New Physics :

High + Low,

Easy + Hard

N. A-H; SUSY 2011

New Physics —

Batavia / Geneva

Year of SUSY	Topic	$\mathcal{N}_{SUSY}$
1999	Large Extra Dim	0_{brane} ($\to$ Lots Bulk)
2000	RS = Technicolor	0
2003	Little Higgs	0
2011	Amplitudes	4

Little Hierarchy

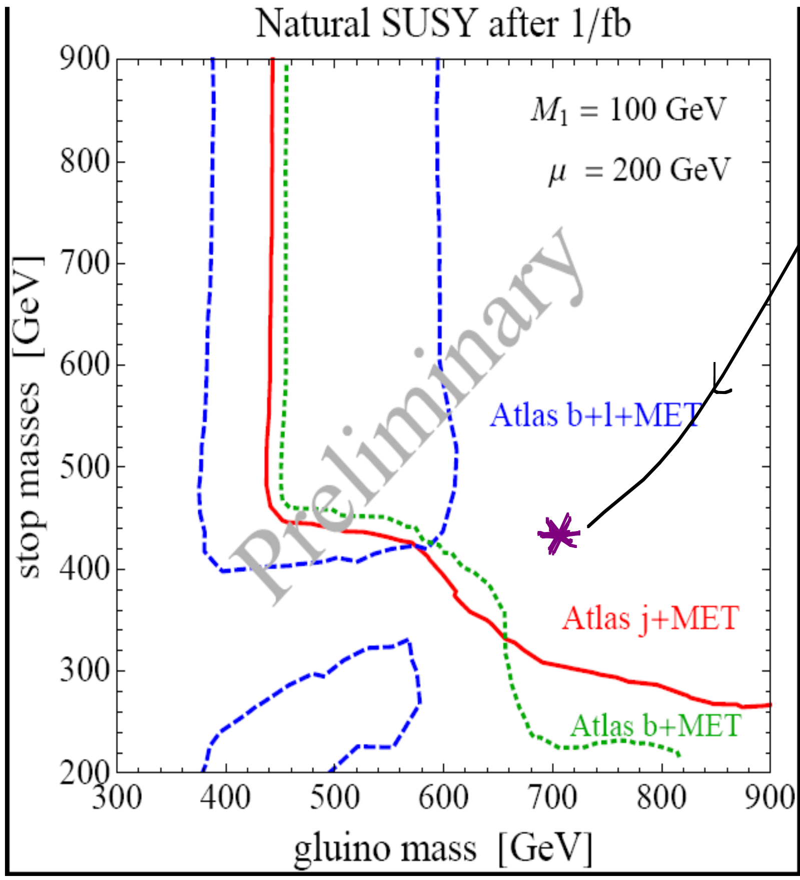
Welcomed back into the fold! $\longrightarrow$ $\mathcal{N}_{average} = 1$ ✓

. . . Some Simple Comments

on L.H.C. SUSY results . . .

A Reported Death
of Low-Energy SUSY
is Wildly Exaggerated

[Papucci, Ruderman, Toro, Weiler]



$m_{\tilde{t}} \sim 400\,\mathrm{GeV}$

$m_{\tilde{g}} \sim 700\,\mathrm{GeV}$

BEAUTIFULLY

NATURAL

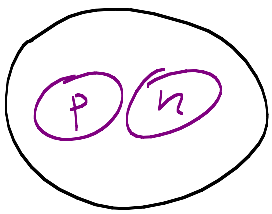
(Bottom-Up)

(B) Nonetheless, a spectrum with somewhat heavy superpartners not at all surprising to those of us worried about little hierarchy problem. [No more worried after first round of L.H.C. analyses].

My own view for ~7 years has been that the weak scale may well be somewhat fine-tuned...

What is "Somewhat"?

Next EFT down:



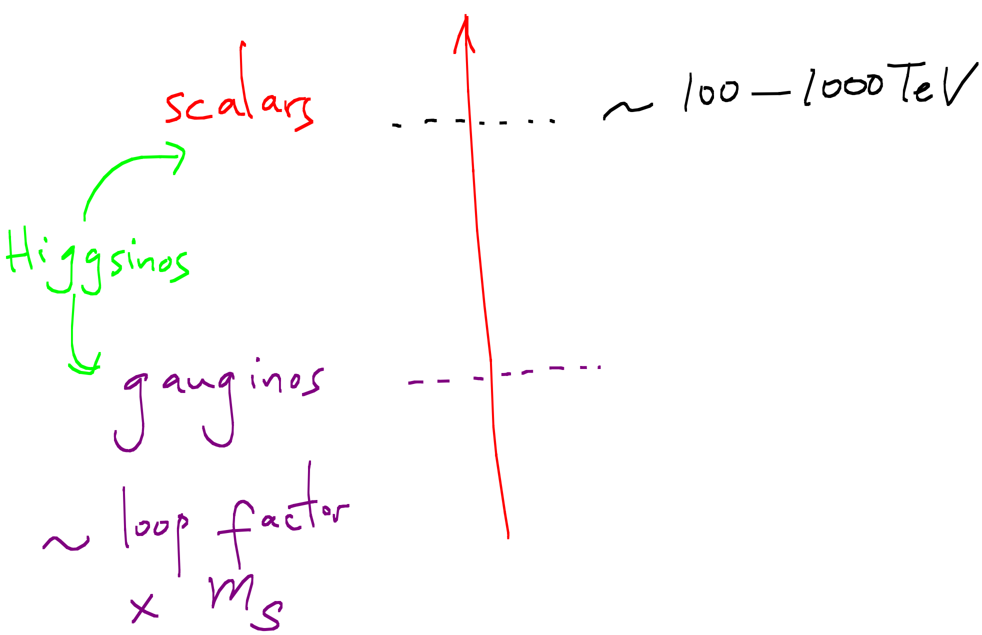
De binding energy
$\sim 2\,\mathrm{MeV}$
1 in 20 tuning

Not bound by
60 keV (!!)
1 in 1000 tuning

This level tuning in SUSY $\Longrightarrow$ LHC accessible superpartners

Might it be more tuned?

Simplest Split SUSY

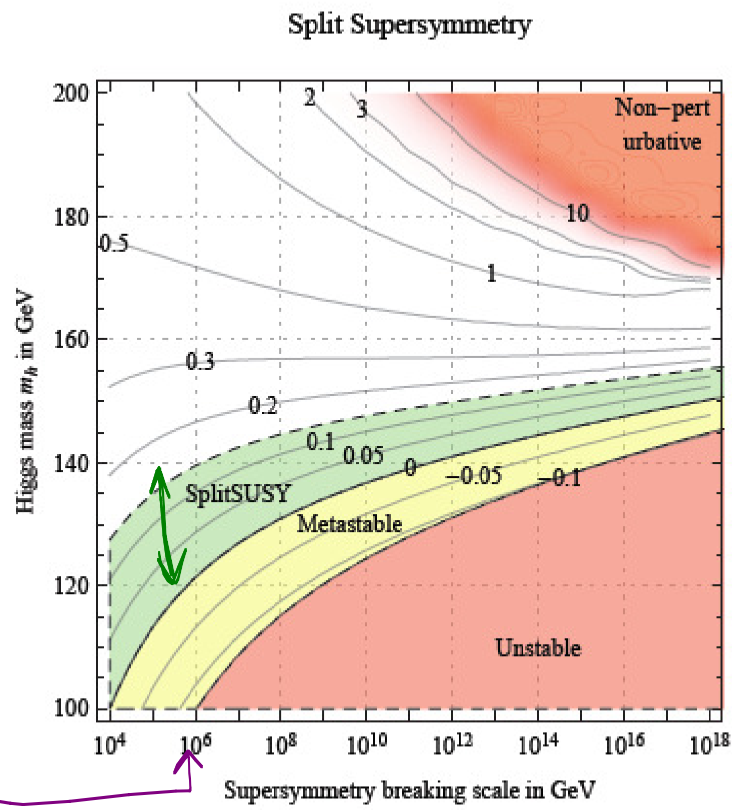


- Tuning ~ 10^{-4}–10^{-6}
- Unification ✓
- DM ✓
- No flavor, CP, moduli, ... problems

Higgs mass in (moderate) SplitSUSY

t_β 1.5 – 4

$m_h \sim 115$-140 GeV



1000 TeV scalars

[Giudice, Strumia hep-ph/last night]

(Similar: Wacker et.al.)

[Back in '04]

((

Here we have outlined an alternate viewpoint, where the usual problems of SUSY vanish, unification is evidence for *high-energy* SUSY, and where accelerators can convincingly demonstrate the presence of fine tuning in the electroweak sector.

The first sign of this proposal at the LHC should be the Higgs, in the mass range of $\sim 120 - 150$ GeV. No other scalar should be present, since it would indicate a second, needless, fine-tuning. Next will be the gluino, whose long lifetime will be crucial evidence that the scale of supersymmetry breaking is too large for the hierarchy problem, and a fine-tuning is at work. A measurement of the gluino lifetime can yield an estimate for the large SUSY breaking scale m_S. Next will come the electroweak gauginos and higgsinos, whose presence will complete the picture, and give supporting evidence that the colored octets of the previous sentence are indeed the gluinos. Further precise measurements of the gaugino-higgsino-higgs couplings, presumably at a linear collider, will accurately determine m_S and provide several unambiguous quantitative cross-checks for high-scale supersymmetry.

))

- Discovering Gluino is key — long lifetime smoking gun for heavy scalars . . .

★ For moderate split SUSY, look for stopped gluinos, ~ cm displaced decays, + for unlucky shorter lifetimes: gluinonium; spin/hadronization . . .

[As amazingly exciting as discovery of conventional low-E SUSY would be, I think Split-SUSY would teach us even more.... still UV SUSY! But another blow to naturalness after $\Lambda \rightsquigarrow$ much stronger (but still circumstantial) push towards landscape picture....]

