Recent developments in the renormalisation of the
Chargino and Neutralino sector of the complex MSSM

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Outline

- Why MSSM with complex phases?
- Why calculate 1-loop corrections?
- Our approach to renormalising the chargino and neutralino sector:
    - Parameter Renormalization
    - Field Renormalization
- Affect on 1-loop results for:
    - $h_2 \to \tilde{\chi}_1{}^+ \tilde{\chi}_2{}^-$@LHC
    - $e^+ e^- \to \tilde{\chi}_i{}^+ \tilde{\chi}_j{}^-$@LC

Motivation

- cMSSM⇒BSM CP violation⇒ Baryon asymmetry
- Strong bounds on certain phases via EDMs (n, e, Hg, Tl) [1]

Important contributing phases:

$$\phi_{A_{t/b/\tau}},\ \phi_{\mu}{}^{a},\ \phi_{M_{1/3}}$$

[a] Note that higgsino phase is also strongly constrained by the EDM's

- 1-loop corrections large in MSSM: important for precision measurements
- On-shell scheme ⇒ correct **IR** properties

[1] for review see J. R. Ellis, J. S. Lee and A. Pilaftsis, [arXiv:0808.1819 [hep-ph]].

Chargino and Neutralino Sector

$$\mathcal{L}_{\tilde{\chi}} = \overline{\tilde{\chi}_i^-}(\not{p}\,\delta_{ij} - \omega_L(U^* X V^\dagger)_{ij} - \omega_R(V X^\dagger U^T)_{ij})\tilde{\chi}_j^- + \frac{1}{2}\overline{\tilde{\chi}_i^0}(\not{p}\,\delta_{ij} - \omega_L(N^* Y N^\dagger)_{ij} - \omega_R(N Y^\dagger N^T)_{ij})\tilde{\chi}_j^0$$

2

$$X = \begin{pmatrix} M_2 & \sqrt{2} M_W \sin\beta \\ \sqrt{2} M_W \cos\beta & \mu \end{pmatrix}$$

diagonalised via $\mathbf{M}_{\tilde{\chi}^+} = U^* X V^\dagger$

[2] where we define $\omega_{L/R} = \frac{1}{2}(1 \mp \gamma_5)$

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diagonalised via $\mathbf{M}_{\tilde{\chi}^+} = U^* X V^\dagger$

$$Y = \begin{pmatrix} M_1 & 0 & -M_Z c_\beta s_W & M_Z s_\beta s_W \\ 0 & M_2 & M_Z c_\beta c_W & -M_Z s_\beta c_W \\ -M_Z c_\beta s_W & M_Z c_\beta c_W & 0 & -\mu \\ M_Z s_\beta s_W & -M_Z s_\beta c_W & -\mu & 0 \end{pmatrix}$$

diagonalised via $\mathbf{M}_{\tilde{\chi}^0} = N^* Y N^\dagger$

[2] where we define $\omega_{L/R} = \frac{1}{2}(1 \mp \gamma_5)$

Renormalisation of the Chargino/Neutralino sector[3]

Parameter renormalisation:

- $X + \delta X$, $Y + \delta Y \Rightarrow M_1 + \delta M_1$, $M_2 + \delta M_2$, $\mu + \delta\mu$ etc.
- e.g. $\delta X = \begin{pmatrix} \delta M_2 & \frac{\delta M_W^2 s_\beta}{\sqrt{2} M_W} + M_W s_\beta c_\beta^2 \delta t_\beta \\ \frac{\delta M_W^2 c_\beta}{\sqrt{2} M_W} - M_W c_\beta s_\beta^2 \delta t_\beta & \delta\mu \end{pmatrix}$

  where s_β denotes $\sin\beta$ etc.
- More physical masses than independent parameters $\Rightarrow$ can only choose three masses on-shell:
  - $\tilde{\chi}^{\pm}_{1,2}$, $\tilde{\chi}^0_{1(2/3)}$: NCC(b/c)
  - $\tilde{\chi}^0_{1,2}$, $\tilde{\chi}^{\pm}_2$: NNC
  - $\tilde{\chi}^0_{1,2}$, $\tilde{\chi}^0_3$: NNN

