Grand Unification and Sfermion Mass Spectroscopy for the Light Generations

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Introduction: *The Idea of Grand Unification*

- The Standard Model of Strong and Electroweak interactions is described by the gauge group $G_{SM} = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$
- The main idea is to embed G_{SM} into a larger simple group
- We will consider $SU(5)$, $SO(10)$ and E_6
- As an example we will also look at the E_6SSM (David's Talk)

First and Second Generation Masses: *1-Loop RGEs*

Squark and Slepton Soft Masses RGE

$$16\pi^2 \frac{dm^2_{\tilde{Q}_L}}{dt} = -\frac{32}{3}g_3^2M_3^2 - 6g_2^2M_2^2 - \frac{2}{15}g_1^2M_1^2 + \frac{1}{5}g_1^2S$$
$$16\pi^2 \frac{dm^2_{\tilde{u}_R}}{dt} = -\frac{32}{3}g_3^2M_3^2 - \frac{32}{15}g_1^2M_1^2 - \frac{4}{5}g_1^2S$$
$$16\pi^2 \frac{dm^2_{\tilde{d}_R}}{dt} = -\frac{32}{3}g_3^2M_3^2 - \frac{8}{15}g_1^2M_1^2 + \frac{2}{5}g_1^2S$$
$$16\pi^2 \frac{dm^2_{\tilde{L}_L}}{dt} = -6g_2^2M_2^2 - \frac{6}{5}g_1^2M_1^2 - \frac{3}{5}g_1^2S$$
$$16\pi^2 \frac{dm^2_{\tilde{e}_R}}{dt} = -\frac{24}{5}g_1^2M_1^2 + \frac{6}{5}g_1^2S$$

- No Yukawa and trilinear couplings contribution for the light generations $\rightarrow$ **possible to solve analyticaly**
- $t \equiv \log(Q/Q_0)$, $M_{1,2,3}$ running gaugino masses and $g_{1,2,3}$ are de usual G_{SM} gauge couplings
- S is a D-term contribution
  - $S \equiv Tr(Ym^2) = m^2_{H_u} - m^2_{H_d} + \sum_{families}\left(m^2_{\tilde{Q}_L} - 2m^2_{\tilde{u}_R} + m^2_{\tilde{d}_R} - m^2_{\tilde{L}_L} + m^2_{\tilde{e}_R}\right)$
  - $\frac{dS}{dt} = \frac{66}{5}\frac{\alpha_1}{4\pi}S \Rightarrow S(t) = S(t_G)\frac{\alpha_1(t)}{\alpha_1(t_G)}$

[Ananthanarayan, Pandita, 2005]

Solution of the RGEs

Squark and Slepton Running Masses

$$
\begin{aligned}
m^2_{\tilde{u}_L}(t) &= m^2_{\tilde{Q}_L}(t_G) + C_3 + C_2 + \tfrac{1}{36}C_1 + \Delta_{u_L} - \tfrac{1}{5}K \\
m^2_{\tilde{d}_L}(t) &= m^2_{\tilde{Q}_L}(t_G) + C_3 + C_2 + \tfrac{1}{36}C_1 + \Delta_{d_L} - \tfrac{1}{5}K \\
m^2_{\tilde{u}_R}(t) &= m^2_{\tilde{u}_R}(t_G) + C_3 + \tfrac{4}{9}C_1 + \Delta_{u_R} + \tfrac{4}{5}K \\
m^2_{\tilde{d}_R}(t) &= m^2_{\tilde{d}_R}(t_G) + C_3 + \tfrac{1}{9}C_1 + \Delta_{d_R} - \tfrac{2}{5}K \\
m^2_{\tilde{e}_L}(t) &= m^2_{\tilde{L}_L}(t_G) + C_2 + \tfrac{1}{4}C_1 + \Delta_{e_L} + \tfrac{3}{5}K \\
m^2_{\tilde{\nu}_L}(t) &= m^2_{\tilde{L}_L}(t_G) + C_2 + \tfrac{1}{4}C_1 + \Delta_{\nu_L} + \tfrac{3}{5}K \\
m^2_{\tilde{e}_R}(t) &= m^2_{\tilde{e}_R}(t_G) + C_1 + \Delta_{e_R} - \tfrac{6}{5}K
\end{aligned}
$$

