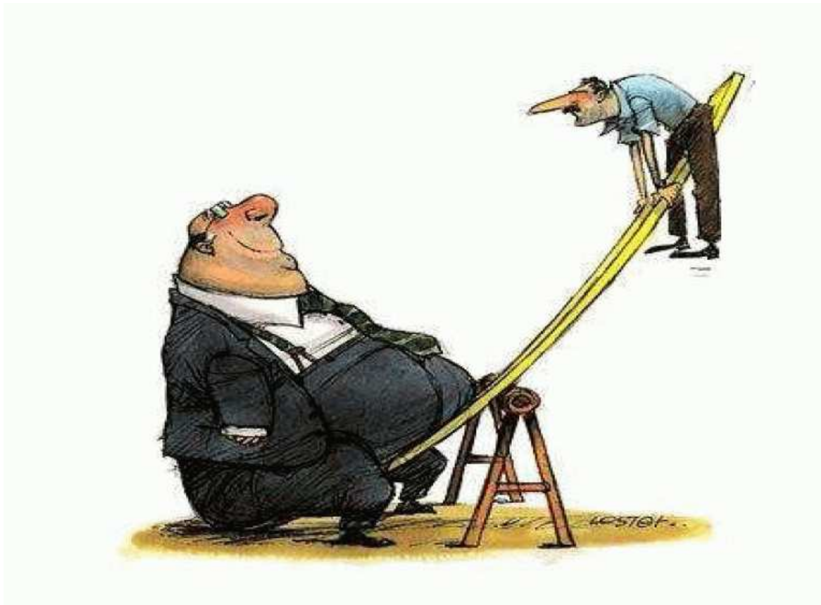
The Light Higgs-Boson Mass in the MSSM-seesaw

Sven Heinemeyer, IFCA (CSIC, Santander)

Fermilab, 08/2011

based on collaborations with
M.J. Herrero, S. Peñaranda and *A. Rodriguez Sanchez*



1. Motivation
2. Calculation
3. Results
4. Conclusions

1. Motivation

MSSM: Enlarged Higgs sector: Two Higgs doublets

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1 + i\chi_1)/\sqrt{2} \\ \phi_1^- \end{pmatrix}$$

$$H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2 + i\chi_2)/\sqrt{2} \end{pmatrix}$$

$$V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.})$$

$$+ \underbrace{\frac{g'^2 + g^2}{8}} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \underbrace{\frac{g^2}{2}} |H_1 \bar{H}_2|^2$$

gauge couplings, in contrast to SM

physical states: $h^0, H^0, A^0, H^\pm$ Goldstone bosons: $G^0, G^\pm$

Input parameters: (to be determined experimentally)

$$\tan\beta = \frac{v_2}{v_1}, \qquad M_A^2 = -m_{12}^2 (\tan\beta + \cot\beta)$$

In lowest order:

Input parameters: (to be determined experimentally)

$$\tan\beta = \frac{v_2}{v_1}, \qquad M_A^2 = -m_{12}^2(\tan\beta + \cot\beta)$$

$\Rightarrow$ m_h, m_H, mixing angle α, $m_{\mathrm{H}^\pm}$: no free parameters, can be predicted

In lowest order:

$$M_h^2 = \frac{1}{2}\left[M_A^2 + M_Z^2 - \sqrt{(M_A^2 + M_Z^2)^2 - 4M_Z^2 M_A^2 \cos^2 2\beta}\right]$$

Keep in mind: higher-order corrections

$\Rightarrow$ Test of the model!

Necessary:

– discover the Higgs(es) at the LHC (or at the ILC)
– measure its mass/characteristics at the LHC (or at the ILC)
– compare with theory prediction for M_h/other characteristics

Our model: **MSSM-seesaw**

1.) MSSM: needs no motivation here!

Our model: **MSSM-seesaw**

1.) MSSM: needs no motivation here!

2.) MSSM-seesaw:

– neutrinos have mass

– Majorana masses are allowed

– seesaw mechanism is an elegant way to create neutrino masses

– possible explanation of BAU via Leptogenesis

– leading corrections: $\Delta M_h^2 \sim m_t^4/M_W^2$
seesaw allows $Y_\nu = \mathcal{O}(1)$
$\Rightarrow$ large effects on M_h?

Our model: **MSSM-seesaw**

1.) MSSM: needs no motivation here!

2.) MSSM-seesaw:

– neutrinos have mass

– Majorana masses are allowed

– seesaw mechanism is an elegant way to create neutrino masses

– possible explanation of BAU via Leptogenesis

– leading corrections: $\Delta M_h^2 \sim m_t^4/M_W^2$
seesaw allows $Y_\nu = \mathcal{O}(1)$
$\Rightarrow$ large effects on M_h?

