Sommerfeld enhancement in neutralino dark matter relic abundance calculations

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Overview

- Motivation
- Sommerfeld enhancement effect
- Effective Field Theory Approach to dark matter annihilations
- Results
- Outlook

Motivation

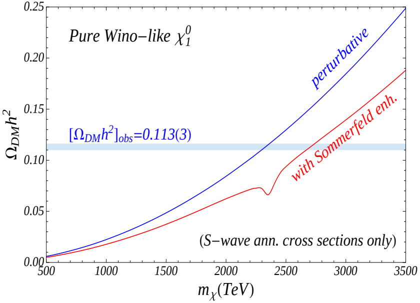
Recent astrophysical measurements allow for the determination of the cold dark matter density at percent level accuracy: $\Omega_{\rm cdm}\, h^2 \;=\; 0.113 \pm 0.003 \quad (68\% CL)$

[K. Nakamura et al. (PDG), (2010)]

Attractive scenario to explain the observed abundance: thermal relic of a WIMP

- Lightest neutralino ($\tilde{\chi}_1^0$) promising candidate within the MSSM
- Requirement to reproduce observed CDM abundance as $\tilde{\chi}_1^0$ relic poses strong constraints on the MSSM parameter space

Sommerfeld enhancement on the annihilation cross sections can significantly shift m_χ consistent with the experimental $\Omega_{\rm cdm} h^2$ value [Hisano et al. (2007)]



Revisit the Sommerfeld enhancement in the $\tilde{\chi}_1^0$ dark matter relic abundance calculations for arbitrary $\tilde{\chi}_1^0$ composition and including P-wave effects

Sommerfeld enhancement effect for $\tilde{\chi}_1^0$ in the MSSM (I)

Generic feature in non-relativistic theories with long-range potential interactions:

Sommerfeld enhancement

Enhancement of cross sections due to distortion of incoming (outgoing) non-relativistic particles' plane wave functions in presence of long range potential interactions

In context of dark matter annihilations: *[J. Hisano et al. (2005/2007); M. Cirelli at al. (2007); N. Arkani-Hamed et al. (2009); R. Iengo (2009); S. Cassel (2010); T. Slatyer (2010);*

A. Hryczuk et al. (2010) (MSSM investigation)]

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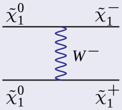
Relevance in $\tilde{\chi}_1^0$ relic abundance calculation:

- $\tilde{\chi}_1^0$ moving at non-relativistic velocities at freeze out: $v \sim 0.2c$
- Wino- or Higgsino-like $\tilde{\chi}_1^0$ must be relatively heavy to produce all observed dark matter as $\tilde{\chi}_1^0$ relic:

$$m_{\tilde{\chi}_1^0} \sim \mathcal{O}(TeV) \quad \rightarrow \quad m_{\chi_1^0} \gg m_W, m_Z$$

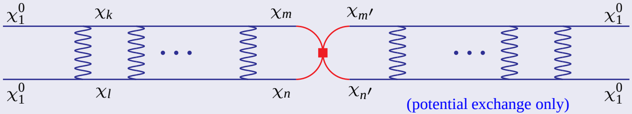
- Mass degeneracies with slightly heavier particles in $\tilde{\chi}^0/\tilde{\chi}^-$ sector generic for heavy SUSY
  $\rightarrow$ Co-annihilations in the relic abundance calculation have to be taken into account



$\rightarrow$ t- and u-channel exchange of the MSSM gauge and Higgs bosons *[$W^\pm, Z, \gamma, h^0, H^0, A^0, H^\pm$]* lead to long-range instantaneous potential interactions $V_{\{ij\}\{kl\}}$

Annihilation cross sections are related to the absorptive part of forward scattering amplitudes:

$$|\mathcal{M}(\vec{p})|^2 = 2\,\Im \sum$$

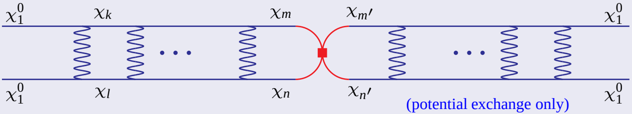


Potentials can not be treated as small perturbations $\longrightarrow$ Need resummation to all orders!

