The Lightest Scalar in Theories with Broken Supersymmetry

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L. Brizi and C. A. Scrucca

Overview

- Subject
  - General criteria for the existence of realistic metastable vacua in generic non-linear σ-models with local gauge invariance;

  - We define an absolute upper bound on the mass of the lightest scalar which depends only on the geometrical properties of the Kähler manifold;
  - The bound can be saturated by tuning the superpotential.
- Motivations

  Useful in string-inspired supergravity models to discriminate among different compactification scenarios;
- Results
  - The bound is defined by looking at all the directions in the scalar field space which do not admit arbitrarily large SUSY masses:

    the Goldstino & the Goldstone directions

  - The largest value for the mass of the lightest scalar (**LS**) is achieved when **LS** is a combination of those two directions.

The General Idea

M. Gomez-Reino, C. Scrucca

- Consider models with a given Kähler potential K (e.g. derived from string compactification) and unspecified superpotential W;
- Realistic models must admit metastable vacua and avoid light scalars with masses $\lesssim 1$ TeV ;
- NOT ALL the scalar masses can be made positive and arbitrarily large by tuning W ! Some masses are constrained by symmetries:
  - The scalar partners of the Goldstino have masses which are entirely controlled by splitting effects due to SUSY breaking;
  - The scalar partners of the would-be Goldstones have the same masses as the gauge bosons in the supersymmetric limit;
- It is not necessary to study the whole mass matrix! The relevant informations to determine the mass of the lightest scalar are contained in some smaller sub-matrices.

Gauged non-linear σ-Model

Most general non-renormalizable model with chiral multiplets Φ^i and vector multiplets V^a:

$$\mathcal{L} = \int d^4\theta\, K(\Phi, \bar{\Phi}, V) + \int d^2\theta \left[W(\Phi) + \frac{1}{4} h_{ab}\, W^{a\alpha} W^b_\alpha \right] + \text{h.c.}\,.$$

The gauge transformations are generated by the holomorphic Killing vectors $X^i_a(\Phi)$ associated to isometries of the Kähler metric $g_{i\bar{\jmath}} = K_{i\bar{\jmath}}$;

The vacuum energy is given by

$$V = g_{i\bar{\jmath}}\, F^i \bar{F}^{\bar{\jmath}} + \frac{1}{2} h_{ab}\, D^a D^b$$

where the expressions for the auxiliary fields at the vacuum are:

$$F^i = -\, g^{i\bar{\jmath}}\, \bar{W}_{\bar{\jmath}}\,, \qquad D^a = -\frac{1}{2}\, h^{ab}\, K_a$$

The stationarity conditions $V_i = 0$ imply that F and D -type SUSY breaking are not independent:

$$\nabla_i W_j\, F^j + ig\, \bar{X}_{ai}\, D^a = 0\,.$$

Gauged non-linear σ-Model

The vector masses are:

$$M_a^2 = 2\,g^2 g_{i\bar{\jmath}}\, X_a^i \bar{X}_a^{\bar{\jmath}}\,,$$

The scalar masses are:

$$m_{i\bar{\jmath}}^2 \;=\; g^{k\bar{l}} \nabla_i W_k \nabla_{\bar{\jmath}} \bar{W}_{\bar{l}} - R_{i\bar{\jmath}k\bar{l}}\, F^k \bar{F}^{\bar{l}} + g^2 h^{ab} \bar{X}_{ai} X_{b\bar{\jmath}} + g\, Q_{ai\bar{\jmath}}\, D^a\,,$$
$$m_{ij}^2 = -\nabla_i \nabla_j W_K\, F^K - g^2 h^{ab} \bar{X}_{ai} \bar{X}_{bj}\,,$$

$R_{i\bar{\jmath}k\bar{l}} = K_{i\bar{\jmath}k\bar{l}} - g^{p\bar{q}} K_{ik\bar{q}} K_{\bar{\jmath}\bar{l}p}$ is the Riemann tensor and $Q_{ai\bar{\jmath}} = i\,\nabla_i\, X_{a\bar{\jmath}}$.

