Introduction to the SM

Yuval Grossman

Cornell

Last time

- Gauge interactions: QED, QCD and EW sectors
- SSB: Must have it
- Charged current interactions

This lecture: Neutral current interaction and flavor

Answer to HW

Write NR operators that violate B and/or L

- For L we get dim 5

$$\frac{HHLL}{\Lambda}$$

Gives neutrino masses. We will talk on it later

- For B we get dim 6

$$\frac{QQQL}{\Lambda^2}$$

Give proton decay. Plan to talk on it tomorrow

Neutral currents

Neutral currents

$$\mathcal{L}_{\rm int} = \frac{e}{\sin\theta\cos\theta}(T_3 - \sin^2\theta_W Q)\,\bar\psi \not{Z} \psi\,,$$

- Photon and Z. The Z is the extra stuff
- Both LH and RH coupling. Still Z is parity violating
- Diagonal couplings. No flavor violation at tree level
- Processes involving the Z can be used to measure $\sin^2\theta_W$
- Together with m_W and $G_F = \sqrt{2}g^2/8m_W^2$ we can get the two parameters of the model, g and g'

Experimental tests

$$\mathcal{L}_{\rm int} \sim (T_3 - \sin^2\theta_W Q)\, \bar\psi \not{Z} \psi \,,$$

Of course, the model was built from experimental data...

- Calculate the ratios (use $\sin^2\theta_W = 0.25$)

$$R_1 = \frac{\Gamma(Z \to e_L \bar e_L)}{\Gamma(Z \to e_R \bar e_R)} \qquad R_2 = \frac{\Gamma(Z \to e \bar e)}{\Gamma(Z \to \nu \bar\nu)}$$

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- We get, in agreement with the data

$$R_1 \approx \frac{(1/2-1/4)^2}{(1/4)^2} = 1 \qquad R_2 \approx \frac{(1/4)^2+(1/4)^2}{3\times(1/2)^2} = \frac{1}{6}$$

- In fact, these Z decays gives $\sin^2\theta_W \approx 0.23$

More tests

- High energy: Open your pdg and check W and Z decays to leptons. What do you expect to see for

$$\frac{\Gamma(W^+ \to e^+\nu)}{\Gamma(W^+ \to \mu^+\nu)} \qquad \frac{\Gamma(Z \to e^+e^-)}{\Gamma(Z \to \mu^+\mu^-)}$$

- Low energy data: For example,
  - tau decays

$$\frac{\Gamma(\tau \to e\nu\bar{\nu})}{\Gamma(\mu \to e\nu\bar{\nu})}$$

  - $\Gamma(\pi \to \ell\nu)$ proof of spin one nature of the weak interaction
  - neutrino scattering: proof of the left-handedness of it

Neutrino scattering

$$\sigma(\nu e^- \to \nu e^-) = \frac{G_F^2 s}{\pi} \qquad \sigma(\bar{\nu} e^- \to \bar{\nu} e^-) = \frac{G_F^2 s}{3\pi}$$

- Note the factor of 3
- Think about backward scattering:
  - νe: Both LH and thus, $J_Z = 0$ before and after. Can go
  - $\bar{\nu} e$: One LH and one RH: $J_Z = +1$ before and $J_Z = -1$ after. Cannot go

Gauge sector: summary

- 3 groups, very different
- A lot tests on the model. Pass them all
- Electroweak precision measurements. A lot was done at LEP and Tevatron

Fermions

Lepton masses

- In a chiral theory fermions are massless
- In the SM they get mass from the interactions with the Higgs
- For leptons only the charged leptons get a mass. We need both LH and RH fields for a mass

$$Y_{ij}\left(\bar{L}_L\right)_i \phi\left(E_R\right)_j \to Y_{ij}\, v\, \bar{e}_L e_R + ...$$

- The mass is proportional to the Yukawa coupling and the vev $m = Yv$
- For leptons we can choose Y to be diagonal in flavor space and we get the known lepton masses

Quarks

$$Y_{ij}^{D}\left(\bar{Q}_{L}\right)_{i}\phi\left(D_{R}\right)_{j}+Y_{ij}^{U}\left(\bar{Q}_{L}\right)_{i}\tilde{\phi}\left(U_{R}\right)_{j}$$

- The Yukawa matrix, Y_{ij}^{F}, is a general complex matrix
- After the Higss acquires a vev, the Yukawa terms give masses to the fermions. Also, after the breaking we can talk about U_L and D_L, not about Q_L
- If Y is not diagonal, flavor is not conserved (soon we will go over the subtleties here)
- If Y carries a phase, CP is violated (soon we will understand). C and P is violated to start with

CP violation

A simple “hand wave” argument of why CP violation is given by a phase

- It is all in the $+h.c.$ term

$$Y_{ij}\left(\bar{Q}_L\right)_i \phi \left(D_R\right)_j + Y_{ji}^* \left(\bar{D}_R\right)_j \phi^\dagger \left(Q_L\right)_i$$

- Under CP

$$Y_{ij}\left(\bar{D}_R\right)_j \phi^\dagger \left(Q_L\right)_i + Y_{ji}^* \left(\bar{Q}_L\right)_j \phi \left(D_R\right)_i$$

- CP is conserved if $Y_{ij} = Y_{ij}^*$
- Not a full proof, since there is still a basis choice...

