Introduction to the SM

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Parameter counting

How many parameters we have?

How many parameters are physical?

- "Unphysical" parameters are those that can be set to zero by a basis rotation
- General theorem

$$N(\text{Phys}) = N(\text{tot}) - N(\text{broken})$$

  - $N(\text{Phys})$, number of physical parameters
  - $N(\text{tot})$, total number of parameters
  - $N(\text{broken})$, number of broken generators

Example: Zeeman effect

A hydrogen atom with weak magnetic field

- The magnetic field add one new physical parameter, B

$$V(r) = \frac{-e^2}{r} \qquad V(r) = \frac{-e^2}{r} + B_x\hat{x} + B_y\hat{y} + B_z\hat{z}$$

- But there are 3 total new parameters
- The magnetic field breaks explicitly: $SO(3) \to SO(2)$
- 2 broken generators, can be “used” to define the z axis

$$N(\text{Phys}) = N(\text{tot}) - N(\text{broken}) \quad \Rightarrow \quad 1 = 3 - 2$$

Back to the flavor sector

Without the Yukawa interaction, a model with N copies of the same field has a $U(N)$ global symmetry

- It is just the symmetry of the kinetic term

$$\mathcal{L} = \bar{\psi}_i D_\mu \gamma^\mu \psi_i, \qquad i = 1, 2, ..., N$$

- $U(N)$ is the general rotation in N dimensional complex space
- $U(N) = SU(N) \times U(1)$ and it has N^2 generators

Two generation SM

First example, two generation SM

- Two Yukawa matrices: Y^D, Y^U, $N_T = 16$
- Global symmetries of the kinetic terms:
  $U(2)_Q \times U(2)_D \times U(2)_U$, 12 generators
- Exact accidental symmetries: $U(1)_B$, 1 generator
- Broken generators due to the Yukawa: $N_B = 12 - 1 = 11$
- Physical parameters: $N_P = 16 - 11 = 5$. They are the 4 quarks masses and the Cabibbo angle

The SM flavor sector

Back to the SM with three generations. Do it yourself

- Total parameters (in Yukawas): $N_T =$
- Symmetry generators of kinetic terms: $N_G =$
- Unbroken global generators: $N_U =$
- Broken generators: $N_B =$
- Physical parameters: $N_P =$

The SM flavor sector

Back to the SM with three generations. Do it yourself

- Total parameters (in Yukawas): $N_T = 2 \times 18 = 36$
- Symmetry generators of kinetic terms: $N_G = 3 \times 9 = 27$
- Unbroken global generators: $N_U = 1$
- Broken generators: $N_B = 27 - 1 = 26$
- Physical parameters: $N_P = 36 - 26 = 10$
- 6 quark masses, 3 mixing angles and one CPV phase

Remark: The broken generators are 17 Im and 9 Re. We have 18 real and 18 imaginary to “start with” so the physical ones are $18 - 17 = 1$ and $18 - 9 = 9$

The CKM matrix

The flavor parameters

- The 6 masses. We kind of know them. There is a lot to discuss, but I will not do it in these lectures
- The CKM matrix has 4 parameters
  - 3 mixing angles (the orthogonal part of the mixing)
  - One phase (CP violating)
- A lot to discuss on how to determined and check them. I will be brief here

The CKM matrix

$$\mathcal{L}_W = \frac{g}{\sqrt{2}} \overline{U_L} V \gamma^\mu D_L W_\mu^+ + \text{h.c.}$$

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- CKM is unitary

$$\sum V_{ij} V_{ik}^* = \delta_{jk}$$

- Experimentally, $V \sim 1$. Off diagonal terms are small
- Many ways to parametrize the matrix

CKM parametrization

- The standard parametrization

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

  where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$.
- In general there are 5 entries that carry a phase
- Experimentally:

$$|V| \approx \begin{pmatrix} 0.97383 & 0.2272 & 3.96 \times 10^{-3} \\ 0.2271 & 0.97296 & 4.221 \times 10^{-2} \\ 8.14 \times 10^{-3} & 4.161 \times 10^{-2} & 0.99910 \end{pmatrix}$$

