# Decoupling the Gravity Multiplet from Supergravity

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C. Cheung, FDE, J. Thaler; arXiv:1104.2598 [hep-ph], arXiv:1104.2600 [hep-ph].

# Motivation

SUSY symmetry of nature $\Rightarrow$ SUGRA

Not all of the SUGRA formalism relevant
for phenomenology at colliders and in cosmology

**Our Goal: Framework to Simplify Supergravity Calculations**

decoupling the gravity multiplet from matter fields
and
accounting for SUGRA effects

# Conformal SUGRA

Minimal SUGRA: gauge fixing of conformal SUGRA

Kaku et al. PLB 69(1977), NPB 129(1977), PRD 17(1978); Gates et al, NPB 147(1979), FP 58(1983).

## Gauge Fixing in Superspace: Conformal Compensator Formalism

Usual superspace of global SUSY + conformal compensator superfield $\mathbf{\Phi}$

$$\mathcal{L}_{\rm SUGRA} = -3 \int d^4\theta \, \mathbf{\Phi}^\dagger \mathbf{\Phi} \, e^{-\boldsymbol{K}/3} + \int d^2\theta \, \mathbf{\Phi}^3 \, \boldsymbol{W} + \text{h.c.}$$

$$+ \frac{1}{4} \int d^2\theta \, \boldsymbol{f}_{ab} \boldsymbol{W}^{a\alpha} \boldsymbol{W}^b_\alpha + \text{h.c.} + \ldots\ldots$$

## Standard Gauge Fixing

Lowest and fermionic components of $\mathbf{\Phi}$ are pure gauge mode:

$$\mathbf{\Phi} = 1 + \theta^2 F_\phi$$

Naive application of standard $\mathbf{\Phi}$:

neglect terms in $\ldots\ldots$ $\Rightarrow$ incorrect answers in many cases

# Problematic Terms

Standard choice for $\Phi$: mixing between gravity and matter multiplets

**Graviton normalization and kinetic mixing**

$C\, R/6$

E.H. action normalization

Graviton/matter mixing

**Gravitino kinetic mixing**

$i\xi\sigma^{\mu\nu}\partial_\mu\psi_\nu + \text{h.c.}$

Gravitino/matter mixing

Non-canonical gravitino

**Grativitino mass phase**

$-z^\dagger\psi_\mu\sigma^{\mu\nu}\psi_\nu + \text{h.c.}$

Gravitino mass real

$C$, $\xi$ and $z$ functions of matter fields

Additional terms must be taken into account in the calculations

Standard gauge fixing:

need to work with component fields

no simple interpretations in terms of superfields

# The Kugo-Uehara gauge

## Gauge Freedoms

Lowest and fermionic components of $\mathbf{\Phi}$ pure gauge modes

They can be fixed to any values by an appropriate gauge choice

## Kugo-Uehara gauge

Enough freedom for $C = -3$, $\xi_\alpha = 0$, $Arg[z] = 0$ to all orders in fields

$$\mathbf{\Phi} = \exp\left[\frac{1}{3}\left(K/2 - i\,\mathrm{Arg}\,W\right)\right] \times \left\{1, \frac{K_i \chi^i}{3}, F_\phi\right\}$$

Kugo and Uehara, NPB B222(1983).

## Hidden problem of KU gauge

Mixing with vector auxiliary field $b_\mu$: $b_\mu \partial^\mu \phi$

Integrating out $b_\mu$ $\Rightarrow$ gravity/matter mixing

Need to work again with component fields

# A Novel Gauge Fixing

## A less stringent gauge choice

$C = -3$, $\xi_\alpha = 0$, $Arg[z] = 0$ to linear order in field fluctuations

Cheung, FDE, Thaler, arXiv:1104.2598 [hep-ph].

$$\mathbf{\Phi} = e^{\mathbf{Z}/3}(1 + \theta^2 F_\Phi), \qquad \mathbf{Z} = \langle K/2 - i\,\mathrm{Arg}\,W\rangle + \langle K_i\rangle \mathbf{X}^i$$

