NNLL resummation for squark-antisquark production at the LHC

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in collaboration with

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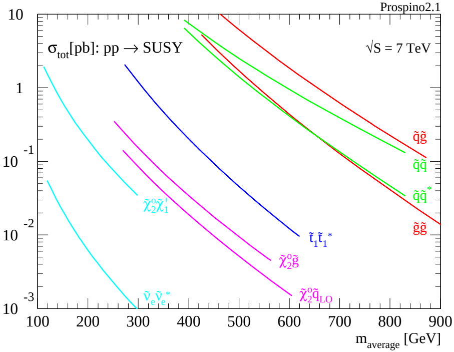
Outline

Production of SUSY particles at hadron colliders

Framework: MSSM with R-parity conservation

LHC:

[Beenakker,Höpker,Krämer,Plehn,Spira,Zerwas'96-'98]

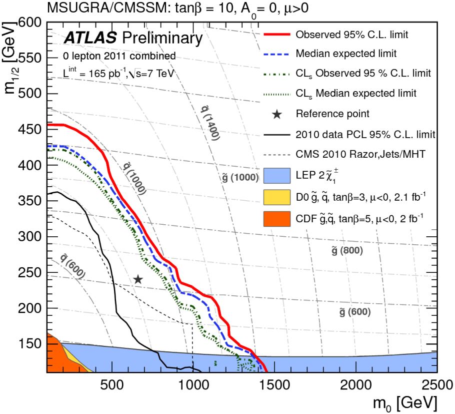


- dominated by processes involving coloured particles in the final state: $\tilde{q}\bar{\tilde{q}}, \tilde{q}\tilde{q}, \tilde{g}\tilde{g}, \tilde{q}\tilde{g}$ and $\tilde{t}\bar{\tilde{t}}$
- Squarks and gluinos are produced with high-production rates

→ offer strongest sensitivity for SUSY searches

Search for squarks and gluinos at the LHC

Total cross section are used to derive exclusion limits



Lower mass-bound for equal squark- and gluino-mass: $\approx$ 1000 TeV

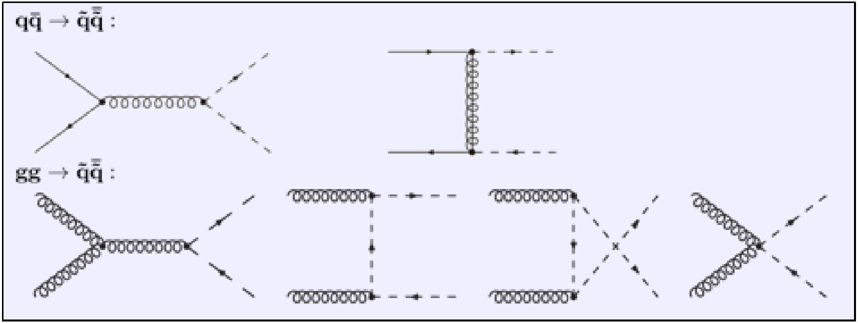
Precise theoretical prediction are necessary.

How NLL+NLO predictions serves to improve on results of SUSY searches
$\rightarrow$ see talk by Anna Kulesza, Tuesday, Parallel Session 5

Production of a squark-antisquark pair

Processes at LO:

Squark-antisquark:



Assume all squarks $\tilde{q} = (\tilde{q}_L, \tilde{q}_R)$ with $\tilde{q} \neq \tilde{t}$ mass degenerate

NLO SUSY-QCD calculation

NLO SUSY-QCD corrections [Beenakker et al. ´96]

- Large positive corrections, depending in detail on squark- and gluino mass
- Significant part can be attributed to the threshold region $\hat{s} \approx 4m^2$

NLO partonic cross section near threshold $\beta = \sqrt{1 - 4m^2/\hat{s}} \to 0$:

$$\hat{\sigma}^{(\mathrm{NLO})} = \hat{\sigma}^{(0)} \left[\alpha_s \{a \log^2(\beta^2) + b \log(\beta^2) + c \log(\beta^2) \log(\mu^2/m^2) + d\,(1/\beta)\}\right]$$