Gauge symmetries are spontaneously broken whenever $M_a^2 \neq 0$;

Each Goldstone multiplet $\boldsymbol{\Phi^i \bar{X}_{ai}}$ contains 2 real scalars:

$$\sigma_a \propto \mathrm{Re}\, \bar{X}_{ai}\phi^i, \qquad \rho_a \propto \mathrm{Im}\, \bar{X}_{ai}\phi^i$$

σ_a are unphysical would-be Goldstones and can be gauged away by choosing the unitary gauge; ρ_a are physical scalars with masses:

$$m_{\rho_a}^2 = M_a^2 + \text{SUSY breaking}$$

Structure of the Scalar Mass Matrix

The quadratic Lagrangian for the scalar fields can be written in the following form:

$$\mathcal{L} = \frac{1}{2} g_{I\bar{J}}\, \partial_\mu \phi^I \partial^\mu \phi^{\bar{J}} - \frac{1}{2} m^2_{I\bar{J}}\, \phi^I \bar{\phi}^{\bar{J}} ,$$

where $\Phi^I = (\, \phi^i, \bar{\phi}^{\bar{\imath}} \,)$ and the wave-function and the mass matrices:

$$g_{I\bar{J}} = \begin{pmatrix} g_{i\bar{\jmath}} & 0 \\ 0 & g_{\bar{\imath}j} \end{pmatrix} , \quad m^2_{I\bar{J}} = \begin{pmatrix} m^2_{i\bar{\jmath}} & m^2_{ij} \\ m^2_{\bar{\imath}\bar{\jmath}} & m^2_{\bar{\imath}j} \end{pmatrix}$$

Physical masses are obtained by performing a local field redefinition to diagonalize the kinetic terms; equivalently one can consider the matrix $m^2_{I\bar{J}}$ in a new basis of vectors $u^I{}_{I'}$ orthonormal w.r.t. the metric $g_{I\bar{J}}$.

The matrix elements in the new basis are:

$$m^2_{I'J'} = u^I{}_{I'}\, m^2_{IJ}\, u^J{}_{J'} \qquad \text{with} \qquad g_{IJ}\, u^I{}_{I'} u^J{}_{J'} = \delta_{I'J'}$$

The eigenvalues of $m^2_{I'J'}$ give the physical masses of the scalar excitations

The Strategy to Define the Absolute Upper Bound

- Use some standard results in linear algebra. Let us call m^2_{min} the lightest eigenvalue of the full mass matrix $m^2_{I\bar{J}}$, then:

the value of $m^2_{I\bar{J}}$ along any particular direction must be larger than m^2_{min};
the eigenvalues of any sub-block of $m^2_{I\bar{J}}$ must be larger than m^2_{min}.

- We can find an upper bound to m^2_{min} by computing the smallest eigenvalue of any principal sub-matrix of $m^2_{I\bar{J}}$;
- The quality of the bound improves by considering larger and larger sub-matrices;
- There exists a limiting situation in which the bound obtained from a sub-matrix saturates m^2_{min}: when the complementary diagonal block has very large eigenvalues w.r.t. the off-diagonal block.
- To detect the obstructions against making m^2_{min} large it is enough to study the mass matrix along those directions where its values cannot be made arbitrarily large by adjusting the superpotential.