Space-Time, Quantum Mechanics

and

Scattering Amplitudes

w/ J. Bourjaily
F. Cachazo
S. Caron-Huot
C. Cheung
A. Hodges
J. Kaplan
J. Trnka

S. Gancherov

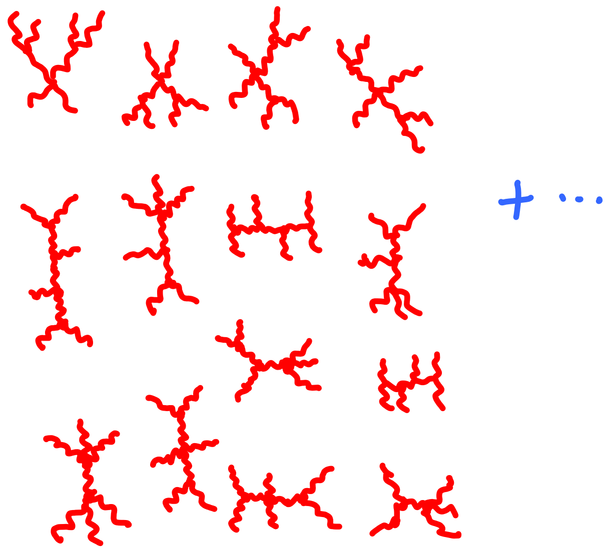
\+

P. Deligne
R. Macpherson
M. Goresky

"New Physics" Under

Our Noses

Feynman Explosion



220 Diagrams

10's of thousands

of terms ...

Result of a brute force calculation:

$$k_1 \cdot k_4\, \varepsilon_2 \cdot k_1\, \varepsilon_1 \cdot \varepsilon_3\, \varepsilon_4 \cdot \varepsilon_5$$

+ 24 more pages ....

$$\mathrm{Amp}(1^+2^-3^+4^-5^+6^+) = \frac{\langle 24\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 56\rangle} \; !$$

"MHV Amplitudes" : $\bar{i}, \bar{j}$, rest plus

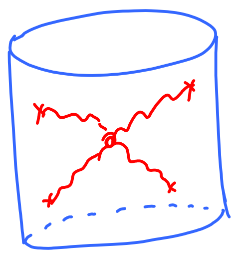
$$\frac{\langle i\, j\rangle^4}{\langle 12\rangle\langle 23\rangle \cdots \langle n1\rangle}$$

[Rest : BCFW Recursion]

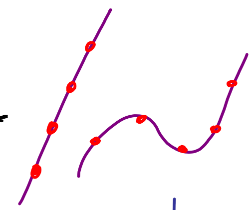
Q. What makes Feynman Diagrams so complicated, obscuring simplicity of answer?

A. Insistence on Manifest Locality + Unitarity!

Sitting Under our Noses for 60 yrs

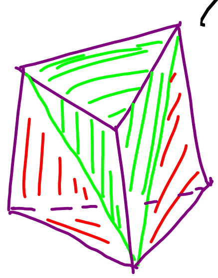


String Theory

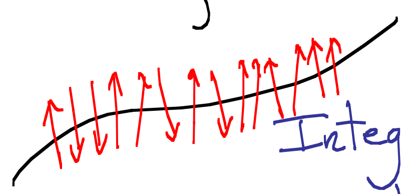


Twistor Theory

Scattering without Spacetime – emergent locality + unitarity



Algebraic Geometry



Integrable systems

Kinematics

$$p^\mu = (p^0, \vec{p}) \longleftrightarrow p_{A\dot{A}} = \begin{pmatrix} p^0+p^3 & p^1+ip^2 \\ p^1-ip^2 & p^0-p^3 \end{pmatrix}$$

$$\det p = p^2 = 0$$

$$\Longrightarrow \quad p_{A\dot{A}} = \lambda_A \tilde{\lambda}_{\dot{A}}$$

Lorentz: $SL(2) \times SL(2)$

Invariants $\langle \lambda_1 \lambda_2 \rangle = \epsilon^{AB} \lambda_{1A} \lambda_{2B}$

$$[\tilde{\lambda}_1 \tilde{\lambda}_2] = \epsilon^{\dot{A}\dot{B}} \tilde{\lambda}_{1\dot{A}} \tilde{\lambda}_{2\dot{B}}$$

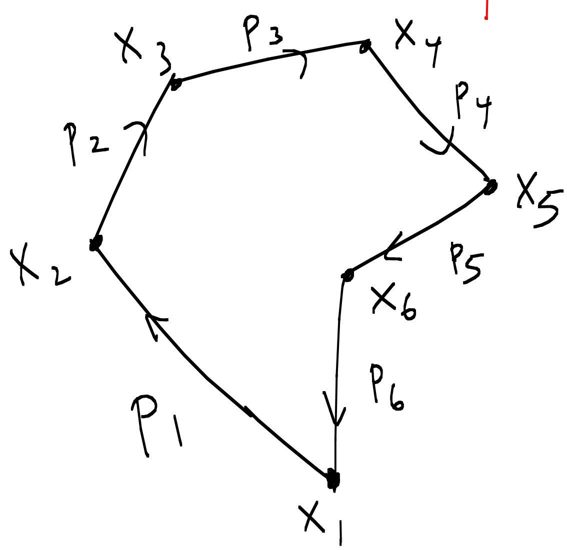
Hidden

Infinite Dimensional

Symmetries

$\mathcal{N}=4$ SYM has an "obvious" (super) conformal symmetry.

Dual (Super) Conformal Symmetry



$$p_a = x_{a+1} - x_a$$

"Experimental" observation

– amplitudes invariant under

Conf. transf. on

<u>this</u> X space!

[Term by term for BCFW form of trees]

(Super) Conformal + Dual (Super) Conformal

$\Downarrow$ generate

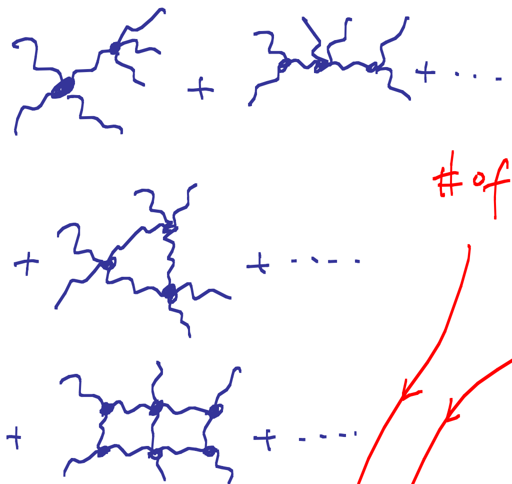
"Yangian Algebra"

Infinite Dimensional Symmetry

Completely Invisible In $\mathcal{L}$

The Grassmannian Formulation

Amplitudes



# of particles

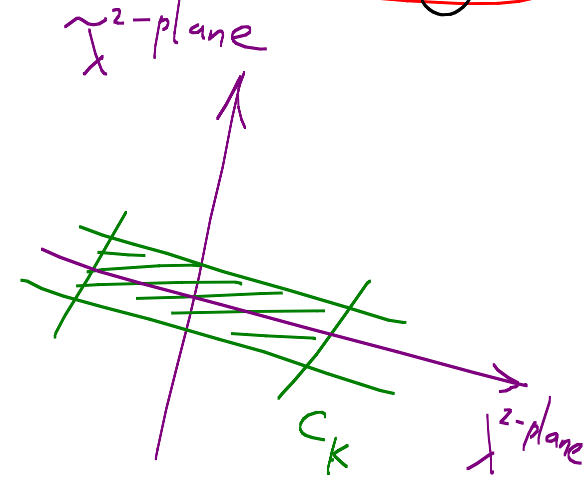
$\#_-$ hel. gluons (PLANAR)

$$\longrightarrow \mathcal{M}_{n,k}[\lambda,\tilde{\lambda},\tilde{\eta}]$$

Manifestly Local, Unitary

Horrendously Complicated

Grassmannian Integral



$$\int \frac{d^{k(n-k)}C}{(12\cdots k)\cdots(n1\cdots k-1)}$$

Non-manifest locality, Unit

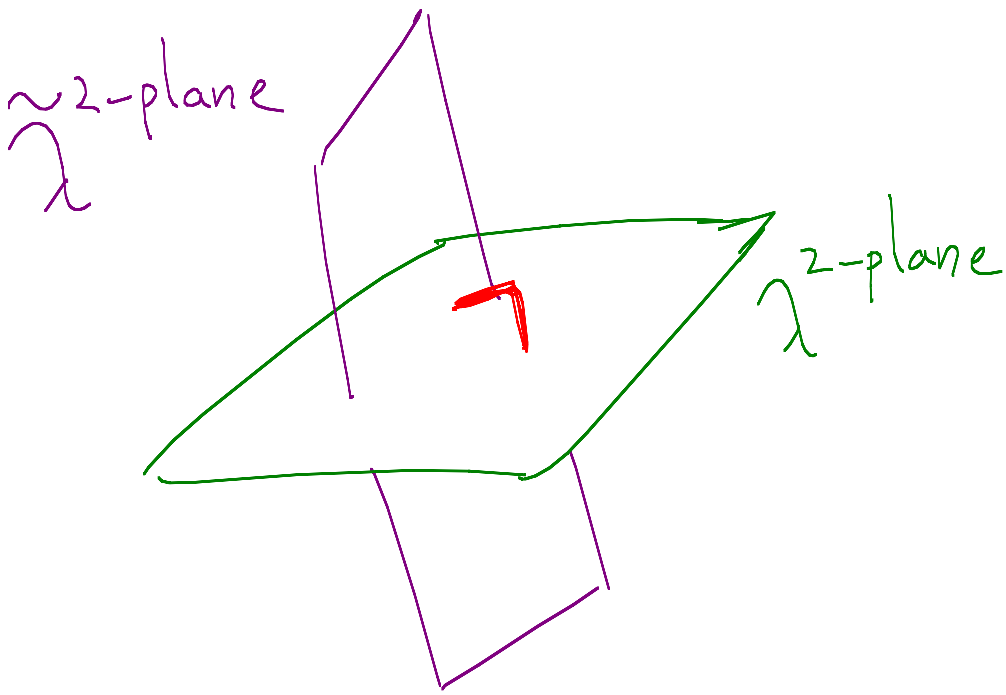
Simple

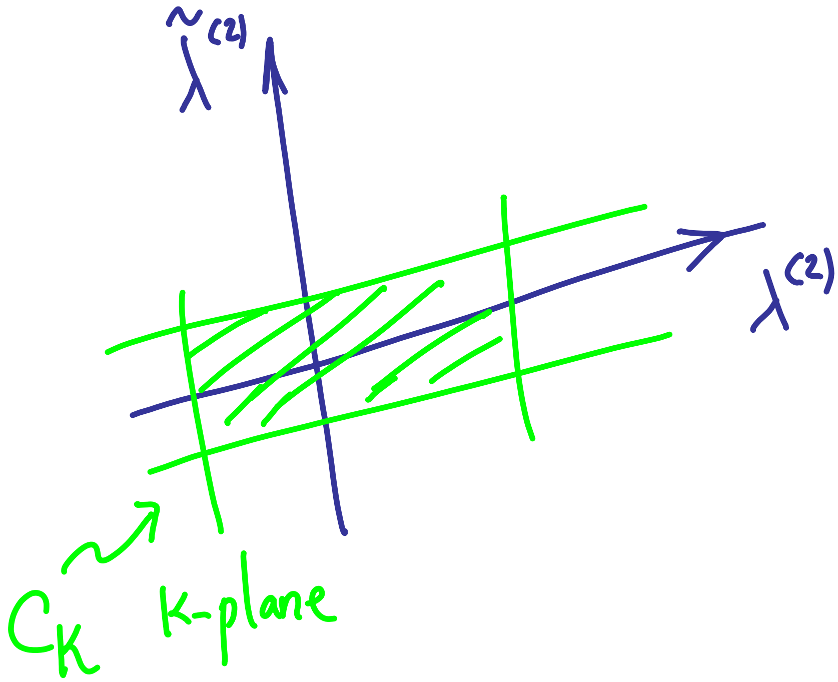
Yangian Invariance Manifest

Geometry of Momentum Conservation

$\lambda_a^A, \tilde{\lambda}_a^{\dot{A}}$

$$\sum_a \lambda_a^A \tilde{\lambda}_a^{\dot{A}} = 0$$





Note: parity invariant since

$\lambda \leftrightarrow \tilde{\lambda}$

k plane $\leftrightarrow$ n-k plane

Note: impossible for $k = 0, 1, n-1, n$.