## Coupling with the vector auxiliary field?

Gauge fixing $\Rightarrow$ $b_\mu = 0 + \mathcal{O}(1/M_{\rm Pl})$ $\Rightarrow$ $1/M_{\rm Pl}^2$ suppressed operators

No need to perform component manipulations

Gravity multiplet decoupled from matter fields calculations

$\langle F_\phi\rangle = m_{3/2}$ $\Rightarrow$ straightforward to identify SUGRA effects $\propto m_{3/2}$

# Phenomenological SUGRA Lagrangian

$$\mathcal{L}_{\text{SUGRA}} = -3 \int d^4\theta \, \boldsymbol{\Phi}^\dagger \boldsymbol{\Phi} \, e^{-\boldsymbol{K}/3} + \int d^2\theta \, \boldsymbol{\Phi}^3 \, \boldsymbol{W} + \text{h.c.} + \frac{1}{4} \int d^2\theta \, \boldsymbol{f}_{ab} \boldsymbol{W}^{a\alpha} \boldsymbol{W}^b_\alpha + \text{h.c.} + \mathcal{O}(1/M_{\text{Pl}})$$

$$\boldsymbol{\Phi} = e^{\boldsymbol{Z}/3}(1 + \theta^2 F_\Phi), \qquad \boldsymbol{Z} = \langle K/2 - i \,\text{Arg}\, W \rangle + \langle K_i \rangle \boldsymbol{X}^i$$

Cheung, FDE, Thaler, arXiv:1104.2598 [hep-ph].

# Application: fermionic spectra of SUGRA theories

SUGRA effects can substantially impact phenomenology

- spectrum of goldstini
- massless modulino in almost no-scale models

Improved compensator allows calculations in superspace without worrying about graviton/gravitino mixing

Cheung, FDE, Thaler, arXiv:1104.2600 [hep-ph].

## Theories with multiples sequestered sectors

Describe physics in terms of $\boldsymbol{\Omega} \equiv -3\exp(-\boldsymbol{K}/3) = \sum_i \boldsymbol{\Omega}^i$

Conformal compensator to compute fermionic spectra

$$\boldsymbol{\Phi} = e^{\langle\Omega_i\rangle X^i/3}\left(1 + \sqrt{2}\theta\frac{\langle\Omega_i\rangle\chi^i}{3} + \theta^2\widetilde{F}_\Phi\right) , \qquad \widetilde{F}_\Phi = m_{3/2} + \frac{\langle\Omega_i\rangle}{3}F^i$$

# The Spectrum of Goldstini

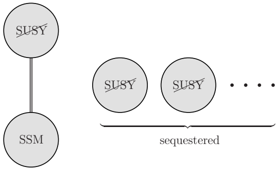
## The Goldstini scenario

$N$ sequestered $\not{\text{SUSY}}$ sectors: $\text{SUSY}^N \equiv \otimes \prod_{i=1}^{N} \text{SUSY}_i$

$N$ goldstini $\eta_i$ in the theory:

- $\eta_{\text{long}}$ eaten by the gravitino
- $N - 1$ goldstini in the spectrum

Cheung, Nomura, Thaler, JHEP 1003 (2010) [arXiv:1002.1967 [hep-ph]].




## Goldstini masses

Minimal scenario: $N$ sequestered sectors, $F$ and $D$ breaking, $\langle \Omega_i \rangle = 0$

$$\boldsymbol{\Omega} = -3 + \sum_A \boldsymbol{\Omega}^A \,, \qquad \boldsymbol{W} = m_{3/2} + \sum_A \boldsymbol{W}^A \,, \qquad \boldsymbol{f}_{ab} = \sum_A \boldsymbol{f}_{ab}^A \,.$$

Cheung, FDE, Thaler, arXiv:1104.2600 [hep-ph].

$$m_\eta = 2\, m_{3/2}$$

# Massless modulino

Improved compensator simplifies study of theories with $\Omega_i \neq 0$

## Minimal almost no-scale model

Single no-scale field $\boldsymbol{T}$, and $N$ sequestered single field, Polonyi-type $\not{SUSY}$

$$\boldsymbol{\Omega} = -3 + \alpha(\boldsymbol{T} + \boldsymbol{T}^\dagger) + \omega_0(\boldsymbol{T}, \boldsymbol{T}^\dagger) + \sum_{a=1}^N \omega_a(\boldsymbol{X}^{a\dagger}\boldsymbol{X}^a) \,, \qquad \boldsymbol{W} = m_{3/2} + \sum_{a=1}^N f_a \boldsymbol{X}^a \,.$$

SUSY breaking in the no-scale sector depends crucially on SUGRA effects!