The Relevant Sub-Matrix

The most relevant directions to consider are the supersymmetry and gauge symmetry breaking directions:

$$f^i = \frac{F^i}{\sqrt{F^k \bar{F}_k}} = \frac{F^i}{F} \qquad x_a^i = \frac{X_a^i}{\sqrt{X_a^k \bar{X}_{ak}}} = \sqrt{2} g \frac{X_a^i}{M_a}$$

These vectors can be used to construct the following orthonormal set:

$$f_\uparrow^I = \frac{1}{\sqrt{2}} \left(f^i, 0 \right), \qquad f_\downarrow^I = \frac{1}{\sqrt{2}} \left(0, \bar{f}^{\bar{i}} \right), \qquad x_a^I = \frac{1}{\sqrt{2}} \left(i\, x_a^i, -i\, \bar{x}_a^{\bar{i}} \right).$$

The sub-matrix of dimension $\#$ of gauge vectors $+ 2$ associated to the subspace spanned by these directions is:

$$m^2_{\alpha\bar{\beta}} = \begin{pmatrix} m^2_{f\bar{f}} & \Delta & -\sqrt{2} i\, m^{2*}_{f\bar{x}_b} \\ \Delta^* & m^2_{f\bar{f}} & \sqrt{2} i\, m^2_{f\bar{x}_b} \\ \sqrt{2} i\, m^2_{f\bar{x}_a} & -\sqrt{2} i\, m^{2*}_{f\bar{x}_a} & 2\, m^2_{x_a\bar{x}_b} \end{pmatrix},$$

where

$$m^2_{f\bar{f}} = m^2_{i\bar{j}} f^i \bar{f}^{\bar{j}}, \quad m^2_{f\bar{x}_b} = m^2_{i\bar{j}} f^i \bar{x}_b^{\bar{j}}, \quad m^2_{x_a\bar{x}_b} = m^2_{i\bar{j}} x_a^i \bar{x}_b^{\bar{j}},$$

$$\Delta = m^2_{ij} f^i f^j.$$

The Relevant Sub-Matrix

The general expressions for $m^2_{f\bar{f}}$, $m^2_{x_a\bar{x}_b}$ and $m^2_{f\bar{x}_b}$ can be simplified by using stationarity and gauge invariance conditions. One obtains:

$$m^2_{f\bar{f}} = -\left[R_{f\bar{f}f\bar{f}} - 4g^2 \sum_c \frac{Q_{cf\bar{f}}\, Q_{cf\bar{f}}}{M_c^2}\right] |F|^2 \,,$$

$$m^2_{x_a\bar{x}_b} = \frac{1}{2} M^2_{ab} - \left[R_{f\bar{f}x_a\bar{x}_b} - 2g^2 \sum_c \frac{Q_{cf\bar{f}}\, Q_{c\,x_a\bar{x}_b}}{M_c^2} - 2g^2 \frac{(Q_a \cdot Q_b)_{f\bar{f}}}{M_a M_b}\right] |F|^2 \,,$$

$$m^2_{f\bar{x}_b} = -\left[R_{f\bar{f}f\bar{x}_b} - 4g^2 \sum_c \frac{Q_{cf\bar{f}}\, Q_{cf\bar{x}_b}}{M_c^2}\right] |F|^2 .$$

Important: the dependence on the second derivatives of the superpotential W'' can be completely eliminated!

The entry Δ has a complicated expression;

Important: the dependence on the third derivatives of the super potential W''' cannot be eliminated:

$$\Delta = \text{generic complex number that can be adjusted by tuning } W''' \,.$$

We may now ask what is the upper bound when $m^2_{f\bar{f}}$, $m^2_{x_a\bar{x}_b}$ and $m^2_{f\bar{x}_b}$ are held fixed and Δ is freely varied. Complicated for large # of symmetries

- When there are only chiral multiplets the Goldstino is the only dangerous direction!
- The average mass of the 2 sGoldstini is controlled by the sectional curvature of the Kähler manifold in the SUSY breaking direction:

$$R = -R_{f\bar{f}f\bar{f}} = -R_{i\bar{\jmath}k\bar{l}}\, F^i \bar{F}^{\bar{\jmath}} F^k \bar{F}^{\bar{l}}$$

- Demanding $R > 0$ is a necessary and sufficient condition for the existence of a metastable vacuum;
- The largest value for the mass of the lightest scalar is obtained by orienting F^i along the direction of largest positive sectional curvature R_{max};
- The remaining scalars can obtain arbitrarily large masses by tuning the second derivatives of the superpotential W''
- This holds true also in the presence of heavy gauge vectors, with:

$$R \rightarrow \tilde{R} = -R_{f\bar{f}f\bar{f}} + \mathcal{O}(g^2)\,.$$

Bound on the Mass of the Lightest Scalar

More in general also the gauge symmetry breaking directions must be considered! We restrict to the simplest case of a single $U(1)$ symmetry.

