Cosmology in p-brane systems

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(M. Minamitsuji, N. Ohta, K. Uzawa, Phys.Rev.D81 (2010) 126005)
(K. Maeda, M. Minamitsuji, N. Ohta, K. Uzawa, Phys.Rev.D82 (2010) 046007)
(M. Minamitsuji, N. Ohta, K. Uzawa, Phys.Rev.D82 (2010) 086002)
(M. Minamitsuji, K. Uzawa, Phys.Rev.D83 (2011) 086002)

[1] Introduction

・ The dynamical solutions of supergravity have a number of important applications: For example, moduli stabilization, analysis of the early universe, BH in expanding universe.

(K. Behrndt, M. Cvetic, Class.Quant.Grav.20:4177-4194,(2003))

・ For the general p-brane system, that the structure of warp factor which depends on the space and time is different from the usual"product type" ansatz.

(H. Kodama, K.Uzawa, JHEP 0507 061 (2005), hep-th/0504193)
(H. Kodama, K.Uzawa, JHEP 0603:053 (2006), hep-th/0512104)
(P. Binetruy, M. Sasaki, K. Uzawa, Phys.Rev. D 80 (2009) 026001)

・More general dynamical brane solutions arise if the gravity is coupled not only to single gauge field but to several combinations of scalars and forms as intersecting brane solutions in the supergravity.

(K. Maeda, N. Ohta, K. Uzawa, JHEP 06 (2009) 051 arXiv:0903.5483 [hep-th])

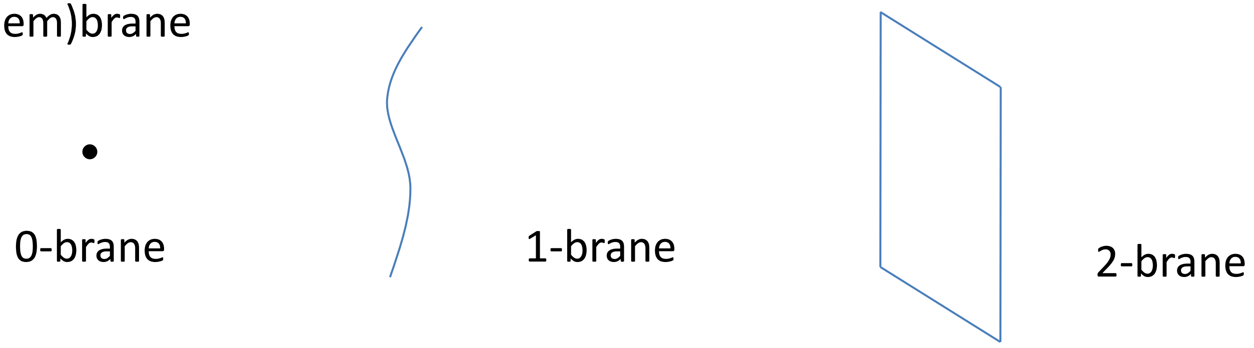
・ We give general dynamical solutions of intersecting brane systems in D-dimensional theories, which may have more general applications to cosmology.

(M. Minamitsuji, N. Ohta, K. Uzawa, Phys.Rev.D82:086002 (2010), arXiv 1007.1762 [hep-th])