- $C_i(t) = M_i^2(t_G)\left[A_i \frac{\alpha_i^2(t_G) - \alpha_i^2(t)}{\alpha_i^2(t_G)}\right] = M_i^2(t_G)\bar{c}_i(t)$, $i = 1,2,3$ [Ananthanarayan, Pandita, 2007]
- $K(t) = \frac{1}{2b_1}S(t_G)\left(1 - \frac{\alpha_1(t)}{\alpha_1(t_G)}\right)$
- $\Delta_\phi = M_Z^2(T_{3\phi} - Q_\phi \sin^2\theta_W)\cos 2\beta$
  - $SU(2)_L \otimes U(1)_Y \to U(1)_{em}$ D-term

Universal Boundary Conditions

- Common scalar mass $m^2_{\tilde{Q}_L}(t_G) = m^2_{\tilde{u}_R}(t_G) = m^2_{\tilde{d}_R}(t_G) = m^2_{\tilde{L}_L}(t_G) = m^2_{\tilde{e}_R}(t_G) = m_0^2$
- $m^2_{H_u} = m^2_{H_d}$
- Common gaugino mass $M_1^2(t_G) = M_2^2(t_G) = M_3^2(t_G) = M^2_{1/2}$
- Since $S(t_G) = 0$, then $S(t)$ is identically 0 at all scales, hence $K = 0$
- We are left with three unknowns: m_0, $M_{1/2}$ and $\cos 2\beta$
    - **Can be determined by measuring three sfermion masses, eg. $\tilde{u}_L$, $\tilde{d}_L$ and $\tilde{e}_R$**

$$\begin{pmatrix} M^2_{\tilde{u}_L} \\ M^2_{\tilde{d}_L} \\ M^2_{\tilde{e}_R} \end{pmatrix} = \begin{pmatrix} 1 & c_{\tilde{u}_L} & \delta_{\tilde{u}_L} \\ 1 & c_{\tilde{d}_L} & \delta_{\tilde{d}_L} \\ 1 & c_{\tilde{e}_R} & \delta_{\tilde{e}_R} \end{pmatrix} \begin{pmatrix} m_0^2 \\ M^2_{1/2} \\ \cos 2\beta \end{pmatrix}$$

- $\Delta_\phi \equiv \delta_\phi \cos 2\beta$
- $c_{\tilde{u}_L} \equiv \bar{c}_3(M_{\tilde{u}_L}) + \bar{c}_2(M_{\tilde{u}_L}) + \frac{1}{36}\bar{c}_1(M_{\tilde{u}_L})$
- $c_{\tilde{d}_L} \equiv \bar{c}_3(M_{\tilde{d}_L}) + \bar{c}_2(M_{\tilde{d}_L}) + \frac{1}{36}\bar{c}_1(M_{\tilde{d}_L})$
- $c_{\tilde{u}_L} \equiv \bar{c}_1(M_{\tilde{e}_R})$

Once m_0, $M_{1/2}$ and $\cos 2\beta$ determined through $M_{\tilde{u}_L}$, $M_{\tilde{d}_L}$ and $M_{\tilde{e}_R}$, it is possible to obtain all the other low scale masses

$SU(5)$ Boundary Conditions

Common $m_{\mathbf{10}}$ for matter in a $\mathbf{10}$

$$m^2_{\tilde{Q}_L}(t_G) = m^2_{\tilde{u}_R}(t_G) = m^2_{\tilde{e}_R}(t_G) = m^2_{\mathbf{10}}$$

Common $m_{\bar{\mathbf{5}}}$ for matter in a $\bar{\mathbf{5}}$

$$m^2_{\tilde{L}_L}(t_G) = m^2_{\tilde{d}_R}(t_G) = m^2_{\bar{\mathbf{5}}}$$

Common gaugino mass $M_{1/2}$

$$M_1^2(t_G) = M_2^2(t_G) = M_3^2(t_G) = M^2_{1/2}$$

Higgs soft masses unrelated

$m^2_{H_u}(t_G) = m^2_{\mathbf{5}'}$ and $m^2_{H_d}(t_G) = m^2_{\bar{\mathbf{5}}'}$