[3] A. C. Fowler and G. Weiglein, "Precise Predictions for Higgs Production in Neutralino Decays in the Complex MSSM," JHEP **1001**, 108 (2010) [arXiv:0909.5165 [hep-ph]]

Parameter renormalisation cont'd

- Requiring these masses to be on-shell, 1-loop correction must vanish,

$$\Delta m_{\tilde{\chi}_i} = \frac{m_{\tilde{\chi}_i}}{2}\mathrm{Re}[\hat{\Sigma}^L_{ii}(m^2_{\tilde{\chi}_i}) + \hat{\Sigma}^R_{ii}(m^2_{\tilde{\chi}_i})] + \frac{1}{2}\mathrm{Re}[\hat{\Sigma}^{SL}_{ii}(m^2_{\tilde{\chi}_i}) + \hat{\Sigma}^{SR}_{ii}(m^2_{\tilde{\chi}_i})] = 0,$$

  results in renormalisation conditions fixing $\delta|M_1|$, $\delta|M_2|$, $\delta|\mu|$
- Here the self energy is written via

$$\Sigma_{ij}(p^2) = \not{p}\omega_L\Sigma^L_{ij}(p^2) + \not{p}\omega_R\Sigma^R_{ij}(p^2) + \omega_L\Sigma^{SL}_{ij}(p^2) + \omega_R\Sigma^{SR}_{ij}(p^2)$$

  and $\omega_{L/R} = \frac{1}{2}(1 \mp \gamma_5)$
- Phases do not require renormalisation $\Rightarrow$ $\delta\phi_{M_1} = \delta\phi_{M_2} = \delta\phi_\mu = 0$

Parameter renormalisation cont'd[4]

	NNN	NNC	NCC	
$\delta\|M_1\|$	-1.468	-1.465	-1.468	
$\delta\|M_2\|$	-9.265	-9.265	-9.410	
$\delta\|\mu\|$	-18.494	-18.996	-18.996	
$\Delta m_{\tilde{\chi}_1^0}$	0	0	0	
$\Delta m_{\tilde{\chi}_2^0}$	0	0	0	
$\Delta m_{\tilde{\chi}_3^0}$	0	-0.5012	-0.5016	
$\Delta m_{\tilde{\chi}_4^0}$	0.3237	-0.1775	-0.1775	
$\Delta m_{\tilde{\chi}_1^\pm}$	0.1446	0.1445	0	
$\Delta m_{\tilde{\chi}_2^\pm}$	0.5012	0	0	

- Finite parts of parameter renormalisation constants (RCs) and mass corrections in GeV for the CPX scenario: $|M_2|$=200 GeV, $M_3 = 1000e^{i\pi/2}$ GeV, $|A_f|$=900 GeV, $\phi_{f1,2} = \pi$, $\phi_{f3} = \pi/2$, M_{SUSY}=500 GeV, $\mu = 2000$ GeV with $M_{H^\pm} = 132.1\,\mathrm{GeV}$ and $\tan\beta = 5.5$
- Last two columns, denoted with an asterisk, show the results for a higgsino-like scenario, with $\mu = 200\,\mathrm{GeV}$, $M_1 = (5/3)(s_W^2/c_W^2)M_2$ and $M_2 = 1000\,\mathrm{GeV}$, and all other parameters the same as the CPX scenario

[4] A. C. Fowler, PhD Thesis, 2010

Parameter renormalisation cont'd[4]

	NNN	NNC	NCC	NCCb	NCCc	
$\delta\lvert M_1\rvert$	-1.468	-1.465	-1.468	2518.7	-3684.6	
$\delta\lvert M_2\rvert$	-9.265	-9.265	-9.410	-9.410	-9.410	
$\delta\lvert\mu\rvert$	-18.494	-18.996	-18.996	-18.996	-18.996	
$\Delta m_{\tilde{\chi}_1^0}$	0	0	0	2518.8	-3681.1	
$\Delta m_{\tilde{\chi}_2^0}$	0	0	0	0	0.356	
$\Delta m_{\tilde{\chi}_3^0}$	0	-0.5012	-0.5016	-0.8446	0	
$\Delta m_{\tilde{\chi}_4^0}$	0.3237	-0.1775	-0.1775	0.6851	-1.439	
$\Delta m_{\tilde{\chi}_1^\pm}$	0.1446	0.1445	0	0	0	
$\Delta m_{\tilde{\chi}_2^\pm}$	0.5012	0	0	0	0	