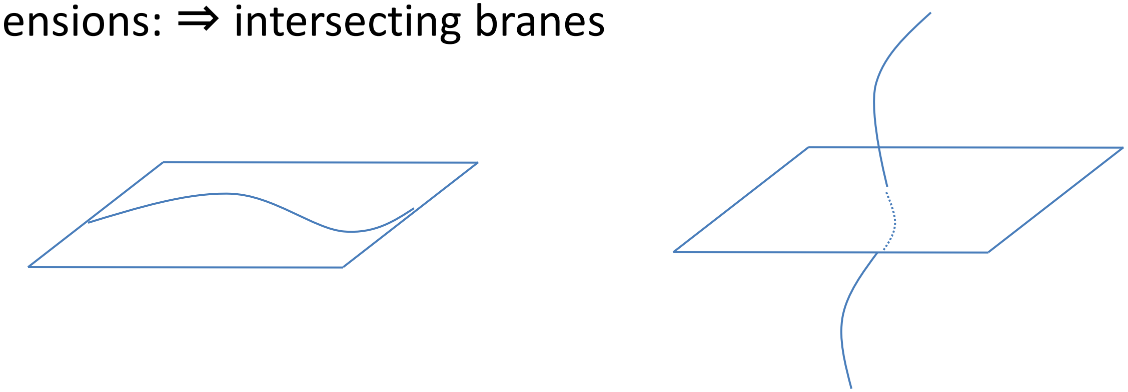
▪String theory, supergravity theory :
There are anti-symmetric tensor fields of higher rank.

▪(p+1)-form gauge field in D-dimensions:
There is (p+2)-form field strength.
⇒A charged objects (p-dim) couples to (p+1)-form gauge field.

p-(mem)brane



These higher dimensional objects (p-brane) intersect each other in D-dimensions: ⇒ intersecting branes



★ Kaluza-Klein compactification

"product type" ansatz

$$
\begin{aligned}
ds^2 &= a(x,y) q_{\mu\nu} dx^\mu dx^\nu + b(x,y) u_{ij} dy^i dy^j , \\
a(x,y) &= a_0(x) a_1(y), \qquad b(x,y) = b_0(x) b_1(y)
\end{aligned}
$$

⇒ The scale a(x, y) is written by the product of the function a_0 (b_0) and a_1 (b_1).

☆ Dynamical p-brane warped compactification

Not "product type" ansatz

$$
\begin{aligned}
ds^2 &= h^a(x,y) q_{\mu\nu} dx^\mu dx^\nu + h^b(x,y) u_{ij} dy^i dy^j , \\
h(x,y) &= h_0(x) + h_1(y)
\end{aligned}
$$

⇒ The scale h(x, y) is written by the linear combination of the function h_0 and h_1.

- expansion law: $a(\tau) \propto \tau^\lambda$, $\lambda = (p+1)/(D+p-1) < 1$ for $D > 2$

[2]Dynamical solution of intersecting brane system in supergravity

(K. Maeda, N. Ohta, K. Uzawa, JHEP 06 (2009) 051)

(M. Minamitsuji, N. Ohta, K. Uzawa, Phys.Rev.D82 (2010) 086002)

◇ D-dimensional action ：

$$S = \frac{1}{2\kappa^2}\int\left[R*\mathbf{1} - \frac{1}{2}d\phi\wedge *d\phi - \frac{1}{2}\frac{1}{(p_r+2)!}e^{\epsilon_r c_r\phi}F_{(p_r+2)}\wedge *F_{(p_r+2)} - \frac{1}{2}\frac{1}{(p_s+2)!}e^{\epsilon_s c_s\phi}F_{(p_s+2)}\wedge *F_{(p_s+2)}\right],$$

R : Ricci scalar, ϕ : scalar field, $F_{(pI+2)}$ (I=r, s) : (p_I+2)-form

κ : gravitational constant

c_I and ε_I (I=r, s) are constants given by

$$c_I^2 = 4 - \frac{2(p_I+1)(D-p_I-3)}{D-2},$$

$$\epsilon_I = \begin{cases} + & \text{if } p_I - \text{brane is electric} \\ - & \text{if } p_I - \text{brane is magnetic} \end{cases}$$

▪ D-dimensional metric : $\alpha = (p_r+1)(D-2)^{-1}, \quad \beta = (p_s+1)(D-2)^{-1}$

$$ds^2 = h_r^{\alpha-1} h_s^{\beta-1} \left[\underbrace{q_{\mu\nu}(\mathrm{X}) dx^\mu dx^\nu}_{(p+1)\text{-dim world volume}} + \underbrace{h_r \gamma_{ij}(\mathrm{Y}_1) dy^i dy^j}_{(p_s-p)\text{-dim relative transverse}} + h_s \underbrace{w_{mn}(\mathrm{Y}_2) dv^m dv^n}_{(p_r-p)\text{-dim relative transverse}} + h_r h_s \underbrace{u_{ab}(\mathrm{Z}) dz^a dz^b}_{(D-p-p_r-p_s-1)\text{-dim transverse}} \right]$$