- $S(t_G) = m^2_{\mathbf{5}'} - m^2_{\bar{\mathbf{5}}'} \Rightarrow K \neq 0$
- Five unkowns: $m_{\bar{\mathbf{5}}}$, $m_{\mathbf{10}}$, $M_{1/2}$, $\cos 2\beta$ and K
- **Can be determined by measuring five sfermion masses, eg. $\tilde{u}_L$, $\tilde{d}_L$, $\tilde{e}_R$, $\tilde{u}_R$ and $\tilde{d}_R$**

$$\begin{pmatrix} M^2_{\tilde{u}_L} \\ M^2_{\tilde{d}_L} \\ M^2_{\tilde{e}_R} \\ M^2_{\tilde{u}_R} \\ M^2_{\tilde{d}_R} \end{pmatrix} = \begin{pmatrix} 0 & 1 & c_{\tilde{u}_L} & \delta_{\tilde{u}_L} & -\frac{1}{5} \\ 0 & 1 & c_{\tilde{d}_L} & \delta_{\tilde{d}_L} & -\frac{1}{5} \\ 0 & 1 & c_{\tilde{e}_R} & \delta_{\tilde{e}_R} & -\frac{6}{5} \\ 0 & 1 & c_{\tilde{u}_R} & \delta_{\tilde{u}_R} & \frac{4}{5} \\ 1 & 0 & c_{\tilde{d}_R} & \delta_{\tilde{d}_R} & -\frac{2}{5} \end{pmatrix} \begin{pmatrix} m^2_{\bar{\mathbf{5}}} \\ m^2_{\mathbf{10}} \\ M^2_{1/2} \\ \cos 2\beta \\ K \end{pmatrix}$$

- $c_{\tilde{u}_R} \equiv \bar{c}_3(M_{\tilde{u}_R}) + \frac{4}{9}\bar{c}_1(M_{\tilde{u}_R})$
- $c_{\tilde{d}_R} \equiv \bar{c}_3(M_{\tilde{d}_R}) + \frac{1}{9}\bar{c}_1(M_{\tilde{d}_R})$

$SO(10)$ Boundary Conditions

- Breaking $SO(10) \to SU(5) \otimes U(1)_x \to G_{SM}$ the rank is reduced from 5 to 4
  - D-term contributions from the additional $U(1)_x$ broken at the high scale
  - Assuming a Higgs type mechanism: additional soft SUSY breaking terms $V_{soft} = m^2|\Phi|^2 + \overline{m}^2|\overline{\Phi}|^2$
  - $\Delta m_a^2 = \sum_I Q_{Ia} d_I$ with $d_I \propto (\overline{m}^2 - m^2)$ [Kolda, Martin, 1995]
  - D-term contribution of the order of $(m_{soft})^2$
- Consider that the Higgs are embedded in a **10** of $SO(10)$

Common sfermion mass $m_{\mathbf{16}}$

$$m^2_{\tilde{Q}_L}(t_G) = m^2_{\tilde{u}_R}(t_G) = m^2_{\tilde{e}_R}(t_G) = m^2_{\mathbf{16}} + g_{10}^2 D$$
$$m^2_{\tilde{L}_L}(t_G) = m^2_{\tilde{d}_R}(t_G) = m^2_{\mathbf{16}} - 3g_{10}^2 D$$

Common Higgs mass $m_{\mathbf{10}}$

$$m^2_{\tilde{H}_u}(t_G) = m^2_{\mathbf{10}} - 2g_{10}^2 D$$
$$m^2_{\tilde{H}_d}(t_G) = m^2_{\mathbf{10}} + 2g_{10}^2 D$$

- $S(t_G) = -4g_{10}^2 D$
- Five unknowns: $m_{\mathbf{16}}$, $g_{10}^2 D$, $M_{1/2}$, $\cos 2\beta$ and K
- **Can be determined by measuring five sfermion masses, eg. $\tilde{u}_L$, $\tilde{d}_L$, $\tilde{e}_R$, $\tilde{u}_R$ and $\tilde{d}_R$**

$$\begin{pmatrix} M^2_{\tilde{u}_L} \\ M^2_{\tilde{d}_L} \\ M^2_{\tilde{e}_R} \\ M^2_{\tilde{u}_R} \\ M^2_{\tilde{d}_R} \end{pmatrix} = \begin{pmatrix} 1 & 1 & c_{\tilde{u}_L} & \delta_{\tilde{u}_L} & -\frac{1}{5} \\ 1 & 1 & c_{\tilde{d}_L} & \delta_{\tilde{d}_L} & -\frac{1}{5} \\ 1 & 1 & c_{\tilde{e}_R} & \delta_{\tilde{e}_R} & -\frac{6}{5} \\ 1 & 1 & c_{\tilde{u}_R} & \delta_{\tilde{u}_R} & \frac{4}{5} \\ 1 & -3 & c_{\tilde{d}_R} & \delta_{\tilde{d}_R} & -\frac{2}{5} \end{pmatrix} \begin{pmatrix} m^2_{\mathbf{16}} \\ g^2_{10}D \\ M^2_{1/2} \\ \cos 2\beta \\ K \end{pmatrix}$$

