Composite Scalars with a Compact Extra Dimension

Don Bunk
In collaboration with Jay Hubisz, Jing Shao and Philip Tanedo

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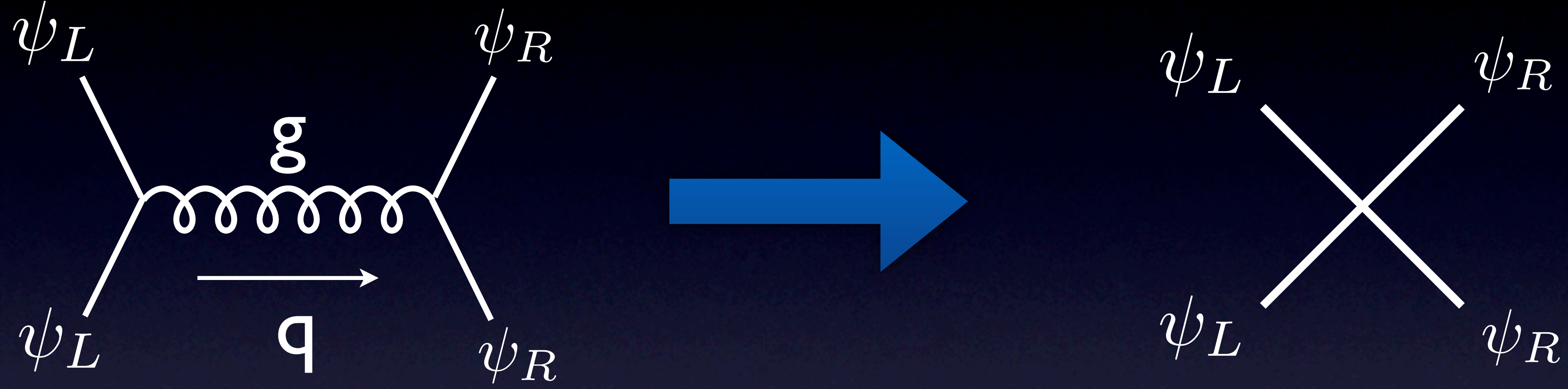
Outline

- Review of ‘traditional’ NJL model
- 5D NJL
- Propagators in a compact dimension
- Loops with a compact dimension
- Results and conclusion

NJL Model

Integrate out massive gluon:



$$\frac{g^2}{2}\left(\bar{\psi}\gamma_\mu\frac{\lambda^A}{2}\psi\right)\frac{\eta^{\mu\nu}}{q^2-\Lambda^2}\left(\bar{\psi}\gamma_\nu\frac{\lambda^A}{2}\psi\right)\Big|_{q=0} \qquad \frac{g^2}{\Lambda^2}\bar{\psi}^a_L\psi^a_R\bar{\psi}^b_R\psi^b_L+\mathcal{O}\left(\frac{1}{N}\right)$$

So we have an effective lagrangian at scale Λ:

$$\mathcal{L}=i\bar{\psi}^a_L\partial\!\!\!/\psi^a_L+i\bar{\psi}^a_R\partial\!\!\!/\psi^a_R+G\bar{\psi}^a_L\psi^a_R\bar{\psi}^b_R\psi^b_L$$

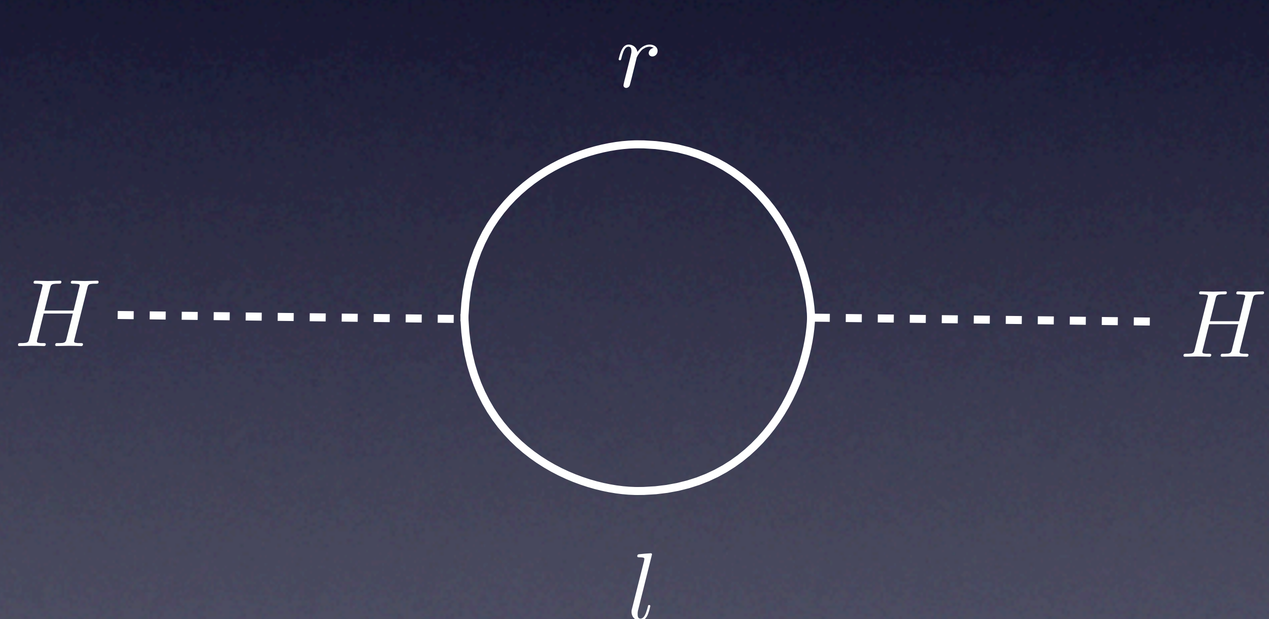
Which has a SU(N) Chiral symmetry.

