Quasinormal Modes of Unparticle Black Holes

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Outlook

- Unparticles Brief
- Ungravity
- Quasinormal modes of Black Holes
- QNM of $\mathcal{U}$-BH

UNPARTICLES

Unparticles $\mathcal{U}$: Basic Idea

H. Georgi, PRL98; PLB650

SM Sector

Weakly interacting

Scale Inv. Sector

Particles with definite masses

NO particles With definite nonzero masses

Unparticle!

Effective Theory for $\mathcal{U}$

energy

$\mathcal{BZ}$

Banks-Zaks($\mathcal{BZ}$) Theory

Massless fermionic gauge theory
With an infrared-stable fixed point.

$\mathbf{M}_{\mathcal{U}}$

$$\mathcal{L}_{\mathcal{BZ}} = \frac{\mathcal{O}_{\rm SM}\mathcal{O}_{\mathcal{BZ}}}{M_{\mathcal{U}}^k}$$

Dimensional transmutation

$\Lambda_{\mathcal{U}}$ ← matching

$$\mathcal{L}_{\mathcal{U}} = C_{\mathcal{U}}\frac{\Lambda_{\mathcal{U}}^{d_{\mathcal{BZ}}-d_{\mathcal{U}}}}{M_{\mathcal{U}}^k}\mathcal{O}_{\rm SM}\mathcal{O}_{\mathcal{U}}$$

Scale inv. emerges.

$\mathbf{M}_{\mathbf{W}}$; EWSB
; scale inv. breaking

SM

Phase Space of $\mathcal{U}$

Production Cross Section

$$d\sigma(p_1, p_2 \to P_{\mathcal{U}}, k_1, k_2, \ldots) = \frac{1}{2s}|\overline{\mathcal{M}}|^2 d\Phi$$

Phase Space

$$d\Phi = (2\pi)^4 \delta^{(4)}(p_1 + p_2 - P_{\mathcal{U}} - k_1 - k_2 - \cdots)$$
$$\times \prod_i \left[2\pi\theta(k_i^0)\delta(k_i^2)\frac{d^4 k_i}{(2\pi)^4} \right]$$
$$\times A_{d_{\mathcal{U}}}\theta(P_{\mathcal{U}}^0)\theta(P_{\mathcal{U}}^2)(P_{\mathcal{U}}^2)^{d_{\mathcal{U}}-2}\frac{d^4 P_{\mathcal{U}}}{(2\pi)^4}$$

$$\lim_{d_{\mathcal{U}}\to 1^+} A_{d_{\mathcal{U}}}(P_{\mathcal{U}}^2)^{d_{\mathcal{U}}-2}\theta(P_{\mathcal{U}}^0)\theta(P_{\mathcal{U}}^2) = 2\pi\theta(P_{\mathcal{U}}^0)\delta(P_{\mathcal{U}}^2)$$

Spectral Function

Two-point function

$$\langle 0|\mathcal{O}_{\mathcal{U}}(x)\mathcal{O}^{\dagger}(0)|0\rangle = \int \frac{d^4P}{(2\pi)^4} e^{-iP\cdot x} \rho_{\mathcal{U}}(P^2)$$

Spectral density function

$$\rho_{\mathcal{U}}(P^2) = A_{d_{\mathcal{U}}} \theta(P^0)\theta(P^2)(P^2)^{d_{\mathcal{U}}-2}$$

Fixed by scale inv.

Normalization factor

$$A_{d_{\mathcal{U}}} = \frac{16\pi^{5/2}}{(2\pi)^{2d_{\mathcal{U}}}} \frac{\Gamma(d_{\mathcal{U}} + \frac{1}{2})}{\Gamma(d_{\mathcal{U}} - 1)\Gamma(2d_{\mathcal{U}})}$$

Unparticles with $d_{\mathcal{U}}$ look like a Non-integral number of massless particles.

UNGRAVITY

Tensor Unparticle

Goldberg & Nath, PRL 100

Tensor unparticle interaction

$$\kappa_* = \Lambda_U^{-1}(\Lambda_U/M_U)^{d_{\rm UV}}$$

$$\kappa_* \frac{1}{\Lambda_U^{d_u-1}} \sqrt{g} T^{\mu\nu} O^{\mathcal{U}}_{\mu\nu}$$

Newtonian gravity modified

$$V(r) = -\frac{m_1 m_2 G}{r}\left[1 + \left(\frac{R_G}{r}\right)^{2d_{\mathcal{U}}-2}\right],$$

$$R_G = \frac{1}{\pi\Lambda_U}\left(\frac{M_{\rm Pl}}{M_*}\right)^{(1/d_{\mathcal{U}}-1)} \times \left(\frac{2(2-\alpha)}{\pi}\frac{\Gamma(d_{\mathcal{U}}+\frac{1}{2})\Gamma(d_{\mathcal{U}}-\frac{1}{2})}{\Gamma(2d_{\mathcal{U}})}\right)^{(1/2d_{\mathcal{U}}-2)},$$

unitary bound

$d_{\mathcal{U}} \geq 4$

Grinstein et al., PLB662

$\mathcal{U}$-enhanced black hole

Mureika, PLB660

Newtonian gravity modified

$$\Phi_{\mathcal{U}}(r) = \Phi_N(r)\left[1 + \Gamma_{d_{\mathcal{U}}}\left(\frac{R_*}{r}\right)^{2d_{\mathcal{U}}-2}\right]$$

$$g_{00} = 1 + 2\Phi(r)$$

$$g_{11} = \left(1 + 2\Phi(r)\right)^{-1}$$

Induced metric

$$ds^2 = \left[1 - \frac{2GM}{r}\left(1 + \Gamma_{d_{\mathcal{U}}}\left(\frac{R_*}{r}\right)^{2d_{\mathcal{U}}-2}\right)\right]dt^2 + \frac{dr^2}{1 - \frac{2GM}{r}\left(1 + \Gamma_{d_{\mathcal{U}}}\left(\frac{R_*}{r}\right)^{2d_{\mathcal{U}}-2}\right)} + r^2 d\Omega^2$$

"Schwarzschild" radius

$$r_H \approx \left(\frac{2M_{\rm BH}\Gamma_{d_{\mathcal{U}}}}{M_{\mathcal{U}}^2 \Lambda_{\mathcal{U}}^{-1}}\right)^{\frac{1}{2d_{\mathcal{U}}-1}} \Lambda_{\mathcal{U}}^{-1}$$

~10^{-5} fm for typical parameters

Geometric BH cross section

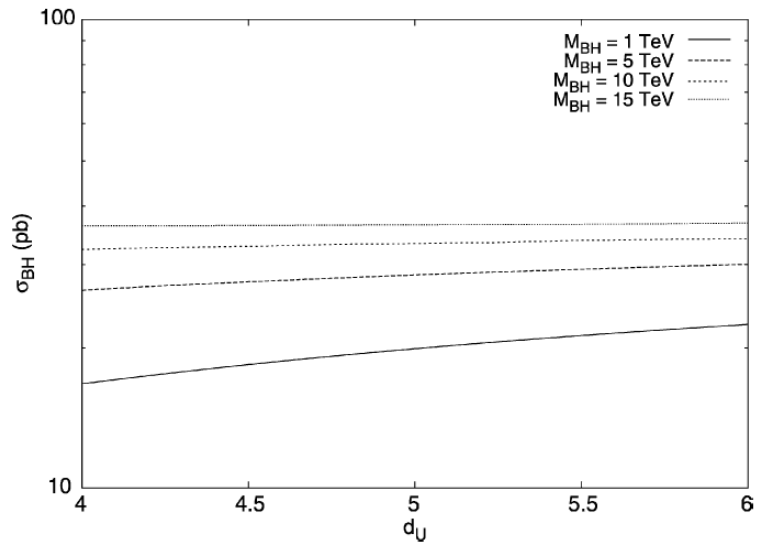


Fig. 2. Geometric black hole cross-section $\sigma_{\rm BH} = \pi r_H^2$ as a function of dimension $d_{\mathcal{U}}$, with $\Lambda_{\mathcal{U}} = 1$ TeV, $M_{\mathcal{U}} = 10$ TeV, and $M_{\rm Pl} = 2.4 \times 10^{15}$ TeV.

σ~10pb~a BH/few sec (30/fb.y)

cf) for XD BH, σ~100pb~1Hz prod.

