Walls of massive Kähler linear sigma models on SO(2N)/U(N) and Sp(N)/U(N)

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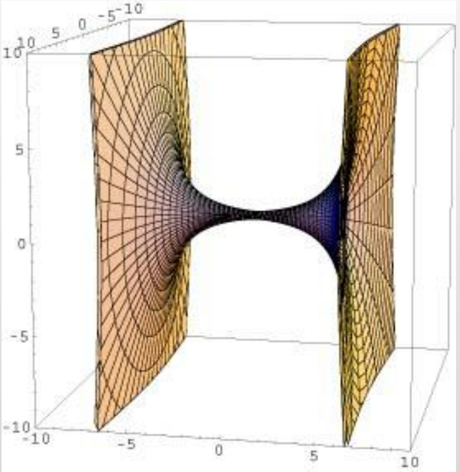
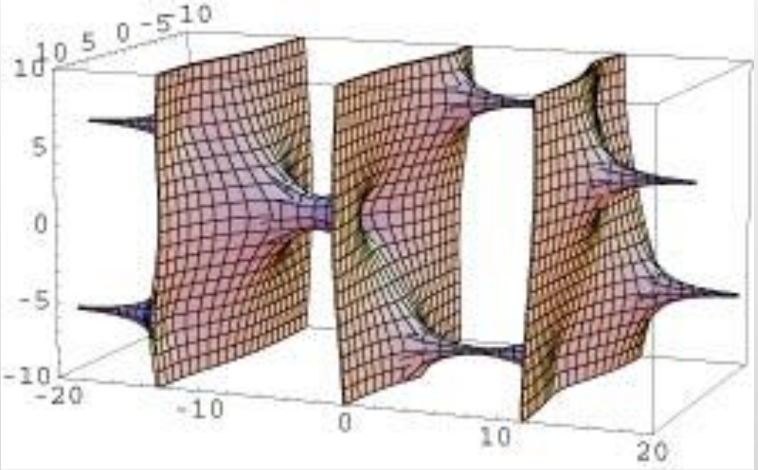
MA & Sunyoung Shin

Introduction

- Soliton - Playing an important role in physics
  - Ex. Domain wall solution ➡ Brane world scenario
- D=5 N=1 SUSY $U(M)$ gauge theory with N massive flavors and FI term

Isozumi, Nitta, Ohashi, Sakai, 2004





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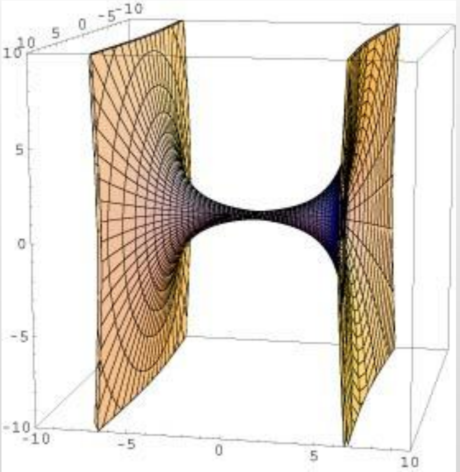

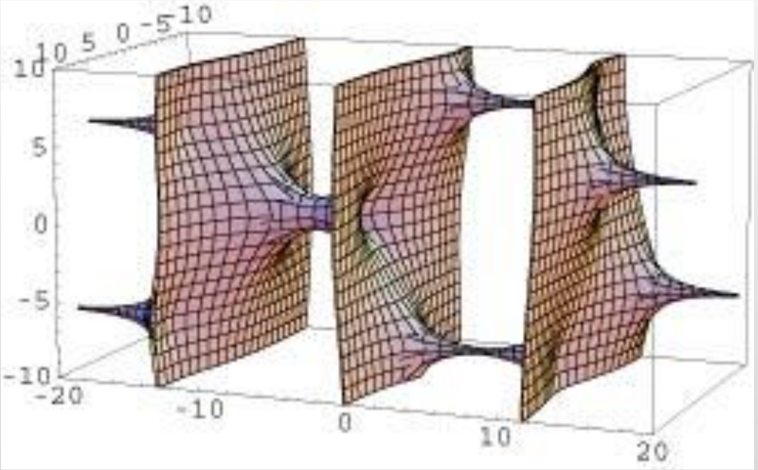

- Infinite gauge coupling limit
  - Massive hyper-Kähler nonlinear sigma model (NLSM) on $T^*G_{N,M}$

