Superconformal Operator Product Expansion and General Gauge Mediation

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based on
arXiv:1107.1721 [hep-th] (JFF, K. Intriligator, A. Stergiou),
work in progress (JFF, K. Intriligator, A. Stergiou)

Outline

(S)CFTs, (S)OPE and real world

- Why (S)CFTs $\Rightarrow$ (S)CFTs are generic
  - $\mathcal{N} = 4$ SYM
  - $\mathcal{N} = 1$ SQCD in conformal window Seiberg (1994)
  - Non-Lagrangian candidates *e.g.* Benini, Tachikawa, Wecht (2009)
- Why (S)OPE $\Rightarrow$ (S)CFT observables
  - Spectrum of operators and dimensions
  - (S)OPE coefficients
  - Non-local Wilson loops

$\Rightarrow$ Highly constrained

(S)CFT model-building applications

- Large anomalous dimensions ⇒ Suppress or enhance otherwise finely-tuned quantities
  - Conformal sequestering Luty, Sundrum (2001)
  - Conformal technicolor Luty, Okui (2004)
  - μ/B_μ problem of gauge mediation Roy, Schmaltz (2007)
  - Flavor hierarchy Poland, Simmons-Duffin (2009)
- RG flows near (S)CFTs
  - Walking technicolor Holdom (1985)
  - Unparticle physics Georgi (2007)

(S)OPE UV/IR applications

- Non-CFTs $\Rightarrow$ Conformally covariant OPEs
- Softly broken symmetries as spontaneously broken symmetries
  - Symmetry breaking seen as IR effect (via field or spurion vev)
  - Symmetry restored in UV theory

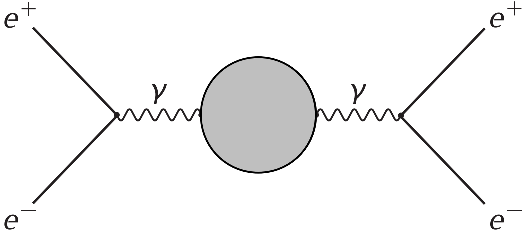
  $\Rightarrow$ (S)OPE selection rules

$\Rightarrow$ Strongly-coupled IR physics described by weakly-coupled UV physics through (S)OPE

- Example: QCD
  - Not conformal (non-trivial RG flow)
  - IR physics $\Rightarrow$ Theory with chiral symmetry breaking ($\langle\bar{Q}Q\rangle \neq 0$) and confinement ($\langle G^A_{\mu\nu}G^{A\mu\nu}\rangle \neq 0$)
  - UV physics $\Rightarrow$ Asymptotically free CFT

  $\Rightarrow$ QCD sum rules Shifman, Vainshtein, Zakharov (1979)

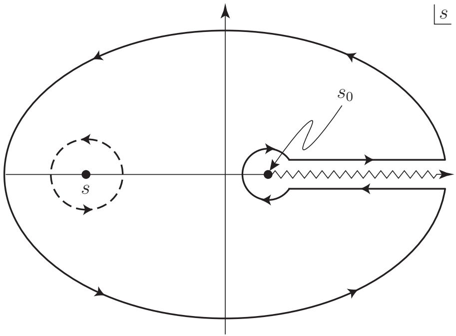
Probe hadron sector from lepton sector through gauge interactions (UV/IR physics separated using OPE)



$$\Pi_{\mathrm{hadron},\mu\nu}(p) = ie^2 \int d^4x\, e^{-ip\cdot x} \langle j_\mu(x) j_\nu(0) \rangle$$

$$\mathcal{O}_i(x)\mathcal{O}_j(0) = \sum_k c_{ij}^k(x) \mathcal{O}_k(0)$$

OPE, analyticity and optical theorem ⇒ Relations between total cross section and OPE coefficients



$$\Pi_{\rm hadron}(s) = \frac{1}{2\pi i}\int_{s_0}^{\infty} ds' \, \frac{{\rm Disc}\,\Pi_{\rm hadron}(s')}{s'-s}$$

$$\sigma_{e^+e^-\to{\rm hadrons}}(s) = \frac{2\pi i\alpha}{s}{\rm Disc}\,\Pi_{\rm hadron}(s)$$