## A curious factor of zero

Fermionic spectrum:
(component SUGRA Lagrangian)

- one gravitino of mass $m_{3/2}$
- N-1 fermion modes with mass $2\widetilde{F}_\Phi \neq 2m_{3/2}$
- one massless fermion mode (modulino)

Modulino puzzling: arises from unexpected cancellation, no hint for its origin

Cheung, Nomura, Thaler, JHEP 1003 (2010) [arXiv:1002.1967 [hep-ph]].

# Enhanced sequestering

## Unitary gauge for the gravitino

Project out of the Lagrangian: $\eta_{\text{eaten}} = \frac{1}{\sqrt{3}} \left( \langle \Omega_T \rangle \chi^T + \frac{\langle W_a \rangle \chi^a}{m_{3/2}} \right)$

To compute fermionic spectrum two equivalent versions of $\mathbf{\Phi}$

$$\mathbf{\Phi}^T = 1 + \sqrt{2}\theta \frac{\langle \Omega_T \rangle \chi^T}{3} + \theta^2 \widetilde{F}_\Phi \,, \qquad \mathbf{\Phi}^X = 1 - \sqrt{2}\theta \frac{\langle W_a \rangle \chi^a}{3 m_{3/2}} + \theta^2 \widetilde{F}_\Phi \,.$$

$$\mathcal{L}_{\text{SUGRA}} = \mathcal{L}^X + \mathcal{L}^T + \ldots$$

$$\mathcal{L}^T = \int d^4\theta \; \mathbf{\Phi}^{T\dagger} \mathbf{\Phi}^T \; \mathbf{\Omega}^T \,, \quad \mathcal{L}^X = \int d^4\theta \; \mathbf{\Phi}^{X\dagger} \mathbf{\Phi}^X \; \mathbf{\Omega}^X + \int d^2\theta \; (\mathbf{\Phi}^X)^3 \; \mathbf{W} + \text{h.c.} \,.$$

The fermions $\chi^T$ and $\chi^a$ are sequestered from each other!

# Modulino as secret Goldstino

## Two levels of sequestering

- sequestering among the $\boldsymbol{X}^a$
- additional sequestering between the $\boldsymbol{X}^a$ and $\boldsymbol{T}$

## Modulino is the Goldstino of a hidden SUSY

Hidden global SUSY in $\mathcal{L}^T$: $\mathcal{L}^T = \int d^2\theta \, \alpha \widetilde{F}_\Phi^\dagger \boldsymbol{T} + \text{h.c.} + \ldots$

$\boldsymbol{T}$ behaves like a chiral multiplet that breaks a hidden global SUSY

$\boldsymbol{T}$ equation of motion: $\langle F^T \rangle = \alpha \widetilde{F}_\Phi$

Modulino as Goldstino of an accidental global SUSY

# Summary

## Supergravity Computations without Gravity Complications
**C. Cheung, FDE, J. Thaler, arXiv:1104.2598 [hep-ph].**

$$\mathcal{L}_{\text{SUGRA}} = -3 \int d^4\theta \, \boldsymbol{\Phi}^\dagger \boldsymbol{\Phi} \, e^{-\boldsymbol{K}/3} + \int d^2\theta \, \boldsymbol{\Phi}^3 \, \boldsymbol{W} + \text{h.c.}$$
$$+ \frac{1}{4} \int d^2\theta \, \boldsymbol{f}_{ab} \boldsymbol{W}^{a\alpha} \boldsymbol{W}^b_\alpha + \text{h.c.} + \mathcal{O}(1/M_{\text{Pl}})$$

$$\boldsymbol{\Phi} = e^{\boldsymbol{Z}/3}(1 + \theta^2 F_\Phi), \qquad \boldsymbol{Z} = \langle K/2 - i \operatorname{Arg} W \rangle + \langle K_i \rangle \boldsymbol{X}^i$$