First step: MSSM with one generation of neutrinos/sneutrinos

The neutrino sector:

The 2×2 neutrino mass matrix is given in terms of
the Dirac mass $m_D \equiv Y_\nu v_2$ and the Majorana mass m_M by:

$$M^\nu = \begin{pmatrix} 0 & m_D \\ m_D & m_M \end{pmatrix}$$

Diagonalization of M^ν: two mass eigenstates (Majorana fermions)
ν, N with mass eigenvalues:

$$m_{\nu,N} = \frac{1}{2}\left(m_M \mp \sqrt{m_M^2 + 4m_D^2}\right)$$

Seesaw limit: $\xi \equiv m_D/m_M \ll 1$:

$$m_\nu = -m_D\xi + \mathcal{O}(m_D\xi^3) \simeq -\frac{m_D^2}{m_M}$$
$$m_N = m_M + \mathcal{O}(m_D\xi) \simeq m_M$$

The sneutrino sector:

$$V_{\text{soft}}^{\tilde{\nu}} = m_{\tilde{L}}^2 \tilde{\nu}_L^* \tilde{\nu}_L + m_{\tilde{R}}^2 \tilde{\nu}_R^* \tilde{\nu}_R + (Y_\nu A_\nu H_2^2 \tilde{\nu}_L \tilde{\nu}_R^* + m_M B_\nu \tilde{\nu}_R \tilde{\nu}_R + \text{h.c.})$$

Two 2×2 mass matrices to describe the $\mathcal{CP}$-even and $\mathcal{CP}$-odd parts of the sneutrino sector:

$$\tilde{M}_{\pm}^2 = \begin{pmatrix} m_{\tilde{L}}^2 + m_D^2 + \frac{1}{2} M_Z^2 \cos 2\beta & m_D(A_\nu - \mu \cot\beta \pm m_M) \\ m_D(A_\nu - \mu \cot\beta \pm m_M) & m_{\tilde{R}}^2 + m_D^2 + m_M^2 \pm 2 B_\nu m_M \end{pmatrix}$$

Diagonalization yields four mass eigenstates
$\tilde{n}_1$, $\tilde{n}_2$, $\tilde{n}_3$, $\tilde{n}_4$ ($\tilde{\nu}_+$, $\tilde{N}_+$, $\tilde{\nu}_-$, $\tilde{N}_-$)

Seesaw limit: $\xi \equiv m_D/m_M \ll 1$:

$$m_{\tilde{\nu}_+,\tilde{\nu}_-}^2 = m_{\tilde{L}}^2 + \frac{1}{2} M_Z^2 \cos 2\beta \mp 2 m_D (A_\nu - \mu \cot\beta - B_\nu)\xi$$
$$m_{\tilde{N}_+,\tilde{N}_-}^2 = m_M^2 \pm 2 B_\nu m_M + m_{\tilde{R}}^2 + 2 m_D^2$$

2. Calculation

M_h, M_H: higher-order corrected $\mathcal{CP}$-even Higgs masses in the MSSM

$M_h^{\nu/\tilde{\nu}}$, $M_H^{\nu/\tilde{\nu}}$: masses in the MSSM-seesaw model

$\rightarrow$ determined as poles of the propagator matrix

Inverse of the propagator matrix:

$$\left(\Delta_{\text{Higgs}}\right)^{-1} = -i \begin{pmatrix} p^2 - m_H^2 + \hat{\Sigma}_{HH}(p^2) & \hat{\Sigma}_{hH}(p^2) \\ \hat{\Sigma}_{hH}(p^2) & p^2 - m_h^2 + \hat{\Sigma}_{hh}(p^2) \end{pmatrix}$$

$\rightarrow$ solve the equation:

$$\left[p^2 - m_h^2 + \hat{\Sigma}_{hh}(p^2)\right]\left[p^2 - m_H^2 + \hat{\Sigma}_{HH}(p^2)\right] - \left[\hat{\Sigma}_{hH}(p^2)\right]^2 = 0$$

with

$$\hat{\Sigma}(p^2) = \hat{\Sigma}^{(1)}(p^2) + \hat{\Sigma}^{(2)}(p^2) + \ldots$$
$$\Sigma(p^2) = \Sigma^{(1)}(p^2) + \Sigma^{(2)}(p^2) + \ldots$$

$\Rightarrow$ calculation of $\nu/\tilde{\nu}$ contributions to $\hat{\Sigma}^{(1)}$

Renormalization and formulas "as usual":

$$\hat{\Sigma}_{hh}(p^2) = \Sigma_{hh}(p^2) + \delta Z_{hh}(p^2 - m_h^2) - \delta m_h^2$$

$$\hat{\Sigma}_{hH}(p^2) = \Sigma_{hH}(p^2) + \delta Z_{hH}(p^2 - \tfrac{1}{2}(m_h^2 + m_H^2)) - \delta m_{hH}^2$$

$$\hat{\Sigma}_{HH}(p^2) = \Sigma_{HH}(p^2) + \delta Z_{HH}(p^2 - m_H^2) - \delta m_H^2$$