Good!

Grassmannian $G(k,n)$: k- planes in n-dimensions.

- $$C_{\alpha a} = \begin{array}{c} \longleftarrow n \longrightarrow \\ \begin{bmatrix} (& &) \\ (& &) \\ (& &) \\ (& \vdots &) \end{bmatrix} \end{array} \updownarrow k \quad , \quad C_{\alpha a} \sim L_{\alpha}^{\beta} C_{\beta a}$$

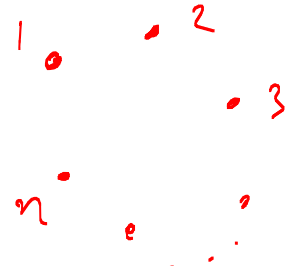
  $GL(K)$ redund.

- $(m_1 \cdots m_K) = \epsilon^{\alpha_1 \cdots \alpha_K} C_{\alpha_1 m_1} \cdots C_{\alpha_K m_K} =$ "minor"

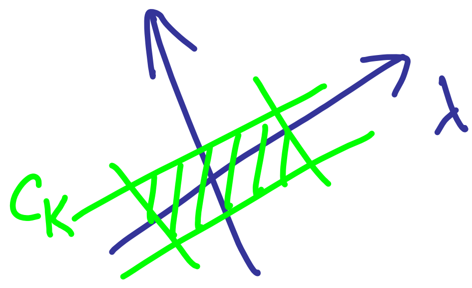
- $$C_{\alpha a} = \begin{bmatrix} \vec{c}_1 & \vec{c}_2 & \cdots & \vec{c}_n \end{bmatrix}$$

  Little group $c_i \to t_i c_i$

  each $c_i \in P^{k-1}$

So, $C_{\alpha a} \sim$ 

Space of points in $P^{k-1} / GL(K)$



$$\int d^{2\times k}\rho_\alpha \; \underbrace{\delta^2[C_{\alpha a}\rho_\alpha-\lambda_a]}_{C \text{ contains } \lambda} \; \underbrace{\delta^2[C_{\alpha a}\tilde\lambda_a]}_{C \text{ orthogonal to } \tilde\lambda} \; \underbrace{\delta^4[C_{\alpha a}\tilde\eta_a]}_{\text{SUSY partner}}$$

Preserve $GL(k)$

This object is very simple in Twistor Space:

$$\prod_{\alpha=1}^{k} \delta^{4|4}\left[C_{\alpha a} W_a \right]$$

Manifests (Super) conformal symmetry

For $k=0,1,n-1,n$, no planes.

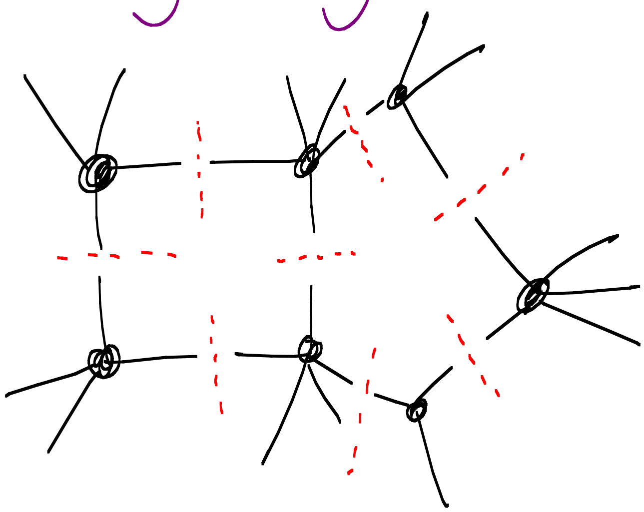
For $k=2, n-2$, unique plance

General k – integrate over planes!

$$\frac{1}{\mathrm{vol}\,GL(k)} \int \frac{d^{k\times n} C}{(12..k)\cdots(n1..k-1)} \leftarrow \underbrace{GL}(k) \text{ invariance}$$

$(m_1 \dots m_k)$ minor : det of columns $m_1, \dots, m_k$

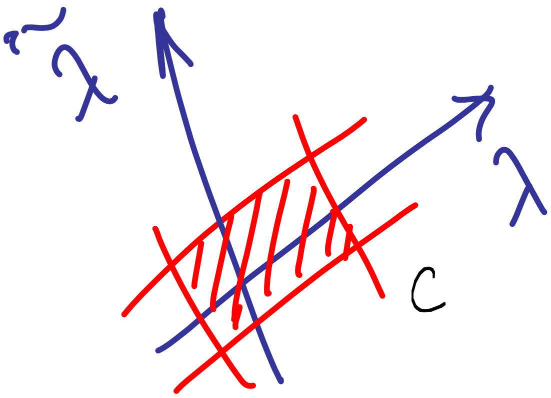
"Leading Singularities"



$\longleftrightarrow$ Residues of $\mathcal{L}_{n,k}$

[Relations giving locality, Unitarity $\longleftrightarrow$ Residue Thms]

Manifest Dual Superconformal Invariance



C contains λ plane:

so really an integral over

$(k-2)$ planes in n dimensions!

Natural linear transformation mapping $k\times k$ minors to $(k-2)\times(k-2)$

minors ...

$$\mathcal{L}_{n,k} \to \int \frac{d^{p\times(n-p)} D_{\alpha a}}{(12\ldots p)\ldots(n1\ldots(p-1))} \times \prod_{\alpha=1}^{p} \delta^{4|4}[D_{\alpha a} \mathcal{Z}_a]$$

— momentum twistor variables

Identical Structure!

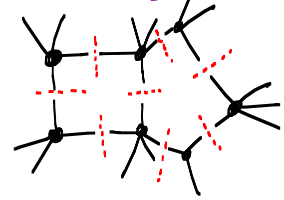
Dual superconformal symmetry manifest

The Grassmannian Formulation makes no mention of locality or Unitarity — but makes all symmetries — The Yangian — manifest.

$$\int \frac{d^{k(n-k)} C_{\alpha a}}{C(12\cdots k)\cdots(n1\cdots k-1)} \prod_{\alpha=1}^{k} \delta^{4|4}\left[C_{\alpha a} W_a\right] \quad \longleftrightarrow \quad \text{All-loop planar integrand in manifestly Yangian Invariant form.}$$

$$\Big\downarrow \ \text{Residues}$$

Grassmannian Residues ⟷ "Leading Singularities"



Generate all Yangian invariants

+ Relations between them ⟷ Locality, Unitarity

The "Positive Part" of the Grassmannian

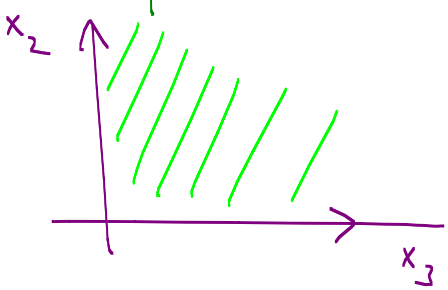
[c.f. Lustig, Postnikov et. al., Fock + Goncharov, ... 90's → 2000's]

- Generalize Simplex in $P^{n-1} = G(1,n)$:

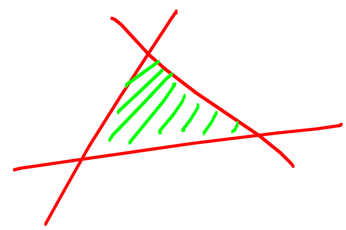
Choose co-ordinates

$X = (x_1 \; x_2 \; x_3)$

"positive part" $x_i > 0$



Closure is Simplex (123)



- Denote by $(123..n)$, boundaries just put diff. $x_i \to 0$, $\partial(123..n) = (23..n) - (13..n) + \cdots (12..n-1)$ is just "deletion".