- $K(t) = \frac{-4g^2_{10}D}{2b_1}\left(1 - \frac{\alpha_1(t)}{\alpha_1(t_G)}\right)$
- Masses are further constrained throught this relation

More explicitly and given that $X_5 = c_{\tilde{d}_L} - c_{\tilde{e}_R} + c_{\tilde{u}_L} - c_{\tilde{u}_R}$

$$\begin{aligned} K &= \frac{1}{6X_5(\sin^2\theta_W - 1)}\Big[3c_{\tilde{u}_R}(M^2_{\tilde{d}_L} - 2M^2_{\tilde{e}_R} + M^2_{\tilde{u}_L}) + 3(c_{\tilde{d}_L} + c_{\tilde{u}_L})\,(M^2_{\tilde{e}_R} - M^2_{\tilde{u}_R}) \\ &-3c_{\tilde{e}_R}(M^2_{\tilde{d}_L} + M^2_{\tilde{u}_L} - 2M^2_{\tilde{u}_R}) + 2\Big(c_{\tilde{u}_R}(M^2_{\tilde{d}_L} + 3M^2_{\tilde{e}_R} - 4M^2_{\tilde{u}_L}) - c_{\tilde{d}_L}(4M^2_{\tilde{e}_R} - 5M^2_{\tilde{u}_L} + M^2_{\tilde{u}_R}) \\ &+c_{\tilde{u}_L}(-5M^2_{\tilde{d}_L} + M^2_{\tilde{e}_R} + 4M^2_{\tilde{u}_R}) + c_{\tilde{e}_R}(4M^2_{\tilde{d}_L} - M^2_{\tilde{u}_L} - 3M^2_{\tilde{u}_R})\Big)\sin^2\theta_W\Big] \\ g^2_{10}D &= \frac{1}{20X_5}\Big[-c_{\tilde{u}_R}(2M^2_{\tilde{d}_L} - 5M^2_{\tilde{d}_R} + M^2_{\tilde{e}_R} + 2M^2_{\tilde{u}_L}) - c_{\tilde{e}_R}(-3M^2_{\tilde{d}_L} + 5M^2_{\tilde{d}_R} - 3M^2_{\tilde{u}_L} + M^2_{\tilde{u}_R}) \\ &+(c_{\tilde{d}_L} + c_{\tilde{u}_L})(5M^2_{\tilde{d}_R} - 3M^2_{\tilde{e}_R} - 2M^2_{\tilde{u}_R}) + 5c_{\tilde{d}_R}(M^2_{\tilde{d}_L} - M^2_{\tilde{e}_R} + M^2_{\tilde{u}_L} - M^2_{\tilde{u}_R})\Big] \end{aligned}$$

- This was obtained for a particular choice of the Higgs in a **10**-plet
- If Higgs in a **120**, **126** or combinations? Different constrains?

E_6 Boundary Conditions

- Consider the simple scenario of the direct breaking to the SM without extra matter
- Breaking $E_6 \to SO(10) \otimes U(1)_S \to SU(5) \otimes U(1)_S \otimes U(1)_X \to G_{SM}$ the rank is reduced from 6 to 4
- Two D-term contributions from the breaking of $U(1)_S$ and $U(1)_X$ at the high scale

Common scalar mass $m_{\mathbf{27}}$

$$m^2_{\tilde{Q}_L}(t_G) = m^2_{\tilde{u}_R}(t_G) = m^2_{\tilde{e}_R}(t_G) = m^2_{\mathbf{27}} - g_6^2 D_S + g_6^2 D_X$$
$$m^2_{\tilde{L}_L}(t_G) = m^2_{\tilde{d}_R}(t_G) = m^2_{\mathbf{27}} - g_6^2 D_S - 3g_6^2 D$$
$$m^2_{\tilde{H}_u}(t_G) = m^2_{\mathbf{27}} + 2g_6^2 D_S - 2g_6^2 D_X$$
$$m^2_{\tilde{H}_d}(t_G) = m^2_{\mathbf{27}} + 2g_6^2 D_S + 2g_6^2 D_X$$

- $S(t_G) = -4g_6^2 D_X$
- Six unknowns: $m_{\mathbf{27}}$, $g_6^2 D_S$, $g_6^2 D_X$, $M_{1/2}$, $\cos 2\beta$ and K
- The system is not invertible $\to$ reduced to the $SO(10)$ analysis.