$$\begin{aligned}
\delta m_h^2 &= \delta M_A^2 c_{\beta-\alpha}^2 + \delta M_Z^2 s_{\alpha+\beta}^2 + \delta\tan\beta\, s_\beta c_\beta (M_A^2 s_{2\alpha-2\beta} + M_Z^2 s_{2\alpha+2\beta}) \\
&\quad + \frac{e}{2M_Z s_W c_W}(\delta T_H c_{\beta-\alpha} s_{\beta-\alpha}^2 - \delta T_h s_{\beta-\alpha}(1 + c_{\beta-\alpha}^2)) \\
\delta m_{hH}^2 &= \tfrac{1}{2}(\delta M_A^2 s_{2\alpha-2\beta} - \delta M_Z^2 s_{2\alpha+2\beta} - \delta\tan\beta\, s_\beta c_\beta (M_A^2 c_{2\alpha-2\beta} + M_Z^2 c_{2\alpha+2\beta}) \\
&\quad + \frac{e}{2M_Z s_W c_W}(\delta T_H s_{\alpha-\beta}^3 - \delta T_h c_{\alpha-\beta}^3) \\
\delta m_H^2 &= \delta M_A^2 s_{\alpha-be}^2 + \delta M_Z^2 c_{\alpha+\beta}^2 - \delta\tan\beta\, s_\beta c_\beta (M_A^2 s_{2\alpha-2\beta} + M_Z^2 s_{2\alpha+2\beta}) \\
&\quad - \frac{e}{2M_Z s_W c_W}(\delta T_H c_{\alpha-\beta}(1 + s_{\alpha-\beta}^2) + \delta T_h s_{\alpha-\beta} c_{\alpha-\beta}^2)
\end{aligned}$$

Field and tan β renormalization:

"Normal" : $\overline{\text{DR}}$:

$$\delta Z_{\mathcal{H}_1}^{\overline{\text{DR}}} = -\left[\text{Re}\Sigma'_{HH\ |\alpha=0}\right]^{\text{div}}$$

$$\delta Z_{\mathcal{H}_2}^{\overline{\text{DR}}} = -\left[\text{Re}\Sigma'_{hh\ |\alpha=0}\right]^{\text{div}}$$

$$\delta\tan\beta^{\,\overline{\text{DR}}} = \frac{1}{2}\left(\delta Z_{\mathcal{H}_2}^{\overline{\text{DR}}} - \delta Z_{\mathcal{H}_1}^{\overline{\text{DR}}}\right)$$

"More appropriate here: m$\overline{\text{DR}}$:

$$\delta Z_{\mathcal{H}_1}^{\text{m}\overline{\text{DR}}} = -\left[\text{Re}\Sigma'_{HH\ |\alpha=0}\right]^{\text{mdiv}}$$

$$\delta Z_{\mathcal{H}_2}^{\text{m}\overline{\text{DR}}} = -\left[\text{Re}\Sigma'_{hh\ |\alpha=0}\right]^{\text{mdiv}}$$

$$\delta\tan\beta^{\,\text{m}\overline{\text{DR}}} = \frac{1}{2}\left(\delta Z_{\mathcal{H}_2}^{\text{m}\overline{\text{DR}}} - \delta Z_{\mathcal{H}_1}^{\text{m}\overline{\text{DR}}}\right)$$

$[\ ]^{\text{mdiv}}$: $\propto \Delta_m \equiv \Delta - \log(m_M^2/\mu_{\overline{\text{DR}}}^2)$

$\Rightarrow$ decoupling "by hand" of large logs

Calculation of Self-energies:

– all diagrams created with FeynArts $\longrightarrow$ T

$\longrightarrow$ model file with all $\nu/\tilde{\nu}$ interactions

– further evaluation with FormCalc

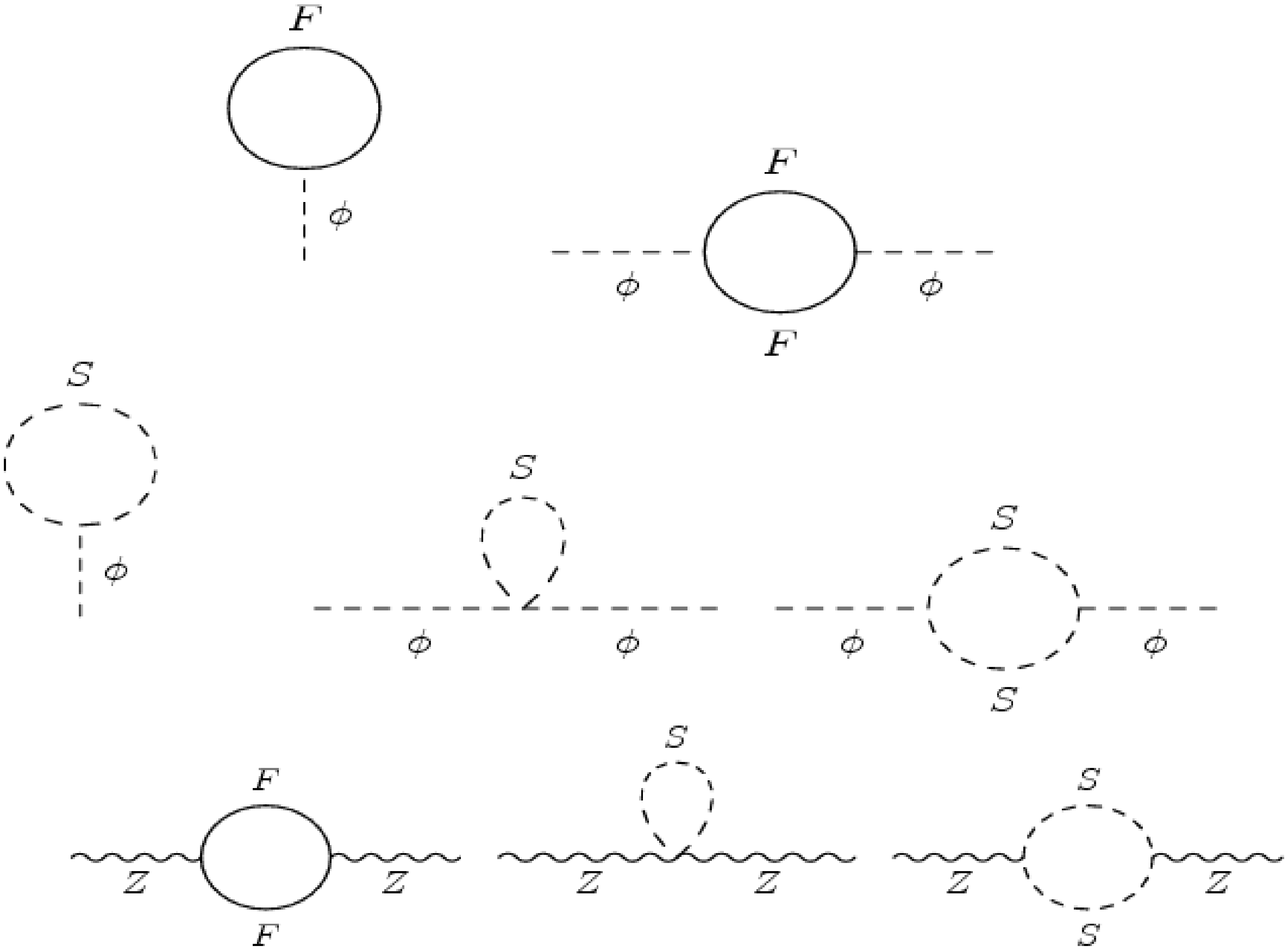
– Dimensional REDuction

– all UV divergences cancel

– results will be included into FeynHiggs (www.feynhiggs.de)

$\longrightarrow$ example plots will focus on $\hat{\Sigma}_{hh}(p^2)$ and $\Delta m_h^{\mathrm{m}\overline{\mathrm{DR}}} := M_h^{\nu/\tilde{\nu}} - M_h$

Feynman diagrams:

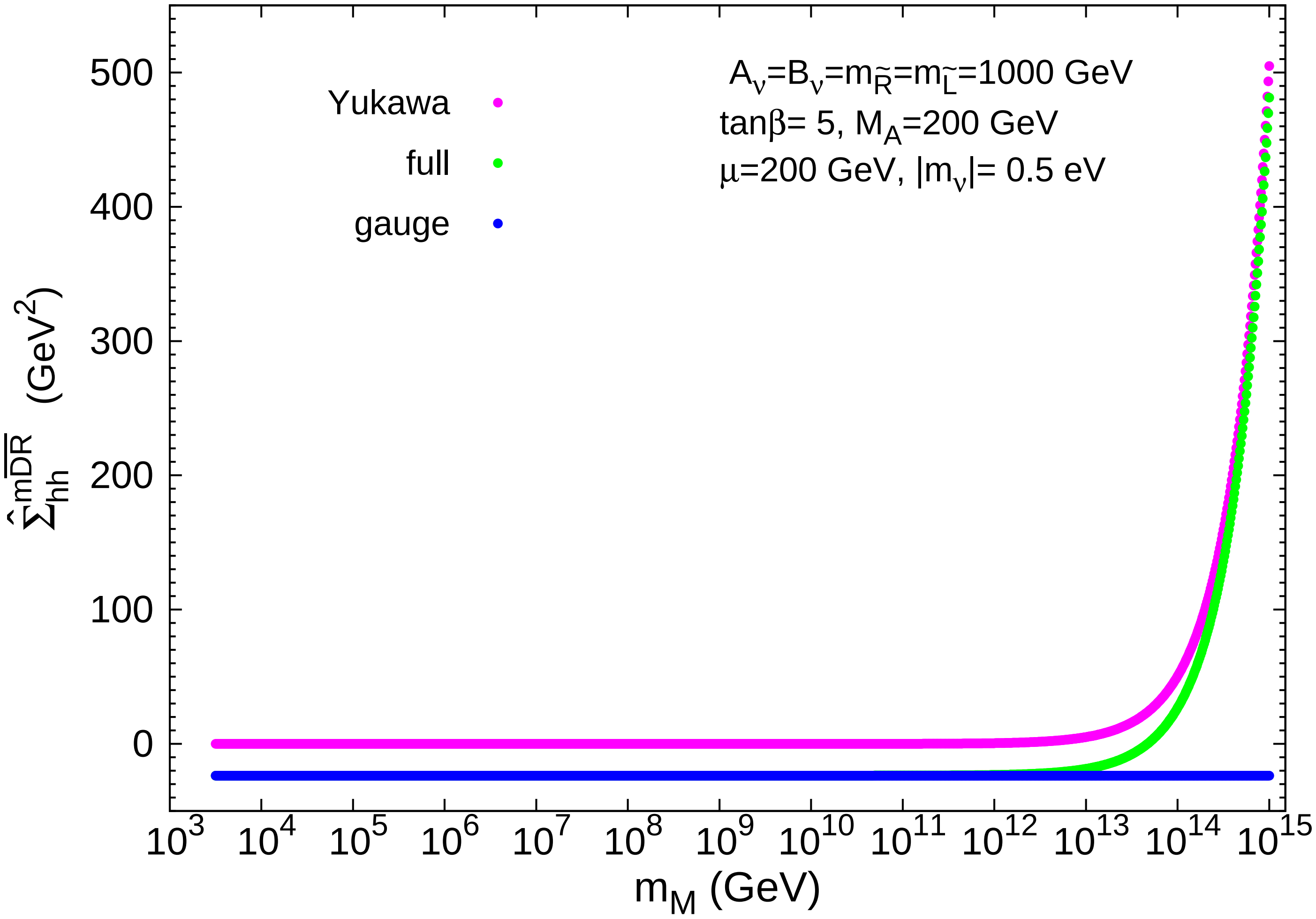


3. Results

Expansion of $\hat{\Sigma}(p^2)$ in powers of $\xi \equiv m_D/m_M$:

$$\hat{\Sigma}(p^2) = \underbrace{\left(\hat{\Sigma}(p^2)\right)_{|m_D^0}}_{\text{gauge MSSM}} + \underbrace{\left(\hat{\Sigma}(p^2)\right)_{|m_D^2} + \left(\hat{\Sigma}(p^2)\right)_{|m_D^4} + \ldots}_{\text{Yukawa}}$$

Gauge part vs. Yukawa part:



3. Results

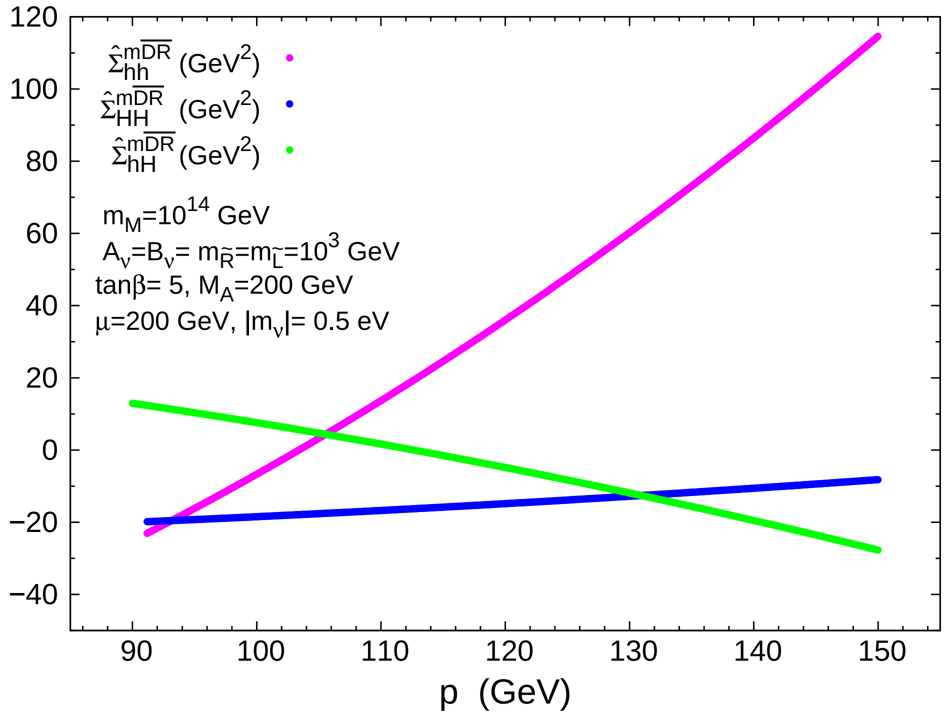
Expansion of $\hat{\Sigma}(p^2)$ in powers of $\xi \equiv m_D/m_M$:

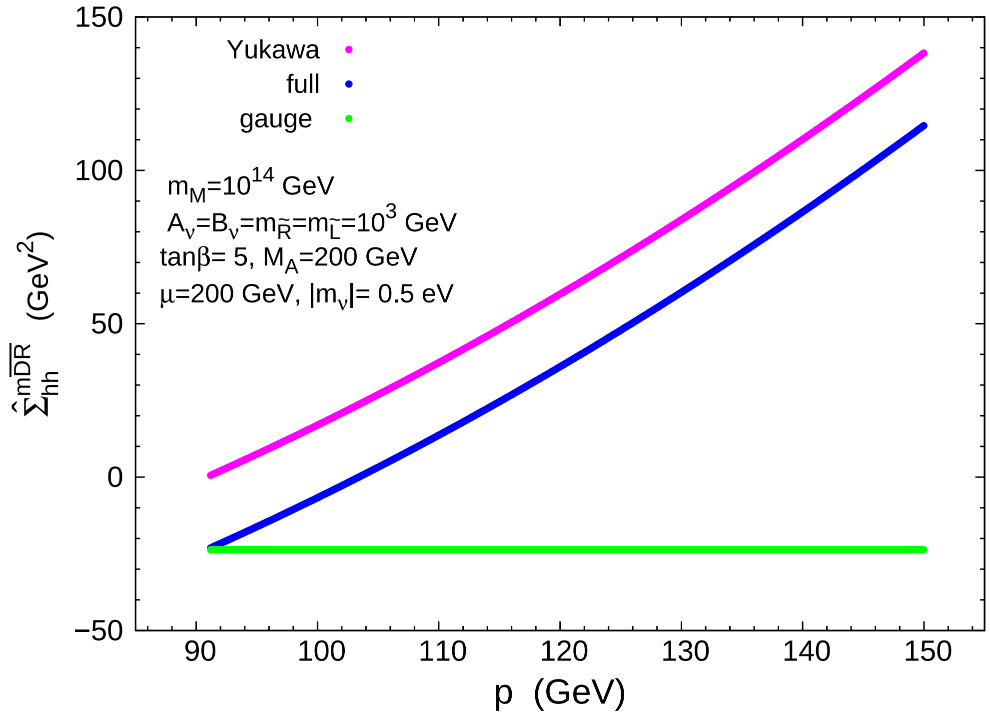
$$\hat{\Sigma}(p^2) = \underbrace{\left(\hat{\Sigma}(p^2)\right)_{|m_D^0}}_{\text{gauge MSSM}} + \underbrace{\left(\hat{\Sigma}(p^2)\right)_{|m_D^2} + \left(\hat{\Sigma}(p^2)\right)_{|m_D^4} + \ldots}_{\text{Yukawa}}$$

$\Rightarrow$ the gauge contribution dominates for $m_M < 10^{12}$ GeV

MSSM-seesaw $\sim$ MSSM ($\oplus$ Dirac neutrinos)

Momentum dependence:





⇒ strong momentum dependence

⇒ only present in the Yukawa part

⇒ (contrary to $\mathcal{O}\left(m_t^4\right)$ corrections) $\mathcal{O}\left(m_D^2\right)$ term dominates

3. Results

Expansion of $\hat{\Sigma}(p^2)$ in powers of $\xi \equiv m_D/m_M$:

$$\hat{\Sigma}(p^2) = \underbrace{\left(\hat{\Sigma}(p^2)\right)_{|m_D^0}}_{\text{gauge MSSM}} + \underbrace{\left(\hat{\Sigma}(p^2)\right)_{|m_D^2} + \left(\hat{\Sigma}(p^2)\right)_{|m_D^4} + \ldots}_{\text{Yukawa}}$$