Operator product expansion: Review

Operator product expansion

$$\begin{aligned}
\mathcal{O}_i(x)\mathcal{O}_j(0) &= \sum_k \frac{c_{ij}^k}{x^{\Delta_i+\Delta_j-\Delta_k}}\mathcal{O}_k(0) \\
&= \sum_{\text{primary } k} \frac{c_{ij}^k}{x^{\Delta_i+\Delta_j-\Delta_k}} F_{\Delta_i\Delta_j}^{\Delta_k}(x,P)\mathcal{O}_k(0)
\end{aligned}$$

- Short distance physics expressed in terms of local operators
- Wilson coefficients $\Rightarrow$ UV physics
- Vacuum expectation values $\Rightarrow$ IR physics
- OPE constrained by conformal symmetry $\Rightarrow$ Wilson coefficients of descendants determined by Wilson coefficients of primaries Ferrara, Gatto, Grillo (1971)

CFT correlation functions

2-point and 3-point correlation functions

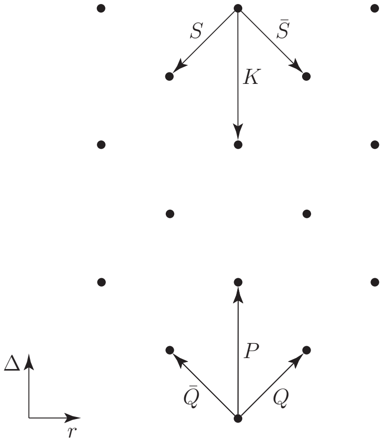
$$\langle \mathcal{O}_i(x_1)\mathcal{O}_j(x_2)\rangle = \frac{g_{ij}}{x_{12}^{\Delta_i+\Delta_j}}$$

$$\langle \mathcal{O}_i(x_1)\mathcal{O}_j(x_2)\mathcal{O}_k(x_3)\rangle = \frac{c_{ijk}}{x_{12}^{\Delta_i+\Delta_j-\Delta_k}\, x_{31}^{\Delta_k+\Delta_i-\Delta_j}\, x_{23}^{\Delta_j+\Delta_k-\Delta_i}}$$

- 2-point function coefficients $g_{ij} = c_{ij}^{0}$ (Zamolodchikov metric)
- 3-point function coefficients $c_{ijk} = c_{ij}^{\ell} g_{\ell k}$

Superconformal operator product expansion

Superconformal modules



Superconformal operator product expansion

$$\mathcal{O}_i(x)\mathcal{O}_j(0) \overset{?}{=} \sum_{\text{superprimary } k} \frac{c_{ij}^k}{x^{\Delta_i+\Delta_j-\Delta_k}} F_{ij}^k(x, P, Q, \bar{Q})\mathcal{O}_k(0)$$

$$\begin{aligned} \mathcal{T}_\mu(z) &= j_\mu^R(x) + \theta^\alpha \mathcal{S}_{\alpha\mu}(x) + \bar{\theta}^{\dot{\alpha}} \bar{\mathcal{S}}_{\dot{\alpha}\mu}(x) + 2\theta\sigma^\nu\bar{\theta} T_{\nu\mu}(x) + \cdots \\ \mathcal{J}(z) &= J(x) + i\theta j(x) - i\bar{\theta}\bar{j}(x) - \theta\sigma^\mu\bar{\theta} j_\mu(x) + \cdots \end{aligned}$$

- OPE constrained by superconformal symmetry
- Sum over superprimaries instead of primaries

$\Rightarrow$ Wilson coefficients of superdescendants NOT fully determined by Wilson coefficients of superprimaries (existence of superconformal 3-point invariants) ! Osborn (1998)

Current-current SOPE

Superconformal 3-point correlation functions for supercurrents

$$\langle \mathcal{J}(z_1)\mathcal{J}(z_2)\mathcal{O}^{\mu_1\cdots\mu_\ell}(z_3)\rangle = \frac{1}{x_{\bar{1}3}{}^2 x_{\bar{3}1}{}^2 x_{\bar{2}3}{}^2 x_{\bar{3}2}{}^2}\, t^{\mu_1\cdots\mu_\ell}_{\mathcal{J}\mathcal{J}\mathcal{O}_\ell}(X_3, \Theta_3, \bar{\Theta}_3)$$