Sommerfeld enhancement effect for $\tilde{\chi}^0$ in the MSSM (II)

Annihilation cross sections are related to the absorptive part of forward scattering amplitudes:

$$|\mathcal{M}(\vec{p})|^2 \;=\; 2\,\Im\,\sum$$



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Factorization of long-range effects and short-distance annihilation reactions:

1. Long-range effects related to interactions through matrix valued potentials $V^s_{\{ij\}\{kl\}}$
2. Short-distance annihilation reactions encoded in the absorptive part of $\chi_i\chi_j \rightarrow X_A X_B \rightarrow \chi_k\chi_l$ scattering (can be written as annihilation matrix $\Gamma_{\{ij\}\{kl\}}$)
   - Correct treatment requires determination of absorptive part of all (off-)diagonal reactions $\chi_i\chi_j \rightarrow X_A X_B \rightarrow \chi_k\chi_l$
   - Diagonal entries of $\Gamma_{\{ij\}\{kl\}}$ are related to coefficients a and $b = b_{S\text{-wave}} + b_{P\text{-wave}}$ in

$$\sigma^{\text{tree}}_{\{ij\}\rightarrow\{AB\}}\; v_{\text{rel}} \;=\; a \,+\, b\, v_{\text{rel}}^2 \,+\, \mathcal{O}(v_{\text{rel}}^4) \qquad\qquad v_{\text{rel}} \;=\; |\vec{v}_i - \vec{v}_j|$$

→ a and b may be obtained numerically from codes as DarkSUSY or micrOMEGAS

× off-diagonal reactions not accessible

× No decomposition of $b = b_{S\text{-wave}} + b_{P\text{-wave}}$ (needed to study Sommerfeld enh. at $\mathcal{O}(v^2)$)

Non-relativistic effective theory approach: NRMSSM

(Co-) annihilation processes of non-relativistic $\tilde{\chi}_i^0 s$ and $\tilde{\chi}_i^{\pm} s$ are characterized by well separated scales:

- hard scale: m_i [associated with the short distance annihilation reactions]
- potential momenta: $E_i \sim m_i \vec{v}^2$, $|\vec{p}| \sim m_i |\vec{v}|$ [associated with long range effects]
- additional scale introduced by small mass differences $\delta m_i = m_{\chi_i} - m_{\tilde{\chi}_1^0} < m_{\tilde{\chi}_1^0}$

$\rightarrow$ Use methods of non-relativistic Effective Field Theories (EFTs) to integrate out all MSSM modes down to the scale of potential momenta

$$\mathcal{L}_{\text{eff}} = \xi_i^\dagger \left(i\partial^0 + \frac{\vec{\partial}^2}{2m_0} - \delta m_i \right) \xi_i + \int d^3\vec{r} \left[\xi_i^\dagger \xi_j^c \right](x, \vec{r})\, V_{\{ij\}\{kl\}}(\vec{r}) \left[\xi^{c\,\dagger}_l \xi_k \right](x)$$

$$+ f_{\{ij\}\{kl\}}\left({}^{2s+1}L_J \right) \mathcal{O}_{\{ij\}\{kl\}}\left({}^{2s+1}L_J \right) + \ldots$$

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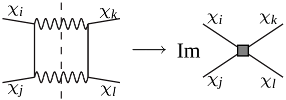


- Short-distance ($\sim 1/m_i$) effects:
  Accounted for by imaginary parts of Wilson coefficients of local four-fermion operators $\mathcal{O}_{\{ij\}\{kl\}}$

$$\mathcal{O}({}^1S_0) = \xi_l^\dagger \xi_k^c \quad {\xi_j^c}^\dagger \xi_i\,,$$
$$\mathcal{O}({}^3S_1) = \xi_l^\dagger \vec{\sigma} \xi_k^c \quad {\xi_j^c}^\dagger \vec{\sigma} \xi_i\,, \quad \cdots\cdots$$

[see *G.T. Bodwin et al., (1995)*]

Four-fermion operators - MSSM matching calculation

Analytical calculation of absorptive parts of all S-wave (including $\mathcal{O}(v^2)$) and leading order P-wave matching coefficients of the relevant four-fermion operators has been performed