For $G(k,n)$:

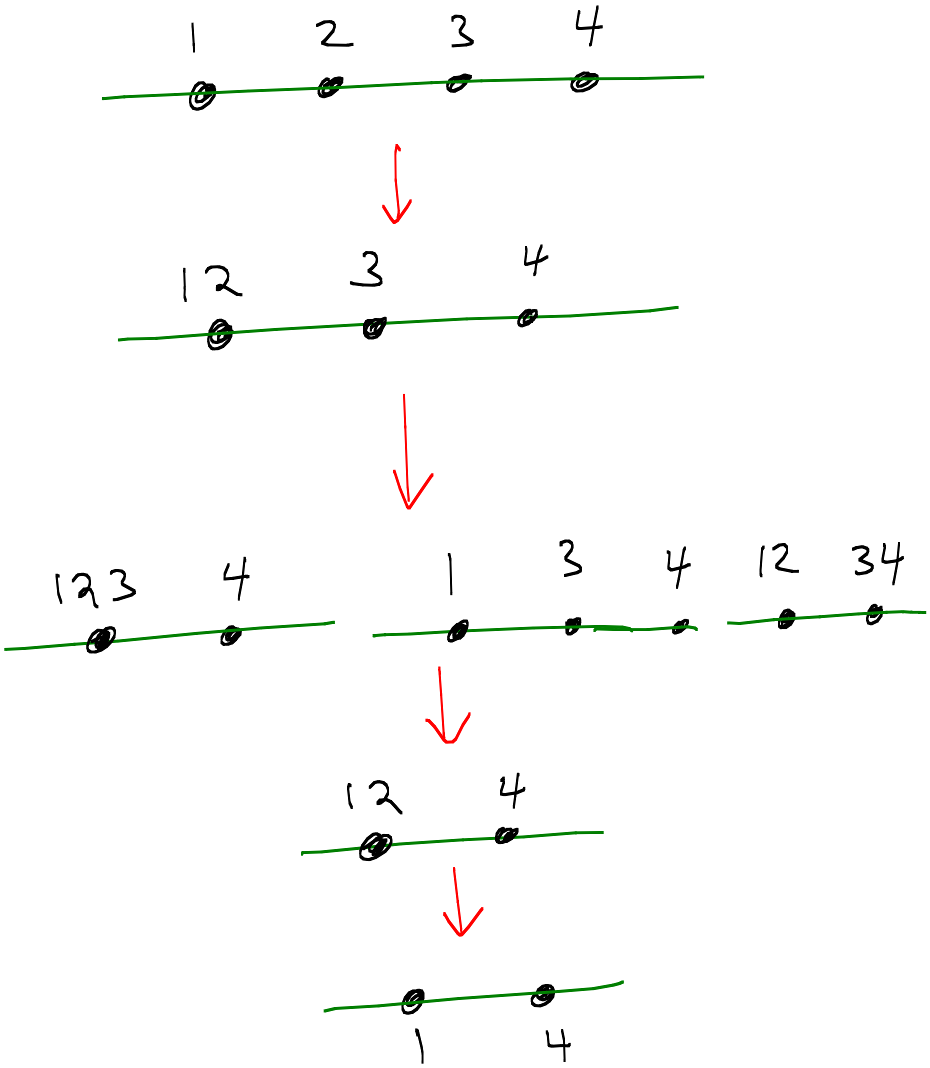
$$C = \begin{array}{c} \xleftrightarrow{\quad n \quad} \\ \begin{bmatrix} c_1 & c_2 & \cdots & c_n \end{bmatrix} \end{array} \updownarrow k \qquad n \text{ } k\text{-vectors.}$$

"Positive Part": $(c_{i_1} \cdots c_{i_k}) > 0$ for $i_k > \cdots > i_1$.

["All minors positive"].

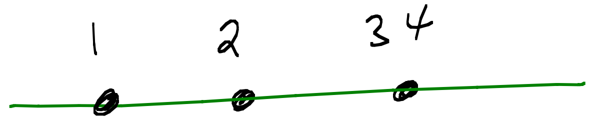
Note: (twisted) cyclic structure

$c_1 \to c_2,\ c_2 \to c_3,\ \ldots,\ c_n \to (-1)^{k+1} c_1$

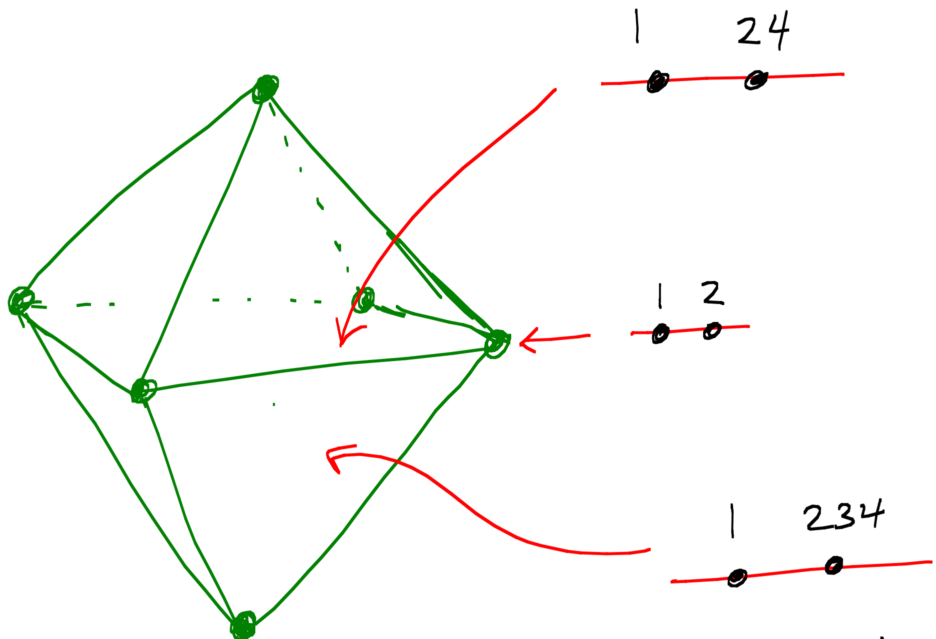


∂ operator is merge + delete

[not just delete!]

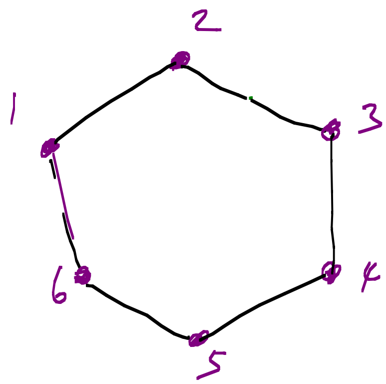


: 3-dim face



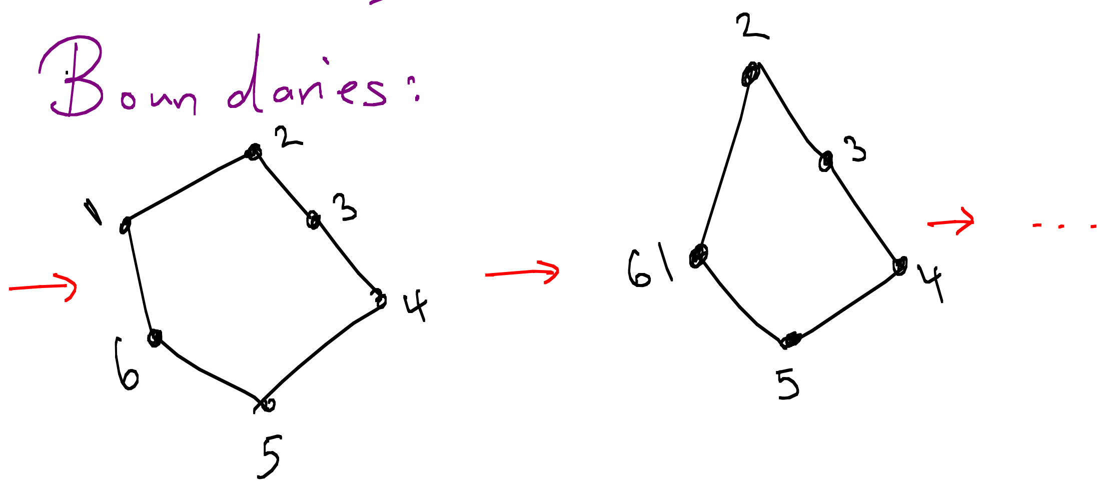
· [Whole 4 d Polytope is topologically a ball].

For $G(3,n)$: $(i_1 i_2 i_3) > 0$



Convex Polygon

Boundaries:



Our measure :

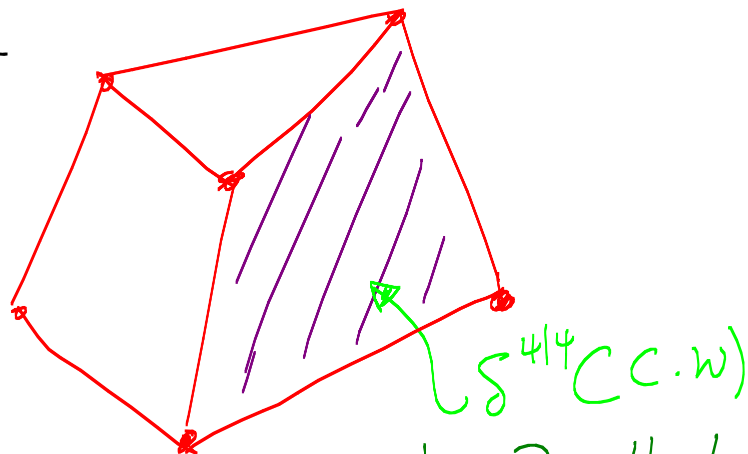
$$\frac{d^{k\times n} C}{(12..k)\ \cdots\ (n1 \cdots k-1)}$$

is unique one
smooth inside polytope
Only sing on boundaries.

Yangian invariance $\longleftrightarrow$ nothing but diffs, preserving positive part!

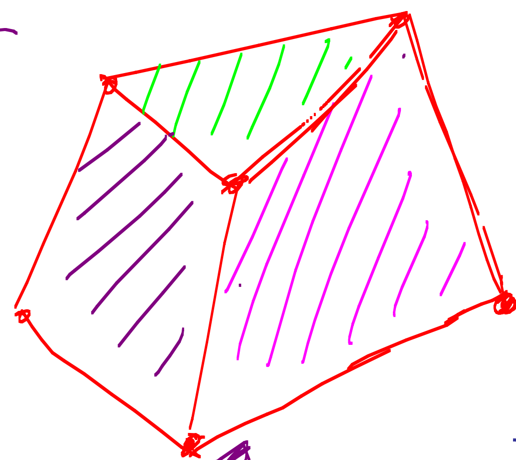
[Deep connections to "theory of motives"]