Three possible upper bounds can be defined by considering sub-matrices of dimension 1, 2 and 3.

- Dimension 1:

$$m^2_{\min} \leq m^2_{(1)} \,, \qquad m^2_{(1)} = \min\{m^2_{f\bar{f}}, 2\, m^2_{x\bar{x}}\} \,.$$

- Dimension 2:

$$\begin{pmatrix} m^2_{f\bar{f}} & \sqrt{2} i\, m^2_{f\bar{x}} \\ -\sqrt{2} i\, m^{2*}_{f\bar{x}} & 2\, m^2_{x\bar{x}} \end{pmatrix} .$$

  This sub-block defines a bound which is stronger (or equal) than the previous one! It takes into account the level repulsion effects induced by off-diagonal elements $m^2_{f\bar{x}}$

$$m^2_{\min} \leq m^2_{(2)} \,, \qquad m^2_{(2)} = \frac{1}{2}\left(m^2_{f\bar{f}} + 2\, m^2_{x\bar{x}}\right) - \frac{1}{2}\sqrt{\left(m^2_{f\bar{f}} - 2\, m^2_{x\bar{x}}\right)^2 + 8\, |m^2_{f\bar{x}}|^2} \,.$$

Bound on the Mass of the Lightest Scalar

- Dimension 3:

$$\begin{pmatrix} m^2_{f\bar{f}} & \Delta & -\sqrt{2}i\, m^{2*}_{f\bar{x}} \\ \Delta^* & m^2_{f\bar{f}} & \sqrt{2}i\, m^2_{f\bar{x}} \\ \sqrt{2}i\, m^2_{f\bar{x}} & -\sqrt{2}i\, m^{2*}_{f\bar{x}} & 2\, m^2_{x\bar{x}} \end{pmatrix} .$$

For generic Δ, the eigenvalues are quite complicated (roots of a cubic characteristic polynomial)!

However, one can verify that the optimal value of the phase of Δ is:

$$\arg \Delta = 2 \arg m^2_{f\bar{x}} + \pi .$$

With this choice the characteristic polynomial simplifies and the 3 eigenvalues can be found analytically.

The optimal value of $|\Delta|$ is:

$$|\Delta| = \frac{1}{2}\left(m^2_{f\bar{f}} - 2\, m^2_{x\bar{x}}\right) + \frac{1}{2}\sqrt{\left(m^2_{f\bar{f}} - 2\, m^2_{x\bar{x}}\right)^2 + 8\left|m^2_{f\bar{x}}\right|^2} ,$$

The lightest (degenerate) eigenvalue is found to be identical to $m^2_{(2)}$

The 3-dimensional block does not give any new bound! $m^2_{(3)} = m^2_{(2)}$

Bound on the Mass of the Lightest Scalar

- These bounds hold for a fixed Kahler geometry at a fixed vacuum.
- They depend on the direction f^i and on the vacuum coordinates ϕ_0^i, which determine the direction x^i and the values of $R_{i\bar{j}k\bar{l}}$ and $Q_{i\bar{j}}$.
- One can then optimize the superpotential W to maximize the lightest mass. The strongest version of the bound is:

$$m_{\min}^2 \leq \max\left\{ \frac{1}{2}\left(m_{f\bar{f}}^2 + 2\, m_{x\bar{x}}^2\right) - \frac{1}{2}\sqrt{\left(m_{f\bar{f}}^2 - 2\, m_{x\bar{x}}^2\right)^2 + 8\,\left|m_{f\bar{x}}^2\right|^2} \right\}.$$

In practice one can tune

> the $(n-1)$ independent first deriv. W' to adjust f^i ;

> $(n-1)$ second deriv. W'' to adjust the values of $n-1$ of the ϕ_0^i compatibly with stationary conditions in the non-Goldstone directions;

> 1 third deriv. W''' to adjust Δ to its optimal value.