- world-volume: Both branes are extended.
- relative transverse : Only one of the two branes is extended.
- transverse : Both branes are not extended.
- α and β are constants.

	x^0	x^1	$\cdots$	x^p	y^1	$\cdots$	y^{p_s-p}	v^1	$\cdots$	v^{p_r-p}	z^1	$\cdots$	$z^{D+p-p_r-p_s-1}$
$\mathrm{D}p_r$	○	○	○	○				○	○	○			
$\mathrm{D}p_s$	○	○	○	○	○	○	○						

world volume ($x^0 \ldots x^p$) — relative transverse ($y^1 \ldots y^{p_s-p}$, $v^1 \ldots v^{p_r-p}$) — transverse ($z^1 \ldots z^{D+p-p_r-p_s-1}$)

- The field equations of intersecting branes allow for the following three kinds of possibilities on p_r- and p_s-branes in D dimensions

Case I: Both h_r and h_s depend on the overall transverse coordinates:
(K. Maeda, N. Ohta, K. Uzawa, JHEP 0906:051,2009)

$$h_r = h_r(x, z), \qquad h_s = h_s(x, z)$$

Case II (new solution): Only h_s depends on the overall transverse coordinates, but the other h_r does on the corresponding relative coordinates:

$$h_r = h_r(x, y), \qquad h_s = h_s(x, z)$$

Case III (new solution): Each of h_r and h_s depends on the corresponding relative coordinates :

$$h_r = h_r(x, y), \qquad h_s = h_s(x, v)$$

	x^0	x^1	$\cdots$	x^p	y^1	$\cdots$	y^{p_s-p}	v^1	$\cdots$	v^{p_r-p}	z^1	$\cdots$	$z^{D+p-p_r-p_s-1}$
Dp_r	○	○	○	○				○	○	○			
Dp_s	○	○	○	○	○	○	○						

[i] Case I (K. Maeda, N. Ohta, K. Uzawa, JHEP 06 (2009) 051)

- Ansatz:

$$\begin{aligned} e^{\phi} &= h_r^{\epsilon_r c_r/2}\, h_s^{\epsilon_s c_s/2}, \\ F_{(p_r+2)} &= d\left[h_r^{-1}(x,z)\right] \wedge \Omega(\mathrm{X}) \wedge \Omega(\mathrm{Y}_2), \\ F_{(p_s+2)} &= d\left[h_s^{-1}(x,z)\right] \wedge \Omega(\mathrm{X}) \wedge \Omega(\mathrm{Y}_1), \end{aligned}$$

- Ω(X), $\Omega(Y_1)$, $\Omega(Y_2)$ are volume form of the X , Y_1 , Y_2 space.

Field equations can be satisfied it only if there is only one function depending on both transverse space coordinates and time.

For $u_{ij} = \delta_{ij}$ and $h_r = h_r(x,z)$, the solution can be obtained explicitly as

$$\begin{aligned} h_r(x,z) &= A_\mu x^\mu + B + \sum_l \frac{M_l}{|z^a - z_l^a|^{D+p-p_r-p_s-3}}, \\ h_s(z) &= C + \sum_c \frac{M_c}{|z^a - z_c^a|^{D+p-p_r-p_s-3}} \end{aligned}$$

A_μ, B, C, M_l , M_c are constants

	x^0	x^1	$\cdots$	x^{p+1}	y^1	$\cdots$	y^{p_s-p}	v^1	$\cdots$	v^{p_r-p}	z^1	$\cdots$	$z^{D+p-p_r-p_s-1}$
Dp_r	○	○	○	○				○	○	○			
Dp_s	○	○	○	○	○	○	○						

◆ In order to satisfy the field equations, two branes have to obey the intersection rules

