Introduction to the SM

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Last time

- homeworks and notes
- Model building and axioms
- Once we have $\mathcal{L}$, how do we measure its parameters and probe the theory?

The gauge interactions

The gauge part

$$SU(3)_C \times SU(2)_L \times U(1)_Y \ \to \ SU(3)_C \times U(1)_{EM}$$

Three parts, each look so different...

- QED - photon interaction: Perturbation theory
- QCD - gluon interaction: Confinement and asymptotic freedom
- Electro-weak: SSB and massive gauge bosons

QED

Lets “built” a simple QED model based on our rules

- Gauge group: $U(1)$
- Fields: E_L and E_R with charges -1 and $+1$
- No scalars and no SSB

The most general renormalizable Lagrangian

$$\mathcal{L} = \overline{E_L} i \not{D} E_L + \overline{E_R} i \not{D} E_R - m \overline{E_L} E_R - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

$$= \overline{E_L} (i \not{\partial} - q \not{A}) E_L + \overline{E_R} (i \not{\partial} - q \not{A}) E_R - m \overline{E_L} E_R - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

$$= \overline{E} (i \not{\partial} - q \not{A} - m) E - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

Remarks

$$\mathcal{L} = \overline{E}(i\not{D} - m)E - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

- The interaction term is part of the kinetic term. Universality!
- In QED we can work with 4-components fields
- The electron has a mass
- We call such theory “vector”. This is in contrast to a “chiral” theory
- Can you think about a chiral theory of QED?

An aside: small electron mass

In QED the electron mass is a free parameter. So we measure it. What do we expect?

- It is a free parameter. We do not expect anything
- Well, we know there is a "UV cutoff" where new theory come in (BTW, what is this new theory?)
- The electron mass is "technically natural." If it were zero we will have an enhanced symmetry
- The enhance symmetry is "chiral symmetry." E_L and E_R rotate differently

QCD

Lets “built” a simple QCD model based on our rules

- Gauge group: $SU(3)$
- Fields: q_L and q_R. Both are triplets of $SU(3)$
- No scalars and no SSB

The most general renormalizable Lagrangian

$$\mathcal{L} = \overline{q}(i\not{D} - m_q)q - \frac{1}{4}G^{\mu\nu}G_{\mu\nu}$$

QCD: remarks

$$\mathcal{L} = \overline{q}(i\not{D} - m_q)q - \frac{1}{4}G^{\mu\nu}G_{\mu\nu}$$

- It looks just like QED. And yes, it is very much the same
- There are 8 gluons DOFs. Can we tell them apart?
- There are gluon self interactions. Very important
- Running is important. Asymptotic freedom and confinement
- Dynamical generated scale, $\Lambda_{QCD} \sim \text{few} \times 10^2\ \text{MeV}$
- Question: What about the color singlet “gluon”?

$SU(2) \times U(1)$ and leptons

Electroweak theory

Lets “built” a simple EW model for leptons

- Gauge group: $SU(2) \times U(1)$
- Fields:

$$L_L(2)_{-1/2} \qquad E_R(1)_{-1}$$

- One scalar $\phi(2)_{1/2}$, with negative $\mu^2\phi^2$ term

The most general renormalizable Lagrangian

$$\mathcal{L} = \mathcal{L}_{\rm kin} + \mathcal{L}_{\rm Yuk} + \mathcal{L}_{\rm SSB}$$

SSB

Very central idea!

- Magnets
- Donkeys
- The lowest energy state “breaks” the symmetry
- QFT: We always expand around the minimum. Very central idea of QFT

Why do we do it all?

- OK, so in the SM we have SSB.
  - Q: Why can't we just write a theory with masses and no Higgs?

Why do we do it all?

- OK, so in the SM we have SSB.
  - Q: Why can't we just write a theory with masses and no Higgs?
  - A: We can get SSB without the Higgs!
  - What we must have is SSB, cannot write explicit mass terms in $\mathcal{L}$

SSB in the SM

The Higgs mechanism (In fact the Englert and Brout; Higgs; Guralnik, Hagen, and Kibble mechanism)

$$\mathcal{L}_{\rm SSB} = -\lambda(\phi^2 - v^2)^2$$

- The minimum is $\phi = v$
- ϕ has 4 DOFs. We can choose

$$\langle\phi_1\rangle = \langle\phi_2\rangle = \langle\phi_4\rangle = 0 \quad \langle\phi_3\rangle = v$$

- We call the remaining symmetry EM. The fact that the vev is for the neutral component is by definition

A bit more about the vev

What is this vev? In a way it is like a Lorentz invariance background

- The old idea of “aether” is coming back
- Think about matter effects. Very similar but only at the quantum level

The gauge sector

$$D^\mu = \partial^\mu + igW_a^\mu T_a + ig' B^\mu Y$$

- There are 4 generators
- There are 2 couplings
- We choose $Q = T_3 + Y$
- Is it a prediction that the vev is for ϕ^0?
- What is Q for the leptons? Recall: $L_L(2)_{-1/2}$, $E_R(1)_{-1}$

$$Q(L_L) = \pm 1/2 - 1/2 = 0, -1 \qquad Q(E_R) = 0 - 1 = -1$$

Nice...