- MSSM matching calculation includes all 1-loop scattering reactions relevant in the thermal relic scenario:

neutral	charged	doubly charged
$\chi^0\ \chi^0 \longrightarrow \chi^0\ \chi^0$	$\chi^0\chi^- \longrightarrow \chi^0\chi^-$	$\chi^-\chi^- \longrightarrow \chi^-\chi^-$
$\chi^-\chi^+ \longrightarrow \chi^-\chi^+$	$\chi^0\chi^+ \longrightarrow \chi^0\chi^+$	$\chi^+\chi^+ \longrightarrow \chi^+\chi^+$
$\chi^-\chi^+ \longrightarrow \chi^0\ \chi^0$		
$\chi^0\ \chi^0 \longrightarrow \chi^-\chi^+$		

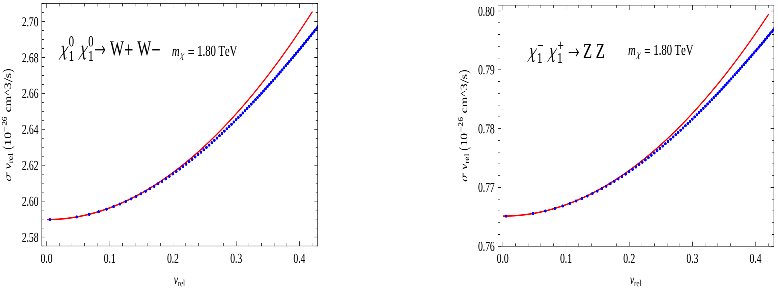
- Includes determination of absorptive parts of all possible off-diagonal scattering reactions $\chi_i\chi_j \to X_A X_B \to \chi_k\chi_l$ (off diagonal terms in $\Gamma_{\{ij\}\{kl\}}$)

- Allows to give analytical expressions for a, $b_{S-\text{wave}}$ and $b_{P-\text{wave}}$ in

$$\sigma^{\text{tree}}_{\{ij\}\to\{X_A X_B\}} v_{\text{rel}} = a + (b_{S-\text{wave}} + b_{P-\text{wave}}) v_{\text{rel}}^2 + \cdots$$

Born level $\sigma v_{\rm rel}$: numerical comparisons

Comparison with numerically determined tree level expressions for $\sigma_{\{ij\}\to\{X_A X_B\}} v_{\rm rel}$



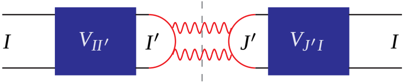
red: $\sigma v_{\rm rel} = a + b\, v_{\rm rel}^2$ determined within the EFT
blue: numerically determined tree-level cross section *[SloopS]*

EFT approach provides good approximation in the non-relativistic regime:
accuracy $\sim 0.4\%$ at $v_{\rm rel} \sim 0.4$ ($\sim 0.03\%$ at $v_{\rm rel} \sim 0.2$)

$\rightarrow$ reliable in $\tilde{\chi}_1^0$ relic abundance calculation ($v_{\rm rel} \sim 0.4$ at freeze out)

$\sigma v_{\rm rel}$ including long range potential interactions

Consider case of N non-relativistic two-particle states of same charge:



Enhancement of annihilation rate relative to corresponding Born level expression Γ_{II} is described by

$$S_I = \frac{\left[\vec{\psi}_I^{*}\,(\vec{r}{=}0)\right]_{I'}\,\Gamma_{I'J'}\,\left[\vec{\psi}_I\,(\vec{r}{=}0)\right]_{J'}}{\left[\vec{\psi}_I^{(0)*}\,(\vec{r}{=}0)\right]_{I'}\,\Gamma_{I'J'}\,\left[\vec{\psi}_I^{(0)}\,(\vec{r}{=}0)\right]_{J'}} \qquad \text{(leading order } S\text{-wave)}$$

- $\vec{\psi}_I, I = 1, \ldots, N$ are the regular scattering solutions ($\psi_I^{(0)}$ corresp. free solutions) of the Schrödinger equation

$$\left(-\frac{\vec{\partial}^2}{\mu_J}\,\delta_{IJ} + \left[V_{IJ}(|\vec{r}|) + \left(M_J - 2m_{\tilde{\chi}_1^0}\right)\delta_{IJ}\right]\right)\vec{\psi}_J = E\vec{\psi}_I$$