☆Intersections rules involving the two M-branes

$$\bar{p}+1=\frac{1}{9}(p_r+1)(p_s+1), \qquad p_r \cap p_s = \bar{p}$$

⇒ M2∩M2=0, M5∩M5=3, M2∩M5=1

	0	1	2	3	4	5	6	7	8	9	10	$\tilde{I}$
M2	○	○	○									√
M5	○		○	○	○	○	○					
	t	x^1	x^2	y^3	y^4	y^5	y^6	z^1	z^2	z^3	z^4	

★ Intersections rules involving the two D-branes

$$\bar{p}=\frac{1}{2}(p_r+p_s-4)$$

⇒ Dp∩Dp=p-2, D(p-2)∩Dp=p-3, D(p-4)∩Dp=p-4

F1∩NS5=1, NS5∩NS5=3

F1∩Dp=0, Dp∩NS5=p-1, (1≦p≦6)

[ii] Case II (M. Minamitsuji, N. Ohta, K. Uzawa, Phys.Rev.D82 (2010) 086002)

- Ansatz:

$$
\begin{aligned}
e^{\phi} &= h_r^{\epsilon_r c_r/2}\, h_s^{\epsilon_s c_s/2}, \\
F_{(p_r+2)} &= d\left[h_r^{-1}(x,y)\right] \wedge \Omega(\mathrm{X}) \wedge \Omega(\mathrm{Y}_2), \\
F_{(p_s+2)} &= d\left[h_s^{-1}(x,z)\right] \wedge \Omega(\mathrm{X}) \wedge \Omega(\mathrm{Y}_1),
\end{aligned}
$$

- Ω(X), Ω(Y_1), Ω(Y_2) are volume form of the X , Y_1 , Y_2 space.

Field equations can be satisfied it only if there is only one function depending on both transverse space coordinates and time.

For $u_{ij} = \delta_{ij}$ and $h_r = h_r(x,y)$, the solution can be obtained explicitly as

$$
\begin{aligned}
h_r(x,y) &= A_\mu x^\mu + B + \sum_l \frac{M_l}{|y^i - y_l^i|^{p_s-p-2}}, \\
h_s(z) &= C + \sum_c \frac{M_c}{|z^a - z_c^a|^{D+p-p_r-p_s-3}},
\end{aligned}
$$

Intersections rules involving the two D-branes and M-branes are the same as in the case I.

[iii] Case III (M. Minamitsuji, N. Ohta, K. Uzawa, Phys.Rev.D82 (2010) 086002)

▪Ansatz:

$$\begin{aligned} e^{\phi} &= h_r^{\epsilon_r c_r/2}\, h_s^{\epsilon_s c_s/2}, \\ F_{(p_r+2)} &= h_s\, d\left[h_r^{-1}(x,y)\right] \wedge \Omega(\mathrm{X}) \wedge \Omega(\mathrm{Y}_2), \\ F_{(p_s+2)} &= h_r\, d\left[h_s^{-1}(x,v)\right] \wedge \Omega(\mathrm{X}) \wedge \Omega(\mathrm{Y}_1), \end{aligned}$$

▪Ω(X), Ω(Y_1), Ω(Y_2) are volume form of the X , Y_1 , Y_2 space.

Field equations can be satisfied it only if there is only one function depending on both transverse space coordinates and time.

For $u_{ij} = \delta_{ij}$ and $h_r = h_r(x,y)$, the solution can be obtained explicitly as

$$\begin{aligned} h_r(x,y) &= A_\mu x^\mu + B + \sum_l \frac{M_l}{|y^i - y_l^i|^{p_s-p-2}}, \\ h_s(v) &= C + \sum_c \frac{M_c}{|v^m - v_c^m|^{p_r-p-2}}, \end{aligned}$$

	x^0	x^1	$\cdots$	x^{p+1}	y^1	$\cdots$	y^{p_s-p}	v^1	$\cdots$	v^{p_r-p}	z^1	$\cdots$	$z^{D+p-p_r-p_s-1}$
Dp_r	○	○	○	○				○	○	○			
Dp_s	○	○	○	○	○	○	○						