The Wolfenstein parametrization

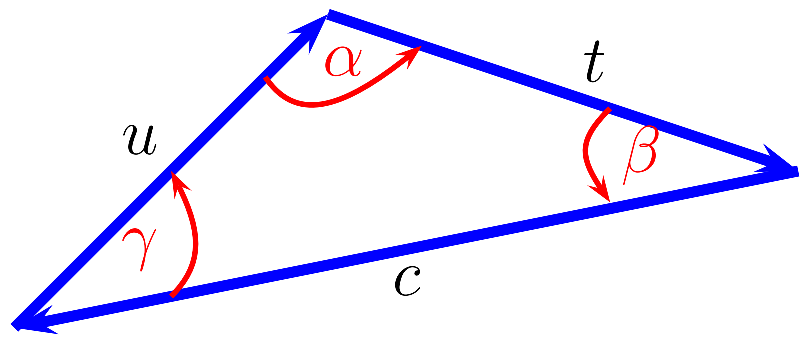
- Since $V \sim 1$ it is useful to expand it

$$V \approx \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

- One small parameter $\lambda \sim 0.2$, and three (A, ρ, η) that are roughly $O(1)$
- As always, be careful (unitarity...)
- Note that to this order only V_{13} and V_{31} have a phase

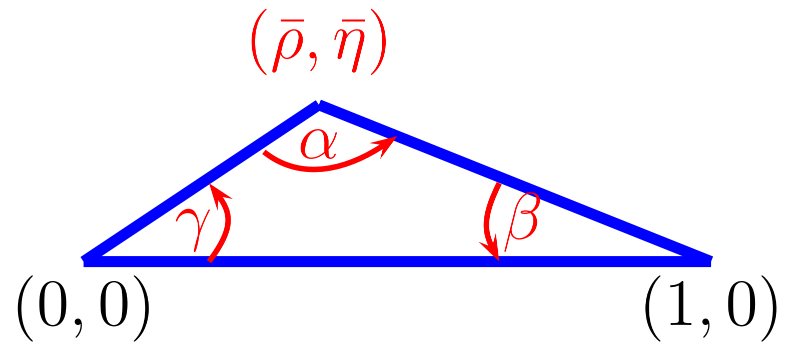
The unitarity triangle

A geometrical presentation of $V_{ub}^* V_{ud} + V_{tb}^* V_{td} + V_{cb}^* V_{cd} = 0$



Rescale by the c size and rotated

$$A\lambda^3\left[(\rho + i\eta) + (1 - \rho - i\eta) + (-1)\right] = 0$$



CKM determination

CKM determination

- Basic idea: Measure the 4 parameters in many different ways. Any inconstancy is a signal of NP
- Problems: Experimental errors and theoretical errors
- Have to be smart...
  - Smart theory to reduce the errors
  - Smart experiment to reduce the errors
- There are cases where both errors are very small

Measuring sides: examples

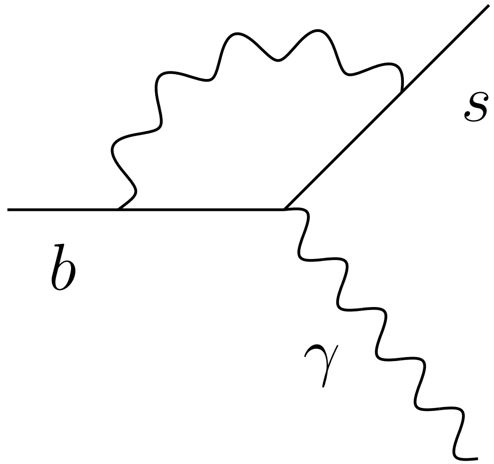
- β-decay, $d \to ue\bar{\nu} \propto V_{ud}$; Isospin
- K-decay, $s \to ue\bar{\nu} \propto V_{us}$; Isospin and SU(3)
- D-decay, $c \to qe\bar{\nu} \propto V_{cq}\ q = d, s$; HQS
- B-decays $b \to ce\bar{\nu} \propto V_{cb}$; HQS
- Not easy with top. Cannot tag the final flavor, low statistics

Loop decays

- We have sensitivity to magnitude of CKM elements in loops
- More sensitive to V_{tq} that is harder to get in tree level decays
- But at the same time it may be modified by new heavy particles
- This is a general argument. NP is likely to include “heavy” particles, that can affect loop processes much more than tree level decays

Loop: example

$$A(b \to s\gamma) \propto \sum V_{ib} V_{is}^*$$



What is $\sum V_{ib} V_{is}^*$?

