Constraints on realistic Gauge-Higgs unified models

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SUSY11

Basic idea

- Identify the Higgs with one (or several) components of a higher-dimensional gauge boson [1]

$$A_M(x,y) \rightarrow \begin{array}{l} A_\mu(x) = \text{4D gauge bosons} \\ A_m(x) = \text{4D scalars} \end{array}$$

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- 5D models predict a very light Higgs [Hall, Nomura, Tucker-Smith (2002); Burdman, Nomura (2003); Haba, Shimizu (2003); Gogoladze, Mimura, Nandi (2003) ...]
- 6D models improve this prediction:

  presence of a quartic coupling at tree level [Csaba Csaki, Grojean, Murayama (2003); Scrucca, Serone, Silvestrini, Wulzer (2004); Gogoladze, Okada, Shafi (2008); ...]

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An example

Consider an SU(3) gauge theory compactified on $T^2/\mathcal{Z}_\mathcal{N}$ [2]

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$$\mathcal{L}_{6D} = -\frac{1}{4} F^{MN} F_{MN} \ , \quad M,\ N \ = \ 0,\ 1,\ 2,\ 3,\ 5,\ 6$$

$$F^{MN} = \partial_M A_N - \partial_N A_M - ig_6 [A_M, A_N]$$

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Take the case $\mathcal{N} = 2$:

Spacetime: $A^\mu \rightarrow A^\mu$, $A^m \rightarrow -A^m$, $m = 5, 6$.

Gauge: for the generators of SU(3), $t_a \rightarrow \Theta^{-1} t_a \Theta$

$$\Theta = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

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$$H_m = \sum_{a=4,5,6,7} A_m^{(a)} \frac{\lambda_a}{2}\ ;\ \ m = 5\,,6$$

An example.. continuation

which in matrix form become

$$A_\mu = \frac{1}{2}\begin{pmatrix} A_\mu^{(3)} + \frac{1}{\sqrt{3}}A_\mu^{(8)} & A_\mu^{(1)} - iA_\mu^{(2)} & 0 \\ A_\mu^{(1)} + iA_\mu^{(2)} & -A_\mu^{(3)} + \frac{1}{\sqrt{3}}A_\mu^{(8)} & 0 \\ 0 & 0 & -\frac{2}{\sqrt{3}}A_\mu^{(8)} \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} W_\mu^{(3)} + \frac{1}{\sqrt{3}}A_\mu^{(8)} & \sqrt{2}W_\mu^{+} & 0 \\ \sqrt{2}W_\mu^{-} & -W_\mu^{(3)} + \frac{1}{\sqrt{3}}A_\mu^{(8)} & 0 \\ 0 & 0 & -\frac{2}{\sqrt{3}}A_\mu^{(8)} \end{pmatrix}$$

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$$H_m = \frac{1}{2}\begin{pmatrix} 0 & 0 & A_m^{(4)} + iA_m^{(5)} \\ 0 & 0 & A_m^{(6)} + iA_m^{(7)} \\ A_m^{(4)} - iA_m^{(5)} & A_m^{(6)} - iA_m^{(7)} & 0 \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & H_m^+ \\ 0 & 0 & H_m^0 \\ H_m^- & H_m^{0*} & 0 \end{pmatrix}$$

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  [Haba, Shimizu (2003); Cacciapaglia, Csaki, Park (2006); AA, Diaz-Cruz (2006) ...]

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$$\to \frac{m_H}{m_W} = \frac{2\sqrt{2\lambda}}{g} = 2$$

Constraints ...

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- Is it possible to obtain a gauge-unification scenario that *automatically* gives [3]
  - $\rho = 1 \rightarrow$ contains SU(2) doublets?
  - Light gauge boson spectrum of the SM?
  - a correct value for $\cos^2\theta_W$?
  - the necessary hypercharge assignments for all matter fields?