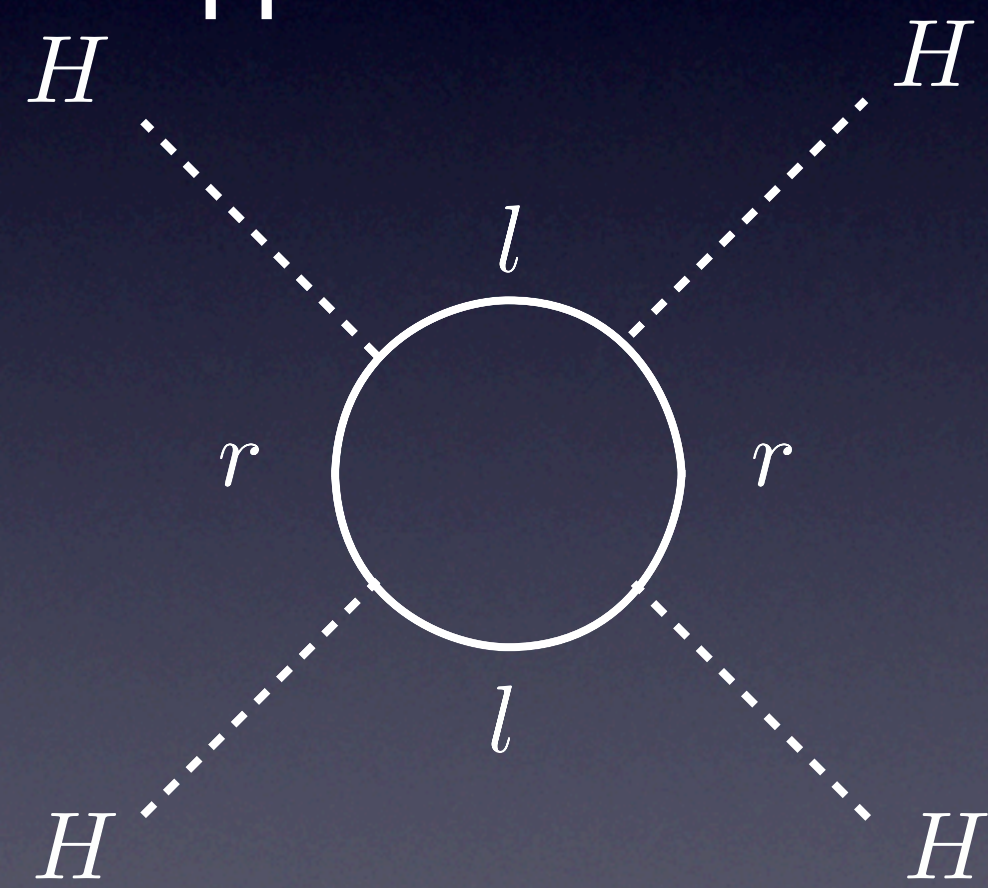
NJL Model

Perform a field redefinition with an auxiliary field H:

$$\mathcal{L} = i\bar{\psi}_L \partial\!\!\!/ \psi_L + i\bar{\psi}_R \partial\!\!\!/ \psi_R + g\bar{\psi}_L \psi_R H + \text{h.c.} - \Lambda^2 |H|^2$$

and work with fermion bubble approximation:





Which produces an effective Lagrangian

$$\mathcal{L}_\mu = i\bar{\psi}_L \partial\!\!\!/ \psi_L + i\bar{\psi}_R \partial\!\!\!/ \psi_R + \tilde{g} H \bar{\psi}_L \psi_R + \text{h.c.} + \partial_\mu H \partial^\mu H^\dagger - \tilde{m}^2 |H|^2 - \tilde{\lambda} |H|^4$$

Top Condensation

At the low scale μ the higgs mass is:

$$\tilde{m}^2 = \frac{2}{\log \frac{\Lambda^2}{\mu^2}} \left(\frac{8\pi^2}{2g^2 N_c}\Lambda^2 - (\Lambda^2 - \mu^2) \right) \approx \frac{2}{\log \frac{\Lambda^2}{\mu^2}} \left(\frac{8\pi^2}{g^2 N_c} - 1 \right) \Lambda^2$$

For $\mu \ll \Lambda$ H develops a vev for critical values of the gauge coupling:

$1 - \frac{g^2 N_c}{8\pi^2} < 0$ → Chiral Symmetry spontaneously broken.

Up to RG corrections $m_H = 2m_f$ so for $m_H \sim \mathcal{O}(100\mathrm{GeV})$ the top at $m_t = 170\mathrm{GeV}$ works best.

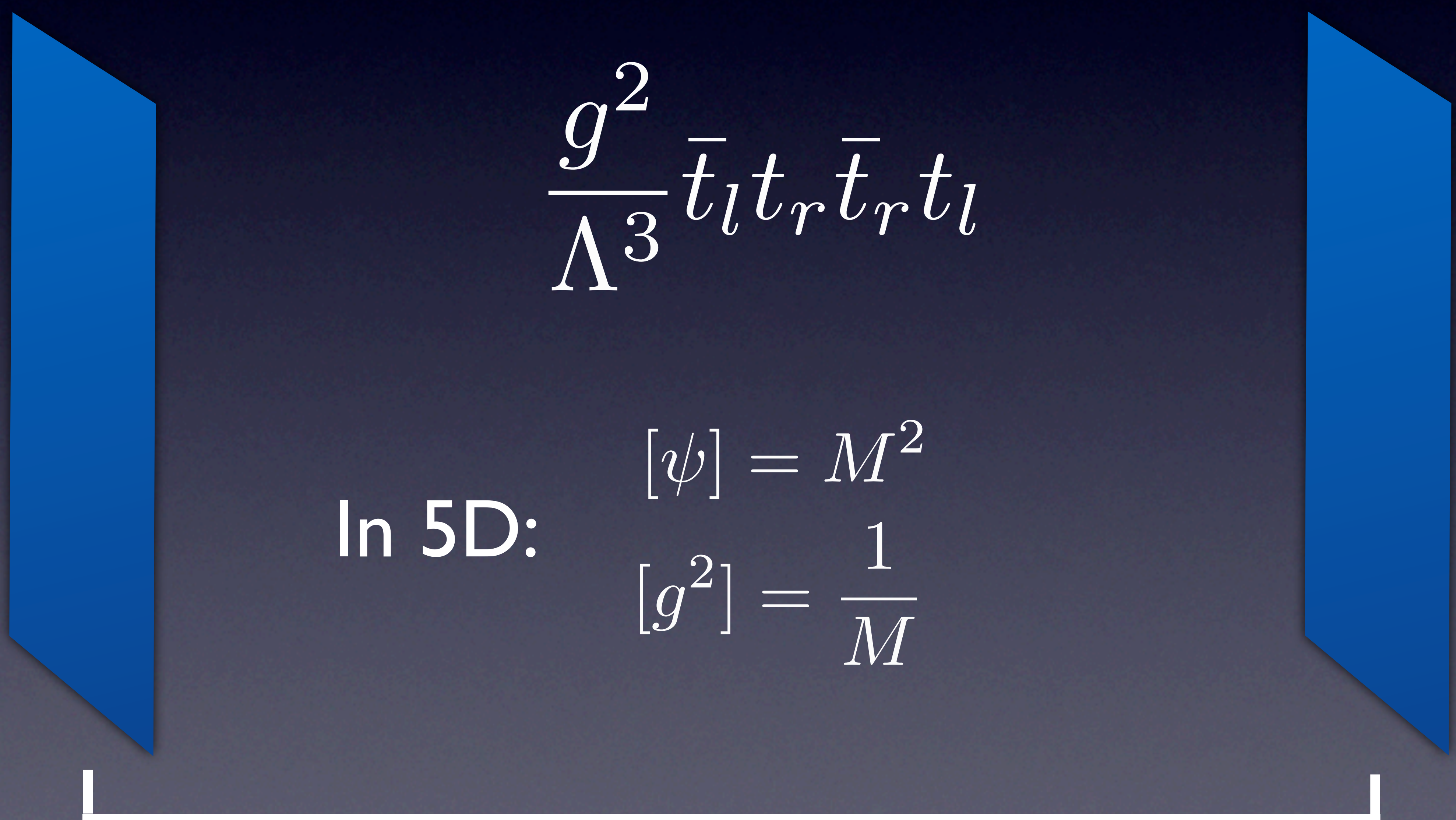
NJL model: Nambu, Jona-Lasino
Top Condensate model: Miransky, Tanabashi, Yamawaki, Bardden, Hill and Linder