$\mathcal{U}$ vs XD

Hawking Temperature

$$\log\left(\frac{T_{\rm ED}}{M_{\rm D}}\right) = -\left(\frac{1}{n+1}\right)\log\left(\frac{M_{\rm BH}}{M_{\rm D}}\right) + \mathcal{T}_{\rm ED}.$$

$$\log\left(\frac{T_{\mathcal{U}}}{\Lambda_{\mathcal{U}}}\right) = -\left(\frac{1}{2d_{\mathcal{U}}-1}\right)\log\left(\frac{M_{\rm BH}}{\Lambda_{\mathcal{U}}}\right) + \mathcal{T}_{\mathcal{U}}$$

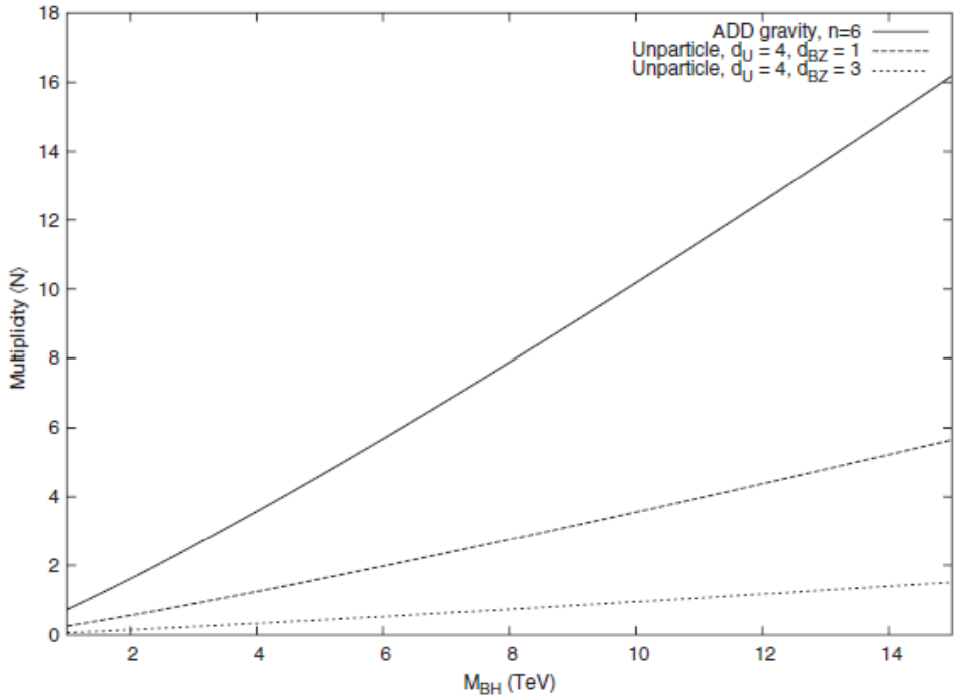
$$\mathcal{T}_{\mathcal{U}} = \mathcal{T}_{\rm ED} + \delta_{\mathcal{T}}.$$

$$\delta_{\mathcal{T}} = -\left(\frac{1}{n+1}\right)\log\left[\frac{n+2}{n+1}\cdot\frac{1}{\Gamma(1+\frac{n}{2})}\right] + \left(\frac{2d_{\rm BZ}}{n+1}\right)\log\left(\frac{M_{\mathcal{U}}}{\Lambda_{\mathcal{U}}}\right) + \log[2\sqrt{\pi}^{(n/(n+1))}].$$

Multiplicity

$$\langle N\rangle_{\rm ED} \approx \frac{M_{\rm BH}}{2T_{\rm ED}} = \frac{2\sqrt{\pi}}{n+1}\left(\frac{M_{\rm BH}}{M_{\rm D}}\right)^{(n+2)/(n+1)}\left[\frac{8\Gamma(\frac{n+3}{2})}{n+1}\right]^{1/(n+1)}$$

$$\langle N\rangle_{\mathcal{U}} = \frac{2\pi}{2d_{\mathcal{U}}-1}\left(\frac{M_{\rm BH}}{\Lambda_{\mathcal{U}}}\right)^{2d_{\mathcal{U}}/(2d_{\mathcal{U}}-1)} \times\left[2\Gamma_{d_{\mathcal{U}}}\left(\frac{M_{\mathcal{U}}}{\Lambda_{\mathcal{U}}}\right)^{2d_{\rm BZ}}\right]^{1/(2d_{\mathcal{U}}-1)}$$

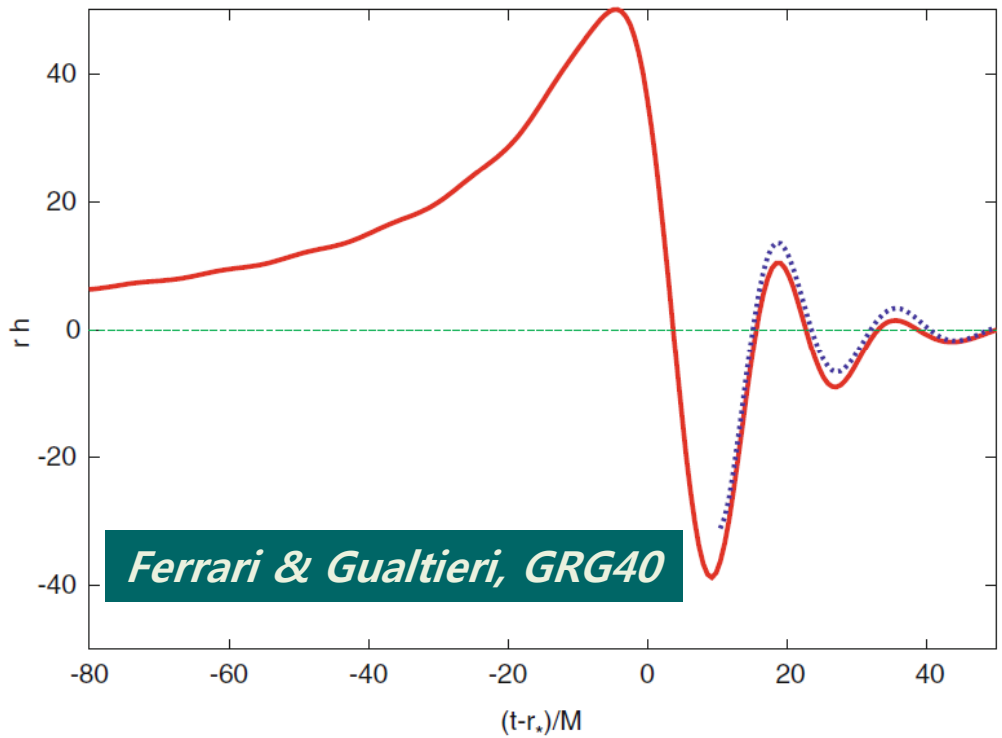


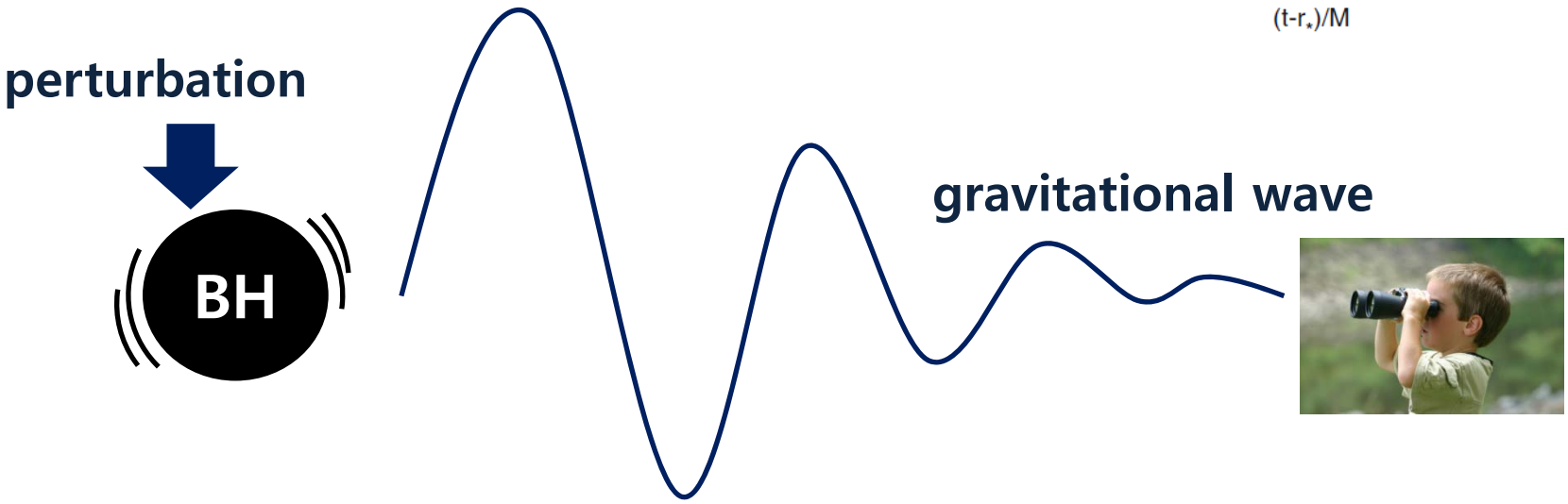
QNM OF BH

What are quasinormal modes?