The stationarity condition in the Goldstone direction: $Q_{ai\bar{j}}F^i\bar{F}^{\bar{j}} - \frac{1}{2}\, g^{-1} M_a^2 \delta_b^a\, D^b = 0$ is satisfied by tuning the gauge coupling g to achieve the desired values of $|F|$ and M

Renormalizable Gauge Theories

The scalar masses undergo relevant simplifications:

$$m_{f\bar{f}}^2 = \Big[Q_{x\bar{x}}^{-1}\, Q_{f\bar{f}}\Big] M^2,$$

$$m_{x\bar{x}}^2 = \frac{1}{2}\Big[1 + Q_{x\bar{x}}^{-1}\, Q_{x\bar{x}} + Q_{x\bar{x}}^{-1}\, (Q_{f\bar{f}})^{-1}\, Q_{f\bar{f}}^2\Big] M^2,$$

$$m_{f\bar{x}}^2 = \Big[Q_{x\bar{x}}^{-1}\, Q_{f\bar{x}}\Big] M^2.$$

- The masses depend on the vacuum point only through the orientation of the direction x^i and the size of M.
- The orientation of the directions f^i and x^i must be optimized to maximize the lightest scalar mass.
- The optimal choice is to have f^i and x^i eigenvectors of $Q^i{}_j$ with largest and smallest eigenvalues, namely $q_{\rm max}$ and $q_{\rm min}$. One finds:

$$m_{f\bar{f}}^2 \to \left|\frac{q_{\rm max}}{q_{\rm min}}\right| M^2\,, \qquad 2\,m_{x\bar{x}}^2 \to \left[2 + \left|\frac{q_{\rm max}}{q_{\rm min}}\right|\right] M^2\,, \qquad m_{f\bar{x}}^2 \to 0\,.$$

Important: the sGoldstinos are the LP and the optimal configuration corresponds to saturate the inequality $|D|/|F| \le |q_{\rm max}/q_{\rm min}|$.

Non-Trivial Kähler Geometry

This situation is much more complicated than the renormalizable case:

$$m^2_{f\bar{f}} = \Big[-\frac{1}{4} g^{-2} M^2 R_{f\bar{f}f\bar{f}}\, \tilde{Q}^{-1}_{x\bar{x}} (Q_{f\bar{f}})^{-1} + \tilde{Q}^{-1}_{x\bar{x}}\, Q_{f\bar{f}} \Big] M^2,$$

$$m^2_{x\bar{x}} = \frac{1}{2} \Big[1 - \frac{1}{2} g^{-2} M^2 R_{f\bar{f}x\bar{x}}\, \tilde{Q}^{-1}_{x\bar{x}} (Q_{f\bar{f}})^{-1} + \tilde{Q}^{-1}_{x\bar{x}}\, Q_{x\bar{x}} + \tilde{Q}^{-1}_{x\bar{x}} (Q_{f\bar{f}})^{-1} Q^2_{f\bar{f}} \Big] M^2,$$

$$m^2_{f\bar{x}} = \Big[-\frac{1}{4} g^{-2} M^2 R_{f\bar{f}f\bar{x}}\, \tilde{Q}^{-1}_{x\bar{x}} (Q_{f\bar{f}})^{-1} + \tilde{Q}^{-1}_{x\bar{x}}\, Q_{f\bar{x}} \Big] M^2.$$

where we introduced the new quantity $\tilde{Q}^i{}_j = iX^i K_j / (K^m K_m)$. In general $\tilde{Q}^i{}_j$ differs from $Q^i{}_j$ and is not constant!