Flat 5D

Goal: Study 4 fermion operator in bulk of compactified 5th dimension and possible bound states

$$\frac{g^2}{\Lambda^3}\bar{t}_l t_r \bar{t}_r t_l$$

In 5D: $$[\psi] = M^2$$
$$[g^2] = \frac{1}{M}$$



Fermions in 5d

- In 5D dimensions $\{\gamma^M, \gamma^N\} = 2\eta^{MN}$
- To get a chiral spectrum on an interval choose B.C.

For example: $\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$ with

$$\psi_R|_{z=0,L} = 0$$

$$(\partial_z - m)\psi_L|_{z=0,L} = 0$$

massless Weyl fermion

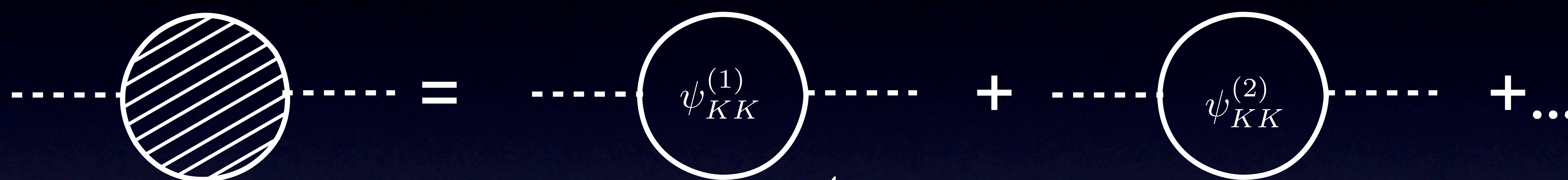
$$\Psi_{5D} = \begin{pmatrix} \chi_{(0)} \\ 0 \end{pmatrix} + \sum_{n=1} \begin{pmatrix} \chi_{(n)} \\ \bar{\psi}_{(n)} \end{pmatrix}$$

Tower starting with mass $\sim \frac{1}{L} \sim TeV$

So for $\frac{g^2}{\Lambda^3}\bar{t}_l t_r \bar{t}_r t_l$ these are 5D Dirac fermions whose (massless) zero modes are standard model tops

5D calculation

In KK picture:



$$= \; g^2 \sum_{n,n'} \int \frac{d^4q}{(2\pi)^4} \mathrm{Tr} \; \frac{i}{\not{q} - m_n} \frac{i}{\not{q} - m_{n'}}$$

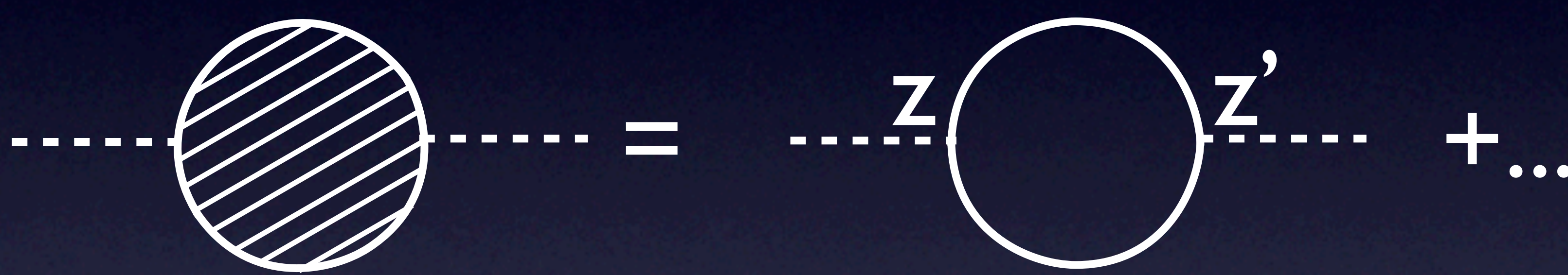
For some case the mass spectrum might be simple:

$$= \; g^2 \sum_{n,n'} \int \frac{d^4q}{(2\pi)^4} \mathrm{Tr} \; \frac{i}{\not{q} - \frac{n\pi}{L}} \frac{i}{\not{q} - \frac{n'\pi}{L}}$$

