HCPSS-2010, Introduction to the SM: PS3+4

## **Question 1**: Exotic light quarks

We consider a model with the gauge symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$ spontaneously broken by a single Higgs doublet into $SU(3)_C \times U(1)_{EM}$. However, quark representations differ from the standard model. The quark sector consists of three quark flavors. Of the left handed quarks, $Q_L = (u_L, d_L)$ form a doublet of $SU(2)_L$ while $s_L$ is a singlet. All the right handed quarks are singlets. All color representations and electric charges are the same as in the standard model. That is, the fermion irreps are

$$Q_L(3,2)_{-1/6}, \qquad s_L(3,1)_{-1/3}, \qquad d_R(3,1)_{-1/3}, \qquad s_R(3,1)_{-1/3}, \qquad u_R(3,1)_{2/3} \tag{1}$$

1. Write down (a) the gauge interactions of the quarks with the charged $W$ bosons (before SSB); (b) the Yukawa interactions (before SSB); (c) the bare mass terms (before SSB); (d) the mass terms after SSB.

   Answer: The charged interaction is

   $$-\mathcal{L}_W = \frac{g}{\sqrt{2}} \left[\bar{u}_L \gamma^\mu d_L\right] W_\mu^+ + h.c. \tag{2}$$

   The Yukawa interactions are

   $$-\mathcal{L}_Y = G_{dd} \left( \bar{u}_L \quad \bar{d}_L \right) \phi \, d_R + G_{ds} \left( \bar{u}_L \quad \bar{d}_L \right) \phi \, s_R + G_{uu} \left( \bar{u}_L \quad \bar{d}_L \right) \tilde{\phi} \, u_R + +h.c. \tag{3}$$

   The bare mass terms are

   $$-\mathcal{L}_m^{bare} = m_{sd} \bar{s}_L d_R + m_{ss} \bar{s}_L s_R + h.c. \tag{4}$$

   After SSB the mass terms are

   $$-\mathcal{L}_m = \left( \bar{d}_L \quad \bar{s}_L \right) \begin{pmatrix} G_{dd} v/\sqrt{2} & G_{ds} v/\sqrt{2} \\ m_{sd} & m_{ss} \end{pmatrix} \begin{pmatrix} d_R \\ s_R \end{pmatrix} + G_{uu} v/\sqrt{2} \bar{u}_L u_R + h.c. \tag{5}$$

2. How many physical flavor parameters are in this model? Separate them into masses, mixing angles and phases. Is there CP violation in this model?

   Answer: Lets count: The global symmetry of the kinetic term is $SU(2) \times U(1)^3$ ($d_R$ and $s_R$ has teh same QNs and the other fileds are different). So we have 7 generators. 6 are broken by the Yukawa and mass terms. We have a total of 10 parameters, the three Yukawas and 2 bare mass terms, each is a complex number, so we have a total of $10 - 6 = 4$ physical parameters. They are the 3 masses and one mixing angle. There is no CPV in this model.

3. Write down the gauge interactions of the quarks with the $Z$ boson in both the interaction basis and the mass basis. (You do not have to rewrite terms that do not change when you rotate to the mass basis. Write only the terms that are modified by the rotation to the mass basis.) Are there generally tree level $Z$ exchange FCNCs? (If needed, you can assume CP conservation from now on.)
   Answer:

$$
\begin{aligned}
-\mathcal{L}_Z = \frac{g}{\cos\theta_W} \quad & \left[\bar{u}_L\gamma^\mu\left(\frac{1}{2}-\frac{2}{3}\sin^2\theta_W\right)u_L + \bar{u}_R\gamma^\mu\left(-\frac{2}{3}\sin^2\theta_W\right)u_R\right. \\
& \bar{d}_L\gamma^\mu\left(-\frac{1}{2}+\frac{1}{3}\sin^2\theta_W\right)d_L + \bar{d}_R\gamma^\mu\left(\frac{1}{3}\sin^2\theta_W\right)d_R \\
& \left.\bar{s}_L\gamma^\mu\left(\frac{1}{3}\sin^2\theta_W\right)s_L + \bar{s}_R\gamma^\mu\left(\frac{1}{3}\sin^2\theta_W\right)s_R\right]Z_\mu
\end{aligned} \tag{6}
$$

   In the mass basis the $T_3$ part of the down type left handed quarks is modified. We have

$$
-\mathcal{L}_Z = \frac{-g}{2\cos\theta_W}\begin{pmatrix}\bar{d}_L & \bar{s}_L\end{pmatrix}\begin{pmatrix}\cos^2\theta_L & \cos\theta_L\sin\theta_L \\ \cos\theta_L\sin\theta_L & \sin^2\theta_L\end{pmatrix}\begin{pmatrix}d_L \\ s_L\end{pmatrix} \tag{7}
$$

   For the right handed part, as well as for the up type quarks the couplings are diagonal.

4. Are there photon and gluons FCNCs? Support your answer by an argument based on symmetries.
   Answer: The photon and gluon couplings are always flavor symmetric. This is because they are the gauge bosons of an exact local symmetry.

5. Is there Higgs exchange FCNCs?
   Answer: Yes. The Higgs couplings are not align with the mass matrix because there are bare mass terms.

# **Question 2**: The GIM mechanism: $b \to s\gamma$ decay

1. Explain why it is a loop decay and drew the one loop diagrams in the SM.
   Answer: The photon couplings is always diagonal so it must be a loop decay. The diagrams are a simple one loop with $W$ and up-type quark in the loop. The photon then can go out of any of the lines, that is, $b$, $s$, $W$ and $u_i$.

2. These diagrams naively diverge. Show it.
   Answer: Lets look at the diagram where the photon is connected to the internal up type

quark. Consider the diagram with internal top. The diagram is roughly proportional to

$$V_{ts}V^{*}tb \int d^4k \, \frac{(\not{k} - m_t)^2}{(k^2 - m_t^2)^2 (k^2 - m_W^2)} \tag{8}$$

The term that go like $\not{k}$ vanish (as it is antisymmetric) and thus in the UV we end up with

$$V_{ts}V^{*}tb \int d^4k \, \frac{k^2 + m_t^2}{k^6} \tag{9}$$

and it is log divergent.

3. Yet, once we add all the diagrams and make use of the CKM unitarity we get a final result. Show that the UV divergent cancel (that is, you can put all masses the same and find that the answer is zero).

   Answer: When we add it all the UV part is

$$\sum_i V_{is}V^{*}ib \int d^4k \, \frac{k^2 + m_i^2}{k^6} \tag{10}$$

   Using unitarity we see that the divergent term vanish (as it does not depend on the external mass) The only one that is there is the one that proportional to $m_i^2$, and it is finite.

4. If we add a vector like quarks to the SM we can have tree level FCNC in $Z$ exchange and the CKM is not unitary. Yet, since we do not have $b \to s\gamma$ at tree level also in this case the one loop diagrams must be finite. Show that it is indeed the case. (Hint: there are more one loop diagrams in that case.)

   Answer: Now there is also a one loop diagram with $Z$ in the loop. The $bs$ FCNC coupling of the $Z$ is proportional to $V_{is}V_{ib}^{*}$. Thus, once you include this diagram we regain unitarity and the sum is finite.