☆ Intersections rules involving the two M-branes

(J. Gauntlett, D. Kastor, J. Traschen, Nucl.Phys.B478:544-56 (1996), hep-th/9604179)
(M. Minamitsuji, N. Ohta, K. Uzawa, Phys.Rev.D82 (2010) 086002)

$$\bar{p} + 3 = \frac{1}{9}(p_r + 1)(p_s + 1), \qquad p_r \cap p_s = \bar{p}$$

⇒ M5∩M5=1

Intersections rules involving the two D-branes

(M. Minamitsuji, N. Ohta, K. Uzawa, Phys.Rev.D82 (2010) 086002)

$$\bar{p} = \frac{1}{2}(p_r + p_s - 8)$$

⇒ Dp∩Dp=p-4, D(p-2)∩Dp=p-5, D(p-4)∩Dp=p-6 ,

D(p-6)∩Dp=p-7, D(p-8)∩Dp=p-8 , NS5∩NS5=1

Dp∩NS5=p-3, (3≦p≦8)

★ Cosmology:

We assume an isotropic and homogeneous three space in the four-dimensional spacetime. Note that the time dependence in the metric comes from only one brane in the intersections.

- Solutions in the original higher-dimensional theory (10D or 11D).

- For each case, the scale factor of 4-dimensional universe is given by $a(\tau)\propto\tau^{\lambda}$, where τ denotes the cosmic time.

- Since the three-dimensional spatial space of our universe stays in the transverse space to the brane, D-dimensional theory gives the fastest expansion of our universe.

- The power of the scale factor becomes $\lambda=(p+1)/(D+p-1) <1$ for $D>2$. It is impossible to find the cosmological model that our universe is accelerating expansion .

■ M2-M5 brane

▪ Let us consider the dynamics of 4-dim universe. To find an expanding universe, we have to smear and compactify the bulk space as well as the brane world volume (fixing our universe at some position in Z space).

Compactification

	0	1	2	3	4	5	6	7	8	9	10	$\tilde{I}$
M2	○	○	○									√
M5	○		○	○	○	○	○					
	t	x^1	x^2	y^3	y^4	y^5	y^6	z^1	z^2	z^3	z^4	

The three-dimensional spatial space of our universe stays in the transverse space to the branes.

▪ If we compactify d dimensions to fit our universe, the power of the scale factor is $\lambda<1$ for $D>2$. There are no expanding universe solutions.

★ The fastest expanding case for each brane system :

(1) M-brane : M2-M5 for case I in 11D theory ⇒ $\lambda=2/5$

(2) M-brane : M5-M5 for case I in 9D effective theory ⇒ $\lambda=6/13$

(3) D-brane : D2-D6 and D4-D6 (case I, II, III) and D6-D6 (case I & II) in 10D theory
⇒ $\lambda=7/15$

(4) F1 string : F1-D6 for case I & II in 10D theory ⇒ $\lambda=7/15$

(5) NS5-brane : NS5-D6 for case I & II in 10D theory ⇒ $\lambda=7/15$

- For the fastest expanding case, since the power is small, it cannot give a realistic expansion law such as that in the matter dominated era $a(\tau)\propto\tau^{2/3}$ or that in the radiation dominated era $a(\tau)\propto\tau^{1/2}$. Hence we conclude that in order to find a realistic expansion of the universe in this type of models, one have to include additional matter fields.