- $\mathcal{O}^{\mu_1\cdots\mu_\ell}$ real spin-ℓ superfield with vanishing R-charge
- $t_{\mathcal{J}\mathcal{J}\mathcal{O}}(X, \Theta, \bar{\Theta})$ fully determined by superconformal symmetry and supercurrent conservation JFF, Intriligator, Stergiou (2011)

Superconformal 3-point correlation functions for supercurrents

$$
\begin{aligned}
t^{\mu_1\ldots\mu_\ell}_{\mathcal{J}\mathcal{J}\mathcal{O}_{\ell=\mathrm{even}}}(X,\Theta,\bar{\Theta}) &= c_{\mathcal{J}\mathcal{J}\mathcal{O}_\ell}\frac{X_+^{(\mu_1}\cdots X_+^{\mu_\ell)}}{(X\cdot\bar{X})^{2-\frac{1}{2}(\Delta-\ell)}} \\
&\times\left[1-\frac{1}{4}(\Delta-\ell-4)(\Delta+\ell-6)\frac{\Theta^2\bar{\Theta}^2}{X\cdot\bar{X}}\right] \\
t^{\mu_1\ldots\mu_\ell}_{\mathcal{J}\mathcal{J}\mathcal{O}_{\ell=\mathrm{odd}}}(X,\Theta,\bar{\Theta}) &= c_{\mathcal{J}\mathcal{J}\mathcal{O}_\ell}\frac{X_+^{(\mu_1}\cdots X_+^{\mu_{\ell-1}}}{(X\cdot\bar{X})^{2-\frac{1}{2}(\Delta-\ell)}} \\
&\times\left[X_-^{\mu_\ell)}-\frac{\ell(\Delta-\ell-4)}{\Delta-2}\frac{(X_-\cdot X_+)X_+^{\mu_\ell)}}{X\cdot\bar{X}}\right]
\end{aligned}
$$

$\Rightarrow$ Relations in current-current SOPE from superconformal symmetry and supercurrent conservation

From superprimaries to superdescendants (applications to GGM)

$$
\begin{aligned}
j_\alpha(x)\bar{j}_{\dot\alpha}(0) &= \frac{1}{x^4}\Big[(S\, ix\cdot\sigma)_{\dot\alpha}(ix\cdot\sigma\bar{S})_\alpha - x^2\bar{Q}_{\dot\alpha}(ix\cdot\sigma\bar{S})_\alpha \\
&\qquad +2\Delta_J x^2(ix\cdot\sigma)_{\alpha\dot\alpha}\Big]\big(J(x)J(0)\big) \\
j_\mu(x)j_\nu(0) &= \frac{1}{16x^8}\Big[(x^2\eta_{\mu\rho}-2x_\mu x_\rho)(S\sigma^\rho\bar{S}-\bar{S}\sigma^\rho S) \\
&\qquad \times x^4(\bar{Q}\bar{\sigma}_\nu Q - Q\sigma_\nu\bar{Q})+\cdots\Big]\big(J(x)J(0)\big) \\
j_\alpha(x)j_\beta(0) &= \frac{1}{x^2}Q_\beta(ix\cdot\sigma\bar{S})_\alpha\big(J(x)J(0)\big) \\
j_\mu(x)J(0) &= \frac{x^2\eta_{\mu\nu}-2x_\mu x_\nu}{4x^4}\Big[S\sigma^\nu\bar{S}-\bar{S}\bar{\sigma}^\nu S\Big]\big(J(x)J(0)\big)
\end{aligned}
$$

$$
S^\alpha S^\beta\big(J(x)J(0)\big) = 0
$$

$$
\Big[x^2 Q_\alpha Q_\beta + Q_\alpha(ix\cdot\sigma\bar{S})_\beta - Q_\beta(ix\cdot\sigma\bar{S})_\alpha\Big]\big(J(x)J(0)\big) = 0
$$