E_6SSM First and Second Generation Sfermion Masses

[King, Moretti, Nevzorov, 2005 and 2007]

- Extended $G_{SM} \otimes U(1)_N$ at the low scale
- The extra $U(1)_N$ breaks close to the EW scale by the vev of an Higgs type singlet
- Extra H' and $\overline{H}'$ form incomplete $\mathbf{27}'$ and $\overline{\mathbf{27}}'$ (David's Talk)
- RGEs with an extra S' D-term contribution, additional fields contributing to the loops and a D-term from $U(1)_N$ breaking

Solution of the $E6SSM$ 1-Loop RGEs

$$m^2_{\tilde{u}_L}(t) = m^2_{\tilde{Q}_L}(t_G) + C_3^{E_6} + C_2^{E_6} + \frac{1}{36}C_1^{E_6} + \frac{1}{4}C_1' + \Delta_{u_L} - \frac{1}{5}K - \frac{1}{20}K' - g_1'^2 D$$
$$m^2_{\tilde{d}_L}(t) = m^2_{\tilde{Q}_L}(t_G) + C_3^{E_6} + C_2^{E_6} + \frac{1}{36}C_1^{E_6} + \frac{1}{4}C_1' + \Delta_{d_L} - \frac{1}{5}K - \frac{1}{20}K' - g_1'^2 D$$
$$m^2_{\tilde{u}_R}(t) = m^2_{\tilde{u}_R}(t_G) + C_3^{E_6} + \frac{4}{9}C_1^{E_6} + \frac{1}{4}C_1' + \Delta_{u_R} + \frac{4}{5}K - \frac{1}{20}K' - g_1'^2 D$$
$$m^2_{\tilde{d}_R}(t) = m^2_{\tilde{d}_R}(t_G) + C_3^{E_6} + \frac{1}{9}C_1^{E_6} + C_1' + \Delta_{d_R} - \frac{2}{5}K - \frac{1}{10}K' - 2g_1'^2 D$$
$$m^2_{\tilde{e}_L}(t) = m^2_{\tilde{L}_L}(t_G) + C_2^{E_6} + \frac{1}{4}C_1^{E_6}C_1' + \Delta_{e_L} + \frac{3}{5}K - \frac{1}{10}K' - 2g_1'^2 D$$
$$m^2_{\tilde{\nu}_L}(t) = m^2_{\tilde{L}_L}(t_G) + C_2^{E_6} + \frac{1}{4}C_1^{E_6}C_1' + \Delta_{\nu_L} + \frac{3}{5}K - \frac{1}{10}K' - 2g_1'^2 D$$
$$m^2_{\tilde{e}_R}(t) = m^2_{\tilde{e}_R}(t_G) + C_1^{E_6}C_1' + \Delta_{e_R} - \frac{6}{5}K - \frac{1}{20}K' - g_1'^2 D$$

- $C_i^{E_6}(t) = M_i^2(t_G)\left[A_i^{E_6}\frac{\alpha_i^2(t_G)-\alpha_i^2(t)}{\alpha_i^2(t_G)}\right] = M_i^2(t_G)\bar{c}_i^{E_6}(t)$
- $D_N = \frac{1}{20}K' + g_1'^2 D$

- Common scalar mass $m^2_{\tilde{Q}_L}(t_G) = m^2_{\tilde{u}_R}(t_G) = m^2_{\tilde{d}_R}(t_G) = m^2_{\tilde{L}_L}(t_G) = m^2_{\tilde{e}_R}(t_G) = m^2_{27}$
- Five unknowns: $m_{\mathbf{27}}$, D_N, $M_{1/2}$, $\cos 2\beta$ and K
- **Can be determined by measuring five sfermion masses, eg. $\tilde{u}_L$, $\tilde{d}_L$, $\tilde{e}_R$, $\tilde{u}_R$ and $\tilde{d}_R$**