Introduction

- $G_{N,M}$: one of compact Hermitian symmetric spaces (HSS)

$$G_{N,M} = \frac{U(N)}{U(N-M) \times U(M)} \quad Q^N = \frac{SO(N+2)}{SO(N) \times U(1)} \quad \frac{SO(2N)}{U(N)} \quad \frac{Sp(N)}{U(N)}$$

$$\frac{E_6}{SO(10) \times U(1)} \quad \frac{E_7}{E_6 \times U(1)}$$

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  - Different vacuum structure
  - Expected interesting configurations as well as $T^*G_{N,M}$

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- Question: How about a NLSM on $T^*\mathcal{M}$? ($\mathcal{M}$ is the HSS except $G_{N,M}$)
  - Different vacuum structure
  - Expected interesting configurations as well as $T^*G_{N,M}$

- Consider domain walls of a NLSM on $T^*\mathcal{M}$ except $T^*G_{N,M}$
  - Difficult to construct such an action because of requirement of N=2 SUSY

Introduction

- Simplifying setup
  - Observation: The cotangent part is trivial for vacua and domain wall configuration in $T^*G_{N,M}$

Isozumi, Nitta, Ohashi, Sakai, 2004

$$\mathcal{L} = \int d^4\theta K(\underbrace{\Phi, \Phi^\dagger}_{\text{Base manifold part}}, \underbrace{\Psi, \Psi^\dagger}_{\text{Cotangent part}}) + \text{potential term}$$

K: Kähler potential of $T^*G_{N,M}$

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  - The result is respected as one of the massive Kähler NLSM on $G_{N,M}$.

$$\mathcal{L} = \int d^4\theta K(\Phi, \Phi^\dagger) + \text{potential term}$$

  - The situation would be the same for other NLSMs on $T^*\mathcal{M}$ ($\mathcal{M}$ is the HSS).

Purpose of our work

- Domain walls in massive kähler NLSM on $\frac{SO(2N)}{U(N)}$ and $\frac{Sp(N)}{U(N)}$ in 3D.
  - Construction of Lagrangians for models
  - BPS equations
  - Solving BPS equations & investigating properties
  - Conclusion

Setup

- Starting with the massless NLSM on $\dfrac{SO(2N)}{U(N)}, \dfrac{Sp(N)}{U(N)}$ in 4 dimensions.

  Higashijima, Nitta, 1999

  - 4D N=1 U(N) gauge theory coupled to 2N flavors including FI term (with $g \to \infty$)

$$\mathcal{L} = \int d^4\theta (\phi_a{}^i \bar{\phi}_i{}^b (e^V)_b{}^a - r^2 V_a{}^a) + \left(\int d^2\theta \, \phi_0^{ab} (\phi_b{}^i J_{ij} \phi^{\mathrm{T}j}{}_a) + \text{c.c.} \right)$$

$$(i = 1, \cdots, 2N, \quad a = 1, \cdots, N)$$

$$J = \mathbf{1} \otimes \begin{pmatrix} 0 & 1 \\ \epsilon & 0 \end{pmatrix}, \quad \epsilon = \begin{cases} +1 & SO(2N)/U(N) \\ -1 & Sp(N)/U(N) \end{cases}$$

  - ϕ : vector rep. of U(N), V: vector superfield
  - $\phi_0^T = \epsilon \phi_0$: symmetric (SO(2N))/anti-symmetric (Sp(N)) rep.
  - Eqs. of motion for V & ϕ_0 give constraints

$$\phi_a{}^i \bar{\phi}_i{}^b - \delta_a{}^b = 0 \quad \text{(D-term)} \qquad \phi_a{}^i J_{ij} \phi^{\mathrm{T}j}{}_b = 0 \quad \text{(F-term)}$$

Setup

- Dimensional reduction
  - Getting a non-trivial scalar potential

$$\mathcal{L}_{\text{bos}} = -D_\mu \phi^i D^\mu \bar{\phi}^i - 4|(\phi_0)^{ab}\phi_b{}^i|^2 + \cdots$$

$$\frac{\partial \phi_a{}^i}{\partial x^3} = i\phi_a{}^j M_j{}^i$$

Cartan subalgebra of $SO(2N), Sp(N)$

$$M_j{}^i = \text{diag}(m_1, m_2, \cdots, m_N) \otimes \sigma_3$$

$$\mathcal{L}_{\text{bos}} = -D_m \phi^i D^m \bar{\phi}^i - |i\phi_a{}^j M_j{}^i - i\Sigma_a{}^b \phi_b{}^i|^2 - 4|(\phi_0)^{ab}\phi_b{}^i|^2 + \cdots$$

$$\Rightarrow -V \qquad (\Sigma = v_3)$$

Giving rise to discrete vacua

Vacua

- Vacuum condition

$$0 = V = |i\phi_a{}^j M_j{}^i - i\Sigma_a{}^b \phi_b{}^i|^2 + 4|(\phi_0)^{ab}\phi_b{}^i|^2$$

with constraints

$$\phi_a{}^i \bar{\phi}_i{}^b - \delta_a{}^b = 0, \quad \phi_a{}^i J_{ij} \phi^{\mathrm{T}j}{}_b = 0$$