## The Spectrum of Goldstini and Modulini
**C. Cheung, FDE, J. Thaler, arXiv:1104.2600 [hep-ph].**

Fermionic spectra calculation directly in superspace

# Backup slides

BACKUP
SLIDES

# More on Conformal Supergravity

## Gauge redundancies of conformal SUGRA

Poincaré SUGRA

diffeomorphisms;
local Lorentz transformations;
local supersymmetry.

Additional gauge redundancies

local dilatations $\hat{D}$;
local $U(1)_R$ chiral transformations $\hat{A}$;
conformal supersymmetry $\hat{S}_\alpha$;
special conformal transformation $\hat{K}_\mu$.

Special conformal transformations fixed by setting dilatation gauge field to zero

## Transformation of the conformal compensator $\Phi = \{\sigma, \sigma\zeta_\alpha, \sigma F_\Phi\}$

Dilatation and chiral transformation: $\sigma \to e^\lambda \sigma \qquad (\lambda \in \mathbb{C})$

Conformal supersymmetry: $\zeta_\alpha \to \zeta_\alpha + \rho_\alpha$

Lowest and fermionic components of $\Phi$ are pure gauge mode

# SUGRA action

We must know the conformal weights $w$ of all the fields in the theory

chiral matter superfields $\boldsymbol{X}^i$: $w_{\boldsymbol{X}^i} = 0$

vector superfields $\boldsymbol{W}^a$: $w_{\boldsymbol{W}^a} = 0$

conformal compensator $\boldsymbol{\Phi}$: $w_{\boldsymbol{\Phi}} = 1$

gauge field strengths $\boldsymbol{W}^a_\alpha$: $w_{\boldsymbol{W}^a_\alpha} = 3/2$

We can couple to conformal gravity only:

- real $w = 2$ multiplets: $\boldsymbol{\Xi}_{(w=2)} = \{C, \xi_\alpha, M, A_\mu, \lambda_\alpha, D\}$
- chiral $w = 3$ multiplets: $\boldsymbol{\Sigma}_{(w=3)} = \{z, \chi_\alpha, F\}$

and construct superconformally invariant $D$-term and $F$-terms

$$[\boldsymbol{\Xi}]_D = \frac{1}{2} eD - \frac{1}{2} e \left(\lambda \sigma^\mu \overline{\psi}_\mu - i\xi \sigma^{\mu\nu} D^c_\mu \psi_\nu + \text{h.c.}\right) + \frac{C}{3} \left(\frac{1}{2} eR - \mathcal{L}_{\text{RS}}\right) + \dots$$

$$[\boldsymbol{\Sigma}]_F = e \left(F - i\sqrt{2}\chi \sigma^\mu \overline{\psi}_\mu - z \overline{\psi}_\mu \overline{\sigma}^{\mu\nu} \overline{\psi}_\nu\right)$$

Kugo and Uehara, NPB B222 and NPB 226 (1983).

## Superconformally invariant action

$$\mathcal{L}_{\text{SUGRA}} = \left[-3\, \boldsymbol{\Phi}^\dagger \boldsymbol{\Phi}\, e^{-K/3}\right]_D + \left\{ \left[\boldsymbol{\Phi}^3\, \boldsymbol{W}\right]_F + \left[\frac{1}{4} \boldsymbol{f}_{ab} \boldsymbol{W}^{a\alpha} \boldsymbol{W}^b_\alpha\right]_F + \text{h.c.} \right\}$$

# Gauge freedoms to remove problematic terms

Gauge freedoms of $\hat{D}$, $\hat{A}$ and $\hat{S}_\alpha$ to fix lowest and fermionic components of $\mathbf{\Phi}$
(to any desired values, even to field-dependent functions of the matter fields)