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Parameter renormalisation cont'd[4]

	NNN	NNC	NCC	NCCb	NCCc	NCCb*	NCCc*
$\delta\|M_1\|$	-1.468	-1.465	-1.468	2518.7	-3684.6	-355.6	-4.642
$\delta\|M_2\|$	-9.265	-9.265	-9.410	-9.410	-9.410	10.683	10.683
$\delta\|\mu\|$	-18.494	-18.996	-18.996	-18.996	-18.996	-5.136	-5.136
$\Delta m_{\tilde{\chi}_1^0}$	0	0	0	2518.8	-3681.1	-11.44	-0.636
$\Delta m_{\tilde{\chi}_2^0}$	0	0	0	0	0.356	0	-0.671
$\Delta m_{\tilde{\chi}_3^0}$	0	-0.5012	-0.5016	-0.8446	0	-339.5	0
$\Delta m_{\tilde{\chi}_4^0}$	0.3237	-0.1775	-0.1775	0.6851	-1.439	-0.0794	-0.0328
$\Delta m_{\tilde{\chi}_1^\pm}$	0.1446	0.1445	0	0	0	0	0
$\Delta m_{\tilde{\chi}_2^\pm}$	0.5012	0	0	0	0	0	0

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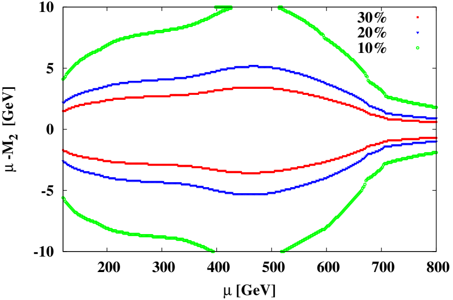

Contours of constant radiative corrections to the higgsino-like neutralino mass (left) and the wino-like chargino mass (right)[5]

[5] A. Chatterjee, M. Drees, S. Kulkarni, Q. Xu, "On the On-Shell Renormalization of the Chargino and Neutralino Masses in the MSSM," [arXiv:1107.5218 [hep-ph]].

Field renormalisation

Require correct on-shell properties for **renormalised two point vertex functions** $\hat{\Gamma}^{(2)}_{ij}(p^2) = i(\not{p} - m_i)\delta_{ij} + i\hat{\Sigma}_{ij}(p^2)$ and **propagator** $\hat{S}^{(2)}_{ij}(p^2) = (\hat{\Gamma}^{(2)}_{ij}(p^2))^{-1}$

- $\hat{\Gamma}^{(2)}_{ij}$ should be diagonal, e.g. $\hat{\Gamma}^{(2)}_{ij}\tilde{\chi}_i(p)|_{p^2=m^2_{\tilde{\chi}_j}} = 0$
- $\hat{S}^{(2)}_{ij}$ should have a unity residue, e.g. $\lim_{p^2 \to m^2_{\tilde{\chi}_i}} \frac{1}{\not{p} - m_{\tilde{\chi}_i}}\hat{\Gamma}^{(2)}_{ii}\tilde{\chi}_i(p) = i\tilde{\chi}_i$
- Demand renormalised propagator to have same Lorentz structure as at tree-level in on-shell limit, i.e. $\hat{\Sigma}^{SL}_{ii}(m^2_{\tilde{\chi}_i}) = \hat{\Sigma}^{SR}_{ii}(m^2_{\tilde{\chi}_i})$

Where does our approach differ?