Soft-gluon corrections

Coulomb corrections

Generic form of higher-order corrections near threshold:

$$\hat{\sigma} = \hat{\sigma}^{(0)} \times \left[1 + \alpha_s(L^2 + L + \ldots) + \ldots + \alpha_s^n(L^{2n} + L^{2n-1} + \ldots)\right] \qquad L = \log(\beta^2)$$

- Logarithmic terms become large near threshold
- Spoil convergent behaviour of perturbative series in α_s

→ Requires all-order summation

→ Soft-gluon resummation

Theoretical Status: Squark-antisquark production

- NLO SUSY-QCD corrections [Beenakker et al.'96][Beenakker et al.'97]
- NLL-resummed corrections using Mellin space approach [Kulesza, Motyka '08,'09, Beenakker, SB, Krämer, Kulesza, Laenen, Niessen, '10]
- Resummation of Coulomb-corrections [Kulesza, Motyka '09]
- Combined (soft-gluon & Coulomb) NLL-resummed corrections using SCET [Beneke, Falgari, Schwinn '09]
- Approximate NNLO contributions [Langenfeld, Moch '09]

- NLO EW corrections [Hollik, Mirabella '08]
- LO EW and QCD-EW interference [Bozzi, Fuks, Klasen '05][Alan, Cankocak, Demir '07] [Bornhauser et al '07][Hollik, Mirabella '08]

Outline

Soft-gluon resummation

[Contopanagos, Kidonakis, Laenen, Oderda, Sterman, Bonciani, Catani, Mangano, Nason '96 – '98]

- Perform resummation of soft-gluon contributions using approach in Mellin-space

$$\sigma_{h_A h_B \to kl}\big(N, \{m^2\}\big) \equiv \int_0^1 d\rho\, \rho^{N-1}\, \sigma_{h_A h_B \to kl}\big(\rho, \{m^2\}\big)$$

- Hadronic cross section for the production of two massive coloured sparticles *k,l*

$$\sigma_{h_A h_B \to kl}\big(\rho, \{m^2\}\big) = \sum_{i,j} \int dx_1\, dx_2\, dz\, \delta\left(z - \frac{\rho}{x_1 x_2}\right) f_{i/h_A}(x_1, \mu^2)\, f_{j/h_B}(x_2, \mu^2)\, \hat{\sigma}_{ij \to kl}\big(z, \{m^2\}, \mu^2\big)$$

with $\rho = \frac{(m_k + m_l)^2}{S}$ and $z = \frac{(m_k + m_l)^2}{\hat{s}}$

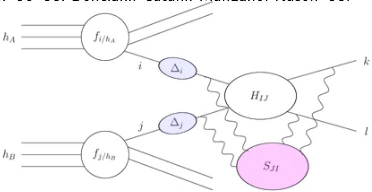
$$\rightarrow \quad \sigma_{h_A h_B \to kl}\big(N, \{m^2\}\big) = \sum_{i,j} f_{i/h_A}(N+1, \mu^2)\, f_{j/h_B}(N+1, \mu^2)\, \hat{\sigma}_{ij \to kl}\big(N, \{m^2\}, \mu^2\big)$$

- Form of soft-gluon corrections $\alpha_s^n \log^m(N)$ $\quad m \leq 2n \qquad N \to \infty$

- Resummation: $\hat{\sigma}^{(\mathrm{res})}_{ij \to kl}(N) = \exp\Big[\underbrace{L g_1(\alpha_s L)}_{\text{LL}} + \underbrace{g_2(\alpha_s L)}_{\text{NLL}} + \underbrace{\alpha_s\, g_3(\alpha_s L)}_{\text{NNLL}} + \ldots\Big] \times P(\alpha_s) \qquad L = \log(N)$

Soft-gluon resummation for coloured heavy (s)particles

[Contopanagos, Kidonakis, Laenen, Oderda, Sterman '96-'98; Bonciani, Catani, Mangano, Nason '98]



