$\mathcal{N} = (0,2)$ Heterotic $\mathbb{CP}^{N-1}$ Sigma Models and a Nonrenormalization Theorem

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Joint work with Mikhail Shifman

XC and M. Shifman, PRD **82**, 105022(2010), arXiv:1009.4421.

XC and M. Shifman, arXiv:1105.5107.

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Heterotic nonlinear sigma models

$$\begin{array}{rcl}
\text{4D gauge theories} & \Leftrightarrow & \text{2D NLSM} \\
\mathcal{N}=2\,,\ U(N)^N \text{ theories} & \Leftrightarrow & \mathcal{N}=(2,2)\,,\ \mathbb{CP}^{N-1} \\
\mathcal{N}=1 \text{ mass deformation} & \Leftrightarrow & \mathcal{N}=(0,2) \text{ heterotic models}
\end{array}$$

Objective: to understand the perturbative aspects of the deformed $\mathbb{CP}^{N-1}$ models.

Heterotic nonlinear sigma models

$$\mathcal{L}_{(2,2)} = G\left\{\partial^\mu \phi \partial_\mu \phi^\dagger + i\bar{\psi}\not{D}\psi + \text{four fermion interactions}\right\} ,$$

$$\begin{aligned}\mathcal{L}_{(0,2)} \;&=\; \mathcal{L}_{(2,2)} + \zeta_R^\dagger\, i\partial_{LL}\,\zeta_R + \left[\gamma\,\zeta_R\, G\left(i\,\partial_{LL}\phi^\dagger\right)\psi_R + \mathrm{H.c.}\right]\\ &\quad+\text{four fermion interactions}\,.\end{aligned}$$

- theory with two couplings.

$$\beta_{g^2} = -\frac{Ng^4}{2\pi} + \cdots , \quad \beta_\gamma = \frac{N\gamma}{2\pi}(\gamma^2 - g^2) + \cdots .$$

- define $\rho = \frac{\gamma^2}{g^2}$, we have

$$\beta_\rho = \frac{Ng^2}{2\pi}\rho(2\rho - 1) + \cdots .$$

- The factorization sustains to all loop level. [XC and M. Shifman, 2010]

$\mathcal{N} = (0, 2)$ supersymmetry

- $\mathcal{N} = (0, 2)$ superspace:

$$\begin{array}{cccc} x_L & & x_R & \\ \theta_L & \theta_L^\dagger & \theta_R & \theta_R^\dagger \end{array} \Rightarrow \begin{array}{cccc} x_L & & x_R & \\ & & \theta_R & \theta_R^\dagger \end{array}$$

- super-derivatives

$$D\,,\bar{D} \Rightarrow \{D, \bar{D}\} = 2i\partial_{LL}$$

  ∂_{RR} commutes with everything!

- chiral superfields

$$\begin{array}{lccc} \text{bosonic} & \phi & \psi_L & \\ \text{fermionic} & & \psi_R & F \end{array}$$

A simplified model with flat target space

In a bid to understand the deformation and to get some higher loop correction, we consider the following model, which corresponds to the limiting case $\frac{\gamma^2}{g^2} \to \infty$ of the previous model.

$$\mathcal{L}_{\rm linear} = \frac{1}{2}\int d^2\theta_R \, iA^\dagger \overleftrightarrow{\partial_{RR}} A + B^\dagger B + \mathcal{B}^\dagger \mathcal{B} - \left(\gamma \mathcal{B} B A^\dagger + \mathrm{H.c.}\right) .$$

- background symmetry

$$A \to A + \alpha + \alpha^\dagger A^2 \;\Rightarrow\; A \to A + \alpha \, .$$

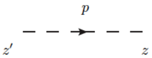
- fermion flavor symmetry:

$$\begin{pmatrix} \mathcal{B} \\ \frac{B}{1+A^\dagger A} \end{pmatrix} \Rightarrow \begin{pmatrix} \mathcal{B} \\ B \end{pmatrix} .$$

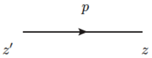
Supergraph as a probe

Feynman rules to remember:

- Propagators:

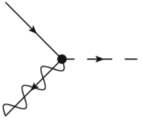
$T[A_z, A^\dagger_{z'}] =$ 

$= -\frac{i}{p^2}\delta^1(\theta_R - \theta'_R)$

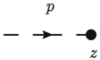
$T[B_z, B^\dagger_{z'}] =$ 

$= -\frac{i}{p_L}\delta^1(\theta_R - \theta'_R)$

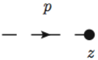
- vertices:


$= \frac{i\gamma}{2}$

- (anti)chiral projectors:

A 

$= \bar{D}_R(p, \theta_R, \theta_R^\dagger) A_z$

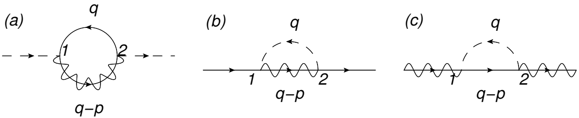
$A^\dagger$ 

$= A_z^\dagger \overleftarrow{\bar{D}}_R(p, \theta_R, \theta_R^\dagger)$

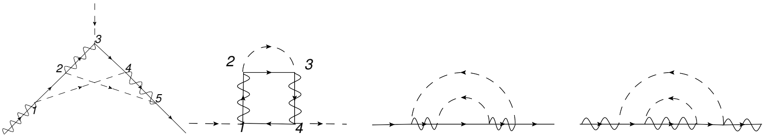
Supergaph as a probe

Feynman diagrams to calculate:

- one-loop diagrams:



- two-loop diagrams:



Finally, two loop contribution to $\beta(\gamma)$ vanishes! [XC and M. Shifman, 2011]

Background field method as a proof

Key idea: shift symmetry of $\theta_R^\dagger$ preserved classically and quantum mechanically $\Rightarrow \int d\theta_R d\theta_R^\dagger$ constant = vanishing loop correction!
Need: nontrivial background!
Upshot:

- something $\bar{Q}_R$-closed, ie,

$$[\bar{Q}_R, X] = 0\,.$$

- F term:

$$\int d^2x\, [Q_R, X]|_{\theta_R=\theta_R^\dagger=0} \neq 0\,,$$

  we end up at the usual nonrenormalization theorem.

- D term:

$$\int d^2x\, \{Q_R, [\bar{Q}_R, X]\}|_{\theta_R=\theta_R^\dagger=0} = 0\,!$$

Background field method as a proof

The way out: target space symmetry helps!

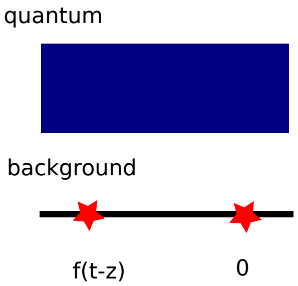
- consider an equivariant version of the previous story,

$$[\bar{Q}_R, \bar{X}] = \bar{0}\,, \quad \int d^2x \, \{Q_R, [\bar{Q}_R, X]\}|_{\theta_R=\theta_R^\dagger=0} \neq 0\,.$$

- Target space symmetry is given by

$$A_{bk} \to A_{bk} + f(x_R)\,, \quad A_{qu} \to A_{qu}\,.$$

Quantum correction is the same for all possible backgrounds in the equivalent class, but the background could be nontrivial!



Nonperturbative regime

Following Seiberg's argument [Seiberg, 93]

Good news:

- can promote γ to a superfield $\Rightarrow$ chirality is preserved!
- can assign R-charges!

Bad news:

- no F term, no holomorphic function!

So you will get,

$$f\left(\gamma\mathcal{B}BA^{\dagger}, \frac{\gamma\mathcal{B}B}{A}, |\mathcal{B}|^2, |B|^2, |\gamma|^2\right) .$$

Now apply target space symmetry, the holomorphic piece will save our day. Similarly one could analyze Z_A. [XC and M. Shifman, 2011]

Conclusion

Lessons we've learned.

- Flat target space, so quantum correction is uniformal, in the sense of respecting target space symmetry.
- $\mathcal{N} = (0, 2)$ makes sure that the translation is powerful.

$\Rightarrow$ generalization to other linear models will be straightforward.
What about nonlinear models?

- For heterotic $\mathbb{CP}^1$ model, Z_A and Z_γ are free from $|\gamma|^{2n}$-correction at n loop order.
- We shall explore the mixing contribution of g and γ at higher loop order.
- For generic non-linear $\mathcal{N} = (0, 2)$ models, can use topologically non-trivial backgrounds to obtain NRT. [XC and M. Shifman, in progress]

Thank you!