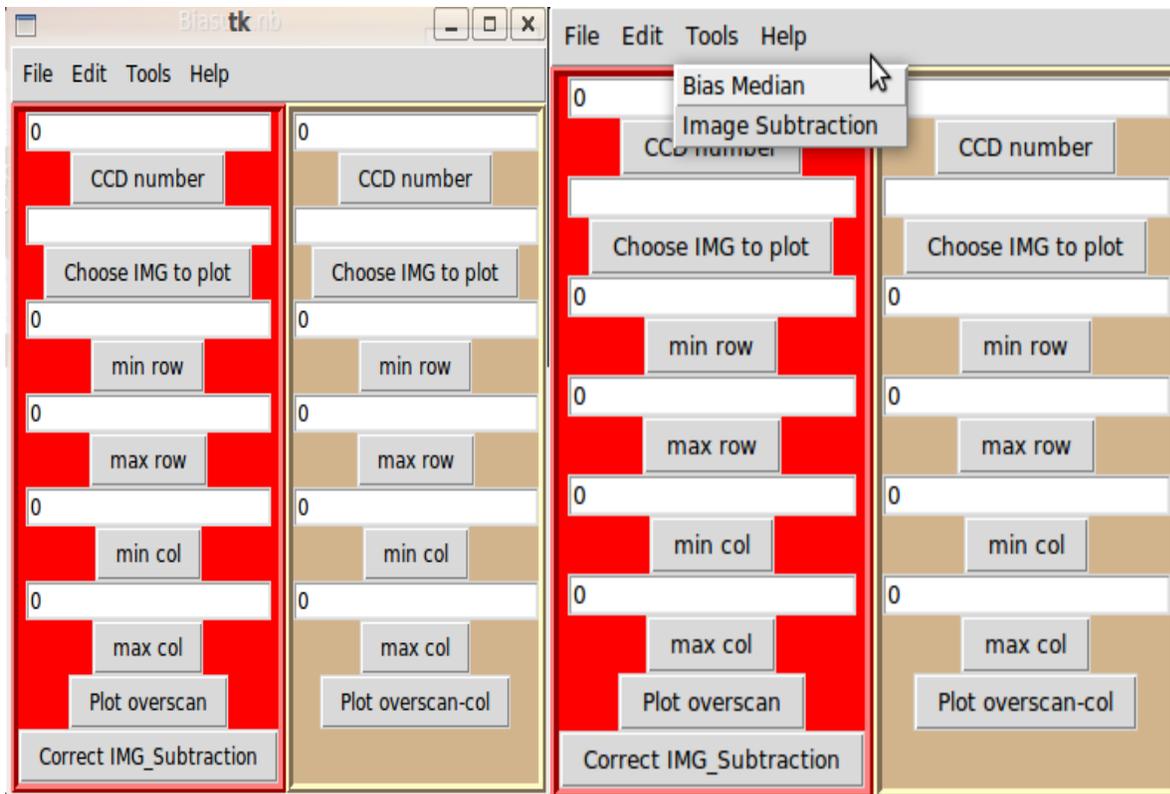


Figure 3: Snap shots of the main window of the GUI for analyzing images from CCD

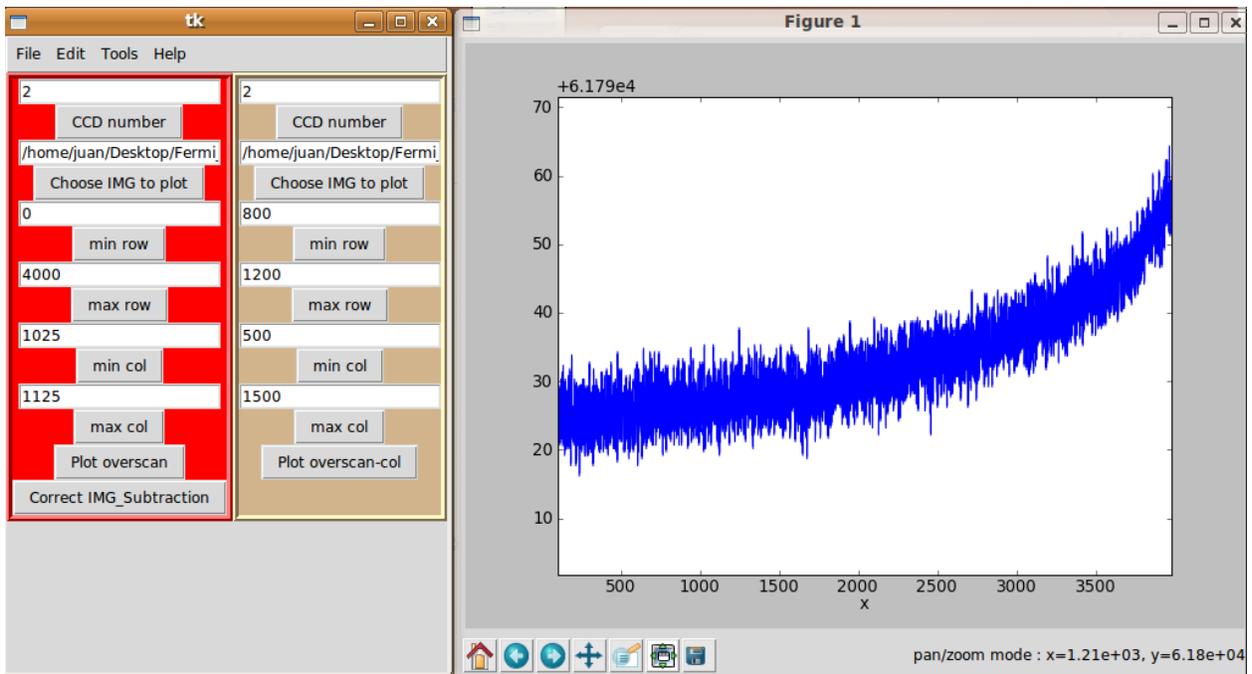


Figure 4: Snap shot of the over scan plot generated by the GUI