- $V_{IJ}(|\vec{r}|)$ calculated at leading order arising from potential $W^{\pm}, Z, \gamma$, and $h^0, H^0, A, H^{\pm}$ exchange (for both the spin of the scattering two particle states being s=0 and s=1)

$\sigma v_{\rm rel}$ including long range potential interactions

Determination of $\sigma v_{\rm rel}$ within the NRMSSM:

- For given SUSY spectrum identify all two particle channels in $\tilde{\chi}^0/\tilde{\chi}^\pm$ sector participating in (co-)annihilation processes during $\tilde{\chi}^0_1$ freeze-out (criterion: $m_{\rm channel} - 2m_{\tilde{\chi}^0_1} < m_{\tilde{\chi}^0_1}/20$)
- Solve the corresponding multi-state Schrödinger equation numerically for given V^s_{IJ} and $\Gamma_{IJ}(^{2s+1}L_J)$ for different values of the non-relativistic energy E of the respective annihilating two particle system [done separately for each of the charge sectors (neutral, single and double charged)]

Sommerfeld enhanced $\sigma_I v_{\rm rel}$ for annihilating two particle channel I, composed of χ_i and χ_j:

$$\sigma_I|\vec{v}_i - \vec{v}_j| = \sum_{^1S_0,\,^3S_1} S_I(^{2s+1}L_J)\,\Gamma_{II}(^{2s+1}L_J) + \vec{p}_i^{\,2}\left[\sum_{^1P_1,\,^3P_J} S_I(^{2s+1}L_J)\,\Gamma_{II}(^{2s+1}L_J) + \sum_{^1S_0,\,^3S_1} S_I(^{2s+1}L_J)\,\Gamma^{p^2}_{II}(^{2s+1}L_J)\right]$$

Relic abundance calculation

Precise method available to determine the $\tilde{\chi}^0$ relic abundance in presence of co-annihilations:

[K. Griest, D. Seckel, (1991); P. Gondolo, G. Gelmini, (1991)]

Consider the Boltzmann equation for the yield $Y \equiv \frac{n}{s}$, where s is the entropy density of the Universe

$$\frac{dY}{dx} = \frac{\langle \sigma_{\rm eff} v_{\rm rel} \rangle}{H\,x} \left(1 - \frac{x}{3\,g_{*s}} \frac{d\,g_{*s}}{d\,x}\right) s \left(Y^2 - (Y^{eq})^2\right)$$

- $n = \sum_i n_{\chi_i}$ is the sum of all (co-)annihilating $\tilde{\chi}^0/\tilde{\chi}^\pm$ species at $\tilde{\chi}^0_1$ freeze out
- $x = m_{\tilde{\chi}^0_1}/T$

$$\langle \sigma_{\rm eff}\, v_{\rm rel} \rangle = \sum_{i,j} \langle \sigma_{ij}\, v_{\rm rel} \rangle \frac{4}{g_{\rm eff}(x)} \left[1 + \Delta_i\right]^{3/2} \left[1 + \Delta_j\right]^{3/2} \exp\left(-x\left(\Delta_i + \Delta_j\right)\right)$$

- $\Delta_i = m_{\chi_i} - m_{\tilde{\chi}^0_1}$
- $g_{\rm eff}(x)$ describes number of effective degrees of freedom in $\tilde{\chi}^0/\tilde{\chi}^\pm$ sector during freeze-out

$\longrightarrow$ Relic abundance determined as $\Omega_{\chi_1} h^2 = \rho^0_{\chi_1}/\rho_{crit}\, h^2 = m_\chi s_0 Y_0/\rho_{crit}\, h^2$

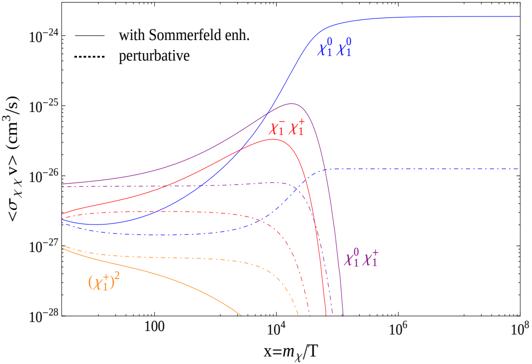
Results for a wino-like $\tilde{\chi}_1^0$