So while some sums have closed forms, not every one does and a bulk fermion mass is not easily incorporated

5D mixed propagator loops

Alternatively work in a 'mixed' basis:



$$= \sum_n$$

+...

$$= g^2 \int_0^L dz \int_0^L dz' \int \frac{d^4q}{(2\pi)^4} \mathrm{Tr}\ \Delta_l(z,z';q)\Delta_r(z',z;q)$$

5D Mixed propagators

Need to solve: $(\not{k} + i\gamma^5\partial_z - m)\Delta(z, z'; k) = i\delta(z - z')$

Defining $\Delta = \begin{pmatrix} \Delta_{LR} & \Delta_{LL} \\ \Delta_{RR} & \Delta_{RL} \end{pmatrix} = \begin{pmatrix} (-\partial_z + m)F_R & k_\mu\sigma^\mu F_L \\ k_\mu\bar{\sigma}^\mu F_R & (\partial_z + m)F_L \end{pmatrix}$

The F's satisfy: $(\partial_z^2 + (k^2 - m^2))F_{L,R} = i\delta(z - z')$

For example, for a left-handed chiral zero mode

$$F_{l,R} = \frac{-i}{2\chi \sin \chi L}\left[\cos \chi((L - |z - z'|) - \cos \chi((L - (z + z'))\right]$$

Where $\chi \equiv \sqrt{k^2 - m^2}$

5D loops

Nonlocal terms appear in the effective lagrangian:

$$\int_0^L dz \int_0^L dz' f(q) e^{iq|z-z'|} H(z) H^\dagger(z')$$

We are interested in the running of the parameters so we take the high energy limit to find dependance on the cutoff

$$\int_0^L dz \int_0^L dz' f(q_E) e^{-q_E|z-z'|} H(z) H^\dagger(z')$$

$$\Rightarrow m_0(\Lambda) H H^\dagger(0) + m_L(\Lambda) H H^\dagger(L) + m(\Lambda) \int_0^L dz H H^\dagger(z)$$

- All divergences are *local*.
- All divergences are parameterized by 4D cutoff.

Effective Lagrangian

In bulk and on branes we naively expect divergent structure of noncompact 5 and 4 dimensions, respectively.

$$\delta Z_{5d}^2 = g^2(a_1\Lambda + \text{finite})$$

$$\delta m_{5d}^2 = g^2(b_1\Lambda^3 + b_2\Lambda + \text{finite})$$

$$\delta Z_{4d}^2 = g^2(c_1 \log\Lambda + \text{finite})$$

$$\delta m_{4d}^2 = g^2(d_1\Lambda^2 + d_2\log\Lambda + \text{finite})$$

But in effective brane Lagrangian not all the naive corrections appear

Effective Lagrangian

At one fermion loop we have:

$$\mathcal{L}_{\text{brane}} = \frac{g^2}{32\pi^2}\Big\{ \log(L\Lambda)\Big[(3m_l^2 + 2m_l m_r + 3m_r^2)H(0)H^\dagger(0)$$

$$+ (m_l - m_r)\left(H(0)H^\dagger(0)\right)' + H'(0)H'^\dagger(0) + 0 \leftrightarrow L\Big] +$$

