Accelerator Homework, Part 1

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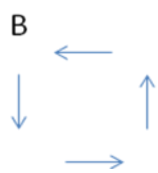
1. Quadrupoles focus in one plane and defocus in the other. Sketch the type of field you would need to focus in both planes and use Maxell's Equations to show that this is impossible using only *external* currents.

For a positively charged particle coming out of the page, we would need a field that looks like this



But by Ampere's Law

$$\oint \vec{H} \cdot d\vec{l} = \oint \frac{\vec{B}}{\mu} \cdot d\vec{l} \neq 0 = I$$

Therefore there would have to be a current contained within the integration path. This is the principle behind neutrino horns and lithium lenses.

2. Beam line or accelerator "admittance" represents the area o f the maximum ellipse in phase space that can be transported. It is calculated from the limiting aperture in the same way the emittance is calculated from the transverse beam size. The radius of the aperture in the final focus quadrupoles of the LHC is 35 mm. Using the optics plots on slide 46, estimate the normalized admittance of these magnets at injection and collision energy. Recalculate the admittance at injection if they tried to inject with the *collision* optics (that is, no "squeeze" later).

We use a formula similar to that used for calculating the beam RMS

$$\sigma = \sqrt{\frac{\beta_T \varepsilon}{\beta\gamma}} \rightarrow d = \sqrt{\frac{\beta_T A}{\beta\gamma}} \rightarrow A = \frac{d^2 \beta\gamma}{\beta_T}$$

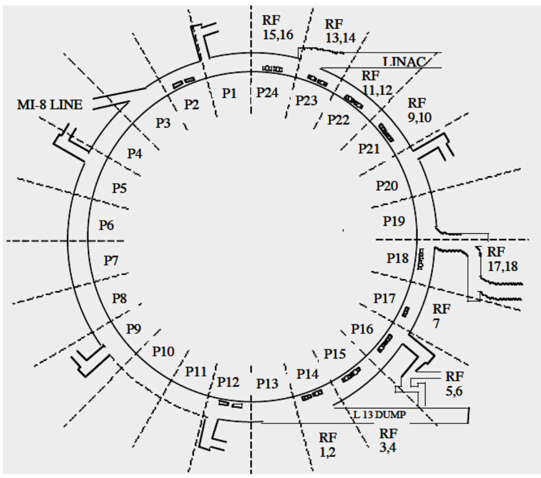
where *d* and *A* are the half aperture and admittance, respectively. At collision, we have

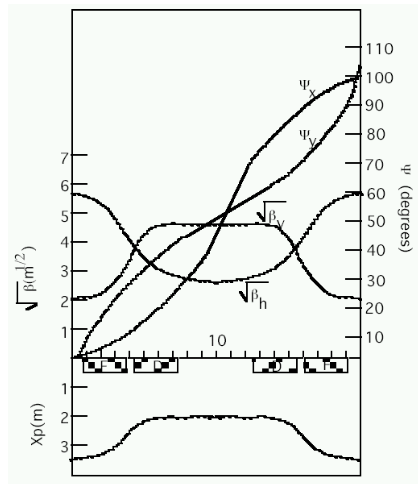
- $d = .035$ m
- $\beta\gamma \sim 7000$
- $\beta_T \sim 2200$ m (from figure)

So $A = .0039 = 3900$ π-mm-mr

If these optics were kept at injection, then the admittance would shrink by 450/700, to roughly 250 π-mm-mr

3. The Fermilab "Booster" is a rapid cycling synchrotron that accelerates protons from 400 MeV to 8GeV. It has a 24-fold periodicity, where each period has the lattice functions shown below





Each period has a corrector dipoles located at the maximum beta points in *x* (horizontal) and *y* (vertical).

   a. Using the "3-bump" equation on slide 29, calculate the vertical angular bends I would need at the maximum beta points in periods 1, 2, and 3 to create a localized 1 cm vertical bump at the maximum beta point in period 2. (hint, for a *localized* bump, you can use the first equation on the slide to calculate the beam deflection).
   b. Calculate the integrated magnetic fields it would take to produce these angles at the extraction energy of 8 GeV.

The transverse deflection going from one period to the next is

$$\Delta y = \theta\sqrt{\beta(s)\beta}\sin\psi(s)$$

Note that the plot is in units of $\beta^{1/2}$ so the maximum vertical β is about 4.5^2 or 20 m and the phase advance between cells is about 102 degrees so

$$\theta = \frac{.01}{20\sin 102°} = .51\text{ mr}$$

Because the β functions are the same at each period, the three bump ratios simplify to

$$\theta_2 = -\theta_1 \left(\frac{\beta_1}{\beta_2}\right)^{1/2} \frac{\sin\psi_{13}}{\sin\psi_{23}} = -\theta_1 \frac{\sin 204^\circ}{\sin 102^\circ} = +.21\,\text{mr}$$

$$\theta_3 = \theta_1 \left(\frac{\beta_1}{\beta_3}\right)^{1/2} \frac{\sin\psi_{12}}{\sin\psi_{23}} = \theta_1 = .51\,\text{mr}$$

The kinetic energy of the Booster is 8 GeV, so the momentum is $p = \frac{1}{c}\sqrt{(K+mc^2)^2 - (mc^2)^2}$ =8.9 GeV/c, which has a stiffness of (Bρ)=8.9/.3=29.7 T-m, so the integrated field is given by

$$BL \approx \theta(B\rho)$$

So $(Bl)_1 = (Bl)_3 = .015\ \text{Tm}$ and $(Bl)_2 = .006\ \text{Tm}$

4. Express the luminosity formula on slide 43 in terms of the maximum beam-beam tune shift (slide 44). Discuss the strategy for increasing luminosity if this is the limiting factor.

(NOTE: there was a minor error in the tune shift formula in my original lecture. See the updated slides at the Indico site for the correct verion).

This situation is known as the "tune shift limit". In this case, we case rewrite the formula as

$$L = \left(\frac{\gamma f_{rev}}{4\pi}\right) \frac{n_b N_b}{\beta^*} \left[\left(\frac{N_b}{\varepsilon_N}\right) R_\phi\right]$$

And substitute the tune shift limit for the brightness

$$L = \left(\frac{\gamma f_{rev}}{4\pi}\right) \frac{n_b N_b}{\beta^*} \left[\left(\frac{4\Delta\nu_{\max}}{n_{collisions} r_0}\right) R_\phi\right]$$

We see that if tune shift is the limit, this strategy will be to increase the emittance proportional to the bunch size to maintain the same tune shift, until the aperture limit is reached, then increase the number of bunches (total current) until current related effects become the limit.

5. In looking for new uses for the Tevatron tunnel, some people (briefly!!) discussed building a "Z Factory" – a higher luminosity version of LEP.

   a. Assuming a uniform bend radius, show how the power lost to synchrotron radiation would scale as one went from the LEP tunnel (C=27km) to the Fermilab tunnel (C=~~6π km~~ 2π km) [corrected from original]

The power equation

$$P = \frac{1}{6\pi\varepsilon_0}\frac{e^2 c}{\rho^2}\left(\frac{E}{m}\right)^4$$

scales with the inverse square of the radius, so it would have $(27/2\pi)^2$=18.5 times more power loss (this ignores one factor of radius, discussed below).

   b. Calculate the power loss for a 1A beam current

Need to calculate the total number of electron the ring and multiply by the power formula. The number of electrons is

$$n = I\frac{2\pi R}{ec}$$

Giving a total power of

$$P = \frac{Ie}{3\varepsilon_0 R}\left(\frac{E}{m}\right)^4 = 360 \text{ MW}$$