Consider wino-like $\tilde{\chi}_1^0$ scenario with $m_{\tilde{\chi}_1^0} = 2.75$ TeV:

- $\tilde{\chi}_1^\pm$ with $\delta m_{\tilde{\chi}_1^-} = 0.21$ GeV
- next-to next-to lightest particle in $\tilde{\chi}^0/\tilde{\chi}^\pm$ sector with $\delta m_{\tilde{\chi}_2^0} \sim 200$ GeV

$\rightarrow$ Perform relic abundance calculation within the NRMSSM with three non-relativistic species $\tilde{\chi}_1^0$ and $\tilde{\chi}_1^\pm$

Thermally averaged and weighted cross sections

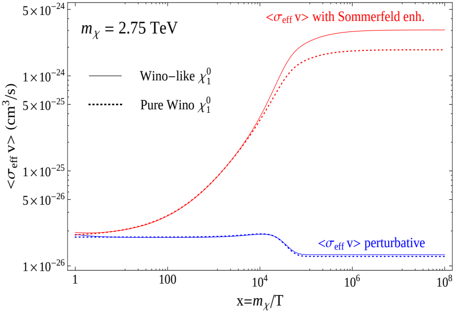


Cross section calculation includes

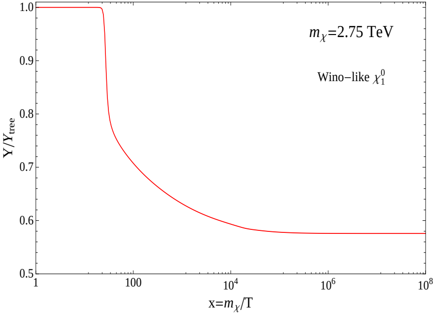
- full mixing matrix effects
- effects of $\mathcal{O}(v^2)$ S- and P- waves
- taking channel $\tilde{\chi}_1^0\tilde{\chi}_2^0$ perturbatively into account ($E \ll \delta M \ll m_{\tilde{\chi}_1^0}$)

Results for a wino-like $\tilde{\chi}_1^0$

$\langle\sigma_{\rm eff} v\rangle$ $[m_\chi = 2.75 TeV,\ \delta m_{\tilde{\chi}_1^-} = 0.21\,\text{GeV}]$



Evolution of yield $Y/Y_{\rm tree}$



- Including full MSSM mixing matrix effects in Wino-like $\tilde{\chi}_1^0$ scenario compared to pure Wino $\tilde{\chi}_1^0$ limit leads to $\sim$ 40% effect on $\langle\sigma_{\rm eff} v\rangle$ for $x \sim 10^6$ [$\sim$ 8% effect for $x \sim 10^4$]

$\longrightarrow$ large x region however gives negligible effect in the final relic abundance

- Sommerfeld enhancement effect in this MSSM scenario leads to a significant 56.7% reduction of the relic abundance relative to the calculation without taking Sommerfeld corrections into account (in agreement with previous investigations *[Hisano et al. (2007), Cirelli et al. (2007)]*)

$\longrightarrow$ Reduction essentially through Sommerfeld enhancement on leading order S-wave (as expected for this particular scenario!)

Outlook

- Non-relativistic EFT approach represents a suitable method to address the Sommerfeld enhancement effect in dark matter relic abundance calculations, providing
  - factorization of long range effects and short distance reactions
  - determination of off-diagonal elements in the annihilation matrix $\Gamma_{\{ij\}\{kl\}}$
  - separation of S- and P- wave annihilation rates at $\mathcal{O}(v^2)$

  for a generic MSSM parameter-space point
- New features in our work
  - analytical expressions for absorptive parts of $\mathcal{O}(v^2)$ S- and P- wave Wilson coefficients encoding all possible (off-)diagonal scattering reactions $\chi_i\chi_j \to X_A X_B \to \chi_k\chi_l$
  - investigation of $\mathcal{O}(v^2)$ effects and effects of heavier states
- Work in progress: with the tools now available we can study the Sommerfeld enhancement effect in $\tilde{\chi}_1^0$ relic abundance calculations for a generic MSSM scenario including higher order effects that have not been taken into account before