Grassmannian residues are associated with facets of big polytope



$D =$ dim $2n-4$ twistor/mom space
dim $4K$ mom-twistor space

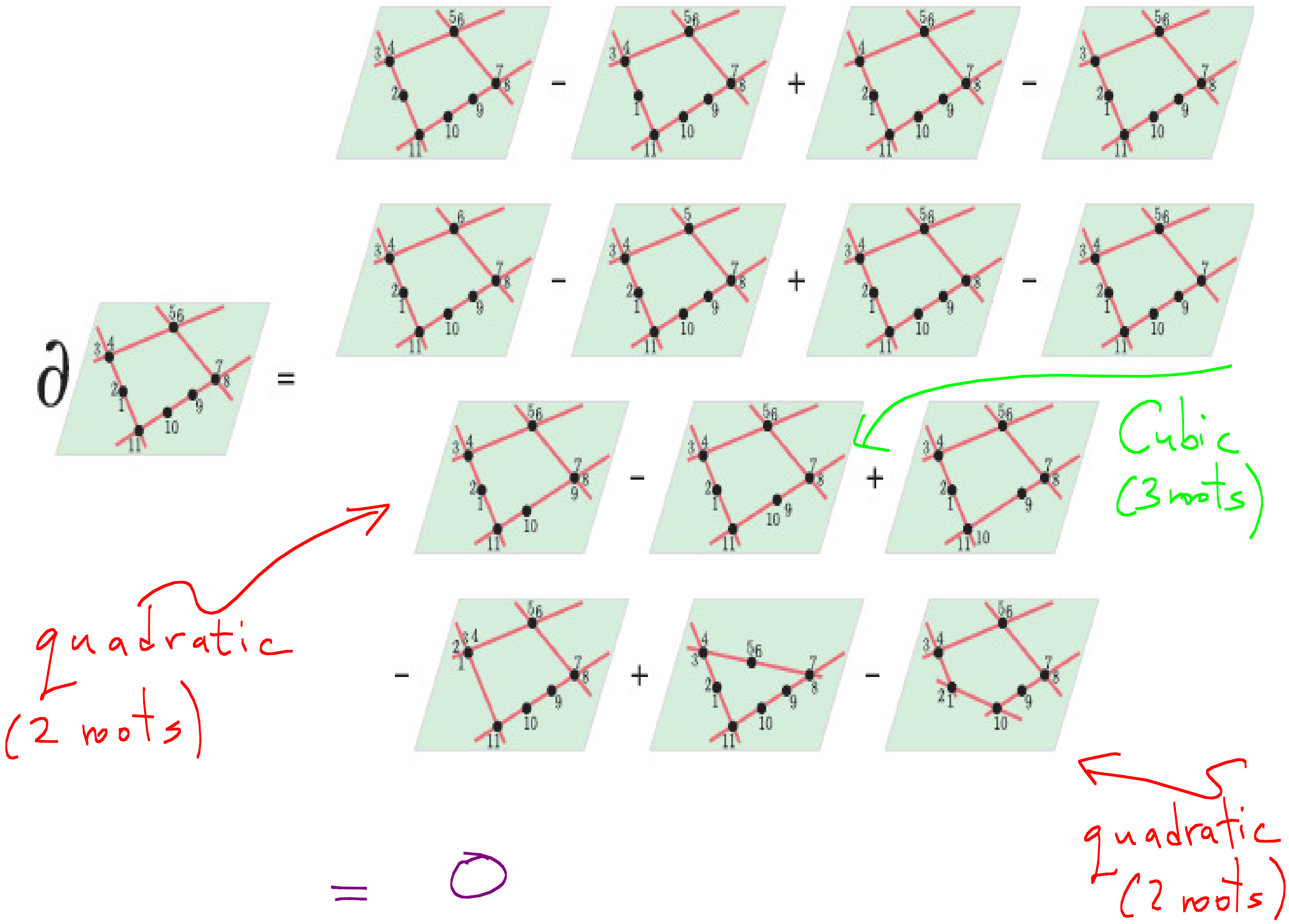
Relations between residues:

$\partial \Big[$  $\Big] = 0$

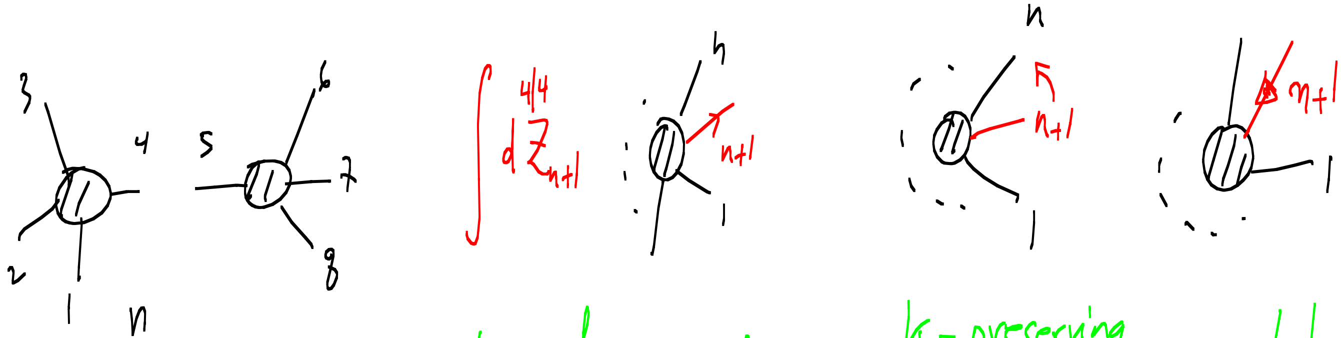
Face of dim $D+1$

[$\longleftrightarrow$ Encode locality, Unitarity]

Ex: 14-term identity involving rationals, $\sqrt{\ }$'s, $\sqrt[3]{\ }$'s:



Basic Operations on Yangian Invariants

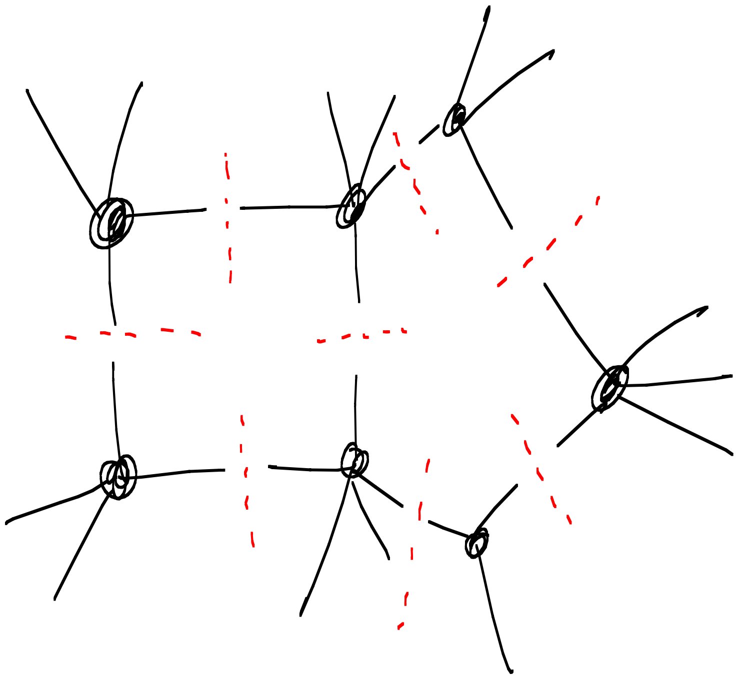


Fuse

K-decreasing remove

k-preserving remove

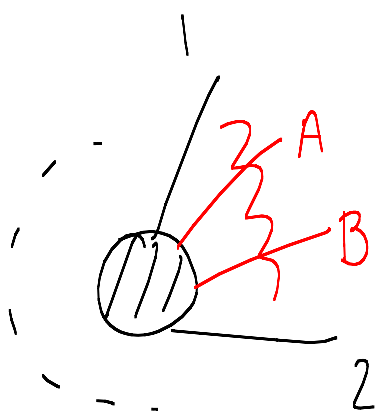
add



$\longleftrightarrow$ Combination of "Fuse", "Merge", "Remove"

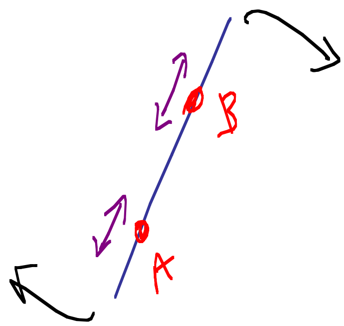
$\longrightarrow$ Leading Sing. are Grassmannian Residues / Yangian Invariant.

Origin of Loops



$$\int d^{4|4}Z_A \, d^{4|4}Z_B$$

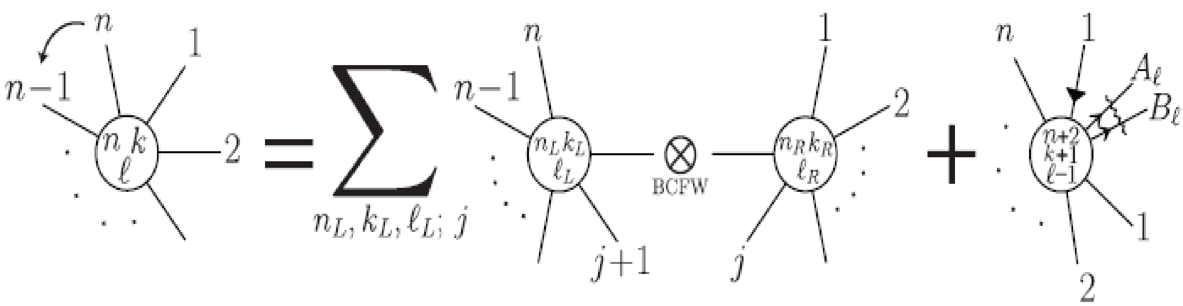
[momentum-twistor form]



"Entangled removal" of a pair of particles

[Non-trivial loop amps only arise in (3,1) signature!]

All-Loop Recursion



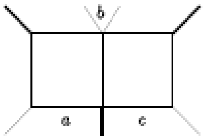
"Classical"

"Quantum"

Complete definition with Yangian symmetry manifest.

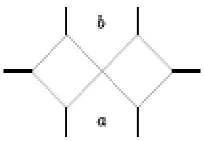
The words "spacetime", "Lagrangian", "Path Integral", "Gauge Symmetry" ....

make no appearance.