QNM?:
"damping oscillation"

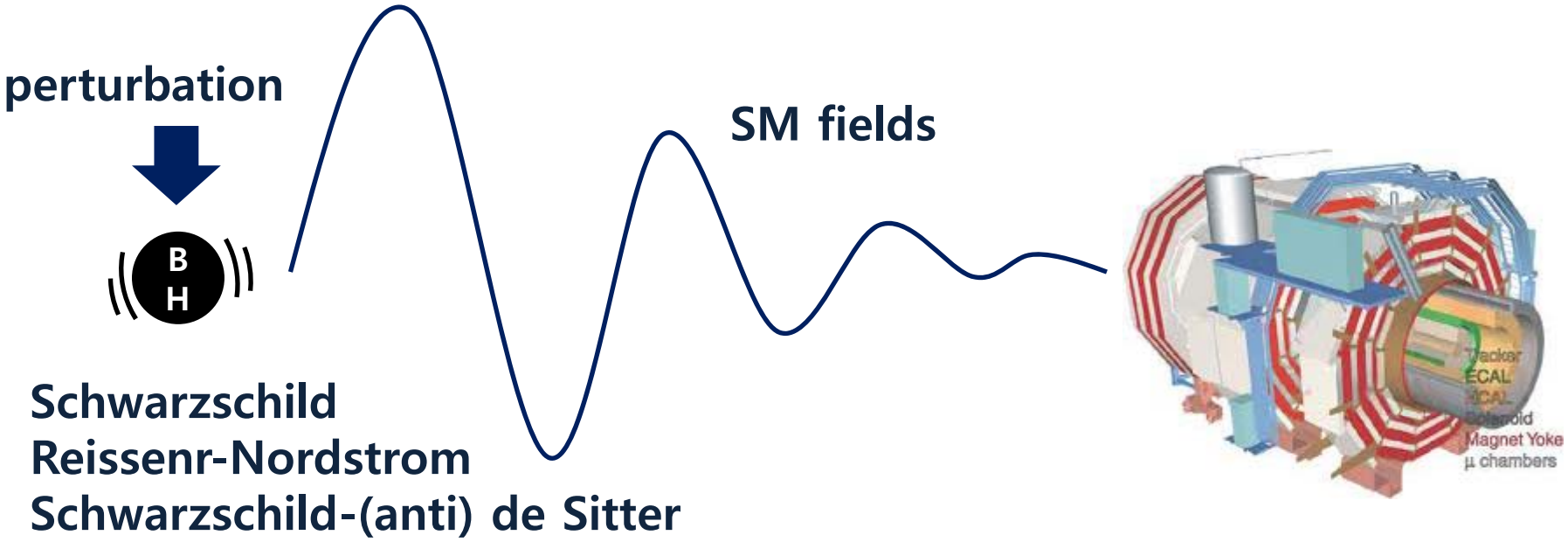
$$\Phi \sim e^{-i\omega t} \sim e^{(\mathrm{Im}\ \omega)t}$$





QNM for black holes in Xtr Dim

Kanti & Konoplya, PRD73



Schwarzschild
Reissenr-Nordstrom
Schwarzschild-(anti) de Sitter

Small Im w means slower decaying

→ Dominant during the "final ring-down" stage

QNM OF $\mathcal{U}$−BH

$\mathcal{U}$-induced metric

Mureika, PLB660

Modified grav. potential

$$\Phi_{\mathcal{U}}(r) = \Phi_N(r)\left[1 + \Gamma_{\mathcal{U}}\left(\frac{R_*}{r}\right)^{2d_{\mathcal{U}}-2}\right]$$

$$\Gamma_{\mathcal{U}} = \frac{2}{\pi^{2d_{\mathcal{U}}-1}}\frac{\Gamma(d_{\mathcal{U}}+\frac{1}{2})\Gamma(d_{\mathcal{U}}-\frac{1}{2})}{\Gamma(2d_{\mathcal{U}})}$$

$$R_* = \frac{1}{\Lambda_{\mathcal{U}}}\left(\frac{M_{\rm Pl}}{\Lambda_{\mathcal{U}}}\right)^{\frac{1}{d_{\mathcal{U}}-1}}\left(\frac{\Lambda_{\mathcal{U}}}{M_{\mathcal{U}}}\right)^{\frac{d_{\rm UV}}{d_{\mathcal{U}}-1}}$$

$\mathcal{U}$-induced metric

$$-g_{00} = 1 + 2\Phi_{\mathcal{U}}(r)$$

$$\Downarrow$$

$$ds^2 = -h_{\mathcal{U}}(r)dt^2 + \frac{1}{h_{\mathcal{U}}(r)}dr^2 + r^2 d\Omega^2$$

$$h_{\mathcal{U}}(r) = 1 - \frac{2GM}{r}\left[1 + \Gamma_{\mathcal{U}}\left(\frac{R_*}{r}\right)^{2d_{\mathcal{U}}-2}\right]$$

$\mathcal{U}$-Strong region

$$h_{\mathcal{U}}(r) \approx 1 - \frac{2GM}{r}\Gamma_{\mathcal{U}}\left(\frac{R_*}{r}\right)^{2d_{\mathcal{U}}-2}$$

$$2d_{\mathcal{U}} - 1 = D - 3$$

Separation of variables

$$\Psi_s(t, r, \theta, \phi) = e^{-i\omega t} e^{im\phi} \Delta^{-s} R_s(r) S_s(\theta)$$

Kanti & Konoplya, PRD73

Radial equation

$$\Delta^s \frac{d}{dr}\left(\Delta^{1-s}\frac{dR_s}{dr}\right) + \left(\frac{\omega^2 r^2}{h} + 2is\omega r - \frac{is\omega r^2 h'}{h} - \tilde{\lambda}\right) R_s = 0$$

$$\tilde{\lambda} = \lambda + 2s = \ell(\ell+1) - s(s-1)$$

Change of variables

$$\frac{dr_*}{dr} = \frac{1}{h}.$$

Schroedinger-like equation

$$\left(\frac{d^2}{dr_*^2} + \omega^2\right) Z_s = V_s Z_s$$

$$R_s = r^{2(s-1/2)} Y_s$$

$$Y_s = hZ_s + 2i\omega\left(\frac{d}{dr_*} - i\omega\right)$$

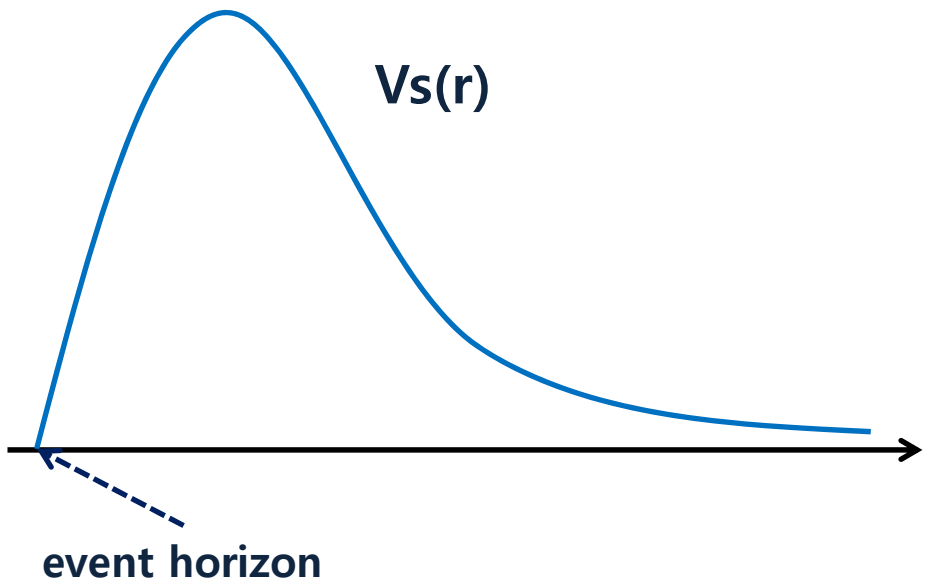
Effective potential

$$V_{s=0} = h\left[\frac{\ell(\ell+1)}{r^2} + \frac{h'}{r}\right] ,$$

$$V_{s=1} = \frac{h}{r^2}\ell(\ell+1) ,$$

$$V_{s=1/2} = hk\left[\frac{k}{r^2} \mp \frac{d}{dr}\left(\frac{\sqrt{h}}{r}\right)\right]$$

$$k = \sqrt{\ell(\ell+1) + 1/4} = 1, 2, 3, \cdots$$



WKB3 for the frequency

Quasinormal modes

$$\Psi_s(r_*) \approx C_\pm e^{\pm i\omega r_*} \quad \text{as} \quad r_* \to \pm\infty$$

" ingoing at the event horizon; outgoing at spatial infinity "

WKB3 Approximation

$$i\frac{\omega^2 - V_0}{\sqrt{-2V_0''}} - L_2 - L_3 = n + \frac{1}{2}$$

"WKB is accurate for $\ell > n$."

n: overtone number=0,1,2,...

$$L_2 = \frac{1}{(-2V_0'')^{1/2}}\left[\frac{1}{8}\left[\frac{V_0^{(4)}}{V_0''}\right]\left[\frac{1}{4}+\alpha^2\right] - \frac{1}{288}\left[\frac{V_0'''}{V_0''}\right]^2(7+60\alpha^2)\right]$$

Iyer, PRD35

$$L_3 = \frac{1}{(-2V_0'')}\left[\frac{5}{6912}\left[\frac{V_0'''}{V_0''}\right]^4(77+188\alpha^2) - \frac{1}{384}\left[\frac{V_0'''^2 V_0^{(4)}}{V_0''^{\,3}}\right](51+100\alpha^2) + \frac{1}{2304}\left[\frac{V_0^{(4)}}{V_0''}\right]^2(67+68\alpha^2)\right.$$
$$\left. + \frac{1}{288}\left[\frac{V_0''' V_0^{(5)}}{V_0''^{\,2}}\right](19+28\alpha^2) - \frac{1}{288}\left[\frac{V_0^{(6)}}{V_0''}\right](5+4\alpha^2)\right].$$