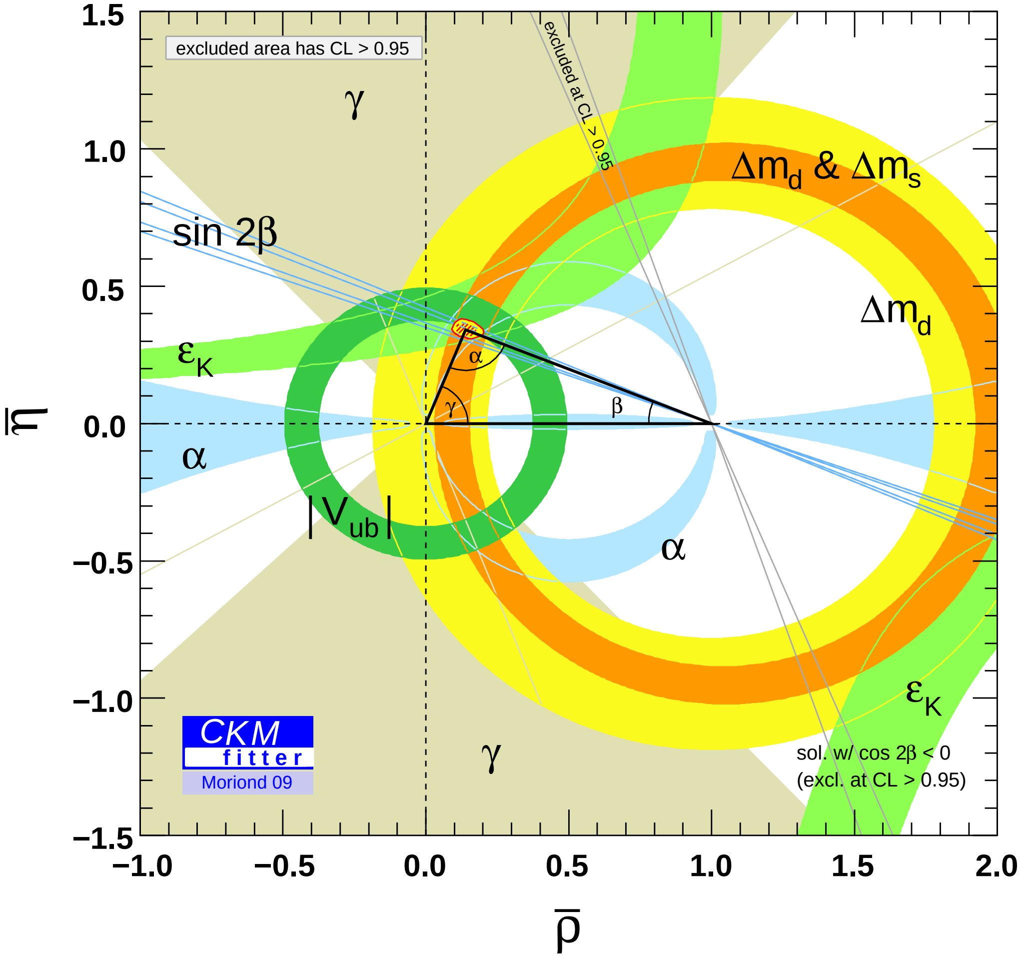
GIM Mechanism

What we really have is

$$A(b \to s\gamma) \propto \sum V_{ib} V_{is}^* f(m_i)$$

- Because the CKM is unitary, the m_i independent term in f vanishes
- Must be proportional to the mass (in fact, m_i^2) so the heavy fermion in the loop is dominant
- In Kaon decay this gives m_c^2/m_W^2 extra suppression. Numerically not important for b decays
- CKM unitarity and tree level Z exchange are related. (Is the diagram divergent?)

All together now



Neutrinos

Fermion masses

- Only discuss the theory of neutrino masses
- There are two types of fermion masses:
  Dirac and Majorana masses

Fermion masses: Dirac mass

Couples left and right handed fields

$$m_D \bar{\psi}_L \psi_R$$

- It always involves two different fields
- The additive QNs of the two fields are opposite
- There are four d.o.f. with the same mass

Fermion masses: Majorana mass

Couples identical left or right handed fields

$$m_M \bar{\psi}_R^c \psi_R, \qquad \psi_R^c = C \bar{\psi}_R^T$$

- There are two d.o.f. with the same mass
- The additive QNs of the two fields are the same. Thus, this term breaks all the U(1) symmetries
- Majorana mass term can be written only for neutral fermions. In particular for neutrinos

Neutrino masses in the SM

The SM implies that the neutrinos are exactly massless

- No ν_R $\Rightarrow$ No Dirac mass $m_D\bar{\nu}_L\nu_R$
- No Higgs triplet $\Rightarrow$ No Majorana mass ΔLL
- The SM is renormalizable $\Rightarrow$ No Majorana $HHLL$ mass term
- $U(1)_{B-L}$ is an accidental non-anomalous global symmetry of the SM $\Rightarrow$ No radiative generated Majorana $HHLL$ mass term