Usual approach: Standard expressions for the wave-function renormalisation e.g. for charginos

$$\delta Z^{L/R}_{-,ij} = \frac{2}{m^2_{\tilde{\chi}^\pm_i} - m^2_{\tilde{\chi}^\pm_j}} \widetilde{\mathrm{Re}} \Big[m^2_{\tilde{\chi}^\pm_j} \Sigma^{L/R}_{-,ij}(m^2_{\tilde{\chi}^\pm_j}) + m_{\tilde{\chi}^\pm_i} m_{\tilde{\chi}^\pm_j} \Sigma^{R/L}_{-,ij}(m^2_{\tilde{\chi}^\pm_j}) + m_{\tilde{\chi}^\pm_i} \Sigma^{SL/SR}_{-,ij}(m^2_{\tilde{\chi}^\pm_j}) + m_{\tilde{\chi}^\pm_j} \Sigma^{SR/SL}_{-,ij}(m^2_{\tilde{\chi}^\pm_j}) - m_{\tilde{\chi}^\pm_{i/j}} \left(U^* \delta X V^\dagger\right)_{ij} - m_{\tilde{\chi}^\pm_{j/i}} \left(V \delta X^\dagger U^T\right)_{ij} \Big],$$

$$\delta \bar{Z}^{L/R}_{-,ij} = \frac{2}{m^2_{\tilde{\chi}^\pm_j} - m^2_{\tilde{\chi}^\pm_i}} \widetilde{\mathrm{Re}} \Big[m^2_{\tilde{\chi}^\pm_i} \Sigma^{L/R}_{-,ij}(m^2_{\tilde{\chi}^\pm_i}) + m_{\tilde{\chi}^\pm_i} m_{\tilde{\chi}^\pm_j} \Sigma^{R/L}_{-,ij}(m^2_{\tilde{\chi}^\pm_i}) + m_{\tilde{\chi}^\pm_i} \Sigma^{SL/SR}_{-,ij}(m^2_{\tilde{\chi}^\pm_i}) + m_{\tilde{\chi}^\pm_j} \Sigma^{SR/SL}_{-,ij}(m^2_{\tilde{\chi}^\pm_i}) - m_{\tilde{\chi}^\pm_{i/j}} (U^* \delta X V^\dagger)_{ij} - m_{\tilde{\chi}^\pm_{j/i}} (V \delta X^\dagger U^T)_{ij} \Big]$$

$\widetilde{\mathrm{Re}} \Rightarrow$ take real part of any loop integrals occurring in the self energies, but not of any complex parameters in coefficients of these integrals

Removes absorptive parts of loop integrals
Additional finite renormalisation term required to restore on-shell properties of external particles

Where does our approach differ?

Our expressions for the wave-function renormalisation e.g. for charginos

$$\delta Z^{L/R}_{-,ij} = \frac{2}{m^2_{\tilde{\chi}^\pm_i} - m^2_{\tilde{\chi}^\pm_j}} \cancel{\widetilde{\mathrm{Re}}} \Big[m^2_{\tilde{\chi}^\pm_j} \Sigma^{L/R}_{-,ij}(m^2_{\tilde{\chi}^\pm_j}) + m_{\tilde{\chi}^\pm_i} m_{\tilde{\chi}^\pm_j} \Sigma^{R/L}_{-,ij}(m^2_{\tilde{\chi}^\pm_j}) + m_{\tilde{\chi}^\pm_i} \Sigma^{SL/SR}_{-,ij}(m^2_{\tilde{\chi}^\pm_j})$$

$$+ m_{\tilde{\chi}^\pm_j} \Sigma^{SR/SL}_{-,ij}(m^2_{\tilde{\chi}^\pm_j}) - m_{\tilde{\chi}^\pm_{i/j}} \left(U^* \delta X V^\dagger\right)_{ij} - m_{\tilde{\chi}^\pm_{j/i}} \left(V \delta X^\dagger U^T\right)_{ij} \Big],$$

$$\delta \bar{Z}^{L/R}_{-,ij} = \frac{2}{m^2_{\tilde{\chi}^\pm_j} - m^2_{\tilde{\chi}^\pm_i}} \cancel{\widetilde{\mathrm{Re}}} \Big[m^2_{\tilde{\chi}^\pm_i} \Sigma^{L/R}_{-,ij}(m^2_{\tilde{\chi}^\pm_i}) + m_{\tilde{\chi}^\pm_i} m_{\tilde{\chi}^\pm_j} \Sigma^{R/L}_{-,ij}(m^2_{\tilde{\chi}^\pm_i}) + m_{\tilde{\chi}^\pm_i} \Sigma^{SL/SR}_{-,ij}(m^2_{\tilde{\chi}^\pm_i})$$