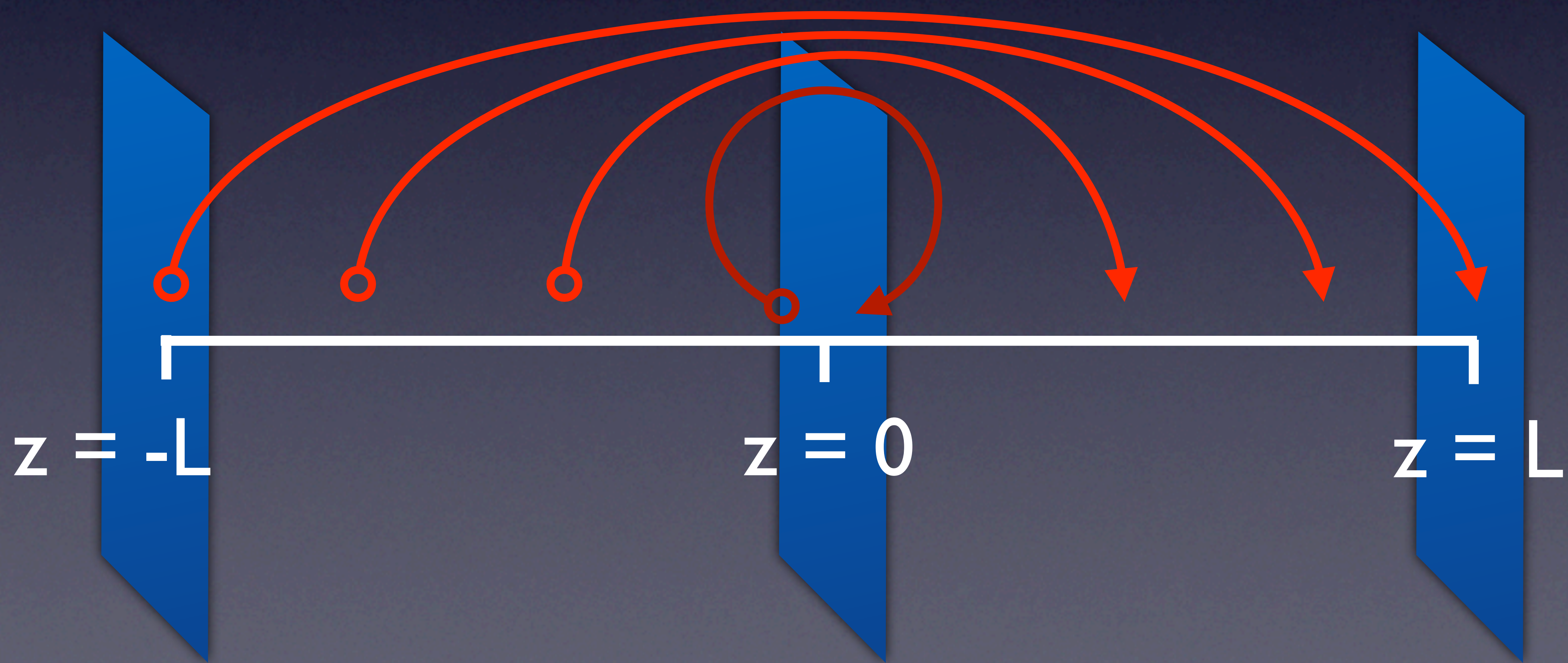
$$+2(m_l - m_r)\Lambda\left(H(0)H^\dagger(0) - H(L)H^\dagger(L)\right)\Big\}$$

All brane localized terms are related to fermion bulk mass:

- No quadratic divergences on the brane
- No 4D kinetic terms on brane
- $(\partial_z H)^2$ terms are trivial for vanishing fermion mass