WKB 6th term

Konoplya, PRD68

$$
\begin{aligned}
\Lambda_6 = \frac{-i}{2022633897984 00 Q_2^{12}\sqrt{2Q_2}}\big(&-171460800Q_{12}Q_2^9+1714608000Q_{11}Q_2^8Q_3-10268596800Q_{10}Q_2^7Q_3^2\\
&+970010662775Q_3^{10}+3772137600Q_{10}Q_2^8Q_4-6262634175525Q_2Q_3^8Q_4+13782983196150Q_2^2Q_3^6Q_4^2\\
&-11954148125850Q_2^3Q_3^4Q_4^3+3449170577475Q_2^4Q_3^2Q_4^4-144528059025Q_2^5Q_4^5+3352602187200Q_2^2Q_3^7Q_5\\
&-12300730092000Q_2^3Q_3^5Q_4Q_5+11994129604800Q_2^4Q_3^3Q_4^2Q_5-2624788605600Q_2^5Q_3Q_4^3Q_5\\
&+2580769643760Q_2^4Q_3^4Q_5^2-3453909784416Q_2^5Q_3^2Q_4Q_5^2+438440697072Q_2^6Q_4^2Q_5^2+260524397952Q_2^6Q_3Q_5^3\\
&-1475306441280Q_2^3Q_3^6Q_6+4329682610400Q_2^4Q_3^4Q_4Q_6-2865128172480Q_2^5Q_3^2Q_4^2Q_6+233443879200Q_2^6Q_4^3Q_6\\
&-1660199804928Q_2^5Q_3^3Q_5Q_6+1281705296256Q_2^6Q_3Q_4Q_5Q_6-87403857408Q_2^7Q_5^2Q_6+231105873600Q_2^6Q_3^2Q_6^2\\
&-68412859200Q_2^7Q_4Q_6^2+552968700480Q_2^4Q_3^5Q_7-1231789749120Q_2^5Q_3^3Q_4Q_7+470726303040Q_2^6Q_3Q_4^2Q_7\\
&+413953400448Q_2^6Q_3^2Q_5Q_7-1262421780 48Q_2^7Q_4Q_5Q_7-91489305600Q_2^7Q_3Q_6Q_7+5619715200Q_2^8Q_7^2\\
&-175752294480Q_2^5Q_3^4Q_8+2717596526 40Q_2^6Q_3^2Q_4Q_8-39736040400Q_2^7Q_4^2Q_8-73378363968Q_2^7Q_3Q_5Q_8\\
&+9773265600Q_2^8Q_6Q_8+47107126080Q_2^6Q_3^3Q_9-43345290240Q_2^7Q_3Q_4Q_9+7400248128Q_2^8Q_5Q_9\big)\\
-\frac{(n+1/2)^2 i}{687970713600Q_2^{12}\sqrt{2Q_2}}\big(&-4551552Q_{12}Q_2^9+60279552Q_{11}Q_2^8Q_3-425036160Q_{10}Q_2^7Q_3^2+73727194625Q_3^{10}\\
&+116743680Q_{10}Q_2^8Q_4-443649208275Q_2Q_3^8Q_4+901144103850Q_2^2Q_3^6Q_4^2-711096726150Q_2^3Q_3^4Q_4^3\\
&+182164306725Q_2^4Q_3^2Q_4^4-6289615575Q_2^5Q_4^5+222467624400Q_2^2Q_3^7Q_5-746418445200Q_2^3Q_3^5Q_4Q_5\\
&+653423900400Q_2^4Q_3^3Q_4^2Q_5-124319674800Q_2^5Q_3Q_4^3Q_5+143980943040Q_2^4Q_3^4Q_5^2-169712521920Q_2^5Q_3^2Q_4Q_5^2\\
&+18188188416Q_2^6Q_4^2Q_5^2+11240861184Q_2^6Q_3Q_5^3-91198200240Q_2^3Q_3^6Q_6+241513732080Q_2^4Q_3^4Q_4Q_6
\end{aligned}
$$

Cont' d

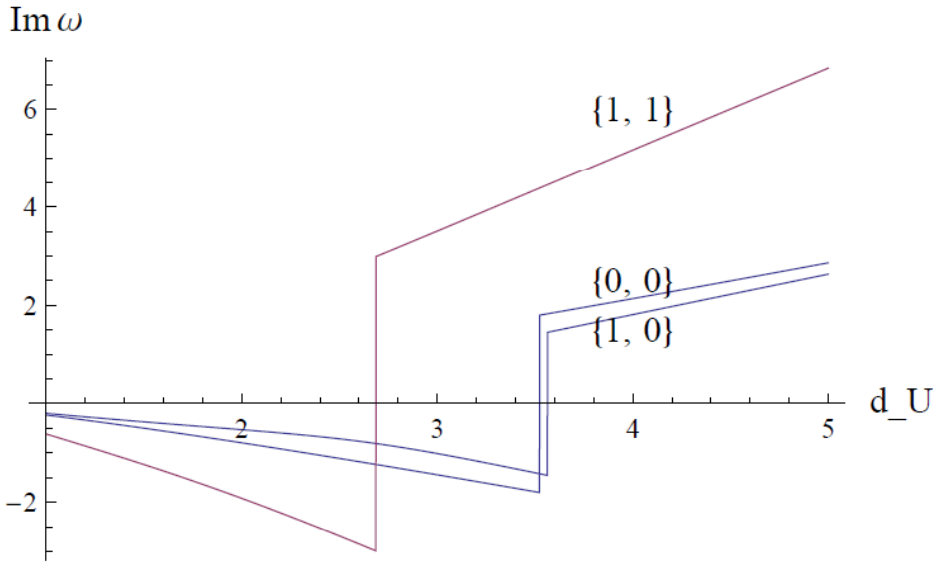
$$-140030897040Q_2^5Q_3^2Q_4^2Q_6+92001031 20Q_2^6Q_4^3Q_6-84218693760Q_2^5Q_3^3Q_5Q_6+55248386688Q_2^6Q_3Q_4Q_5Q_6$$
$$-3173043456Q_2^7Q_5^2Q_6+10464952896Q_2^6Q_3^2Q_6^2-2403421632Q_2^7Q_4Q_6^2+31637744640Q_2^4Q_3^5Q_7$$
$$-62649953280Q_2^5Q_3^3Q_4Q_7+204098227 20Q_2^6Q_3Q_4^2Q_7+188605324 80Q_2^6Q_3^2Q_5Q_7-4693344768Q_2^7Q_4Q_5Q_7$$
$$-3625731072Q_2^7Q_3Q_6Q_7+188054784Q_2^8Q_7^2-9155635200Q_2^5Q_3^4Q_8+12238024320Q_2^6Q_3^2Q_4Q_8$$
$$-14052787 20Q_2^7Q_4^2Q_8-2866700160Q_2^7Q_3Q_5Q_8+303295104Q_2^8Q_6Q_8+2210705280Q_2^6Q_3^3Q_9$$
$$-1685525760Q_2^7Q_3Q_4Q_9+235488384Q_2^8Q_5Q_9)-\frac{(n+1/2)^4 i}{20065812480Q_2^{12}\sqrt{2Q_2}}(-66528Q_{12}Q_2^9+1245888Q_{11}Q_2^8Q_3$$
$$-11158560Q_{10}Q_2^7Q_3^2+4668804525Q_3^{10}+2116800Q_{10}Q_2^8Q_4-25898331375Q_2Q_3^8Q_4+47959232650Q_2^2Q_3^6Q_4^2$$