- Based on near threshold factorisation of the cross section

$$\hat{\sigma}_{ij\to kl}(N) = \underbrace{\Delta_i\,\Delta_j}_{\text{soft-collinear}} \sum_{IJ} H_{ij\to kl,JI}\,\underbrace{S_{ij\to kl,IJ}}_{\text{wide-angle soft}}$$

- Evolution equations
  e.g.:
$$\mu\frac{d}{d\mu}S_{JI} = -\Gamma^\dagger_{JK}\,S_{KI} - S_{JK}\,\Gamma_{KI}$$

- Solving evolution equations ⟶ resummed expressions

$$\tilde{\sigma}^{(\mathrm{res})}_{ij\to\tilde{q}\bar{\tilde{q}}}(N,\{m^2\},\mu^2) = \sum_I \tilde{\sigma}^{(0)}_{ij\to kl,I}(N,\{m^2\},\mu^2)\,\mathcal{C}_{ij\to kl,I}(\{m^2\},\mu^2)\,\tilde{\delta}^{(\mathrm{C})}_{ij\to kl,I}(N,\{m^2\},\mu^2)$$
$$\times\ \Delta_i(N+1,Q^2,\mu^2)\,\Delta_j(N+1,Q^2,\mu^2)\,\Delta^{(\mathrm{s})}_{ij\to kl,I}(N+1,Q^2,\mu^2)$$

- $\mathcal{C}_{ij\to kl,I}$: hard matching coefficients
- $\tilde{\delta}^{(\mathrm{C})}_{ij\to kl,I}$: Coulomb corrections, inclusion based on soft-Coulomb factorisation
  [Bonciani et al. '98, Beneke, Falgari, Schwinn '10]
- Δ_i,Δ_j: resums soft and collinear gluon radiation
- $\Delta^{(\mathrm{s})}_{ij\to kl,I}$: resums wide-angle soft gluon radiation

Soft-radiative factors at NNLL

Soft-radiative factors $\Delta_i \Delta_j \Delta^{(\mathrm{s})}_{ij\to\tilde{q}\bar{\tilde{q}},I}$:

$$\Delta_i(N, Q^2, \mu^2) = \int_0^1 \frac{z^{N-1}-1}{1-z} \int_{\mu^2}^{Q^2(1-z)^2} \frac{dq^2}{q^2} A_i(\alpha_s(q^2))$$

$$\Delta^{(\mathrm{s})}_{ij\to\tilde{q}\bar{\tilde{q}},I} = \int_0^1 \frac{z^{N-1}-1}{1-z} D_{ij\to\tilde{q}\bar{\tilde{q}},I}(\alpha_s((1-z)^2 Q^2))$$

$A_i, D_{ij\to\tilde{q}\bar{\tilde{q}},I}$: power series in α_s

In terms of g_n : ($LL(\mathrm{g}_1)$, NLL(g_2), NNLL(g_3))

$$\Delta_i \Delta_j \Delta^{(\mathrm{s})}_{ij\to\tilde{q}\bar{\tilde{q}},I} \;=\; \exp\left[L g_1(\alpha_\mathrm{s} L) + g_2(\alpha_\mathrm{s} L) + \alpha_\mathrm{s} g_3(\alpha_\mathrm{s} L) + \ldots \right]$$

For NNLL accuracy: $A^{(1)}, A^{(2)}, A^{(3)}$

[Kodaira, Trentadue '82, Catani, D'Emilio, Trentadue '88, Moch, Vermaseren, Vogt '05]

$D^{(1)}_I, D^{(2)}_I$

[Catani et. al '96, Kidonakis, Sterman '96, Czakon, Mitov, Sterman '09, Beneke, Falgari, Schwinn '09]