## Coefficient of the problematic terms: $C$, $\xi_\alpha$ and Arg$[z]$

$$C = -3\sigma^\dagger\sigma e^{-K/3}, \quad \xi_\alpha = 3i\sqrt{2}\sigma^\dagger\sigma e^{-K/3}\left(\zeta_\alpha - \frac{K_i}{3}\chi^i_\alpha\right), \quad \text{Arg}[z] = \text{Arg}[\sigma^3 W].$$

KU gauge: freedom in $\sigma$ and $\zeta_\alpha$ to set $C = -3$, $\xi_\alpha = 0$, and $\text{Arg}\, z = 0$.

## Novel gauge fixing

$C = -3$, $\xi_\alpha = 0$, $Arg[z] = 0$ to linear order in field fluctuations

$$\sigma = \exp\left[\frac{1}{3}\left(\langle K/2 - i\,\text{Arg}\, W\rangle + \langle K_i\rangle X^i\right)\right], \quad \zeta_\alpha = \frac{1}{3}\langle K_i\rangle\chi^i_\alpha .$$

Normalize EH action; prevent matter and graviton/gravitino kinetic mixings; gravitino mass real.

Expression for $\mathbf{\Phi}$ rewritten in terms of superfields (modulo redefinition of $F_\phi$)

# Understanding the VEVs

## Presence of VEVs in $\Phi$ unintuitive

VEVs: vacuum structure of the theory, dependent on $\mathbf{\Phi}$, thus on the VEVs.

## EOM not affected by the appearence of VEVs

Function $f(x)$ and 1st order Taylor expansion $\tilde{f}(x) = \langle f(x) \rangle + \langle f'(x) \rangle (x - \langle x \rangle)$

Solution to $\langle \partial f / \partial x \rangle = 0$ is the same for
$f(x)$ and $\tilde{f}(x)$ and any linear combination

One can self-consistently solve for $\langle K_i \rangle$ terms
by treating $\langle K_i \rangle$ as numbers
determined by the scalar equations of motion

# Superfields in SUGRA

Describe matter supermultiplets in terms of global superspace variables $(\theta, \bar{\theta})$

Kugo and Uehara, NPB B222 and NPB 226 (1983).

## Chiral superfield $\boldsymbol{X} = \{X, \chi_\alpha, F\}$

Package components into $\theta$-dependent superfields, including SUGRA effects

$$\begin{aligned} \boldsymbol{X} \;=\; X \;&+\; \sqrt{2}\theta\chi \;+\; \theta^2 F \\ &+\; i\theta\sigma^\mu\bar{\theta}D^c_\mu X \;-\; \tfrac{i}{\sqrt{2}}\theta\theta D^c_\mu\chi\sigma^\mu\bar{\theta} \\ &+\; \tfrac{1}{4}\theta^4 D^c_\mu D^{c\mu} X. \end{aligned}$$

## SUGRA covariant derivative $D^c_\mu$

$$D^c_\mu X = \left(\partial_\mu - i\tfrac{w}{2}b_\mu\right) X + \ldots\,, \qquad D^c_\mu\chi_\alpha = \left(\partial_\mu - i\left(\tfrac{3}{4} - \tfrac{w}{2}\right) b_\mu\right)\chi_\alpha + \ldots\,.$$

$\ldots =$ graviton and gravitino terms (not problematic).

# Rewriting $\Phi$ in terms of superfields

## Novel gauge fixing for $\Phi = \{\sigma, \sigma\zeta_\alpha, \sigma F_\Phi\}$

$$\sigma = \exp\left[\frac{1}{3}\left(\langle K/2 - i\,\mathrm{Arg}\,W\rangle + \langle K_i\rangle X^i\right)\right] \;, \quad \zeta_\alpha = \frac{1}{3}\langle K_i\rangle \chi^i_\alpha \;.$$

Inconveniently written in components. Rewrite it in terms of superfields?

Subtleties: $\Phi$ has $w_\Phi = 1$. It Couples to the vector auxiliary field $b_\mu$.