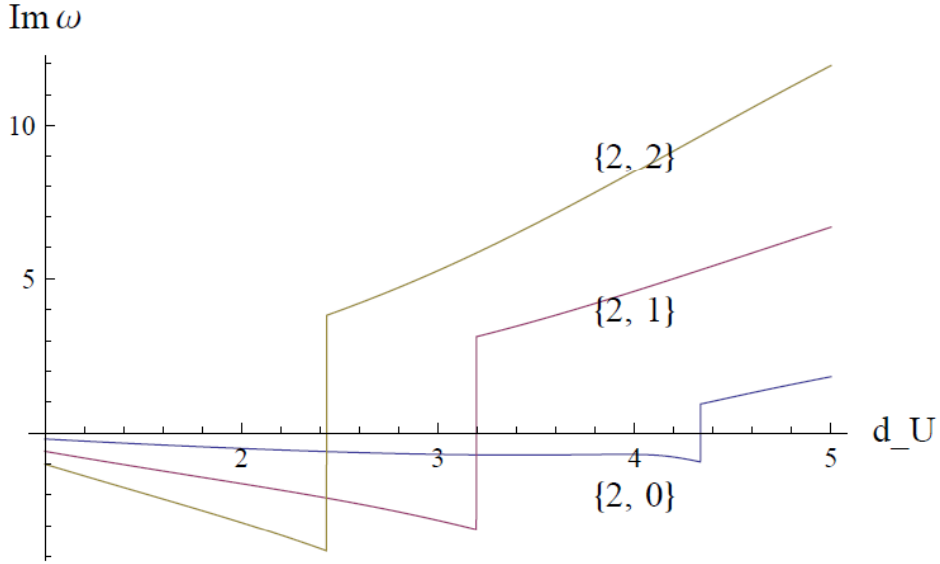
$$-33861927750Q_2^3Q_3^4Q_4^3+7454763225Q_2^4Q_3^2Q_4^4-184988475Q_2^5Q_4^5+11891917800Q_2^2Q_3^7Q_5$$
$$-36105463800Q_2^3Q_3^5Q_4Q_5+27953667000Q_2^4Q_3^3Q_4^2Q_5-4457716200Q_2^5Q_3Q_4^3Q_5+6285855240Q_2^4Q_3^4Q_5^2$$
$$-6471756144Q_2^5Q_3^2Q_4Q_5^2+565259688Q_2^6Q_4^2Q_5^2+380939328Q_2^6Q_3Q_5^3-4375251160Q_2^3Q_3^6Q_6$$
$$+10317018600Q_2^4Q_3^4Q_4Q_6-5113813320Q_2^5Q_3^2Q_4^2Q_6+238888440Q_2^6Q_4^3Q_6-3203871552Q_2^5Q_3^3Q_5Q_6$$
$$+1758685824Q_2^6Q_3Q_4Q_5Q_6-88566912Q_2^7Q_5^2Q_6+335466432Q_2^6Q_3^2Q_6^2-55073088Q_2^7Q_4Q_6^2+1351294560Q_2^4Q_3^5Q_7$$
$$-2341442880Q_2^5Q_3^3Q_4Q_7+626542560Q_2^6Q_3Q_4^2Q_7+619520832Q_2^6Q_3^2Q_5Q_7-123524352Q_2^7Q_4Q_5Q_7$$
$$-96574464Q_2^7Q_3Q_6Q_7+4048704Q_2^8Q_7^2-341160120Q_2^5Q_3^4Q_8+386210160Q_2^6Q_3^2Q_4Q_8-30837240Q_2^7Q_4^2Q_8$$
$$-78073632Q_2^7Q_3Q_5Q_8+5848416Q_2^8Q_6Q_8+70415520Q_2^6Q_3^3Q_9-43424640Q_2^7Q_3Q_4Q_9$$

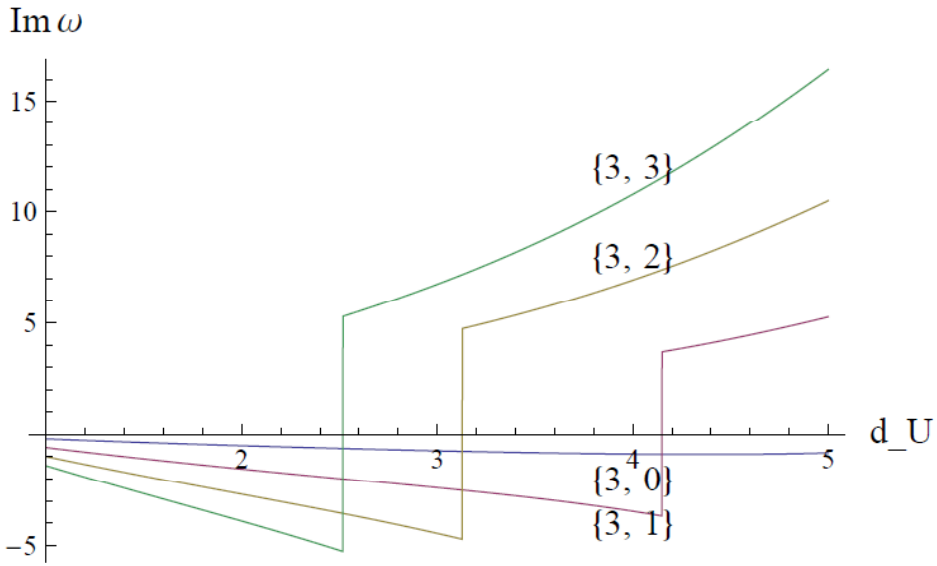
$$+5255712Q_2^8Q_5Q_9)-\frac{(n+1/2)^6 i}{300987187200Q_2^{12}\sqrt{2Q_2}}(-72576Q_{12}Q_2^9+1886976Q_{11}Q_2^8Q_3-22135680Q_{10}Q_2^7Q_3^2$$
$$+27463538375Q_3^{10}+2903040Q_{10}Q_2^8Q_4-141448688325Q_2Q_3^8Q_4+240655765350Q_2^2Q_3^6Q_4^2-152907158250Q_2^3Q_3^4Q_4^3$$
$$+28724479875Q_2^4Q_3^2Q_4^4-413669025Q_2^5Q_4^5+59058073200Q_2^2Q_3^7Q_5-164264209200Q_2^3Q_3^5Q_4Q_5$$
$$+113654696400Q_2^4Q_3^3Q_4^2Q_5-15166342800Q_2^5Q_3Q_4^3Q_5+260611948 80Q_2^4Q_3^4Q_5^2-238762339 20Q_2^5Q_3^2Q_4Q_5^2$$
$$+1767189312Q_2^6Q_4^2Q_5^2+1292433408Q_2^6Q_3Q_5^3-18902165520Q_2^3Q_3^6Q_6+40256773200Q_2^4Q_3^4Q_4Q_6$$
$$-17116974000Q_2^5Q_3^2Q_4^2Q_6+483582960Q_2^6Q_4^3Q_6-11384150400Q_2^5Q_3^3Q_5Q_6+5285056896Q_2^6Q_3Q_4Q_5Q_6$$
$$-246903552Q_2^7Q_5^2Q_6+992779200Q_2^8Q_3^2Q_6^2-101860416Q_2^7Q_4Q_6^2+4966859520Q_2^4Q_3^5Q_7-7661606400Q_2^5Q_3^3Q_4Q_7$$
$$+1683037440Q_2^6Q_3Q_4^2Q_7+1861574400Q_2^6Q_3^2Q_5Q_7316141056Q_2^7Q_4Q_5Q_7-235146240Q_2^7Q_3Q_6Q_7+8895744Q_2^8Q_7^2$$
$$-10423728 00Q_2^5Q_3^4Q_8+1016789760Q_2^6Q_3^2Q_4Q_8-52436160Q_2^7Q_4^2Q_8-189060480Q_2^7Q_3Q_5Q_8$$
$$+9217152Q_2^8Q_6Q_8+175190400Q_2^6Q_3^3Q_9-87816960Q_2^7Q_3Q_4Q_9+10378368Q_2^8Q_5Q_9). \qquad \text{(A3)}$$

Scalar fields for (ℓ, n)