Superconformal blocks

Decomposition into primaries Poland, Simmons-Duffin (2010)

$$\mathcal{O}^{\mu_1\cdots\mu_\ell}(x,\theta,\bar\theta) = A^{\mu_1\cdots\mu_\ell}(x) + \theta\sigma_\mu\bar\theta B^{\mu\mu_1\cdots\mu_\ell}(x) + (\theta\sigma_\mu\bar\theta)^2 D^{\mu_1\cdots\mu_\ell}(x) + \cdots$$

- A and D irreducible spin-ℓ representations
- $B \sim M + N + L$ where M spin-$(\ell+1)$ representation and N spin-$(\ell-1)$ representation

$\Rightarrow$ A_{primary}, M_{primary}, N_{primary}, L_{primary} and D_{primary}

Superconformal 3-point function and SOPE coefficients

$$c_{JJM^{\ell+1;\ell=\text{even}}_{\text{primary}}} = c_{JJN^{\ell-1;\ell=\text{even}}_{\text{primary}}} = c_{JJL_{\text{primary}}} = 0$$

$$c_{JJD^{\ell;\ell=\text{even}}_{\text{primary}}} = -\frac{\Delta(\Delta+\ell)(\Delta-\ell-2)}{8(\Delta-1)} c_{JJA^{\ell;\ell=\text{even}}_{\text{primary}}}$$

$$c_{JJA^{\ell;\ell=\text{odd}}_{\text{primary}}} = c_{JJL_{\text{primary}}} = c_{JJD^{\ell;\ell=\text{odd}}_{\text{primary}}} = 0$$

$$c_{JJN^{\ell-1;\ell=\text{odd}}_{\text{primary}}} = -\frac{(\ell+2)(\Delta-\ell-2)}{\ell(\Delta+\ell)} c_{JJM^{\ell+1;\ell=\text{odd}}_{\text{primary}}}$$

- Consistent with Lorentz symmetry
- Consistent with unitary bound $\Rightarrow D_{\text{primary}}$ and N_{primary} are null states for $\Delta = \ell + 2$ (short representation)

4-point superconformal blocks

$$\langle J(x_1)J(x_2)J(x_3)J(x_4)\rangle = \frac{1}{x_{12}^4 x_{34}^4}\sum_{\mathcal{O}_{\Delta,\ell}\in J\times J}\frac{(c_{JJA_\ell})^2}{c_{A_\ell A_\ell}}\mathcal{G}^{JJ;JJ}_{\Delta,\ell}(u,v)$$

$$\begin{aligned}
\mathcal{G}^{JJ;JJ}_{\Delta,\ell=\mathrm{even}} &= g_{\Delta,\ell} + \frac{(\Delta+\ell)(\Delta-\ell-2)}{16(\Delta+\ell+1)(\Delta-\ell-1)} g_{\Delta+2,\ell} \\
\mathcal{G}^{JJ;JJ}_{\Delta,\ell=\mathrm{odd}} &= \frac{(\ell+1)^2(\Delta+\ell)}{4(\Delta+\ell+1)} g_{\Delta+1,\ell+1} \\
&\quad + \frac{(\ell+2)^2(\Delta-\ell-2)}{\Delta-\ell-1} g_{\Delta+1,\ell-1}
\end{aligned}$$

- $g_{\Delta,\ell}(u,v)$ universal 4-point conformal blocks (accounting for descendants) with u, v usual conformal cross-ratios
- $\mathcal{G}^{JJ;JJ}_{\Delta,\ell}(u,v)$ non-universal 4-point superconformal blocks (accounting for superdescendants)

General gauge mediation: Overview

- SUSY breaking hidden sector connected to visible sector through gauge interactions Buican, Meade, Seiberg, Shih (2008)
  - Decoupled hidden sector in $g \to 0$ limit
  - Universal visible sector SUSY breaking effects introduced via loops

$\Rightarrow$ Current-current correlation functions (even without hidden sector Lagrangian)

$$\mathcal{J}(z) = J(x) + i\theta j(x) - i\overline{\theta}\overline{j}(x) - \theta\sigma^\mu\overline{\theta} j_\mu(x) + \cdots$$

$$\langle J(x)J(0)\rangle,\ \langle j_\alpha(x)\overline{j}_{\dot{\alpha}}(0)\rangle,\ \langle j_\mu(x)j_\nu(0)\rangle,\ \langle j_\alpha(x)j_\beta(0)\rangle$$