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Assume: Γ has a translation subgroup Θ

$$\Theta = \{\{1|\sum k_i \mathbf{t_i}\},\ k_i \in \mathbb{Z}\}$$

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We require ($\forall g \in \Gamma$):

$$
\begin{aligned}
A_a^\mu(x, y') &= \mathbb{V}(g)_{ab} A_b^\mu(x, y) \\
\mathbf{A_a}(x, y') &= \mathbb{V}(g)_{ab} \mathcal{R}(g) \mathbf{A_b}(x, y)
\end{aligned}
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invariance of the F^2 term implies

$$\begin{aligned} r &= R(g);\ g = \{r|\mathbf{I}\} \\ 1 &= R(g)^T R(g) \\ 1 &= \mathbb{V}(g)^\dagger \mathbb{V}(g) \\ f_{a'b'c'} &= f_{abc} \mathbb{V}(g)_{aa'} \mathbb{V}(g)_{bb'} \mathbb{V}(g)^\dagger_{c'c} \end{aligned}$$

Consequences

... for the massless modes:

$$\mathbb{V}(g)_{ab} A^{\mu}_{b}(x,0) = A^{\mu}_{a}(x,0)$$

$$\mathbb{V}(g)_{ab} R(g) \mathbf{A_b}(x,0) = \mathbf{A_a}(x,0)$$

- Light gauge bosons $\leftrightarrow$ associated to trivial subspace where $\mathbb{V} = \mathbb{I}$
- 4D scalars $\leftrightarrow$ subspace where $\mathbb{V} \otimes \mathcal{R} = \mathbb{I}$

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  SU(2) generators: Specified by choosing a root α

$$J^0 = \frac{1}{|\alpha|}\hat{\alpha} \cdot \mathbf{C};\ J^+ = \frac{\sqrt{2}}{|\alpha|}E_\alpha;\ J^- = \frac{\sqrt{2}}{|\alpha|}E_{-\alpha}$$

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  Hypercharge:

$$Y = \mathcal{N}\hat{\mathbf{y}}\cdot\mathbf{C};\ \alpha\cdot\hat{\mathbf{y}} = 0$$

- **C** denote the Cartan generators
- E_β are root generators

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- **Only choices of G with $R = 0$ are phenomenologically viable!.**

Vectors and scalars

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Vector bosons

$$A_\mu = W_\mu^+ E_\alpha + W_\mu^- E_{-\alpha} + W_\mu^0 \hat{\alpha} \cdot \mathbf{C} + \sum_{r=0}^{R} B_\mu^{(r)} \hat{\mathbf{y}} \cdot \mathbf{C} + \ldots$$

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$$A_n = \sum_{\beta>0} \left(\phi_{n,\beta} E_\beta + \phi^*_{n,\beta} E_{-\beta}\right) + \dots$$

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$$\rightarrow \quad - \quad tr\,[A_\mu\,,\,A_n]^2 =$$

$$\sum_{\beta>0} \quad |\phi_{n\beta}|^2 \left\{ \frac{1}{2}\alpha^2 W^+ W^- + (W^0_\mu \hat{\alpha} \cdot \beta + \sum_{r=0}^{R} B^{(r)}_\mu \hat{\mathbf{y}} \cdot \beta)^2 \right\}$$

Massive gauge bosons

Scalars can get vevs if they do not break charge:

$$\langle \phi_{n\beta} \rangle \neq 0 \rightarrow [Q, E_\beta] = 0 \rightarrow \beta \cdot \left(\alpha + |\alpha|^2 \mathbf{y} \right) = 0$$

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$$\tan\theta_w = \frac{1}{|\alpha||\mathbf{y}|}$$

Hypercharge for quarks

Matter: There must be representations for which the standard Y assignments are realized.

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α^i simple roots, μ_k fundamental weights

$$\mu_k \cdot \alpha^i = \frac{1}{2}\delta_i^k |\alpha_i|^2 \rightarrow \mu_k = \sum_j \left(a^{-1}\right)_{jk} \alpha^j\,; \quad a^{ij} = 2\frac{\alpha^i \cdot \alpha^j}{\alpha^i \cdot \alpha^i}$$

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Thus, in order to accommodate quarks:

$$\alpha^i \cdot \mathbf{y} = \frac{N}{6 \times \text{integer}}$$

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- in addition, having acceptable values of $\cos\theta_W \rightarrow$ impossible to obtain

Possibilities

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group	s_w^2	α	y
$SU(3l)$	$3l/(6l-2)$	α^1	$\tilde{\mu}_2/2$
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G_2	$3/4$	α^1	$\tilde{\mu}_2/6$
F_4	$3/4$	α^1	$\tilde{\mu}_2/6$
E_6	$3/8$	$\alpha^{1,5}$	$\tilde{\mu}_{2,3}/2$
E_7	$3/4, 3/5$	$\alpha^{1,7}$	$\tilde{\mu}_{2,3}/6$
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