JPL, arXiv:1106.6157

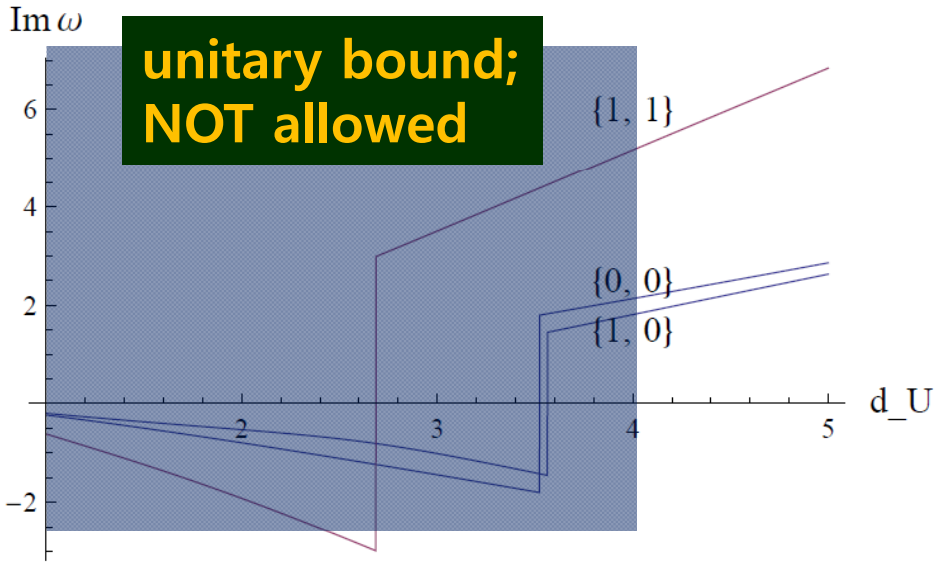


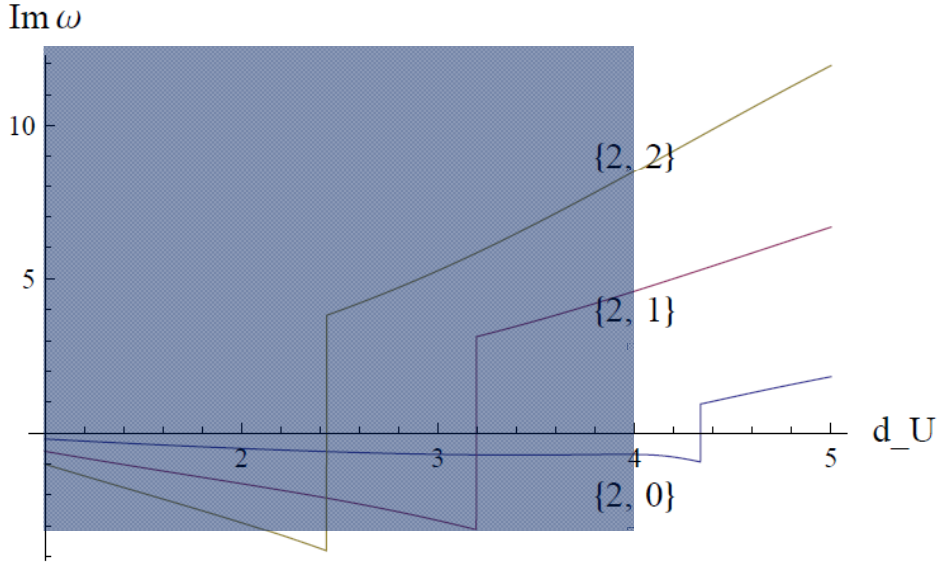


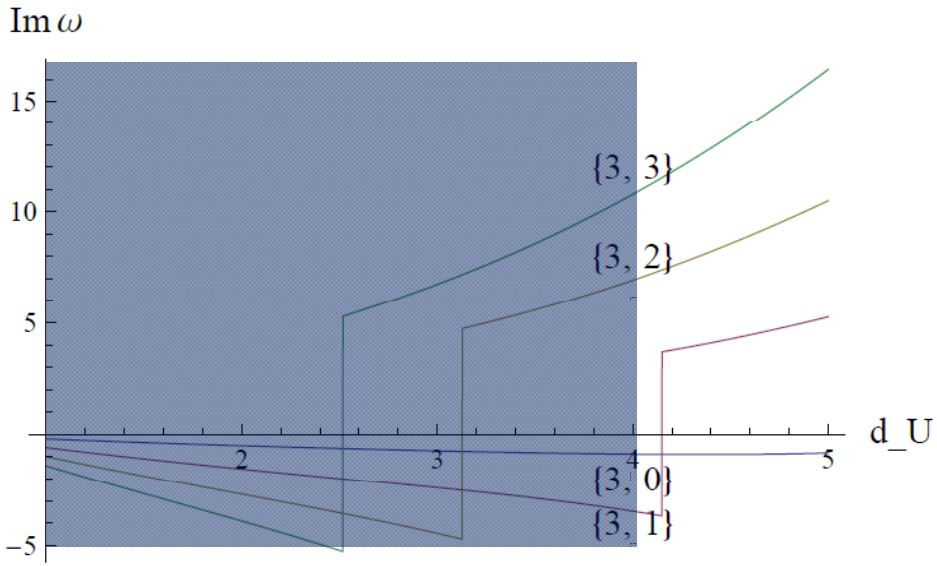


Scalar fields for (ℓ, n)