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$$\Downarrow$$

$$(\phi_0)^{ab}\phi_b{}^i = 0$$

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$$\phi_0 = 0, \quad \phi_a^i \neq 0$$

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$$(\phi_0)^{ab}\phi_b{}^i = 0$$

$$\phi_0 = 0, \quad \phi_a^i \neq 0$$

$$(m_k - \Sigma_a)\phi_a{}^{2k-1} = 0$$
$$(m_k + \Sigma_a)\phi_a{}^{2k} = 0$$

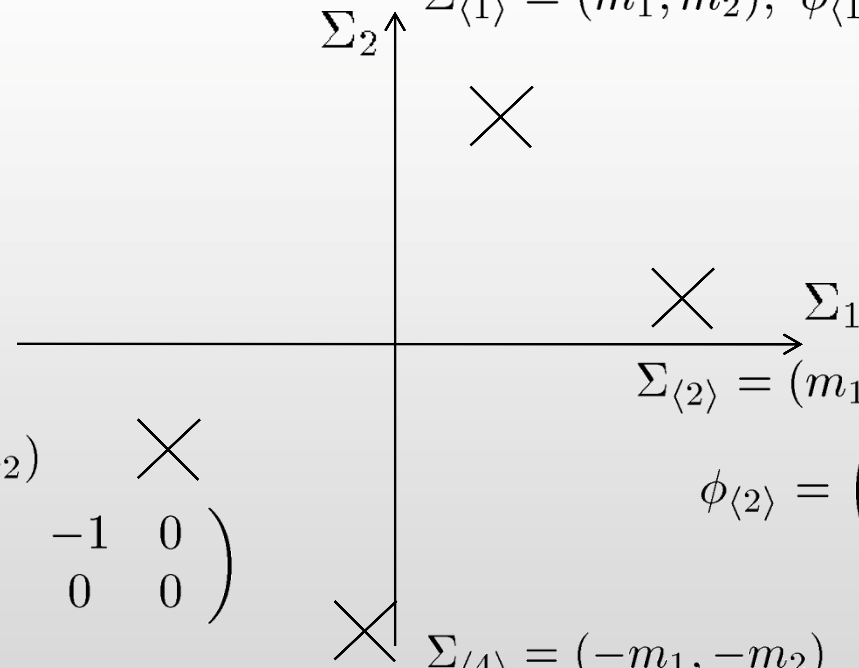
$(\Sigma_a{}^b \to \mathrm{diag}(\Sigma_1, \Sigma_2, \cdots \Sigma_N)$ by gauge trans.)

with constraints

$$\phi_a{}^i \bar{\phi}_i{}^b - \delta_a{}^b = 0, \quad \phi_a{}^i J_{ij} \phi^{\mathrm{T}j}{}_b = 0$$

Vacua

- N=2 case (SO(4)/U(2) & Sp(2)/U(2) – 4 discrete vacua?)



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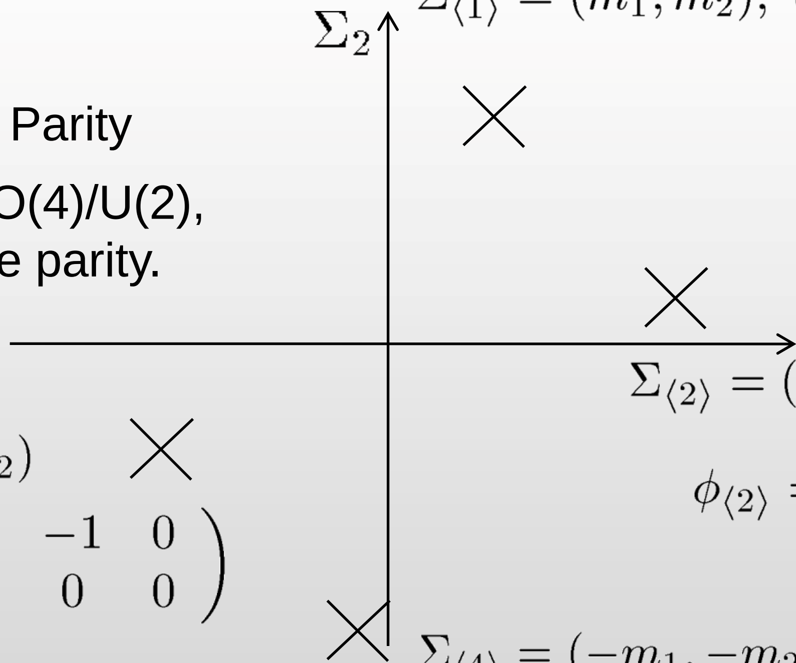
Vacua