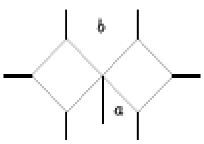


$$-\frac{1}{4}\begin{bmatrix} a & a+1 & b-1 \\ b+1 & c-1 & c \end{bmatrix} \tag{53}$$

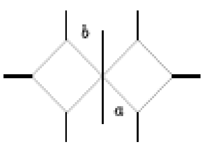
B. Kissing double-box topologies



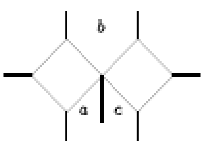
$$\begin{aligned}-\frac{1}{4}\begin{bmatrix} a & a+1 & b-1 & b \\ b & b+1 & a-1 & a \end{bmatrix} + \frac{1}{4}\begin{bmatrix} a & a+1 \\ b-1 & b \end{bmatrix}\begin{bmatrix} b & b+1 \\ a-1 & a \end{bmatrix} = \\ \frac{1}{4}\Big(x^2_{a-1,b+1}x^2_{a+1,b-1}\left(x^2_{ab}\right)^2 - x^2_{a-1,b-1}x^2_{a+1,b+1}\left(x^2_{ab}\right)^2 + \\ + x^2_{a-1,a+1}x^2_{b-1,b+1}\left(x^2_{ab}\right)^2 - x^2_{a-1,b}x^2_{a,b+1}x^2_{a+1,b-1}x^2_{ab} - \\ - x^2_{a-1,b+1}x^2_{a,b-1}x^2_{a+1,b}x^2_{ab} + x^2_{a-1,b-1}x^2_{a,b+1}x^2_{a+1,b}x^2_{ab} + \\ + x^2_{a-1,b}x^2_{a,b-1}x^2_{a+1,b+1}x^2_{ab}\Big)\end{aligned} \tag{54}$$



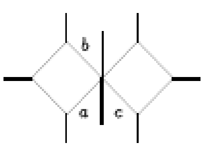
$$-\frac{1}{4}\begin{bmatrix} a+1 & a+2 & b-1 & b \\ b & b+1 & a-1 & a \end{bmatrix} + \frac{1}{4}\begin{bmatrix} a-1 & a \\ b & b+1 \end{bmatrix}\begin{bmatrix} a+1 & a+2 \\ b-1 & b \end{bmatrix} \tag{55}$$



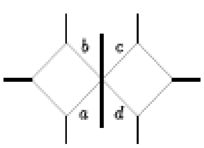
$$-\frac{1}{4}\begin{bmatrix} a+1 & a+2 & b-1 & b \\ b+1 & b+2 & a-1 & a \end{bmatrix} + \frac{1}{4}\begin{bmatrix} a+1 & a+2 \\ b-1 & b \end{bmatrix}\begin{bmatrix} b+1 & b+2 \\ a-1 & a \end{bmatrix} \tag{56}$$



$$-\frac{1}{4}\begin{bmatrix} a & a+1 & b-1 & b \\ b & b+1 & c-1 & c \end{bmatrix} + \frac{1}{4}\begin{bmatrix} a & a+1 \\ b-1 & b \end{bmatrix}\begin{bmatrix} b & b+1 \\ c-1 & c \end{bmatrix} \tag{57}$$



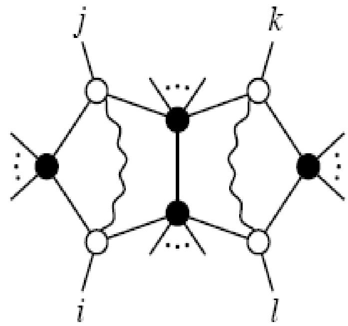
$$-\frac{1}{4}\begin{bmatrix} a & a+1 & b-1 & b \\ b+1 & b+2 & c-1 & c \end{bmatrix} + \frac{1}{4}\begin{bmatrix} a & a+1 \\ b-1 & b \end{bmatrix}\begin{bmatrix} b+1 & b+2 \\ c-1 & c \end{bmatrix} \tag{58}$$



$$-\frac{1}{4}\begin{bmatrix} a & a+1 & b-1 & b \\ c & c+1 & d-1 & d \end{bmatrix} + \frac{1}{4}\begin{bmatrix} a & a+1 \\ b-1 & b \end{bmatrix}\begin{bmatrix} c & c+1 \\ d-1 & d \end{bmatrix} \tag{59}$$

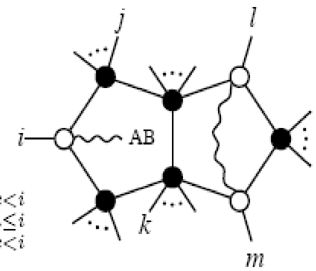
...+7 more pages...

$$\mathcal{A}^{2-\text{loop}}_{\text{MHV}} = \frac{1}{2}\sum_{i<j<k<l<i}$$

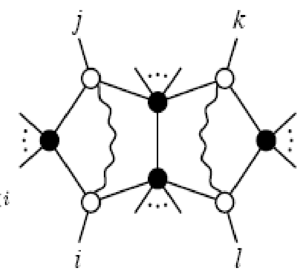


Momentum Twistor Integrals

$$\mathcal{A}^{2-\text{loop}}_{\text{NMHV}} = \sum_{\substack{i<j<l<m\leq k<i \\ i<j<k<l<m\leq i \\ i\leq l<m\leq j<k<i}}$$

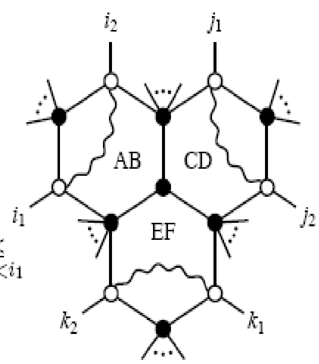


$$\times\, [i, j, j+1, k, k+1] \quad + \frac{1}{2}\sum_{i<j<k<l<i}$$

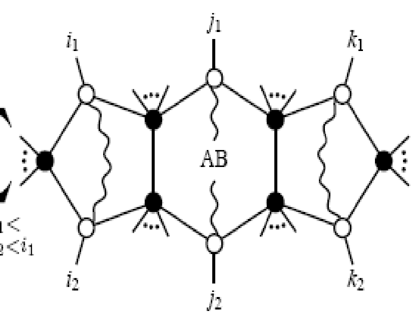


$$\times \left\{ \begin{array}{c} \mathcal{A}^{\text{tree}}_{\text{NMHV}}(j, \ldots, k;\ l, \ldots, i) \\ +\mathcal{A}^{\text{tree}}_{\text{NMHV}}(i, \ldots, j) \\ +\mathcal{A}^{\text{tree}}_{\text{NMHV}}(k, \ldots, l) \end{array} \right\}$$

$$\mathcal{A}^{3-\text{loop}}_{\text{MHV}} = \frac{1}{3}\sum_{\substack{i_1\leq i_2<j_1\leq \\ \leq j_2<k_1\leq k_2<i_1}}$$



$$+\, \frac{1}{2}\sum_{\substack{i_1\leq j_1<k_1< \\ <k_2\leq j_2<i_2<i_1}}$$



$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},\frac{1}{1-u_3},1;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},\frac{1}{1-u_3},\frac{1}{1-u_3};1\right)+$$
$$\frac{1}{2}\mathcal{G}\left(v_{123},0,1,\frac{1}{1-u_1};1\right)+\frac{1}{2}\mathcal{G}\left(v_{123},0,\frac{1}{1-u_1},1;1\right)+\frac{1}{2}\mathcal{G}\left(v_{123},1,0,\frac{1}{1-u_1};1\right)-$$
$$\frac{5}{4}\mathcal{G}\left(v_{123},1,1,\frac{1}{1-u_1};1\right)+\frac{1}{2}\mathcal{G}\left(v_{123},1,\frac{1}{1-u_1},0;1\right)-\frac{5}{4}\mathcal{G}\left(v_{123},1,\frac{1}{1-u_1},1;1\right)+$$
$$\frac{1}{2}\mathcal{G}\left(v_{123},1,\frac{1}{1-u_1},\frac{1}{1-u_1};1\right)+\frac{1}{2}\mathcal{G}\left(v_{123},\frac{1}{1-u_1},0,1;1\right)+\frac{1}{2}\mathcal{G}\left(v_{123},\frac{1}{1-u_1},1,0;1\right)-$$
$$\frac{5}{4}\mathcal{G}\left(v_{123},\frac{1}{1-u_1},1,1;1\right)+\frac{1}{2}\mathcal{G}\left(v_{123},\frac{1}{1-u_1},1,\frac{1}{1-u_1};1\right)+$$
$$\frac{1}{2}\mathcal{G}\left(v_{123},\frac{1}{1-u_1},\frac{1}{1-u_1},1;1\right)-\frac{1}{4}\mathcal{G}\left(v_{132},1,1,\frac{1}{1-u_1};1\right)-\frac{1}{4}\mathcal{G}\left(v_{132},1,\frac{1}{1-u_1},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(v_{132},\frac{1}{1-u_1},1,1;1\right)-\frac{1}{4}\mathcal{G}\left(v_{213},1,1,\frac{1}{1-u_2};1\right)-\frac{1}{4}\mathcal{G}\left(v_{213},1,\frac{1}{1-u_2},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(v_{213},\frac{1}{1-u_2},1,1;1\right)+\frac{1}{2}\mathcal{G}\left(v_{231},0,1,\frac{1}{1-u_2};1\right)+\frac{1}{2}\mathcal{G}\left(v_{231},0,\frac{1}{1-u_2},1;1\right)+$$
$$\frac{1}{2}\mathcal{G}\left(v_{231},1,0,\frac{1}{1-u_2};1\right)-\frac{5}{4}\mathcal{G}\left(v_{231},1,1,\frac{1}{1-u_2};1\right)+\frac{1}{2}\mathcal{G}\left(v_{231},1,\frac{1}{1-u_2},0;1\right)-$$
$$\frac{5}{4}\mathcal{G}\left(v_{231},1,\frac{1}{1-u_2},1;1\right)+\frac{1}{2}\mathcal{G}\left(v_{231},1,\frac{1}{1-u_2},\frac{1}{1-u_2};1\right)+\frac{1}{2}\mathcal{G}\left(v_{231},\frac{1}{1-u_2},0,1;1\right)+$$
$$\frac{1}{2}\mathcal{G}\left(v_{231},\frac{1}{1-u_2},1,0;1\right)-\frac{5}{4}\mathcal{G}\left(v_{231},\frac{1}{1-u_2},1,1;1\right)+\frac{1}{2}\mathcal{G}\left(v_{231},\frac{1}{1-u_2},1,\frac{1}{1-u_2};1\right)+$$
$$\frac{1}{2}\mathcal{G}\left(v_{231},\frac{1}{1-u_2},\frac{1}{1-u_2},1;1\right)+\frac{1}{2}\mathcal{G}\left(v_{312},0,1,\frac{1}{1-u_3};1\right)+\frac{1}{2}\mathcal{G}\left(v_{312},0,\frac{1}{1-u_3},1;1\right)+$$
$$\frac{1}{2}\mathcal{G}\left(v_{312},1,0,\frac{1}{1-u_3};1\right)-\frac{5}{4}\mathcal{G}\left(v_{312},1,1,\frac{1}{1-u_3};1\right)+\frac{1}{2}\mathcal{G}\left(v_{312},1,\frac{1}{1-u_3},0;1\right)-$$
$$\frac{5}{4}\mathcal{G}\left(v_{312},1,\frac{1}{1-u_3},1;1\right)+\frac{1}{2}\mathcal{G}\left(v_{312},1,\frac{1}{1-u_3},\frac{1}{1-u_3};1\right)+\frac{1}{2}\mathcal{G}\left(v_{312},\frac{1}{1-u_3},0,1;1\right)+$$
$$\frac{1}{2}\mathcal{G}\left(v_{312},\frac{1}{1-u_3},1,0;1\right)-\frac{5}{4}\mathcal{G}\left(v_{312},\frac{1}{1-u_3},1,1;1\right)+\frac{1}{2}\mathcal{G}\left(v_{312},\frac{1}{1-u_3},1,\frac{1}{1-u_3};1\right)+$$
$$\frac{1}{2}\mathcal{G}\left(v_{312},\frac{1}{1-u_3},\frac{1}{1-u_3},1;1\right)-\frac{1}{4}\mathcal{G}\left(v_{321},1,1,\frac{1}{1-u_3};1\right)-\frac{1}{4}\mathcal{G}\left(v_{321},1,\frac{1}{1-u_3},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(v_{321},\frac{1}{1-u_3},1,1;1\right)-\frac{3}{4}G\left(0,\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)H\left(0;u_1\right)-$$
$$\frac{3}{4}G\left(0,\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)H\left(0;u_1\right)-\frac{1}{4}G\left(0,\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)H\left(0;u_1\right)-$$
$$\frac{1}{4}G\left(0,\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)H\left(0;u_1\right)-\frac{1}{4}G\left(0,\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}G\left(0,\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2};1\right)H\left(0;u_1\right)-\frac{3}{4}G\left(\frac{1}{u_1},0,\frac{1}{u_1+u_2};1\right)H\left(0;u_1\right)-$$
$$\frac{3}{4}G\left(\frac{1}{u_1},0,\frac{1}{u_1+u_3};1\right)H\left(0;u_1\right)+\frac{1}{2}G\left(\frac{1}{u_1},\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{2}G\left(\frac{1}{u_1},\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)H\left(0;u_1\right)+\frac{1}{4}G\left(\frac{1}{u_1},\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}G\left(\frac{1}{u_1},\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)H\left(0;u_1\right)-\frac{1}{4}G\left(\frac{1}{1-u_2},1,\frac{1}{u_1};1\right)H\left(0;u_1\right)+$$