JPL, arXiv:1106.6157

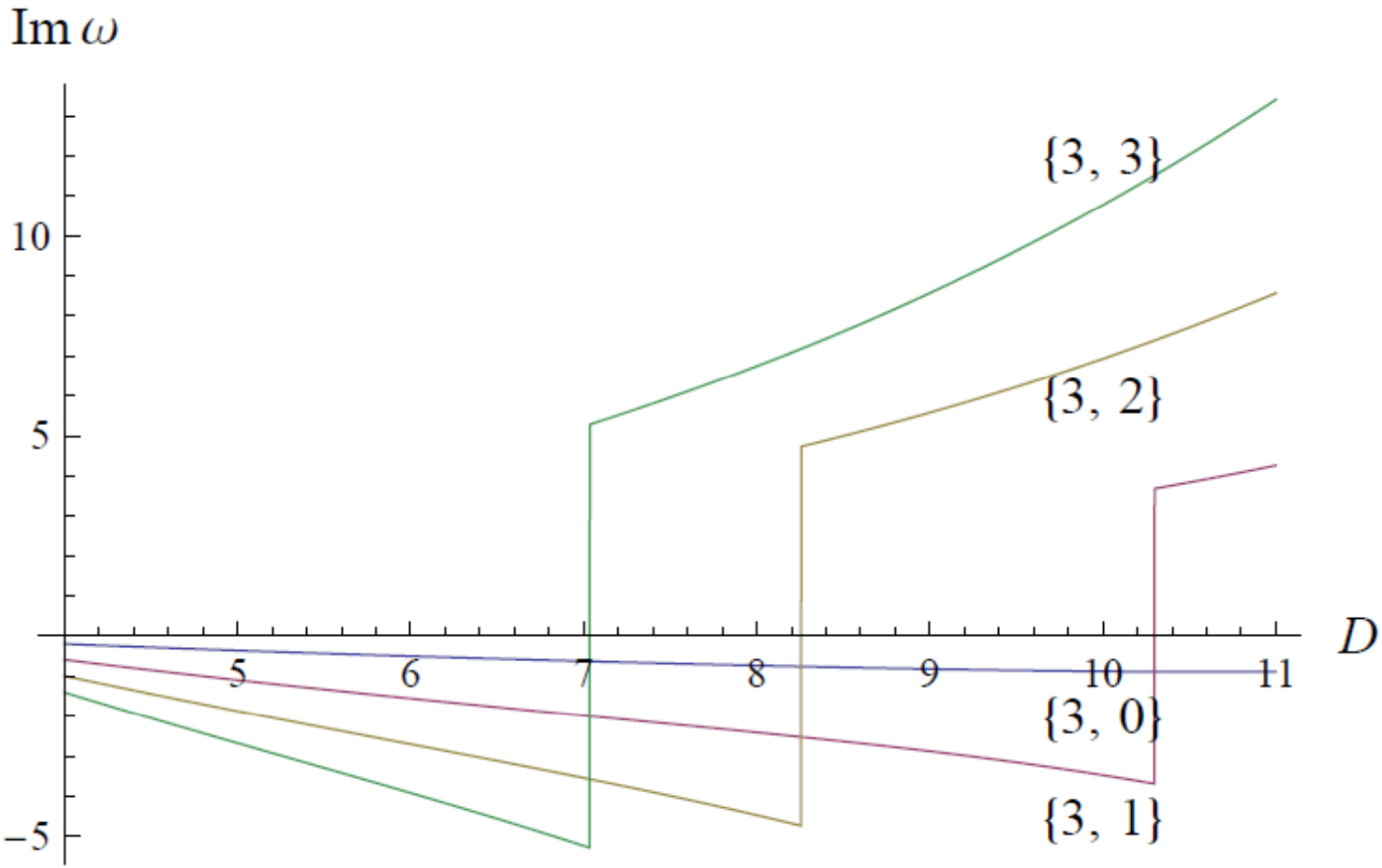






Comparision: scalar in XD

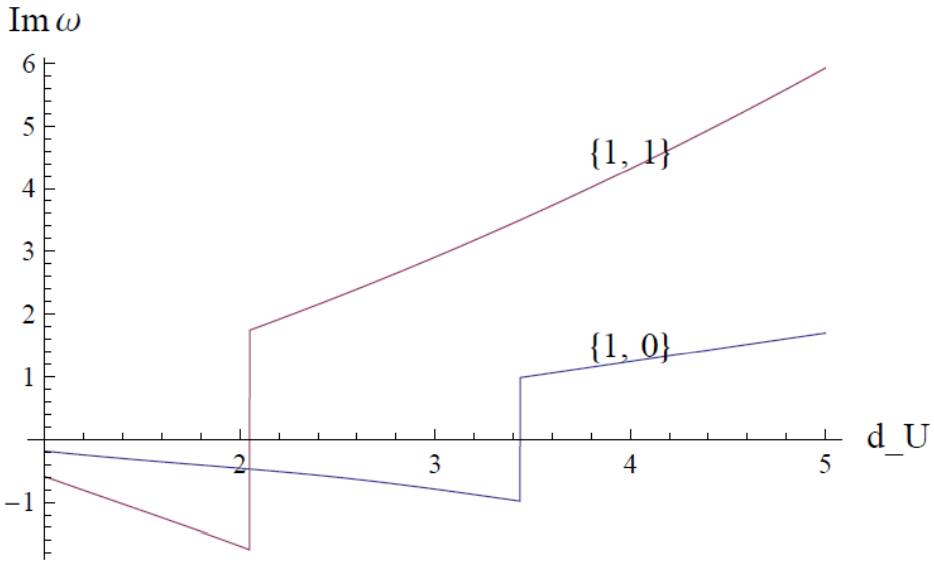
$$2d_{\mathcal{U}} - 1 = D - 3$$

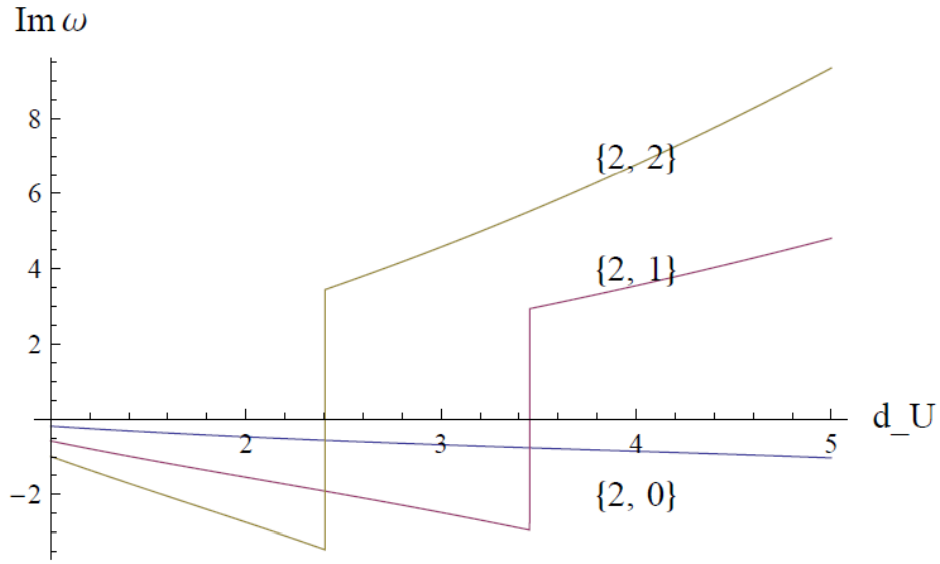


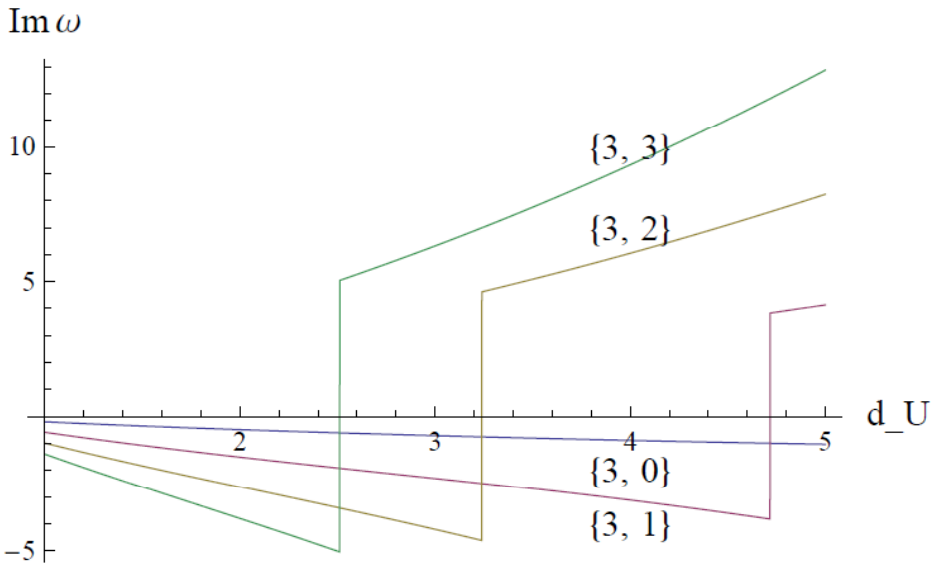
$d_{\mathcal{U}} = 4$	ω	$d_{\mathcal{U}} = 5$	ω
$(\ell, n) = (0, 0)$	$0.116019 + 2.13604i$	$(\ell, n) = (0, 0)$	$0.399872 + 2.86104i$
$(\ell, n) = (1, 0)$	$0.173311 + 1.81192i$	$(\ell, n) = (1, 0)$	$0.530601 + 2.6313i$
$(\ell, n) = (1, 1)$	$0.541964 + 5.17633i$	$(\ell, n) = (1, 1)$	$0.852281 + 6.8386i$
$(\ell, n) = (2, 0)$	$0.609546 - 0.691852i$	$(\ell, n) = (2, 0)$	$0.678472 + 1.81891i$
$(\ell, n) = (2, 1)$	$1.1295 + 4.59508i$	$(\ell, n) = (2, 1)$	$2.41065 + 6.66656i$
$(\ell, n) = (2, 2)$	$3.5565 + 8.48243i$	$(\ell, n) = (2, 2)$	$5.69487 + 11.9428i$
$(\ell, n) = (3, 0)$	$2.14206 - 0.874824i$	$(\ell, n) = (3, 0)$	$1.79741 - 0.820522i$
$(\ell, n) = (3, 1)$	$0.244724 - 3.47487i$	$(\ell, n) = (3, 1)$	$1.62507 + 5.25536i$
$(\ell, n) = (3, 2)$	$2.0898 + 6.93238i$	$(\ell, n) = (3, 2)$	$5.47658 + 10.4901i$
$(\ell, n) = (3, 3)$	$4.98341 + 10.7721i$	$(\ell, n) = (3, 3)$	$10.4015 + 16.4574i$

;