$\Rightarrow$ the gauge contribution dominates for $m_M < 10^{12}$ GeV

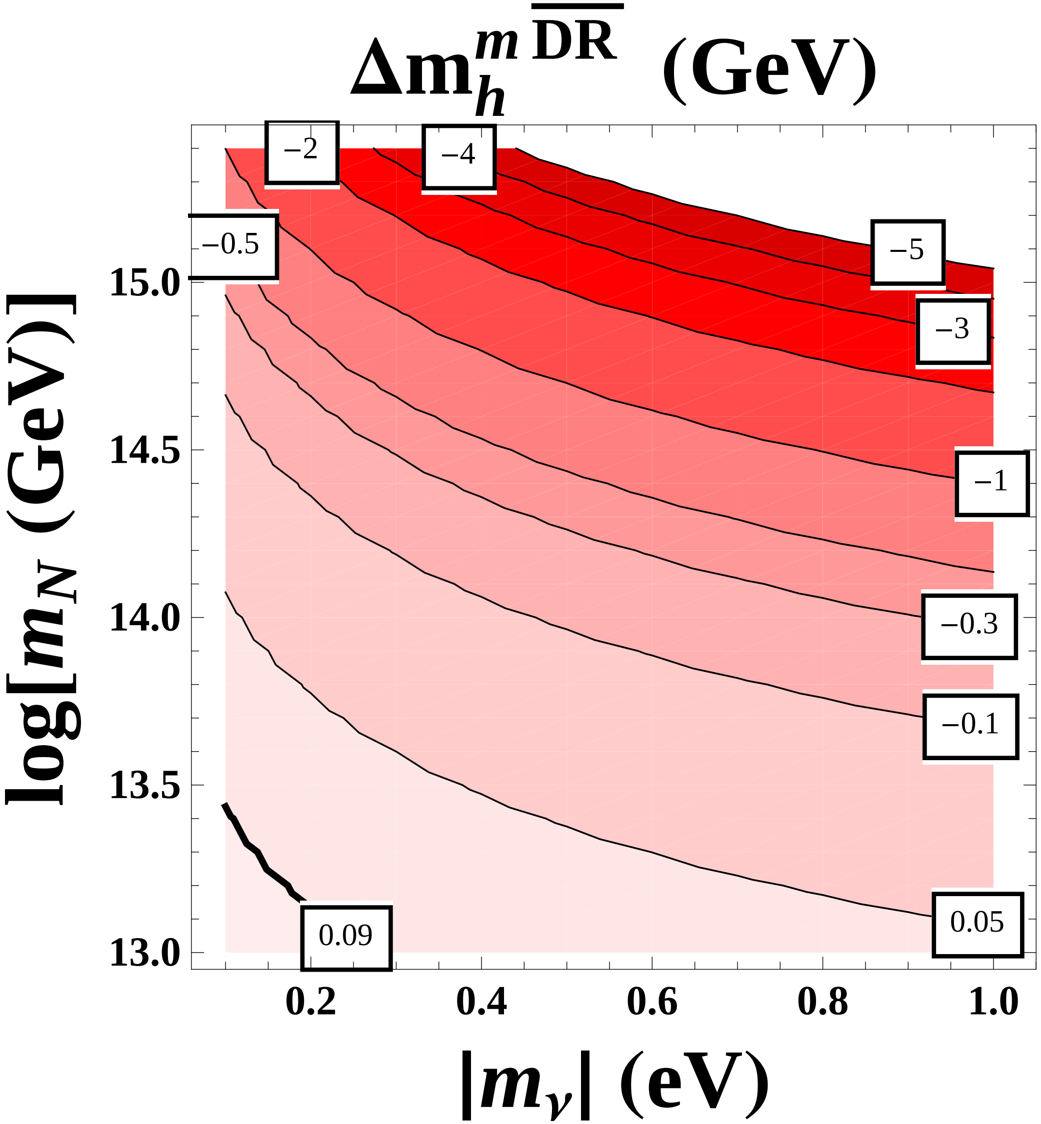
MSSM-seesaw $\sim$ MSSM ($\oplus$ Dirac neutrinos)

Dominant term $\mathcal{O}\left(m_D^2\right)$:

$$\hat{\Sigma}_{hh}^{\overline{\text{DR}}}(p^2) = \frac{g^2 m_D^2 p^2 \cos^2\alpha}{32\pi^2 M_W^2 \sin^2\beta}\left(\frac{1}{2} - \log\frac{m_M^2}{\mu_{\overline{\text{DR}}}^2}\right) + \frac{g^2 m_D^2 p^2 \cos^2\alpha}{64\pi^2 M_W^2 \sin^2\beta}$$

Main result: $\Delta m_h^{\mathrm{m}\overline{\mathrm{DR}}}$:

$A_\nu = B_\nu = m_{\tilde{L}} = m_{\tilde{R}} = 1000$ GeV, $M_A = \mu = 200$ GeV, $\tan\beta = 5$



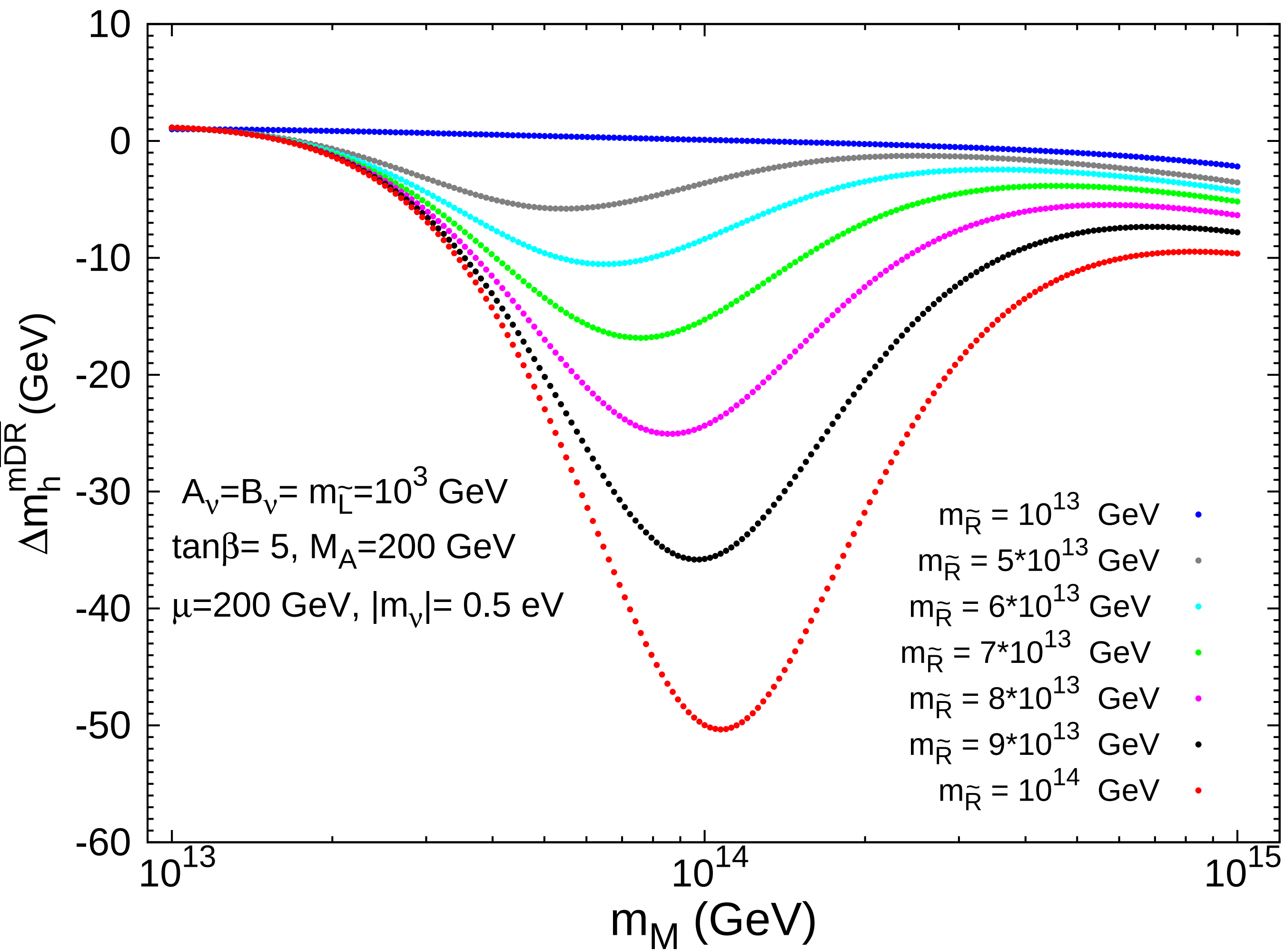
$\Rightarrow$ large corrections possible for large $|m_\nu|$ and m_M