$$+ m_{\tilde{\chi}^\pm_j} \Sigma^{SR/SL}_{-,ij}(m^2_{\tilde{\chi}^\pm_i}) - m_{\tilde{\chi}^\pm_{i/j}} (U^* \delta X V^\dagger)_{ij} - m_{\tilde{\chi}^\pm_{j/i}} (V \delta X^\dagger U^T)_{ij} \Big]$$

In the CP-conserving case one can choose a scheme where (up to purely imaginary terms that do not contribute to squared matrix elements at 1-loop) the hermiticity relation holds: $\delta \bar{Z}_{ij} = \delta Z^\dagger_{ij}$

Keep absorptive parts of loop integrals

Analogy to the CKM Matrix

- On-shell conditions result in inconsistent equations due to branch cuts in self energies[6]
- Ignore absorptive parts $\Rightarrow$ gauge dependence of $\delta V_{\rm CKM}$
- Possible solutions via mass renormalization[7], but not fully on-shell
- Require separate **incoming and out-going** wfr constants [8], 0.5% observable difference

[6] A. Denner and T. Sack, Nucl. Phys. B **347** (1990) 203

[7] B. A. Kniehl and A. Sirlin, Phys. Rev. D **74** (2006) 116003, B. A. Kniehl and A. Sirlin, Phys. Lett. B **673** (2009) 208

[8] D. Espriu, J. Manzano and P. Talavera, Phys. Rev. D **66**, 076002 (2002)

Numerical Example 1: $h_2 \to \tilde{\chi}_1^+ \tilde{\chi}_2^-$ @LHC...

- Detection at LHC via 4 letpon + missing E_T : Discovery potential of H^0/A^0 for masses $\lesssim 800$ GeV[a]
- Neutral Higgs sector (h, H^0, A^0) mix at loop level in the cMSSM, physical states h_{1-3}
- Calculate full 1-loop corrections in the cMSSM, using FeynArts, FormCalc, FeynHiggs

[a] M. Bisset, J. Li, N. Kersting, R. Lu, F. Moortgat and S. Moretti, "Four-lepton LHC events from MSSM Higgs boson decays into neutralino and chargino pairs," JHEP **0908** (2009) 037 [arXiv:0709.1029]

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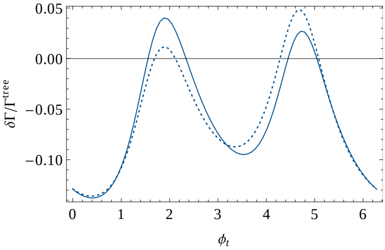
Clear $\sim$ 3% difference between results with/without the absorptive parts



1-loop correction to decay width for $h_2 \longrightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^-$ as a function of ϕ_t

M_{H^+}=800 GeV,$\tan\beta$=20, M_1=100 GeV, μ=420 GeV, M_2=200 GeV, M_3=1 TeV, $M_{\mathrm{SUSY}\,12}$=1000 GeV, $M_{\tilde{t}}$=500 GeV $M_{\tilde{l}_3}$=500 GeV, $M_{\tilde{l}_{12}}$=400 GeV, A_{q_3}=1.3 TeV, $A_{q_{12}}$=1 TeV, A_l=1 TeV

Numerical Example 2: $e^+e^- \rightarrow \tilde{\chi}_{\mathbf{i}}^+\tilde{\chi}_{\mathbf{j}}^-$@LC

- Existing results for the total cross-section for chargino production in the real MSSM at SPS1a'[a], NLO corrections $\mathcal{O}(10\%)$
- Cross section for $e^+e^- \rightarrow \tilde{\chi}_{1,L}^+\tilde{\chi}_{2,R}^-$ as a function of the phase of A_t

[a] W. Oller, H. Eberl and W. Majerotto, "Precise predictions for chargino and neutralino pair production in e+ e- annihilation," Phys. Rev. D **71** (2005) 115002 [arXiv:hep-ph/0504109]