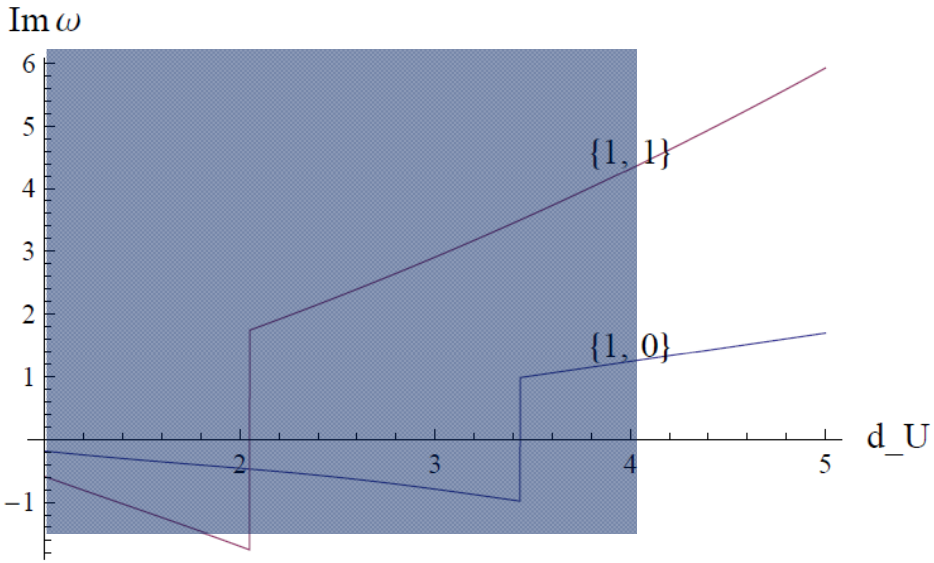
Vector fields

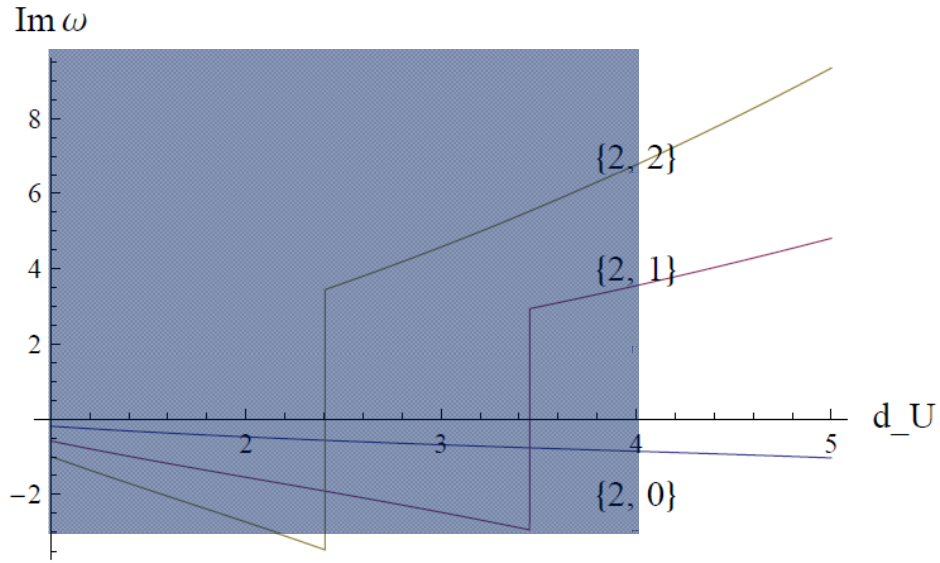


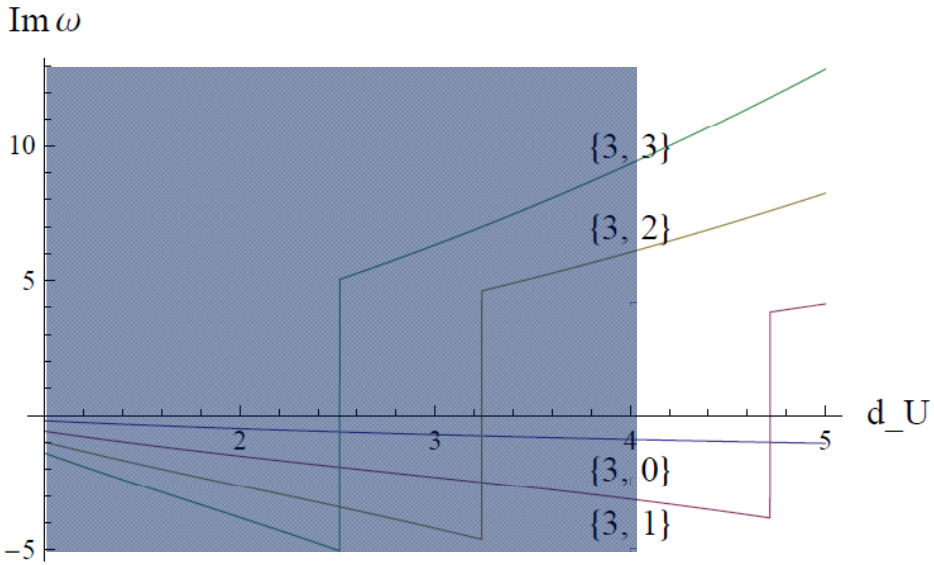




Vector fields

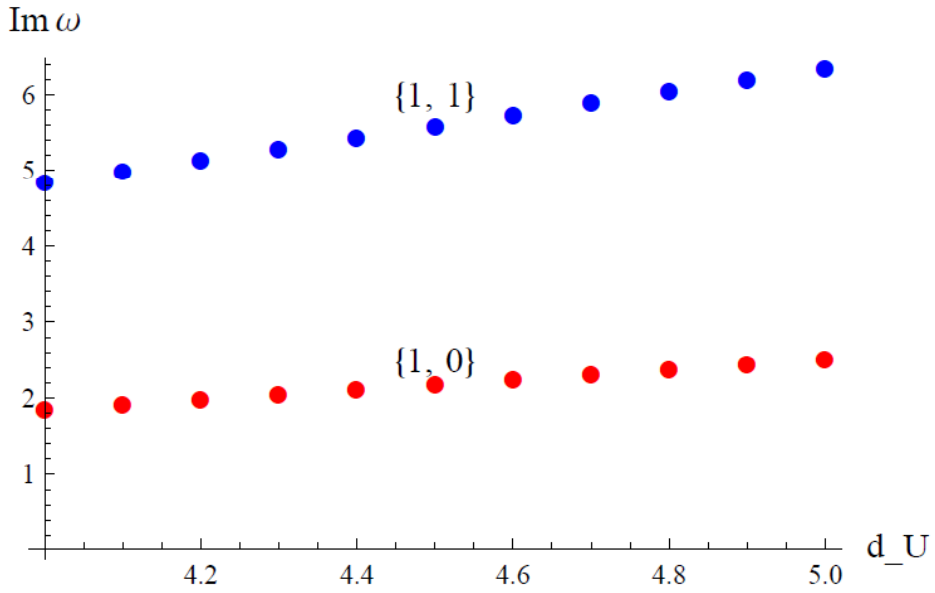


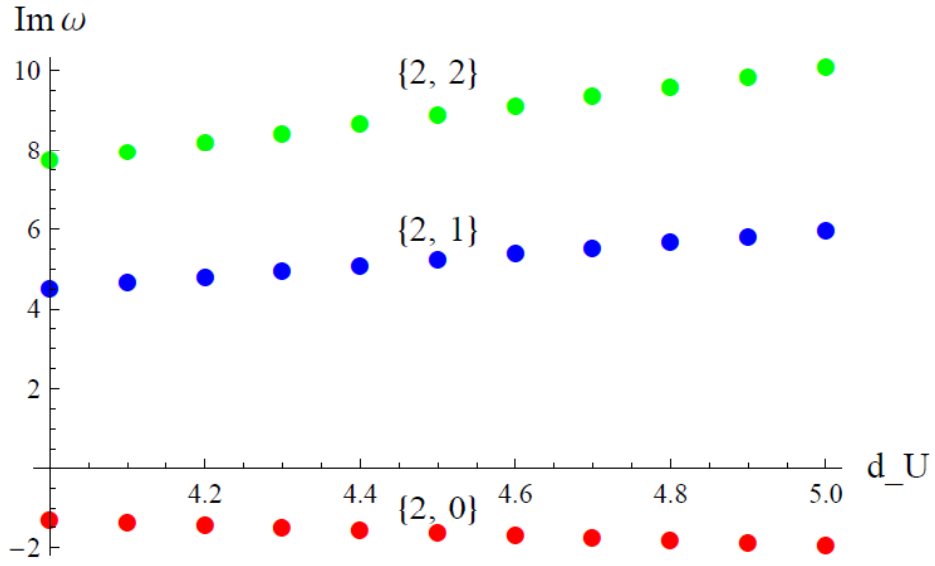


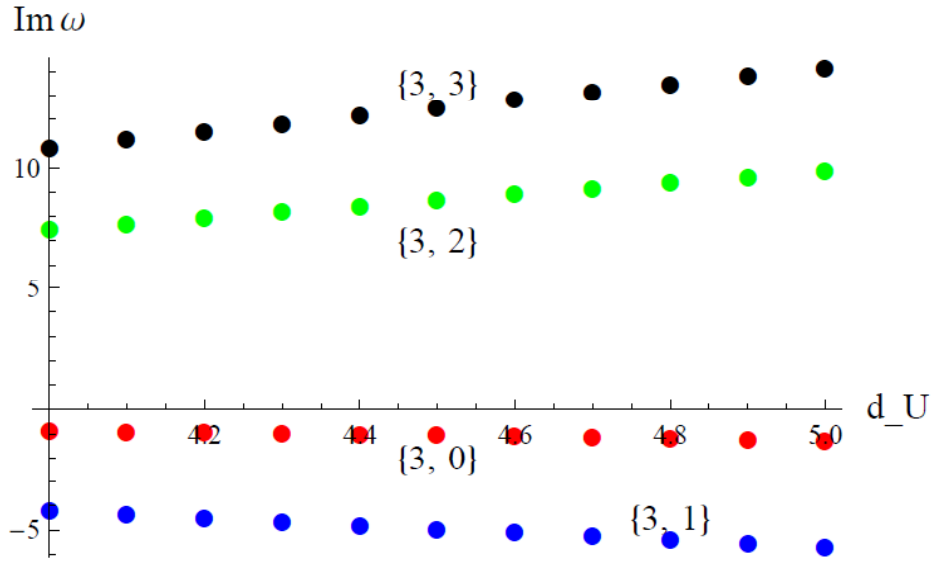


$d_{\mathcal{U}} = 4$	ω	$d_{\mathcal{U}} = 5$	ω
$(\ell, n) = (1, 0)$	$0.19116 + 1.24064i$	$(\ell, n) = (1, 0)$	$0.523096 + 1.6981i$
$(\ell, n) = (1, 1)$	$2.06499 + 4.31569i$	$(\ell, n) = (1, 1)$	$3.5286 + 5.92237i$
$(\ell, n) = (2, 0)$	$1.11899 - 0.854651i$	$(\ell, n) = (2, 0)$	$0.83325 - 1.03127i$
$(\ell, n) = (2, 1)$	$0.525274 + 3.55141i$	$(\ell, n) = (2, 1)$	$1.65328 + 4.80792i$
$(\ell, n) = (2, 2)$	$2.7902 + 6.77041i$	$(\ell, n) = (2, 2)$	$5.17625 + 9.3328i$
$(\ell, n) = (3, 0)$	$2.0822 - 0.881647i$	$(\ell, n) = (3, 0)$	$2.01102 - 1.02585i$
$(\ell, n) = (3, 1)$	$0.690689 - 3.12684i$	$(\ell, n) = (3, 1)$	$0.303258 + 4.13687i$
$(\ell, n) = (3, 2)$	$1.24996 + 6.04807i$	$(\ell, n) = (3, 2)$	$3.26883 + 8.23007i$
$(\ell, n) = (3, 3)$	$3.64909 + 9.32242i$	$(\ell, n) = (3, 3)$	$6.97334 + 12.8731i$

Dirac fields for (k, n)







$d_{\mathcal{U}} = 4$	ω	$d_{\mathcal{U}} = 5$	ω
$(\ell, n) = (1, 0)$	$0.274179 + 1.84419i$	$(\ell, n) = (1, 0)$	$0.62748 + 2.51579i$
$(\ell, n) = (1, 1)$	$0.994088 + 4.83761i$	$(\ell, n) = (1, 1)$	$1.71823 + 6.34767i$
$(\ell, n) = (2, 0)$	$0.467238 - 1.30984i$	$(\ell, n) = (2, 0)$	$0.126116 - 1.95711i$
$(\ell, n) = (2, 1)$	$0.104486 + 4.52932i$	$(\ell, n) = (2, 1)$	$0.56586 + 5.96395i$
$(\ell, n) = (2, 2)$	$1.09812 + 7.72804i$	$(\ell, n) = (2, 2)$	$1.9171 + 10.0892i$
$(\ell, n) = (3, 0)$	$1.45664 - 0.896816i$	$(\ell, n) = (3, 0)$	$0.949156 - 1.26542i$
$(\ell, n) = (3, 1)$	$0.363197 - 4.20139i$	$(\ell, n) = (3, 1)$	$0.0248592 - 5.698i$
$(\ell, n) = (3, 2)$	$0.727909 + 0.746487i$	$(\ell, n) = (3, 2)$	$1.14576 + 9.86467i$
$(\ell, n) = (3, 3)$	$2.25403 + 10.8068i$	$(\ell, n) = (3, 3)$	$2.83694 + 14.119i$

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  - ◆Fractional number of extra dimensions.
  - ◆Hawking temp. and multiplicity.
  - ◆Forbidden QNM modes of SM particles.