Rigid Supersymmetry in curved Superspace

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Why study Susy on nontrivial backgrounds?

- A new handle to understand the dynamics of SUSY theories.
- Introduces new computable observables for known theories.

Questions we want to answer

- Which backgrounds allow for Susy?
- What are the corresponding superalgebras?
- How do we construct Lagrangians?
- How do we compute (I'll not address this question here)?

Outline

- The standard approach.
- A new description in terms of background superfields.
- Examples: Ads, S4, S3xS1, S3.
- An Index.
- Conclusion.

The customary approach

- Start with a Lagrangian in flat space.
- Deform the metric to a curved manifold with size parametrized by r. Generically ~~Susy~~.
- A Deformation of the Susy variations of the fields and the Lagrangian order by order in 1/r to preserve Susy is sometimes possible.
- If lucky the expansion terminates!

A different approach

- Start with off-shell formulation of Supergravity.
- Set arbitrary background values for the metric and auxiliary fields in the gravity multiplet.

The Rigid limit (≠ Linearized limit):

- Set the gravitino $\psi_{\mu\alpha}$ to zero.
- Take $M_P \to \infty$ to decouple the gravity multiplet keeping background values fixed.
- Susy is preserved if there exists ζ_α which sets $\delta_\zeta \psi_{\mu\alpha} = 0$ this requirement gives integrability conditions for the metric and auxiliary fields.

Advantages

- Unified treatment of different cases. The consistency conditions do not depend on the matter content.
- Great simplification.
- The auxiliary fields couple to definite components of the Supercurrent $\Rightarrow$ the deformation of the theory can be described also when a Lagrangian is not available.
- As the gravity multiplet is not on-shell different formulations of Sugra can allow different backgrounds.

Example: AdS [Zumino...]

- In Old Minimal Sugra the auxiliary fields are a complex scalar M, $\bar{M}$ and a real vector b_μ.
- Set the metric to be AdS of radius r.
- Set $M = \bar{M} = 3/r$ and $b_\mu = 0$.
- Can solve $\delta_\zeta \psi_{\mu\alpha} = \nabla_\mu \zeta_\alpha + (iM/3)\,(\sigma_\mu \bar{\zeta})_\alpha = 0$.
- The resulting superalgebra is OSp(1|4).
- Kähler transformations allow to adsorb the superpotential W in the Kähler potential. W is not protected and holomorphy not applicable.

How to get S^4

- Analytically continue $M, \bar{M}$ to $M = \bar{M} = -3i/r$ or equivalently $r \to ir$ in euclidean AdS.
- When not superconformal the theory is not reflection positive at $O(m/r)$. There is no unitary Susy theory on dS!
- The isometry SO(5) does not extend to a real form for OSp(1|4): the Q_α anticommute to a complexified isometry.
- As in AdS the superpotential is not protected because of Kähler transformations. No holomorphy.
- It is still possible to compute in N=2 [Pestun...].

Example: $S^3 \times R$ [D.Sen, Romelsberger...]

- Set M, $\bar{M}=0$ and constant b_μ along the axis.

- The resulting Q_α are time dependent unless ∃ U(1) R-symmetry and a background $U(1)_R$ gauge field.

- Alternatively if $U(1)_R$ present work in new minimal Sugra. The auxiliary fields are the $U(1)_R$ gauge field $A_\mu \sim A_\mu + \partial_\mu \varphi$ and a real 2 form $B_{\mu\nu} \sim B_{\mu\nu} + \partial_\mu a_\nu + \partial_\nu a_\mu$.

- Set A_μ along the axis and $H=dB$ threading S^3.

- The superalgebra is $SU(2|1) \times SU(2) \times U(1)$.

- M, $\bar{M}=0$ $\Rightarrow$ Kahler transformations do not change W.

$S^3 \times S^1$

- With time independent Q_α we can go to euclidean space and compactify euclidean time to get $S^3 \times S^1$.
- This works only for theories with a global U(1) R-symmetry.
- By reducing along the S^1 we get an N=2 theory on S^3 [Kapustin,Willet,Yaakov; Jafferis....].
- On S^3 as on S^4 if the theory is not superconformal there is no reflection positivity.
- If flavor $U(1)_F$ are present can give the relative background gauge fields $A_{F\mu}$ arbitrary complex values. Leads to shifts in the R-charges and real masses in 3D.

An Index

$$Z = \mathrm{Tr}\,(-1)^F \exp\left(-\beta H - \frac{\beta}{r}\sum_F \mu_F Q_F\right)$$

- Compute the partition function on $S^3 \times S^1$.
- Complex chemical potentials μ_F for $U(1)_F$ can be introduced. The dependence on μ_F is holomorphic.
- Only states in short representations of $SU(2|1)$ contribute. This is an index [Romelsberger].

- It does not vary as the 4d Lagrangian parameters are changed. It is the same in the UV and IR. Can be used to test many dualities [Dolan,Osborn; Spiridonov, Vartanov,...].

- If superconformal reduces to superconformal index [Kinney,Maldacena,Minwalla,Raju]. Otherwise the use of this term is misleading.

Conclusions

- There are many interesting examples of Susy theories on curved manifolds.
- They can be described in a unified way.
- New interesting observables can be defined and computed in many cases.
- Another handle to understand dynamics of Susy theories.
- We only started answering the questions at the beginning! Lots to be uncovered?

Thank You !