- N=2 case: SO(4)/U(2)

$$\phi_a{}^i J_{ij} \phi^{\mathrm{T}j}{}_b = 0$$

  - O(4) invariant
    Rotation & Parity
  - To consider SO(4)/U(2), we remove the parity.



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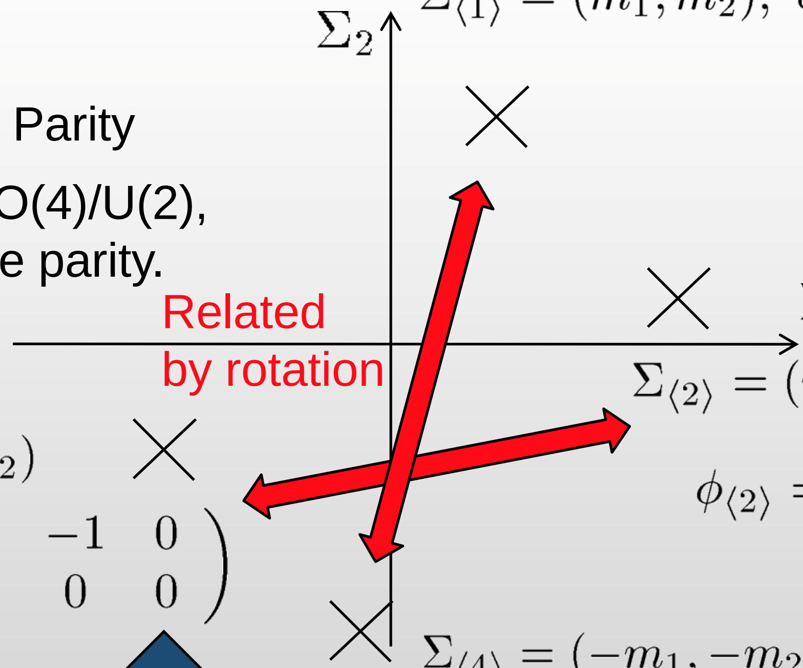

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Vacua

- N=2 case: SO(4)/U(2) – 2 discrete vacua

$$\phi_a{}^i J_{ij} \phi^{\mathrm{T}}{}^j{}_b = 0$$

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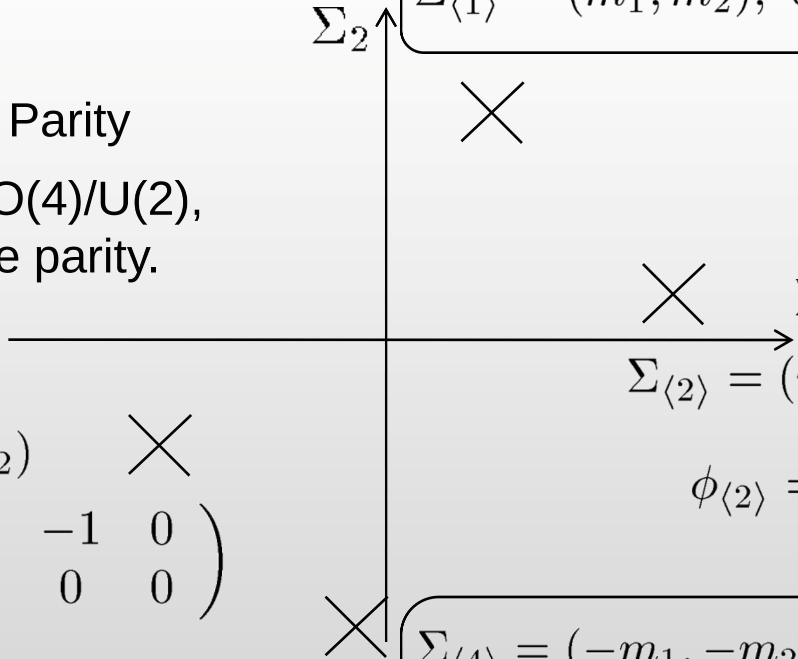
  - To consider SO(4)/U(2), we remove the parity.