Hard matching coefficients

Hard matching coefficients at one-loop are required for NNLL resummation

$$\mathcal{C}_{ij\to\tilde{q}\bar{\tilde{q}},I} = 1 + \mathcal{C}^{(1)}_{ij\to\tilde{q}\bar{\tilde{q}},I} + \ldots = 1 + \frac{\tilde{\sigma}^{(1)}_I\big|_{\mathcal{O}(\beta)}}{\tilde{\sigma}^{(0),\mathrm{thr}}_I} + \ldots$$

$\tilde{\sigma}^{(1)}\big|_{\mathcal{O}(\beta)}$: Mellin transform of $\mathcal{O}(\beta)$ term of NLO cross section
(i.e. leading Coulomb correction ($\mathcal{O}(\beta^0)$) and $\log(N)$ terms omitted)

Calculated at one-loop for $q\bar{q}/gg \to \tilde{q}\bar{\tilde{q}}$ [Beenakker, SB, Krämer, Kulesza, Laenen, Niessen, in prep.]

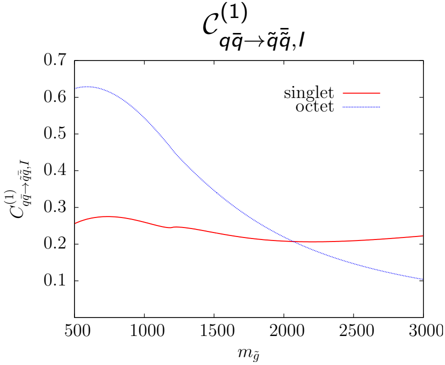
- can be expressed in a compact form
- e.g. for the gluon-gluon initiated process they read:

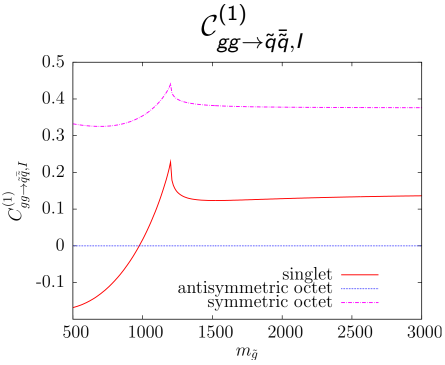
$$\begin{aligned}
\mathcal{C}^{(1)}_{gg\to\tilde{q}\bar{\tilde{q}},I=\mathbf{1},\mathbf{8_S}} = \frac{\alpha_s}{\pi}\mathrm{Re}\Bigg\{ & \pi^2\left(\frac{5N_c}{12} - \frac{C_F}{4}\right) + \gamma_g \log\left(\frac{\mu_R^2}{\mu_F^2}\right) - \frac{m_{\tilde{g}}^2 N_c}{2m_{\tilde{q}}^2}\log^2\left(x_{\tilde{g}\tilde{g}}(4m_{\tilde{q}}^2)\right) \\
& + C_F\left(\frac{m_+^2 m_-^2}{2m_{\tilde{q}}^4}\log\left(\frac{m_+^2}{m_-^2}\right) - \frac{m_{\tilde{g}}^2}{m_{\tilde{q}}^2} - 3\right) + \frac{m_+^2 N_c}{2m_{\tilde{q}}^2}\left(\mathrm{Li}_2\left(-\frac{m_{\tilde{q}}^2}{m_{\tilde{g}}^2}\right) - \mathrm{Li}_2\left(\frac{m_{\tilde{q}}^2}{m_{\tilde{g}}^2}\right)\right) \\
& + \left[\frac{\pi^2}{8} - \frac{1}{2}\mathrm{Li}_2\left(-\frac{m_{\tilde{q}}^2}{m_{\tilde{g}}^2}\right) + \frac{1}{2}\mathrm{Li}_2\left(\frac{m_{\tilde{q}}^2}{m_{\tilde{g}}^2}\right) + \frac{m_{\tilde{g}}^2}{4m_{\tilde{q}}^2}\log^2\left(x_{\tilde{g}\tilde{g}}(4m_{\tilde{q}}^2)\right) + 2\right] C_2(I)\Bigg\}
\end{aligned}$$

$$\mathcal{C}^{(1)}_{gg\to\tilde{q}\bar{\tilde{q}},\mathbf{8_A}} = 0$$

Hard matching coefficients

Numerical results for the hard matching coefficients $\mathcal{C}_I^{(1)}$:





$m_{\tilde{q}} = 1.2$ TeV fixed, $m_{\tilde{g}}$ varied, $m_t = 172.9$ GeV, $\mu_R = \mu_F = m_{\tilde{q}}$

Coulomb corrections

Exchange of Coulomb-gluons between the coloured final state particles

Leading Coloumb corrections have the form:

$$\sigma^{(\mathrm{C},1)}_{ij\to\tilde{q}\bar{\tilde{q}},I} = -\frac{\pi\alpha_{\mathrm{s}}}{2\beta}\kappa_{ij\to\tilde{q}\bar{\tilde{q}},I}\sigma^{(0)}_{ij\to\tilde{q}\bar{\tilde{q}},I}$$

with

$$\kappa_{q\bar{q}/gg\to\tilde{q}\bar{\tilde{q}},\mathbf{1}} = -\frac{4}{3}, \kappa_{q\bar{q}/(gg)\to\tilde{q}\bar{\tilde{q}},\mathbf{8},(\mathbf{8_A},\mathbf{8_S})} = \frac{1}{6}$$

see also [Kulesza, Motyka '10, Beneke, Falgari, Schwinn '10]

In order to include in Mellin-space resummation formula:

$$\tilde{\delta}^{(\mathrm{C})}_{ij\to\tilde{q}\bar{\tilde{q}},I} = 1 + \tilde{\delta}^{(\mathrm{C},1)}_{ij\to\tilde{q}\bar{\tilde{q}},I} + \cdots$$

Calculate Mellin-transform of $\sigma^{(\mathrm{C},1)}_{ij\to\tilde{q}\bar{\tilde{q}},I} \to \tilde{\delta}^{(\mathrm{C},1)}_{ij\to\tilde{q}\bar{\tilde{q}},I}$

Matching with fixed-order calculation

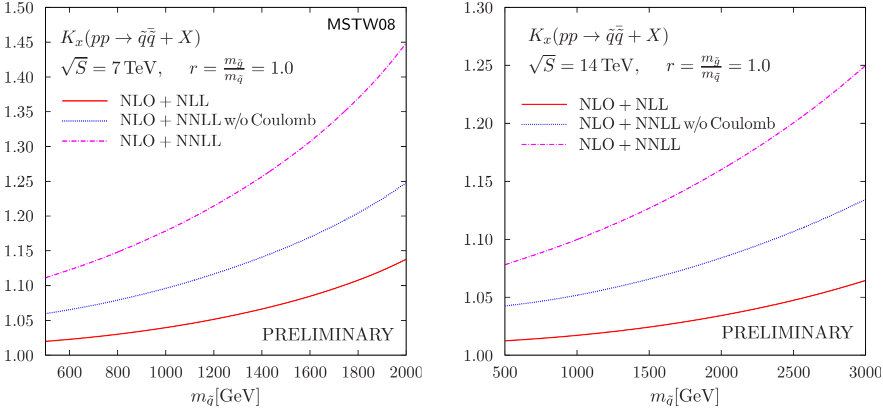
NLO (NNLO-calculation not available) and NNLL-resummed are combined through a matching procedure:

$$
\begin{aligned}
\sigma^{\mathrm{NLO+NNLL}}_{h_A h_B \to kl}(\rho, \{m^2\}, \mu^2) \quad &= \quad \sum_{i,j} \int_{C_{MP}-i\infty}^{C_{MP}+i\infty} \frac{dN}{2\pi i}\, \rho^{-N}\, f_{i/h_A}(N+1, \mu^2)\, f_{j/h_B}(N+1, \mu^2) \\
&\times \quad \left[\hat{\sigma}^{\mathrm{res,NNLL}}_{ij\to kl}(N, \{m^2\}, \mu^2) \,-\, \hat{\sigma}^{\mathrm{res,NNLL}}_{ij\to kl}(N, \{m^2\}, \mu^2)\Big|_{(\mathrm{NLO})} \right] \\
&+ \quad \sigma^{\mathrm{NLO}}_{h_A h_B \to kl}(\rho, \{m^2\}, \mu^2)
\end{aligned}
$$