- $\lambda=(p+1)/(D+p-1)$, (higher dimension)

[3]Dynamical solution of intersecting brane system

(K. Maeda, M. Minamitsuji, N. Ohta, K. Uzawa, Phys.Rev.D82 (2010) 046007)
(M. Minamitsuji, K. Uzawa, Phys.Rev.D83 (2011) 086002)

◇ D-dimensional action :

$$S = \frac{1}{2\kappa^2}\int \left[\left(R - 2\mathrm{e}^{\alpha\phi}\Lambda\right) * \mathbf{1}_D - \frac{1}{2}d\phi \wedge * d\phi - \frac{1}{2\cdot(p+2)!}\mathrm{e}^{\epsilon c\phi}F_{(p+2)} \wedge * F_{(p+2)}\right]$$

$$c^2 = N - \frac{2(p+1)(D-p-3)}{D-2}, \qquad \epsilon = \begin{cases} + & \text{if } \ p-\text{brane is electric} \\ - & \text{if } \ p-\text{brane is magnetic}\,. \end{cases}$$

◆ Solution :

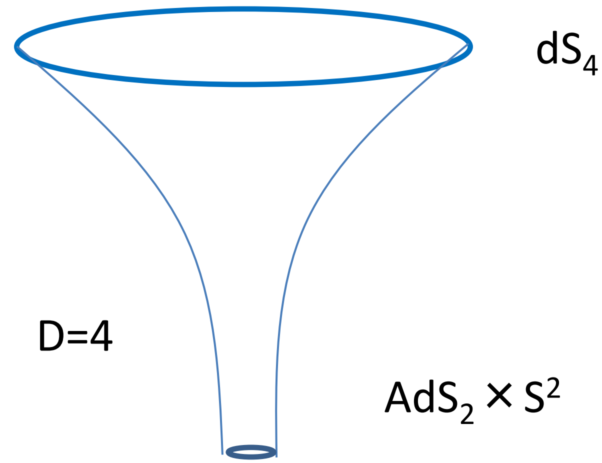
$$ds^2 = h^{-\frac{4(D-p-3)}{N(D-2)}}(x,z)q_{\mu\nu}(\mathrm{X})dx^\mu dx^\nu + h^{\frac{4(p+1)}{N(D-2)}}(x,z)u_{ab}(\mathrm{Z})dz^a dz^b,$$

$$\begin{aligned} \mathrm{e}^{\phi} &= h^{2\epsilon c/N}, \\ F_{(p+2)} &= \frac{2}{\sqrt{N}}d(h^{-1}) \wedge \sqrt{-q}\, dx^0 \wedge \cdots \wedge dx^p \end{aligned}$$

★Asymptotically de Sitter solution

In the case of trivial dilaton

$$N = \frac{2(D-p-3)(p+1)}{D-2}$$



D-dimensional metric for p=0

$$ds^2 = -\left(1 + c_1^{-1} \mathrm{e}^{-c_1 \tau} h_1\right)^{-2} d\tau^2 + \left(1 + c_1^{-1} \mathrm{e}^{-c_1 \tau} h_1\right)^{2/(D-3)} \left(c_1 \mathrm{e}^{c_1 \tau}\right)^{2/(D-3)} u_{ab}(\mathrm{Z}) dz^a dz^b \,,$$

$$c_1 = \pm(D-3)\sqrt{\frac{2\Lambda}{(D-1)(D-2)}}$$

In the limit when the terms with h_1 are negligible, which is realized in the limit $\tau \to \infty$ and for $c_1 > 0$, we find a D-dimensional de Sitter universe. The solution has been discussed by Maki & Shiraishi. Furthermore, for D = 4, the solution is found by Kastor & Traschen.

(T. Maki, K. Shiraishi, Class.Quant.Grav.10:2171-2178,(1993))

(D. Kastor, J. H. Traschen, Phys.Rev.D47:5370-5375,(1993))

[4] Summary :

☆ We give some dynamical intersecting brane solutions in ten- or eleven-dimensional supergravity.

・The cosmological solutions we found have the property that they are genuinely higher-dimensional in the sense that one can never neglect the dependence on the coordinates of transverse space.

☆ We found that these solutions give FLRW universe if we regard the homogeneous and isotropic as our spacetime.

・The power of the scale factor is so small that the solutions of field equations cannot give a realistic expansion law. This means that we have to consider additional matter in order to get a realistic expanding universe.

☆ In the presence of a cosmological constant, we find accelerating solutions if the dilaton is not dynamical.