

$$\int dy \left(\Phi^\dagger \Phi \right)^2 = \sum_{n_1, n_2, n_3, n_4} e^{-i(n_1 - n_2 + n_3 - n_4)y/R} \Phi(n_1)^\dagger \Phi(n_2) \Phi(n_3)^\dagger \Phi(n_4)$$