Accelerator Homework, Part 2

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1. Below is an example of a simple dipole magnet.

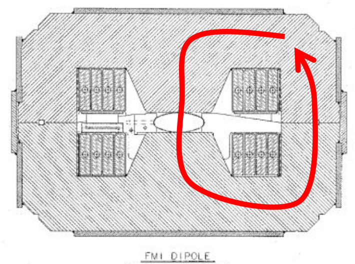

Figure 3.1-2. Cross-section of the Main Injector Dipole Magnet.

It has a pole face width w (ignore the beveling near the coils), a gap g, a length l and is wound with N turns of conductor. Assuming the permeable laminations have $\mu >> \mu_0$, show that the magnetic field in the gap is

$$B \approx \mu_0 \frac{NI}{g}$$

and calculate the inductance.

Integrating along the path shown, Ampere's Law gives us

$$\oint \vec{H} \cdot d\vec{l} = \oint \frac{\vec{B}}{\mu} \cdot d\vec{l} \approx \frac{B}{\mu_0} g = NI$$

$$\Rightarrow B \approx \mu_0 \frac{NI}{g}$$

The inductance will be given by

$$L = \frac{N\Phi}{I} = \mu_0 \frac{N^2 l w}{g}$$

2. In order to extract the beam from the8 GeV Fermilab Booster , one needs to bend it by about 50 mrad to clear the nearest downstream magnet. Let's assume we want to do this with a single two meter long magnet. Use:
   a. A single turn (N=1)
   b. g=w=6 cm to provide the required beam aperture

   Calculate the magnetic field and current required. Also calculate the inductive voltage to ramp to this field with the present rise time of 40 ns.

To extract with a single kicker, we need an integrated field given by

$$Bl \approx \theta(B\rho) = (.050)(29.7) \approx 1.5 \text{ Tm}$$

So a 2 m magnet would need a field of .75 Tm. If the gap is 6 cm, then, using the formula above, this would require a current of about 36 kA.

The inductance of the 2 m magnet is 2.5 μH. The inductive voltage

$$V = L\frac{dI}{dt} = L\frac{I_0}{\tau} = (2.5 \times 10^{-6})\frac{36000}{40 \times 10^{-9}} = 2.25 \text{ MV}$$

Ouch!! The problem is actually even a little bit worse because you would need to increase the vertical aperture by at least a few cm for the extracted beam to clear.

3. Fun with antimatter… The Fermilab antiproton source holds the record for antiproton production, by a good margin. Its record average stacking rate over 1 week is $2.5\text{x}10^{11}$ pBar/hour. At this rate

   a. How long would it take to make enough antiprotons to fill the LHC to the nominal intensity (slide 21); ie use one beam pipe and run it as a p-pBar collider with the same luminosity?

      The nominal fill in the LHC is 2808 bunches of $1.15\text{x}10^{11}$ protons each, or $3.2\text{x}10^{14}$ protons. At $2.5\text{x}10^{11}$ pBar/hour, it would take 1291 hours, or about 54 *days* to collect this many antiprotons.

   b. How long would it take to make the "half gram" of antimatter in Dan Brown's Angels and Demons?

      Each antiproton weighs $1.67\text{x}10^{-27}$ kg, so there are $3\text{x}10^{23}$ of them in a half gram (or half of Avagadro's Number, if you like that sort of thing). At the rate above, it would take $1.2\text{x}10^{12}$ hours or about 136 million years to accumulate that much.

   c. How long would it take to do this, assuming we could magically turn the entire design LHC beam energy into antimatter like they apparently did in the book, optimistically assuming one fill every 2 hours?

      The energy of a half gram of matter is $4.5\text{x}10^{13}$ J. The energy of one beam at maximum energy is $366\text{x}10^{6}$ J, so it would take 123000 fills to make this many antiprotons. At two hours each, this is 246000 hours or 28 years. To satisfy baryon number conservation, we would either need twice that time, or to use both beams ☺.

4. Consider the SLAC B-Factory, which collides electrons and positrons asymmetrically with energies of 9 and 3.1 GeV respectively.

   a. Show that the center of mass energy is, to this accuracy, equal to the Υ_{4S} resonance.

$$\sqrt{s} = \sqrt{E^2 - (pc)^2} = \sqrt{(3.1+9)^2 - (9-3.1)^2} = 10.56 \text{ GeV}$$

      (Υ_{4S}=10.58 GeV)

   b. How fast is the center of mass moving?

$$\beta = \frac{pc}{E} = \frac{(9-3.1)}{(9+3.1)} = .49$$

c. On average, how far will a B_d meson travel before decaying?

The lifetime of a B_d meson is 1.5 ps. The average decay length will be

$$d = c\gamma\beta\tau \approx 250\ \mu\mathrm{m}$$

d. How far would the same meson travel a symmetric machine, assuming it was also produced at the Υ_{4S} resonance?

The kinetic energy of each meson is

$$K = \frac{1}{2}M(\Upsilon_{4S})c^2 - \mathrm{M}(\mathrm{B_d})c^2 = .5 * 10.5794 - 5.2795 = .010\ \mathrm{GeV}$$

This is clearly non-relativistic, so

$$\beta \approx \sqrt{\frac{2K}{Mc^2}} = .062$$

And the path length will be

$$d = c\beta\tau \approx 28\ \mu\mathrm{m}$$

And moving in 3 dimensions.