Orbifold

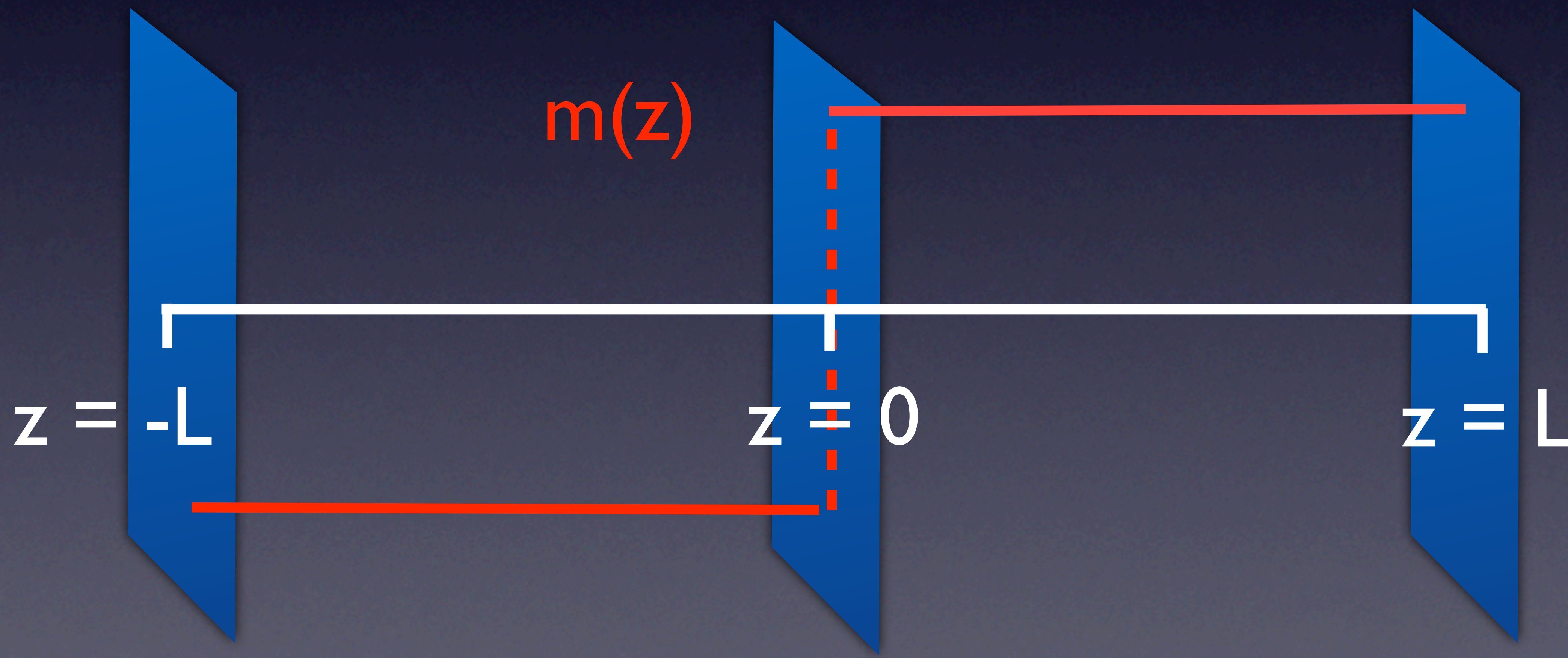
If had instead taken the orbifold as a starting point we would have made an identification $\psi(+z) = \pm\gamma^5\psi(-z)$ in order to produce a chiral spectrum



Orbifold

- In order to have a mass term we need $\int_{-L}^{L} m(z)\bar{\psi}(z)\psi(z)$

  Which is satisfied by $m(z) = m(\theta(z) - \theta(-z))$
  which violates translational invariance.



- Any brane term also violates translational invariance

Effective Lagrangian

Although $(\partial_z H)^2$ brane terms are not proportional to bulk fermion mass, they are trivial in the case of a vanishing bulk fermion mass.

Varying the action we would obtain:

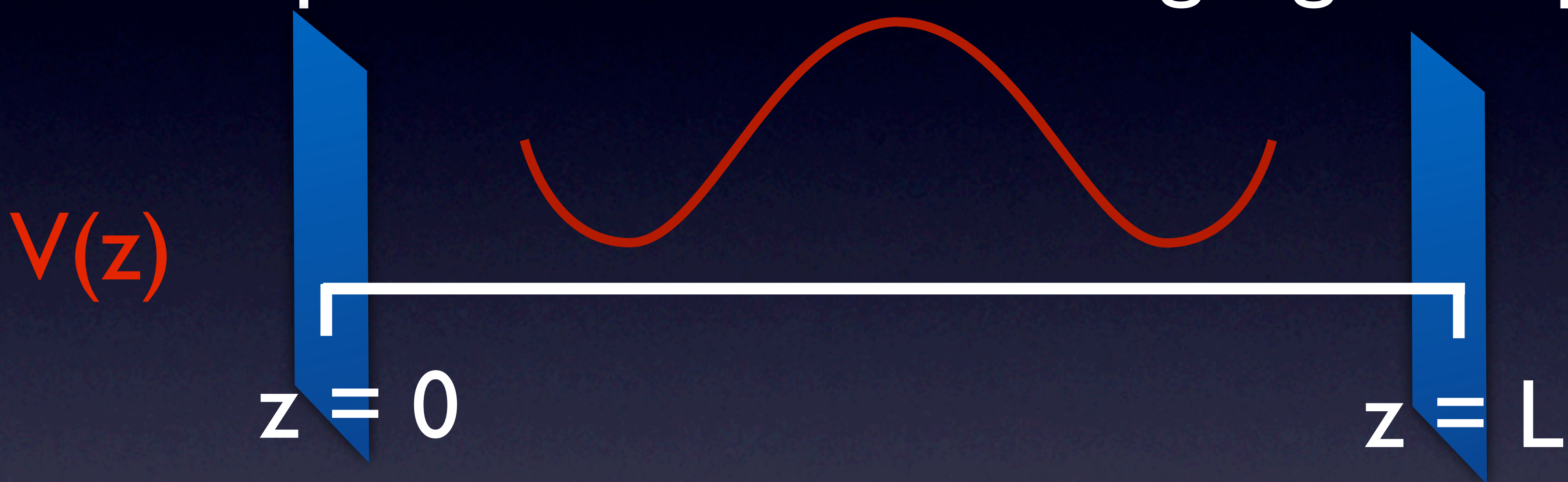
$$(\delta H \partial_z H + \delta(\partial_z H)^2)|_{z=0,L} \stackrel{!}{=} 0$$