- Avoids double counting of logarithmic terms
- Using “minimal prescription” for the contour of the inverse Mellin transform [Catani et al., ’96]
- NLO cross section calculated with PROSPINO [Beenakker, Höpker, Krämer, Plehn, Spira, Zerwas, ’96-’98]

$K_{\mathrm{NNLL}} = \sigma_{\mathrm{NLO+NNLL}}/\sigma_{\mathrm{NLO}}$ at the LHC

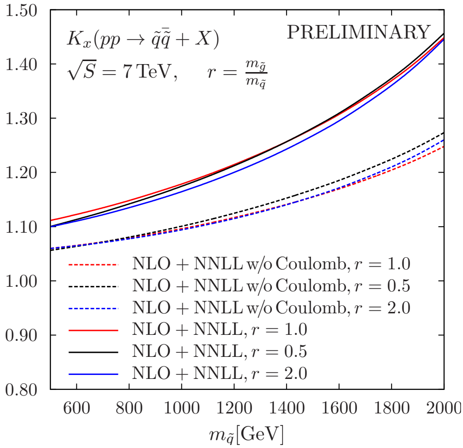
[Beenakker, SB, Krämer, Kulesza, Laenen, Niessen, in prep.]



- K_{NNLL}-factors grow with increasing sparticle mass due to importance of threshold region
- large corrections beyond NLL
  - can be mostly attributed to incorporating hard matching coefficients and leading Coulomb corrections

Mass dependence of K_{NNLL}

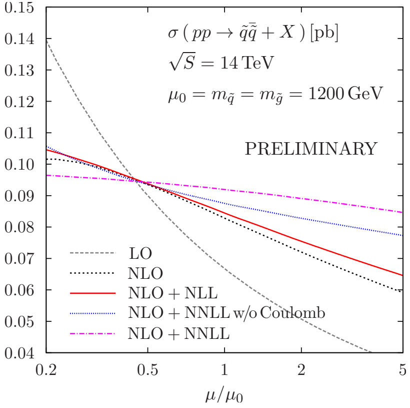
[Beenakker, SB, Krämer, Kulesza, Laenen, Niessen, in prep.]

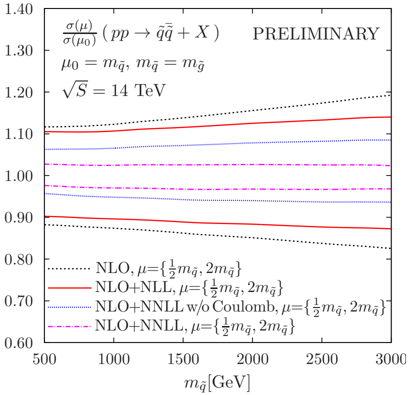


- small r-dependence, also for LHC@14TeV

Scale variation

[Beenakker, SB, Krämer, Kulesza, Laenen, Niessen, in prep.]





- significant reduction of theoretical error due to scale variation
- similar results for LHC@7TeV

Outline

Summary

- Squark-antisquark production is an important channel for sparticle production at the LHC
- Soft-gluon resummation to NNLL accuracy: (non-trivial colour structure)
  - First calculation of hard machting coefficients
    $\rightarrow$ needed to perform resummation to NNLL accuracy
  - Numerical results for NNLL resummed matched to NLO
    $\rightarrow$ significant enhancement of NLO+NLL cross section predictions
    $K_{\mathrm{NNLL}} \sim 25\%(m_{\tilde{q}} \sim 1.5\mathrm{TeV}$, LHC@7TeV)
    $\rightarrow$ significant reduction of scale dependence