- The masses depend on the vacuum point not only through the orientation of the direction x^i and the size of M, but also through the values of $R_{i\bar{j}k\bar{l}}$, $Q_{i\bar{j}}$ and $\tilde{Q}_{i\bar{j}}$ which are in general not constant;
- The optimization of the superpotential does not simply amounts to optimizing the orientation of the directions f^i and x^i;
- the LS is generically a linear combination of Goldstino and Goldstone partners, and its mass is not necessarily positive.

A case-by-case analysis is needed!

Logarithmic Kähler Potential and Shift Isometry

- An example with 2 chiral multiplet and 1 gauged shift symmetry.
- The directions f^i and x^i are rigidly tied and parametrized by 1 angle θ

$$K = -\Lambda_1^2 \log\left(\frac{\Phi^1 + \bar{\Phi}^1}{\Lambda_1}\right) - \Lambda_2^2 \log\left(\frac{\Phi^2 + \bar{\Phi}^2}{\Lambda_2}\right), \quad X^i = i\left(A_1, A_2\right).$$

The vacuum point is parametrized as follows ($\lambda_i = \frac{g\Lambda_i}{M}, \quad a_i = \frac{gA_i}{M}$) :

$$\Phi_0^i = \frac{1}{\sqrt{2}} g^{-1} M \left(a_1 \lambda_1 |\sec\theta|, a_2 \lambda_2 |\csc\theta|\right).$$

this defines a surface of the field space in which the vector mass is constant and equal to M. The relevant directions are:

$$x^i = \sqrt{2} i\left(a_1, a_2\right) \quad \text{and} \quad f^i = \sqrt{2} i\left(a_1 |\tan\theta|, -a_2 |\cot\theta|\right).$$

The relation between $|D|$, $|F|$ and M^2 are in this case:

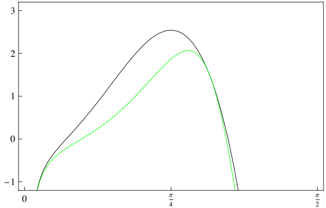
$$|D| = \frac{1}{\sqrt{2}} g^{-1}\left(\lambda_1 |\cos\theta| + \lambda_2 |\sin\theta|\right) M^2,$$

$$|F| = \frac{1}{\sqrt{2}} g^{-1} \sqrt{\lambda_1 \lambda_2}\, |2\cos\theta \sin\theta|^{-1/2} M^2,$$

$$|D/F| = \sqrt{|2\cos\theta\sin\theta|}\left(\sqrt{\lambda_1/\lambda_2}\, |\cos\theta| + \sqrt{\lambda_2/\lambda_1}\, |\sin\theta|\right).$$

Logarithmic Kähler Potential and Shift Isometry

- $m^2_{f\bar{f}}$ (black) and the optimized lightest eigenvalue of the relevant 3×3 mass matrix (green) have the form: $M^2 \times h(\theta, \lambda_1/\lambda_2)$



This situation is different from the renormalizable case!

- The largest possible value of the lightest mass DOES NOT coincide with the maximal value of the sGoldstino mass!
- The discrepancy is larger for manifolds with a large hierarchy between the curvature scales Λ_1 and Λ_2.
- Adjusting the superpotential to maximize the sGoldstino mass is not the optimal choice!

Conclusions

- For general Kähler geometries, the Goldstino is not the only dangerous direction for metastability;
- The upper bound obtained by just looking at the Goldstino direction, turns out to be too optimistic and tends to overestimate the mass of the lightest scalar;
- In the presence of vector multiplets, maximizing the Goldstino mass is not the best thing to do to maximize the LS mass;
- The region of metastability can be substantially reduced when considering also the gauge symmetry breaking directions;
- We believe there exist examples in which vacuum instability is driven by the scalar partners of massive vector bosons;
- Our strategy can be generalized without any conceptual difficulty to non-Abelian symmetries and to the supergravity case; there are however some technical complications which arise

... a more careful study is needed!