Which would be satisfied by $\partial_z H|_{z=0,L} = 0$

Top Condensate in 5D

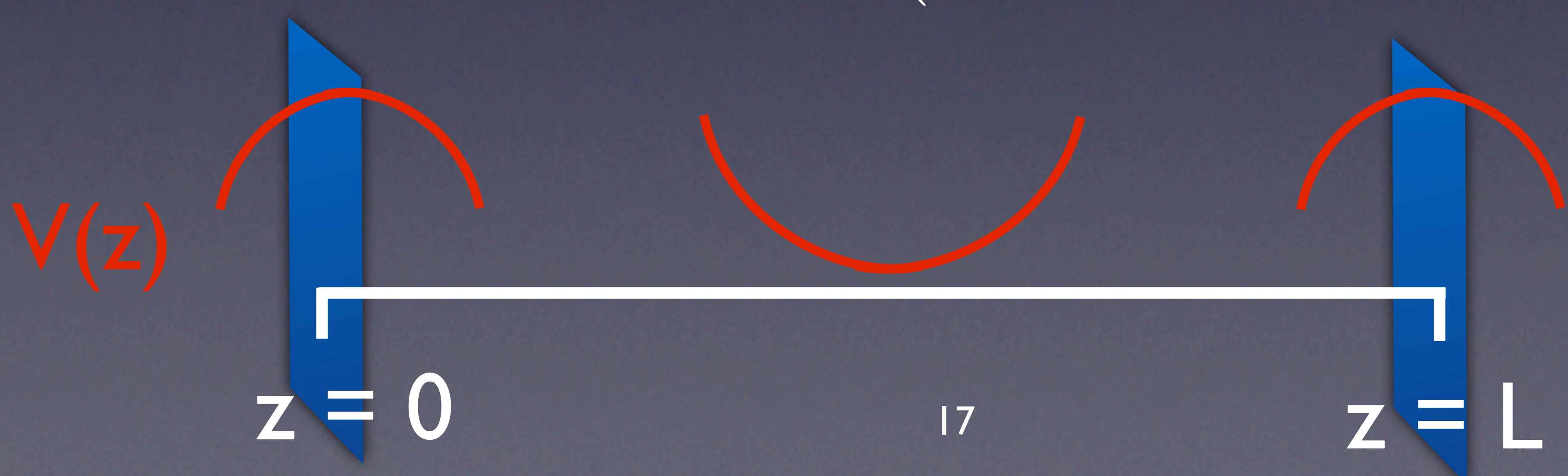
2 ways to break chiral symmetry now:

- Bulk potential with critical gauge coupling g



- Brane potential with fermion masses:

$$\mathcal{L}_{\rm brane} \approx (m_l - m_r)\left(H(0)H^\dagger(0) - H(L)H^\dagger(L)\right)$$



Fermion Condensate in 5D

The vev EOM: $\langle H(z)\rangle \equiv \dfrac{v(z)}{\sqrt{2L}}$ → $v''(z) = m^2 v(z) + \dfrac{\lambda}{L} v(z)^3$

Can be solved exactly:

$$v(z) = \frac{2iv_0}{\kappa_-}\sqrt{\frac{\tilde{\lambda}/L}{\kappa_+}}\,\mathrm{sn}\left(\sqrt{\frac{-(z-z_0)^2\kappa_-}{2L^2}\frac{\kappa_+}{\kappa_-}}\right)$$

Where: $\kappa_\pm = (\tilde{m}L)^2 \pm \sqrt{(\tilde{m}L)^4 - 2\tilde{\lambda}v_0}$

For example for: $m = \dfrac{2}{L} \quad m_0 = -m_L = \dfrac{\sqrt{2.021}}{L}$

We have: $v_0 = 4.8 \cdot 10^{-4}\dfrac{1}{L} \quad z_0 = 1.32L$

Summary

- Analyzed scalar condensate arising from a 4 fermion operator with a compact extra dimension.
- Performed one loop approximation in mixed basis.
- Brane localized divergences are softer than expected.
- Can break chiral symmetry with bulk or brane V.
- There are power law divergences so the model is fine tuned.
- Future work: phenomenology, model in warped (RS) space.