$$\begin{pmatrix} M^2_{\tilde{u}_L} \\ M^2_{\tilde{d}_L} \\ M^2_{\tilde{e}_R} \\ M^2_{\tilde{u}_R} \\ M^2_{\tilde{d}_R} \end{pmatrix} = \begin{pmatrix} 1 & c_{\tilde{u}_L} & \delta_{\tilde{u}_L} & -\frac{1}{5} & -1 \\ 1 & c_{\tilde{d}_L} & \delta_{\tilde{d}_L} & -\frac{1}{5} & -1 \\ 1 & c_{\tilde{e}_R} & \delta_{\tilde{e}_R} & -\frac{6}{5} & -1 \\ 1 & c_{\tilde{u}_R} & \delta_{\tilde{u}_R} & \frac{4}{5} & -2 \\ 1 & c_{\tilde{d}_R} & \delta_{\tilde{d}_R} & -\frac{2}{5} & -1 \end{pmatrix} \begin{pmatrix} m^2_{\mathbf{27}} \\ M^2_{1/2} \\ \cos 2\beta \\ K \\ D_N \end{pmatrix}$$

- Note that $D = \left(Q_d^N v_d^2 + Q_u^N v_u^2 + Q_s^N s^2\right)$
- If able to measure v_d^2, v_u^2 and s^2 independently one can determine K'
  - $S(t_G) = -m^2_{H'} + m^2_{\overline{H}'}$
  - $S'(t_G) = 4m^2_{H'} - 4m^2_{\overline{H}'}$

Sum Rules

From the solution of the 1-loop RGEs, it is possible to obtain the following sum rules: [Ananthanarayan, Pandita, 2005 and 2007]

Sum rules for $SU(5)$, $SO(10)$ and E_6

$$M^2_{\tilde{u}_L} + M^2_{\tilde{d}_L} - M^2_{\tilde{u}_R} - M^2_{\tilde{e}_R} = C_3 + 2C_2 - \frac{25}{18}C_1 = 2.18207\ (GeV)^2$$

$$\frac{1}{2}\left(M^2_{\tilde{u}_L} + M^2_{\tilde{d}_L}\right) + M^2_{\tilde{d}_R} - M^2_{\tilde{e}_R} - \frac{1}{2}\left(M^2_{\tilde{e}_L} + M^2_{\tilde{\nu}_L}\right) = 2C_3 - \frac{10}{9}C_1 = -0.817037\ (GeV)^2$$

Sum rules for the E_6SSM

$$M^2_{\tilde{u}_L} + M^2_{\tilde{d}_L} - M^2_{\tilde{u}_R} - M^2_{\tilde{e}_R} = C_3^{E_6} + 2C_2^{E_6} - \frac{25}{18}C_1^{E_6} - \frac{3}{4}C_1' = 2.82233\ (GeV)^2$$

$$\frac{1}{2}\left(M^2_{\tilde{u}_L} + M^2_{\tilde{d}_L}\right) + M^2_{\tilde{d}_R} - M^2_{\tilde{e}_R} - \frac{1}{2}\left(M^2_{\tilde{e}_L} + M^2_{\tilde{\nu}_L}\right) = 2C_3^{E_6} - \frac{10}{9}C_1^{E_6} - \frac{3}{4}C_1' = 4.49462\ (GeV)^2$$

- Values for $Q = 500\ GeV$

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Summary and Conclusions

- Studied the 1-loop RGEs for the sfermion masses of the light generations for $SU(5)$, $SO(10)$ and E_6 boundary conditions
- For $SO(10)$ with Higgs in a 10-plet we get extra constrains on the low energy masses
- Obained sum rules for different GUT models
- **Parameters obtained from measurement of first and second generations low scale masses will be very useful for the study of the third generation**

BACKUP

- $SO(10) \otimes U(1)_S$ is a maximal subalgebra of E_6
- One can identify $m^2_{\mathbf{16}} = m^2_{\mathbf{27}} - g_6^2 D_S$ and $m^2_{\mathbf{10}} = m^2_{\mathbf{27}} + 2g_6^2 D_S$
- Since we only know $m^2_{\mathbf{16}}$ from $SO(10)$ calculations $\rightarrow$ not possible to determine $m^2_{\mathbf{27}}$ and $g_6^2 D_S$ alone
- Analysis reduced to the case of $SO(10)$
- Values for $Q = 500\ GeV$
  - $C_1 = 0.177807$, $C_2 = 1.36938$, $C_3 = -0.309737$
  - $C_1^{E_6} = 0.122243$, $C_2^{E_6} = 0.342345$, $C_3^{E_6} = 2.32302$, $C_1' = 0.0207902$