Stunning Simplification

$$R_6^{(2)}(u_1,u_2,u_3) = \sum_{i=1}^{3}\left(L_4(x_i^+,x_i^-) - \frac{1}{2}\mathrm{Li}_4(1-1/u_i)\right)$$
$$-\frac{1}{8}\left(\sum_{i=1}^{3}\mathrm{Li}_2(1-1/u_i)\right)^2 + \frac{1}{24}J^4 + \frac{\pi^2}{12}J^2 + \frac{\pi^4}{72}. \quad (3)$$

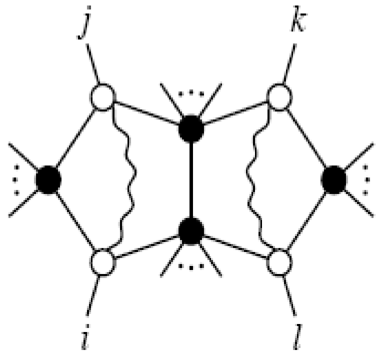
Goncharov
Spradlin
Vergu
Volovich

[Make use of "theory of motives"]

.

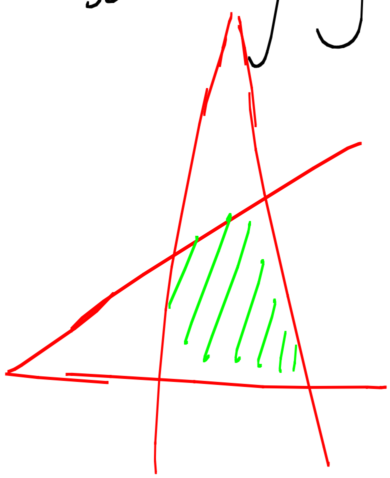
This Picture

$$\mathcal{A}_{\rm MHV}^{2-\rm loop} = \frac{1}{2}\sum_{i<j<k<l<i}$$

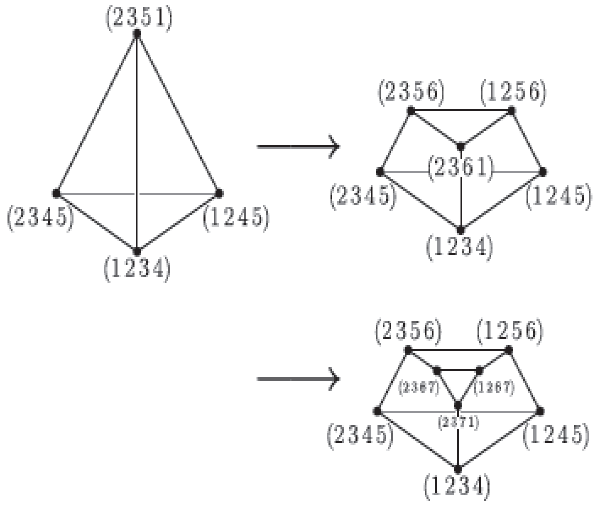


Is telling an algebraic -
geometric story - should allow us us to
just write down the answer.

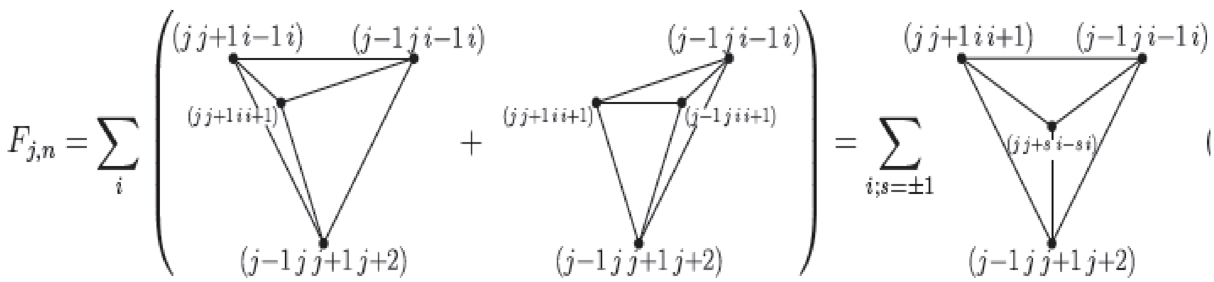
· In a specific sense, amplitudes are to be thought of as "the volume" of some polytope:



Different triangulations make different properties (Yangian, locality, Unitarity...) manifest.



Understood in Simple Cases
⋮

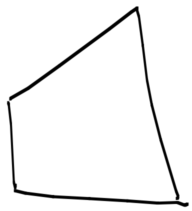
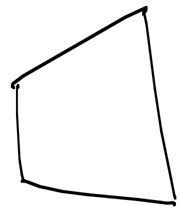


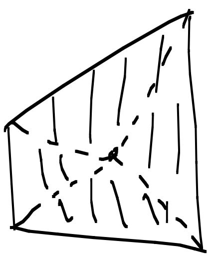
17

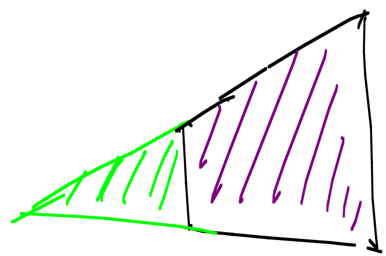
$$M_n^{\text{NMHV}} = \sum_{i,j;s=\pm1} \frac{\langle \eta_j, \{j-1\, j\, j+1\, j+2\, i\}, \{j-1\, j\, j+1\, i-s\, i\}, \{j\, j+s\, i-1\, i\, i+1\}\rangle}{\langle j-1\, j\, j+1\, j+2\rangle \langle j-1\, j\, i-1\, i\rangle, \langle j\, j+1\, i\, i+1\rangle \langle j\, j+s\, i-s\, i\rangle}$$

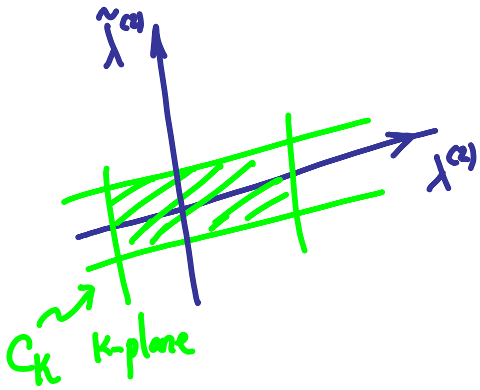
NEW LOCAL FORM!