Note that unlike the $m_\gamma = 0$ prediction, the $m_\nu = 0$ prediction is somewhat “accidental”

m_ν beyond the SM

Generally, NP models predict massive neutrinos

- New light fields
- New heavy fields

m_ν beyond the SM: light NP

Two options:

- Add RH neutrinos $N_R(1,1)_0 \;\Rightarrow$ Dirac mass $m_D \bar{\nu}_L \nu_R$
  - Why the Yukawa couplings is small
  - Why there are no large Majorana mass terms for the RH neutrinos
- Add Higgs triplet $\Delta(1,3)_1 \;\Rightarrow$ Majorana mass ΔLL
  - Why $\langle \Delta \rangle \ll \langle H \rangle$

m_ν beyond the SM: heavy NP

The SM is an effective low energy theory, with NR terms. NP effects are suppressed by powers of the small parameter

$$\frac{m_W}{\Lambda}$$

- Neutrino masses are generated by

$$\frac{\lambda_{ij}}{\Lambda} H H L_i L_j \quad \Rightarrow \quad m_\nu = \lambda_{ij} \frac{v^2}{M}$$

- λ_{ij} are dimensionless couplings
- Λ is the high energy scale

m_ν beyond the SM: heavy NP

$$m_\nu = \lambda_{ij} \frac{v^2}{\Lambda}$$

- Allowing for NR terms implies $m_\nu \neq 0$
- m_ν is small since it arises from NR terms
- Neutrino masses probe the high energy physics
- Both total lepton number and family lepton numbers are broken. We expect lepton mixing and CP violation

Example: the see-saw mechanism

Consider one generation SM with an additional singlet $N_R(1,1)_0$

$$\mathcal{L}_{m_\nu} = \frac{1}{2} M_N NN + Y_\nu HLN$$

- $M_N \gg v$ is a Majorana mass of the RH neutrino
- The second term is a Dirac mass term.
- In the (ν_L, N_R) basis the neutrino mass matrix is

$$m_\nu = \begin{pmatrix} 0 & m_D \\ m_D & M_N \end{pmatrix}$$

where $m_D \equiv Y_\nu v$

The see-saw mechanism

$$m_\nu = \begin{pmatrix} 0 & m_D \\ m_D & M_N \end{pmatrix}$$

Assuming $M_N \gg v$, to first order ($m_D \equiv Y_\nu v$)

$$m_{N_R} = M_N \qquad m_{\nu_L} = \frac{m_D^2}{M_N}$$

- To be compared with the NR term

$$m_\nu = \lambda \frac{v^2}{\Lambda}$$

- The NP scale Λ is identified with M_N
- The NP coupling λ is identified with Y_ν^2

See-saw: more remarks

- One more example of “integrating out” the heavy physics
- At the UV we have a renormalizable theory. At the IR we have NR terms
- Just to emphasis, in general, when RH neutrinos are add we have the see-sea
- See-saw is realized, for example, in GUT and LRS models
- The see-saw can be generalized to three generation

Neutrino mixing

When neutrinos are massive there can be lepton mixing, just like quark mixing in the SM

- For quarks, V_{CKM}

$$\mathcal{L} = \frac{g}{\sqrt{2}} \bar{u}_i V_{ij} \gamma^\mu d_j W_\mu^+ \qquad i = u, c, t \quad j = d, s, b$$

- For lepton, U_{MNS} (Maki, Nakagawa, Sakata)

$$\mathcal{L} = \frac{g}{\sqrt{2}} \bar{\ell} U_{\ell i} \gamma^\mu \nu_i W_\mu^- \qquad \ell = e, \mu, \tau \quad i = 1, 2, 3$$

The charged lepton mass basis is also called the flavor basis

Neutrino mixing

Quarks and charged leptons are identified as mass eigenstates. Neutrinos are identified as flavor eigenstates

There are two equivalent ways to think about the mixing

- Quarks: two mass matrices are diagonal, the W interaction is not diagonal
- Leptons: the charged lepton and the W interactions are diagonal, the neutrino mass is not
- The difference is just because in experiment we measure quark masses and neutrino flavor (not mass)

Neutrino: summary

- We expect to have mass to the neutrino due to high energy physics
- Such a model is called νSM
- See sew is an explicit realization of how to get the NR term
- Of course, then we have to measure the neutrinos parameters and check. Colloquium yesterday