

Question 1: Exotic light quarks

We consider a model with the gauge symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$ spontaneously broken by a single Higgs doublet into $SU(3)_C \times U(1)_{EM}$. However, quark representations differ from the standard model. The quark sector consists of three quark flavors. Of the left handed quarks, $Q_L = (u_L, d_L)$ form a doublet of $SU(2)_L$ while s_L is a singlet. All the right handed quarks are singlets. All color representations and electric charges are the same as in the standard model. That is, the fermion irreps are

$$Q_L(\mathbf{3}, \mathbf{2})_{-1/6}, \quad s_L(\mathbf{3}, \mathbf{1})_{-1/3}, \quad d_R(\mathbf{3}, \mathbf{1})_{-1/3}, \quad s_R(\mathbf{3}, \mathbf{1})_{-1/3}, \quad u_R(\mathbf{3}, \mathbf{1})_{2/3} \quad (1)$$

1. Write down (a) the gauge interactions of the quarks with the charged W bosons (before SSB); (b) the Yukawa interactions (before SSB); (c) the bare mass terms (before SSB); (d) the mass terms after SSB.
2. How many physical flavor parameters are in this model? Separate them into masses, mixing angles and phases. Is there CP violation in this model?
3. Write down the gauge interactions of the quarks with the Z boson in both the interaction basis and the mass basis. (You do not have to rewrite terms that do not change when you rotate to the mass basis. Write only the terms that are modified by the rotation to the mass basis.) Are there generally tree level Z exchange FCNCs? (If needed, you can assume CP conservation from now on.)
4. Are there photon and gluons FCNCs? Support your answer by an argument based on symmetries.
5. Is there Higgs exchange FCNCs?

Question 2: The GIM mechanism: $b \rightarrow s\gamma$ decay

1. Explain why it is a loop decay and draw the one loop diagrams in the SM.
2. These diagrams naively diverge. Show it.
3. Yet, once we add all the diagrams and make use of the CKM unitarity we get a final result. Show that the UV divergent cancel (that is, you can put all masses the same and find that the answer is zero).

4. If we add a vector like quarks to the SM we can have tree level FCNC in Z exchange and the CKM is not unitary. Yet, since we do not have $b \rightarrow s\gamma$ at tree level also in this case the one loop diagrams must be finite. Show that it is indeed the case. (Hint: there are more one loop diagrams in that case.)