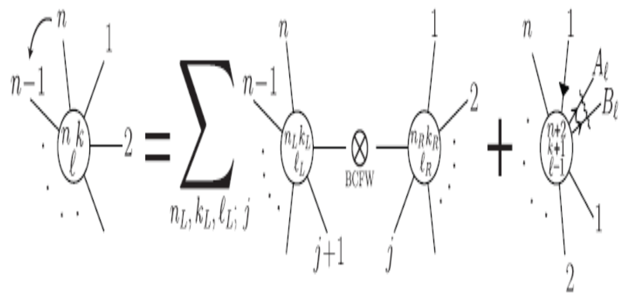
Another ex: MHV 1-loop:

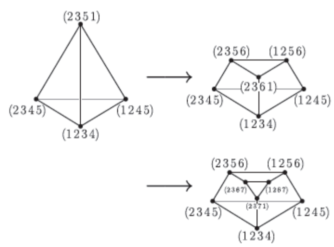
"Vol" [ $\otimes$ ]

 $\rightarrow$ Locality

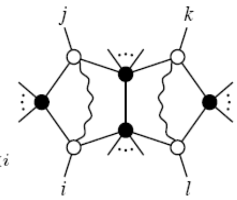
 $\rightarrow$ Unitarity

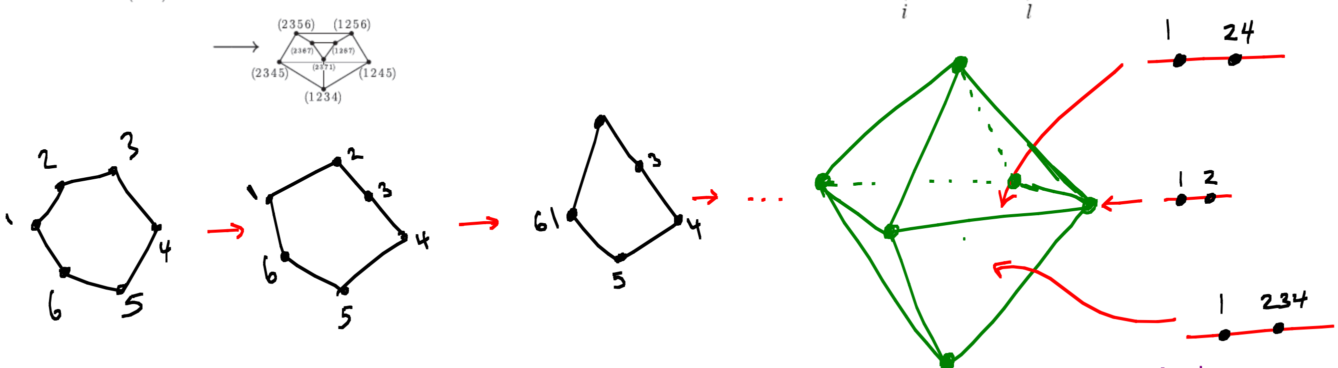






$$\mathcal{A}_{\mathrm{MHV}}^{2-\mathrm{loop}} = \frac{1}{2}\sum_{i<j<k<l<i}$$





PRIMITIVE COMBINATORIAL STRUCTURE UNDERLYING GLUONS

- Extensions to:
  * Massive theories
  * Less SUSY ($\to \mathcal{N}=0$, sorry!)
  * Lower + higher dimensions
  * Beyond planar limit

    ⋮

have been getting off the ground by a number of groups.

20'th Century Revolutions

Space-Time + Quantum Mechanics

Gravity

Intellectual Pinnacle

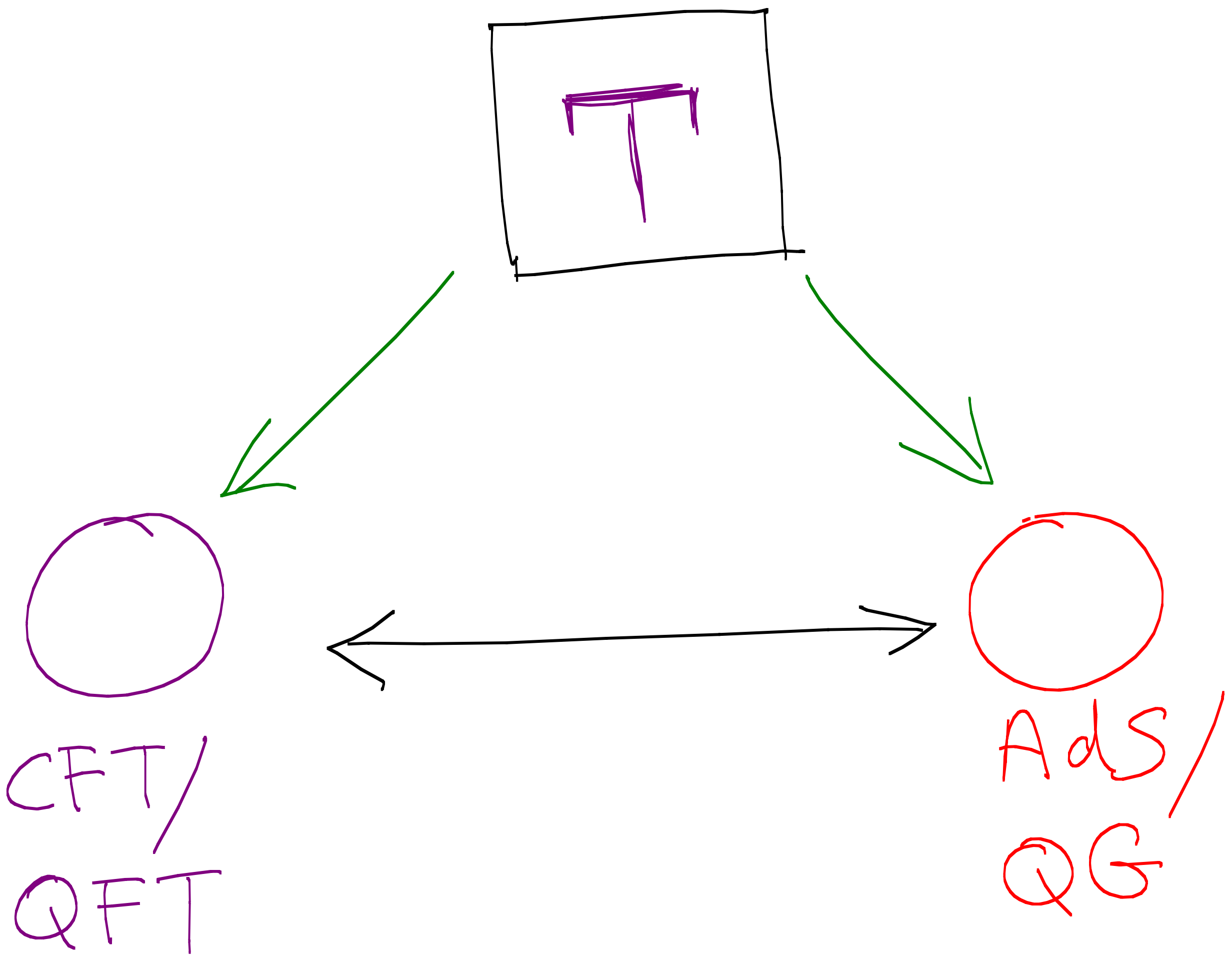
AdS = CFT

21st Century Revolutions

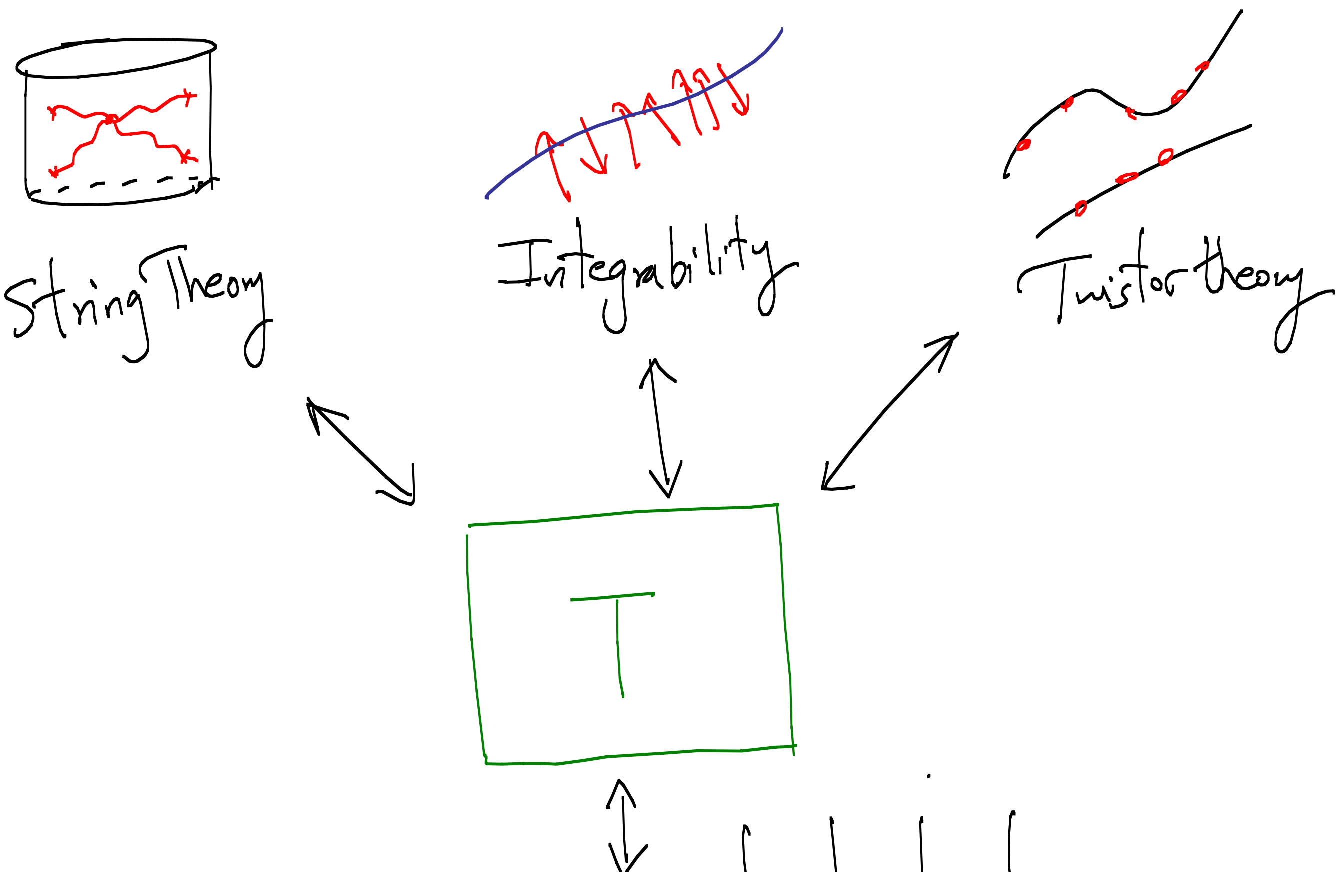
- Emergent space-time
- Deformation of QM? [Only in Cosmology...]

$\Updownarrow$

Must understand emergence of Locality + Unitarity from more primitive notions



T
CFT/
QFT
AdS/
QG



Amazing new mathematical structures:
(Grassmannians, Polylogs, Motivic Galois Theory)

. We are seeing, quite explicitly, primitive building blocks from which locality and Unitarity emerge.

This lets us understand physics invariantly, without the usual redundancies obstructing our view. In particular — hidden symmetries + dualities are being made manifest.

- We are in the middle of an extraordinary period in our understanding of QFT, with possibly deep reprecussions on our picture of Spacetime + QM.
- Grand synthesis yet to come!

An EXHILRATING

For Fundamental Physics —

ON ALL FRONTS !

[Bring on

5 fb^{-1} !]