Gauge boson masses

From the kinetic term of the Higgs we get mass for the gauge bosons

$$|D^\mu \phi|^2 \sim \frac{1}{8} \left| \begin{pmatrix} gW_3 + g'B & g(W_1 - iW_2) \\ g(W_1 + iW_2) & -gW_3 + g'B \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix} \right|^2$$

which gives for mass terms

$$\frac{1}{4} g^2 v^2 W^+ W^- + \frac{1}{8} v^2 (gW_3 - g'B)^2$$

Masses

Define the mass eigenstates

$$W^{\pm} = \frac{1}{\sqrt{2}}(W_1 \mp iW_2)$$
$$Z = \cos\theta_W W_3 - \sin\theta_W B$$
$$A = \sin\theta_W W_3 + \cos\theta_W B$$
$$\tan\theta_W \equiv \frac{g'}{g}$$

The masses are

$$M_W^2 = \frac{1}{4}g^2 v^2 \qquad M_Z^2 = \frac{1}{4}(g^2 + g'^2)v^2 \qquad M_A^2 = 0$$

We have a rotation from W_3, B to the mass basis Z, A

Remarks

- $W^{\pm}$ are charged under EM. A and Z are not
- We have a mechanism for $SU(2)_L \times U(1)_Y \to U(1)_{EM}$
- $M_A^2 = 0$ is not a prediction, it is a consistency check on our calculation
- Note that we get the following testable relation:

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1$$

- Out of the four scalar degrees of freedom, three are the would-be Goldstone bosons eaten by the $W_{\pm}$ and Z, and one is the physical Higgs boson with $m_H^2 = 2\lambda v^2$

$\rho = 1$

Very non-trivial prediction:

$$\frac{M_W^2}{M_Z^2} = \frac{g^2}{g^2 + g'^2}$$

- Tested experimentally
- $\rho = 1$ is a prediction of the SM with a Higss doublet
- Quantum corrections
- Related to a symmetry: Custodial symmetry