## Coupling with the auxiliary field $b_\mu$

SUGRA-covariant derivative acting on $\sigma$: $\quad D^c_\mu \sigma = \partial_\mu \sigma - \frac{i}{2} b_\mu \sigma + \ldots$

Linear terms involving $b_\mu$ appear in the action , since $\langle\sigma\rangle \neq 0$.

## Novel gauge fixing: $b_\mu = 0 + \mathcal{O}(1/M_{\rm Pl})$

Gauge fixing can be written as

$$\Phi = e^{Z/3}(1 + \theta^2 F_\Phi), \qquad Z = \langle K/2 - i\,\mathrm{Arg}\,W\rangle + \langle K_i\rangle X^i$$

up to irrelevant $1/M_{\rm Pl}$-suppressed operators.

# The vector auxiliary field $b_\mu$

## Lagrangian for $b_\mu$

$$3\sigma^\dagger \sigma e^{-K/3} \left( \frac{b^\mu b_\mu}{4} + b^\mu \,\mathrm{Im} \left( \frac{1}{3} K_i \partial_\mu x^i - \frac{\partial_\mu \sigma}{\sigma} \right) \right)$$

## Novel gauge fixing

$$\sigma = \exp \left[ \frac{1}{3} \left( \langle K/2 - i \,\mathrm{Arg}\, W \rangle + \langle K_i \rangle X^i \right) \right] \quad \Rightarrow \quad \frac{\partial_\mu \sigma}{\sigma} = \frac{1}{3} \langle K_i \rangle \partial_\mu x^i$$

In the novel gauge fixing: $b_\mu = 0 + \mathcal{O}(1/M_{\mathrm{Pl}})$

# More on the modulino

## Sequestering between $\chi^T$ and $\chi^a$

$$\mathcal{L}_{\rm SUGRA} = \mathcal{L}^X + \mathcal{L}^T + \ldots$$

$$\mathcal{L}^T = \int d^4\theta\, \boldsymbol{\Phi}^{T\dagger}\boldsymbol{\Phi}^T\, \boldsymbol{\Omega}^T \,, \quad \mathcal{L}^X = \int d^4\theta\, \boldsymbol{\Phi}^{X\dagger}\boldsymbol{\Phi}^X\, \boldsymbol{\Omega}^X + \int d^2\theta\, (\boldsymbol{\Phi}^X)^3\, \boldsymbol{W} + \text{h.c.}\,.$$

## More on $\mathcal{L}^T$

Non-linear parameterization for $\boldsymbol{T}$: $\boldsymbol{T} = \left(\theta + \frac{1}{\sqrt{2}}\frac{\chi^T}{F^T}\right)^2 F^T$

To compute fermionic spectrum: $\boldsymbol{\Omega}^T = \alpha(\boldsymbol{T} + \boldsymbol{T}^\dagger) + \beta\boldsymbol{T}^\dagger\boldsymbol{T} + \ldots$

The Lagrangian for $\boldsymbol{T}$ reads:

$$\mathcal{L}^T = \int d^4\theta\, \left[\alpha\left(\boldsymbol{\Phi}^{T\dagger}\boldsymbol{T} + \boldsymbol{\Phi}^T\boldsymbol{T}^\dagger\right) + \beta\boldsymbol{T}^\dagger\boldsymbol{T}\right],$$
$$= \int d^2\theta\, \alpha\widetilde{F}^\dagger_{\Phi}\boldsymbol{T} + \text{h.c.} + \ldots.$$

# More on the modulino 2

## Modulino massless under two conditions

**The no-scale field is stabilized ($\partial V/\partial T = 0$)** : modulino not protected by a chiral symmetry, massless as a dynamical effect.

**The cosmological constant is tuned to zero ($V = 0$)** : not usually thought of as a symmetry enhanced point.

## Key ingredients in our derivation

- non-linear parametrization for $\boldsymbol{T}$, implicitly assumes that $T$ is stabilized at $\langle T \rangle = 0$;
- identification of the eaten direction only true in flat space, implicitly assume $V = 0$.