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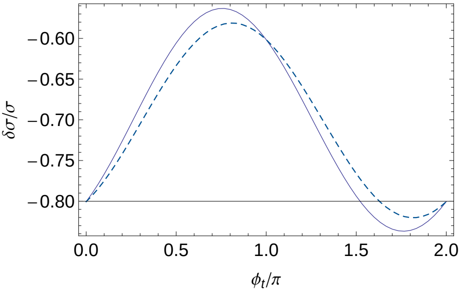


Summary and Outlook

Summary

- On-shell renormalisation procedure for the Chargino and Neutralino sector non-trivial
- Careful choice of 3 on-shell mass conditions crucial
- Demand on-shell conditions fulfilled $\Rightarrow$ Absorptive parts must be included
- Full 1-loop results for $h_{(2/3)} \to \tilde{\chi}_i^+ \tilde{\chi}_j^-$ and $e^+e^- \to \tilde{\chi}_i^+ \tilde{\chi}_j^-$ calculated, inclusion of absorptive parts seen to have an observable effect

Outlook

- Calculation of similar processes e.g. $e^+e^- \to \tilde{\chi}_i^0 \tilde{\chi}_j^0$ at 1-loop
- Tree-level parameter determination possible up to $\mathcal{O}(\%)$ level at a LC via $\tilde{\chi}^0/\tilde{\chi}^\pm$ production[9]
- Goal: match parameter determination with exp. accuracy

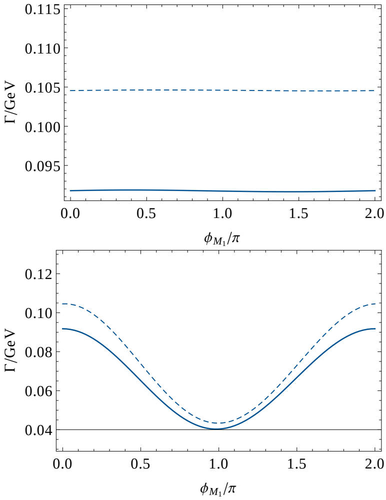
[9] K. Desch, J. Kalinowski, G. A. Moortgat-Pick, M. M. Nojiri and G. Polesello, [arXiv:hep-ph/0312069]

Field renormalisation

Using separate field RCs for incoming and outgoing particles e.g. $\omega_L \tilde{\chi}_i^- \rightarrow (1 + \frac{1}{2}\delta Z_-^L)_{ij} \omega_L \tilde{\chi}_j^-$ and $\overline{\tilde{\chi}_i^-} \omega_L \rightarrow \overline{\tilde{\chi}_i^-}(1 + \frac{1}{2}\delta \bar{Z}_-^R)_{ij} \omega_L$, find renormalised self-energies take the form

$$
\begin{aligned}
\hat{\Sigma}^R_{ij,-}(p^2) &= \Sigma^R_{ij,-}(p^2) + \frac{1}{2}(\delta Z_-^R + \delta \bar{Z}_-^R)_{ij}, \\
\hat{\Sigma}^L_{ij,-}(p^2) &= \Sigma^L_{ij,-}(p^2) + \frac{1}{2}(\delta Z_-^L + \delta \bar{Z}_-^L)_{ij}, \\
\hat{\Sigma}^{SR}_{ij,-}(p^2) &= \Sigma^{SR}_{ij,-}(p^2) - [V \delta X^\dagger U^T + \frac{1}{2} V X^\dagger U^T \delta Z_-^R + \frac{1}{2} \delta \bar{Z}_-^L V X^\dagger U^T]_{ij}, \\
\hat{\Sigma}^{SL}_{ij,-}(p^2) &= \Sigma^{SL}_{ij,-}(p^2) - [U^* \delta X V^\dagger + \frac{1}{2} U^* X V^\dagger \delta Z_-^L + \frac{1}{2} \delta \bar{Z}_-^R U^* X V^\dagger]_{ij}
\end{aligned}
$$

NLO results for $\mathbf{h_2} \to \tilde{\chi}_i^+ \tilde{\chi}_j^-$



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