The CKM matrix

It is all about moving between bases...

- We can diagonalize the Yukawa matrices

$$Y_{diag} = V_L Y V_R^\dagger, \qquad V_L, V_R \text{ are unitary}$$

- The mass basis is defined as the one with Y diagonal, and this is when

$$(d_L)_i \to (V_L)_{ij}(d_L)_j, \qquad (d_R)_i \to (V_R)_{ij}(d_R)_j$$

- The couplings to the photon is not modifies by this rotation

$$\mathcal{L}_\gamma \sim \bar{d}_i \delta_{ij} d_i \;\to\; \bar{d}_i V \delta_{ij} V^\dagger d \sim \bar{d}_i \delta_{ij} d_i$$

CKM, W couplings

- For the W the rotation to the mass basis is important

$$\mathcal{L}_W \sim \bar{u}^i_L \delta_{ij} d^i_L \;\to\; \bar{u}_i V_L^U \delta_{ij} V_L^{D\dagger} d \sim \bar{u}_i V_{CKM} d_i$$

where

$$V_{CKM} = V_L^U V_L^{D\dagger}$$

- The point is that we cannot have Y_U, Y_D and the couplings to the W diagonal at the same basis
- In the mass basis the W interaction change flavor, that is flavor is not conserved

CKM: Remarks

$$V_{CKM} = V_L^U V_L^{D\dagger}$$

- V_{CKM} is unitary
- The CKM matrix violates flavor only in charge current interactions, for example, in transition from u to d

$$V_{us}\, \bar{u}\, s\, W^+,$$

- In the lepton sector without RH neutrinos $V = 1$ since V_L^ν is arbitrary. This is in general the case with degenerate fermions
- When we add neutrino masses the picture is the same as for quarks. Yet, for leptons it is usually not the best to work in the mass basis

FCNC

FCNC=Flavor Changing Neutral Current

- Very important concept in flavor physics
- Important: Diagonal couplings vs universal couplings

FCNCs

In the SM there are no FCNCs at tree level. Very nice! In Nature FCNC are highly suppressed

- Historically, $K \to \mu\nu$ vs $K_L \to \mu\mu$
- The suppression was also seen in charm and B
- In the SM we have four neutral bosons, g, γ, Z, h. Their couplings are diagonal
- The reasons why they are diagonal, and what it takes to have FCNC, is not always trivial
- Of course we have FCNC at one loop (two charged current interactions give a neutral one)

Photon and gluon tree level FCNC

- For exact gauge interactions the couplings are always diagonal. It is part of the kinetic term

$$\partial_\mu \delta_{ij} \to (\partial_\mu + iqA_\mu)\delta_{ij}$$

- Symmetries are nice...
- In any extension of the SM the photon couplings are flavor diagonal

Higgs tree level FCNC

- The Higgs is a possible source of FCNC. With one Higgs doublet, the mass matrix is align with the Yukawa

$$\mathcal{L}_m \sim Y\, v\, \bar{d}_L d_R \qquad \mathcal{L}_{int} \sim Y H \bar{d}_L d_R$$

- With two doublets we have tree level FCNC

$$\mathcal{L}_m \sim \bar{d}_L (Y_1 v_1 + Y_2 v_2) d_R \qquad \mathcal{L}_{int} \sim H_1 \bar{d}_L Y_1 d_R$$

- There are “ways” to avoid it, by imposing extra symmetries

Z exchange FCNC

- For broken gauge symmetry there is no FCNC when: “All the fields with the same irreps in the unbroken symmetry also have the same irreps in the broken part”
- In the SM the Z coupling is diagonal since all $q = -1/3$ RH quarks are $(3,1)_{-1/3}$ under $SU(2) \times U(1)$
- What we have in the couplings is

$$\bar{d}_i\,(T_3)_{ij}\,d_j \;\to\; \bar{d}\,V\,(T_3)_{ij}\,V^\dagger d_j, \qquad VT_3V^\dagger \propto I \text{ if } T_3 \propto I$$

- Adding quarks of different irreps generate tree level FCNC Z couplings
- It is the same for new neutral gauge bosons (usually denoted by Z')

A little conclusion

- In the SM flavor is the issue of the 3 generations of quarks
- Flavor is violated by the charged current weak interactions only
- There is no FCNC at tree level. Not trivial, and very important
- All flavor violation is from the CKM matrix