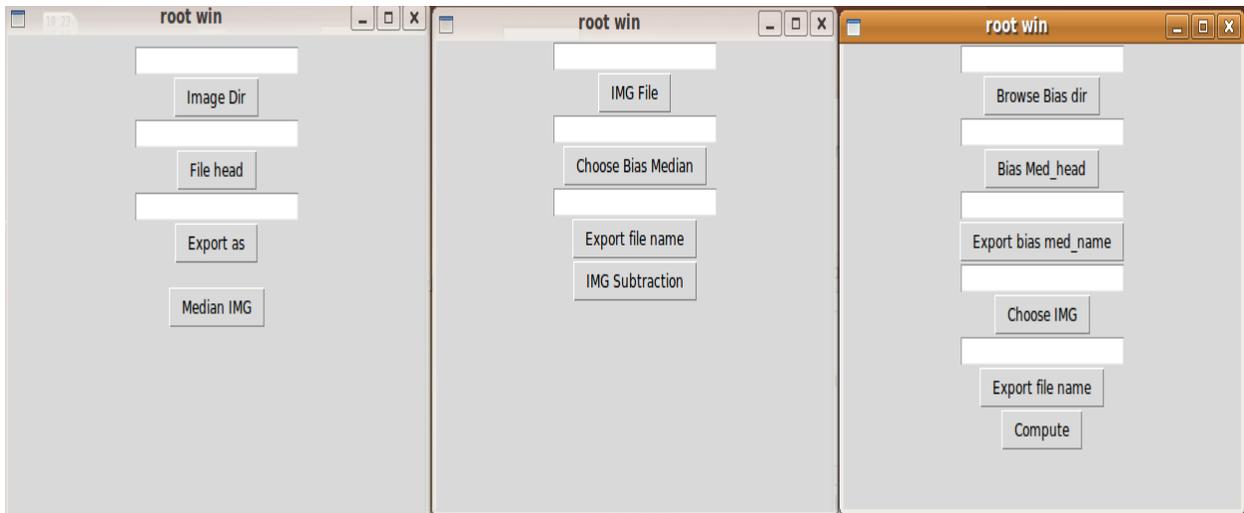


Figure 5: Median creator window, Image Subtraction window given a median image, Image Subtraction window given no median Image (left to right)