Interactions

Interactions

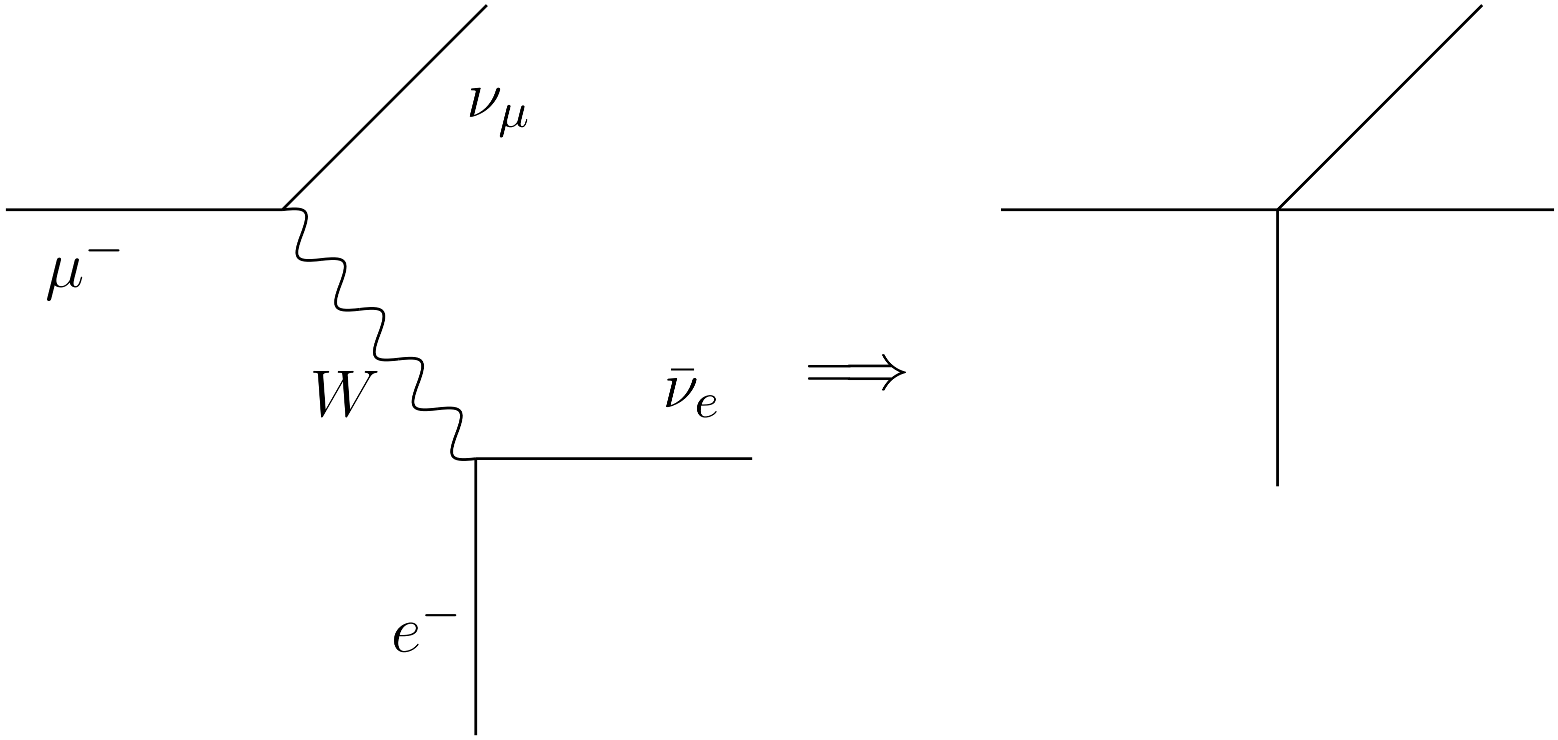
$$-\frac{g}{\sqrt{2}}\,\overline{\nu_{eL}}\,W^{\mu}\gamma_{\mu}e_L^{-}+h.c.$$

- Only left-handed particles take part in charged-current interactions. Therefore the W interaction violate parity
- Universality: the couplings of the W to $\tau\bar{\nu}_\tau$, to $\mu\bar{\nu}_\mu$ and to $e\bar{\nu}_e$ are equal
- At low energy we can “integrate out” the W

$$G_F=\frac{\sqrt{2}g^2}{8M_W^2}=\frac{1}{\sqrt{2}v^2}$$

- Almost direct measurement of the vev, $v = 246\ GeV$
- Instead of g, g', v we can use $G_F, m_Z, \sin^2\theta_W$

Muon decay



$$\mathcal{A} \sim \frac{g^2}{p^2 - m_W^2} \sim \frac{g^2}{m_W^2} \sim G_F$$

NR and the Fermi theory

Muon decay and G_F is a prime example of NR terms.

- The idea of NR terms!
- Cannot calculate loop corrections, but we may not need it

Neutral currents

$$\mathcal{L}_{\rm int} = \frac{e}{\sin\theta\cos\theta}(T_3 - \sin^2\theta_W Q)\,\bar\psi \not{Z} \psi\,,$$

- Photon and Z. The Z is the extra stuff
- Both LH and RH coupling. Still Z is parity violating
- Diagonal couplings. No flavor violation at tree level

Experimental tests

Of course, the model was built from experimental data...

- High energy: Open your pdg and check W and Z decays to leptons. What do you expect to see?
- Z decays to lepton actually measures $\sin^2\theta_W \approx 0.23$
- Based on universality, what do we expect for μ vs τ decays?
- More low energy data:
  - pion decay: proof of spin one nature of the weak interaction
  - neutrino scattering: proof of the left-handedness of it

Neutrino scattering

$$\sigma(\nu e^- \to \nu e^-) = \frac{G_F^2 s}{\pi} \qquad \sigma(\bar{\nu} e^- \to \bar{\nu} e^-) = \frac{G_F^2 s}{3\pi}$$

- Note the factor of 3
- Think about backward scattering:
  - νe: Both LH and thus, $J_Z = 0$ before and after. Can go
  - $\bar{\nu} e$: One LH and one RH: $J_Z = +1$ before and $J_Z = -1$ after. Cannot go.

Some summary

- The SM gauge sector has three parts:
  - QED: perturbation theory
  - QCD: Confinement and asymptotic freedom
  - Electroweak: SSB, masses and parity violation

  Gauge interactions are universal!