Constructing Flat Inflationary Potentials in Supersymmetry

Rouzbeh Allahverdi
University of New Mexico

SUSY11
Fermilab, 31 August 2011

Based on:
R A, Sean Downes, Bhaskar Dutta arXiv:1106.5004

Inflation driven by a scalar field ϕ. Assume:

- Minimal kinetic terms
- Minimal coupling to gravity

$\Rightarrow$ Slow-roll inflation

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

$V(\phi)$: potential

Slow-roll parameters:

$$\varepsilon \equiv \frac{1}{2} M_P^2 \left(\frac{V'}{V} \right)^2 \qquad \eta \equiv M_P^2 \frac{V''}{V}$$

Successful inflation requires $|\varepsilon|, |\eta| << 1$.

How to satisfy the slow-roll conditions?

- Large field inflation: monomial potential at $\phi > M_P$.

Problems with maintaining flatness in realistic UV completions (e.g., supergravity models).

- Small field inflation: flat potential at $\phi << M_P$:

1) $V(\phi)$ lifted by a constant piece (hybrid inflation)

or

2) $V'(\phi), V''(\phi)$ made very small

2) $\Rightarrow$ Inflation around points where derivatives of $V(\phi)$ vanish

Can be naturally accommodated within SUSY

Consider EFT expansion of superpotential for a **D-flat direction** associated with a Yukawa coupling:

$$W(\phi) = \sum_n \frac{\lambda_n}{3n} \frac{\phi^{3n}}{M_P^{3n-3}} \quad (\phi << M_P)$$

Then:

$$V(\phi) = \left| \frac{dW}{d\phi} \right|^2 = \left| \sum_n \lambda_n \frac{\phi^{3n-1}}{M_P^{3n-3}} \right|^2$$

$$\frac{dV}{d\phi} = \frac{dV}{d\phi^*} = 0 \Leftrightarrow \frac{dW}{d\phi} = 0 \;,\; \frac{d^2W}{d\phi^2} = 0$$

$$\frac{dW}{d\phi} = 0 \Rightarrow V = 0$$ (Trivial)

$$\frac{d^2W}{d\phi^2} = 0 \Rightarrow \phi \sum_n (3n-1)\lambda_n z^{n-1} = 0 \;,\; z \equiv \left(\frac{\phi}{M_P}\right)^3$$

Polynomial of complex variable $z \Rightarrow$ has $n-1$ roots

$V' = 0$ for $n-1$ values of $|\phi|$

$n \geq 3 \Rightarrow$ two or more non-trivial extrema

One can obtain points where $V' = V'' = 0$, suitable for inflation

Consider the case where the first 3 terms are dominant:

$$V = |\phi|^4 |\lambda_1 + \lambda_2 z + \lambda_3 z^2|^2$$

The 2 extrema coincide when

$$\lambda_2^2 = \frac{64}{25}\lambda_1\lambda_3$$

$$|\phi| = \phi_0 = \left(\frac{5}{16}\frac{\lambda_2}{\lambda_3}\right)^{1/3} M_P \qquad (\lambda_2 \sim 10^{-3}\lambda_3 \Rightarrow \phi_0 \sim 10^{-1} M_P)$$

Acceptable density perturbations:

$$\delta_H = \frac{1}{5\pi}\frac{1}{\sqrt{2\varepsilon}}\frac{H_{\inf}}{M_P} \approx 1.9\times 10^{-5}$$

$$\lambda_1 < 10^{-8} \Rightarrow H_{\inf} < 3\times 10^{9}\ GeV$$

$$n_s = 1 + 2\eta - 6\varepsilon \quad (0.963 \pm 0.024)$$

$$\alpha \equiv 1 - \frac{25\lambda_2^2}{64\lambda_1\lambda_3} \propto \left(\frac{\phi_0}{M_P}\right)^4$$

Other contributions to $V(\phi)$:

1) Soft SUSY breaking terms: completely negligible for low scale SUSY.

2) Supergravity corrections:

$\propto \frac{|W|^2}{M_P^4}$: known and tiny $\sim (\phi_0/M_P)^2 V$, slightly modifies α

$\propto H_{\inf}^2 \phi^2$: origin of eta-problem, $\propto (\phi_0/M_P)^2 V$, negligible effect on α if suppressed by a factor of $(\phi_0/M_P)^2 \sim 10^{-2}$ (typical requirement)

In general one has $n-1$ extrema at $\phi << M_P$ if:

$$\lambda_1 << ... << \lambda_n \leq 1$$

Many possibilities to have points where $V' = V'' = 0$,
as well as enhanced degeneracy $V' = V'' = ... = V^{n-1} = 0$.

Successful inflation with $H_{\inf} >> O(TeV)$ may occur around any of these points.

One may use Catastrophe Theory to classify the spectrum of extrema for any value of n and their suitability.

For example, various phases of inflation may happen along a single flat direction.
(Work in progress)