Operator product expansion

Strongly-coupled hidden sector and OPE

$$J(x)J(0) = \sum_k c^k_{JJ}(x)\mathcal{O}_k(0)$$

$$i\int d^4x\, e^{-ip\cdot x} J(x)J(0) = \sum_k \tilde{c}^k_{JJ}(p)\mathcal{O}_k(0)$$

- Relations between Wilson coefficients in same OPE and different OPEs
  - $\langle J(x)J(0)\rangle$ overdetermined ✓
  - $\langle j_\alpha(x)\bar{j}_{\dot\alpha}(0)\rangle$, $\langle j_\mu(x)j_\nu(0)\rangle$ and $\langle j_\alpha(x)j_\beta(0)\rangle$ determined in terms of $\langle J(x)J(0)\rangle$ ✓

$\Rightarrow$ Knowledge of strongly-coupled quantities in terms of $J(x)J(0)$ OPE

Cross sections

OPE constraints, analyticity and optical theorem

$$\sigma_{\text{visible}\to D^*\to\text{hidden}}(s) = -\frac{(4\pi\alpha)^2}{s}\sum_k \mathrm{Im}[\tilde{c}^k_{JJ}(s)]\langle\mathcal{O}_k(0)\rangle$$

$$\sigma_{\text{visible}\to\lambda^*_\alpha\to\text{hidden}}(s) = f_{1/2}(\mathrm{Im}[\tilde{c}^k_{JJ}(s)])$$

$$\sigma_{\text{visible}\to A^*_\mu\to\text{hidden}}(s) = f_1(\mathrm{Im}[\tilde{c}^k_{JJ}(s)])$$

- Good approximation with first few terms
- Consistent with direct computation in ordinary gauge mediation

Visible sector SUSY breaking masses

OPE constraints and analyticity

$$M_{\rm gaugino} \approx \sum_k \frac{\alpha\,{\rm Im}[s^{d_k/2}\,\tilde{c}^k_{JJ}(s)]}{2^{d_k-1}d_k M^{d_k}}\langle Q^2(\mathcal{O}_k(0))\rangle$$

$$m^2_{\rm sfermion} \approx 4\pi\alpha Y\langle J(x)\rangle - \sum_k \frac{\alpha^2 c_2\,{\rm Im}[s^{d_k/2}\,\tilde{c}^k_{JJ}(s)]}{2^{d_k+1}\pi d_k^2 M^{d_k}}\langle \bar{Q}^2 Q^2(\mathcal{O}_k(0))\rangle$$

- Reproduce the usual $f(x)$ and $g(x)$ functions of ordinary gauge mediation Martin (1996)

OPE constraints and analyticity

$$J^A(x)J^B(0) = \tau\frac{\delta^{AB}\mathbb{1}}{16\pi^4x^4} + \frac{kd^{ABC}}{\tau}\frac{J^C(0)}{16\pi^2x^2} + w\frac{\delta^{AB}K(0)}{4\pi^2x^{2-\gamma_K}} + c_i^{AB}\frac{\mathcal{O}_i(0)}{x^{4-\Delta_i}} + \cdots$$

$$M_{\text{gaugino}} \approx -\frac{\alpha\pi w\gamma_{Ki}}{8M^2}\langle Q^2(\mathcal{O}_i(0))\rangle$$

$$m^2_{\text{sfermion}} \approx 4\pi\alpha Y\langle J(x)\rangle + \frac{\alpha^2c_2w\gamma_{Ki}}{64M^2}\langle\bar{Q}^2Q^2(\mathcal{O}_i(0))\rangle$$

- Good approximation with first few terms
- $f(x), g(x) \sim 1/2 + \cdots$ instead of $f(x), g(x) \sim 1 + \mathcal{O}(x)$ in ordinary gauge mediation

Features and applications

Features

- Undetermined superconformal 3-point correlation functions
- Fully determined superconformal current-current SOPE
- Relations between current-current OPEs
- Well-defined current 4-point superconformal blocks

Applications to GGM

- All current-current OPEs determined by $J(x)J(0)$ OPE
- Relations between Wilson coefficients
- Cross sections and SUSY breaking masses from $J(x)J(0)$
- Incalculable strongly-coupled models made tractable