Growing of $\Delta m_h^{\mathrm{m}\overline{\mathrm{DR}}}$ with m_M:

ONLY

due to $Y_\nu \propto \frac{1}{v_2}\sqrt{m_M\,|m_\nu|}$

Dependence on $m_{\tilde{R}}$:



⇒ large corrections for $m_{\tilde{R}} \sim m_M$

The main result:

Dominant term $\mathcal{O}\left(m_D^2\right)$:

$$\hat{\Sigma}_{hh}^{\overline{\rm DR}}(p^2) = \frac{g^2 m_D^2 p^2 \cos^2\alpha}{32\pi^2 M_W^2 \sin^2\beta}\left(\frac{1}{2} - \log\frac{m_M^2}{\mu_{\overline{\rm DR}}^2}\right) + \frac{g^2 m_D^2 p^2 \cos^2\alpha}{64\pi^2 M_W^2 \sin^2\beta}$$

$\Rightarrow$ "usually" considered sub^{2}leading

$\Rightarrow$ not present in effective potential approach

$\Rightarrow$ not present in the RGE approach

Easy (and accurate) formula:

$$\Delta m_h^{\rm m\overline{DR}} \simeq -\frac{\hat{\Sigma}_{hh}^{\nu/\tilde{\nu}}(M_h^2)}{2M_h} \approx -\frac{\left(\hat{\Sigma}_{hh}^{\rm m\overline{DR}}(M_h^2)\right)_{m_D^2}}{2M_h}$$

(if corrections are not too large . . .
otherwise full pole determination necessary $\Rightarrow$ FeynHiggs . . .)

4. Conclusinos

- MSSM is well motivated
  neutrinos have mass . . . via seesaw ???
  $\Rightarrow$ good motivation for MSSM seesaw model

- leading corrections: $\Delta M_h^2 \sim m_t^4/M_W^2$
  seesaw: $Y_\nu = \mathcal{O}(1) \Rightarrow$ large effects on M_h?

- First step: MSSM with one generation of neutrinos/sneutrinos

- – Evaluation of $\nu/\tilde{\nu}$ corrections to $\hat{\Sigma}_\phi(p^2)$
  – Renormalization for fields and $\tan\beta$: $\mathrm{m}\overline{\mathrm{DR}}$
  – Leading terms:

$$\Delta m_h^{\mathrm{m}\overline{\mathrm{DR}}} \simeq -\frac{\hat{\Sigma}_{hh}^{\nu/\tilde{\nu}}(M_h^2)}{2M_h} \approx -\frac{\left(\hat{\Sigma}_{hh}^{\mathrm{m}\overline{\mathrm{DR}}}(M_h^2)\right)_{m_D^2}}{2M_h}$$

  Effects of up to -5 GeV for large m_M and $|m_\nu|$

- Effects relevant for all future collider phenomenology!


Higgs Days at Santander 2011
Theory meets Experiment
19.-23. September
IFCA
Instituto de Física de Cantabria
CSIC
UC
UNIVERSIDAD DE CANTABRIA
contact:Sven.Heinemeyer@cern.ch
http://www.ifca.es/HDays11