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Vacua

- N=2 case: SO(4)/U(2) – 2 discrete vacua ➡ 2^{N-1} discrete vacua

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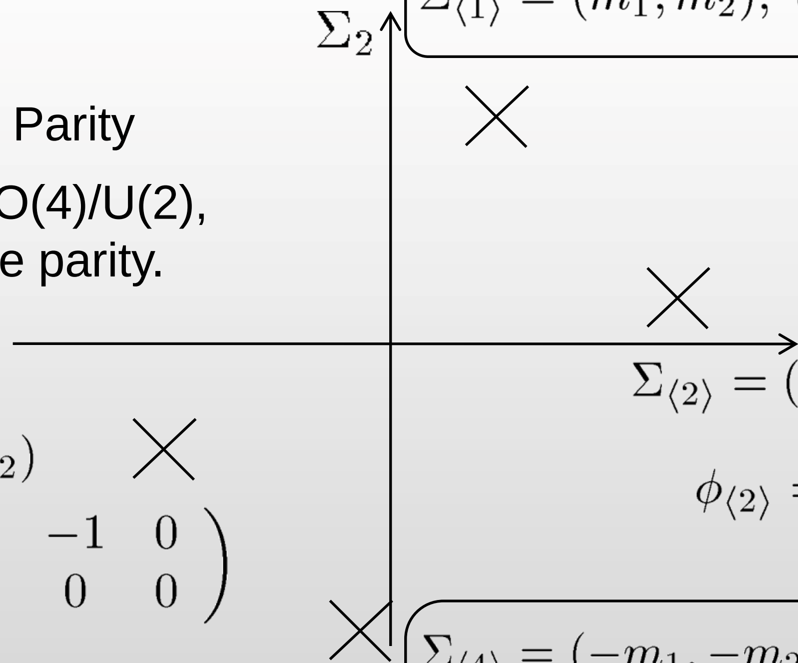

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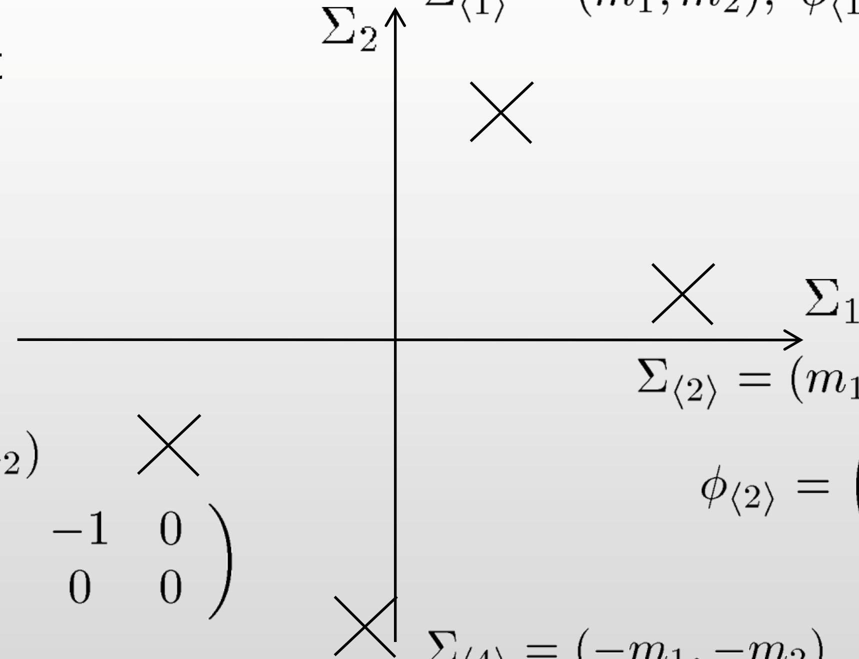
Vacua

- N=2 case: Sp(2)/U(2) – 4 discrete vacua ⟹ 2^N discrete vacua

$$\phi_a{}^i J_{ij} \phi^{\mathrm{T}}{}^j{}_b = 0$$

  - Sp(2) invariant



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BPS equation

- Bogomol'nyi completion
  - Supposing a non-trivial configuration along $x = x_1$ direction, $v_0 = v_2 = 0$

$$E = \int dx \left(|D_1 \phi_a{}^i \mp (\phi_a^i M_j{}^i - \Sigma_a{}^b \phi_b{}^i)|^2 + 4|(\phi_0)^{ab} \phi_b{}^i|^2 \right) \pm T \geq \pm T$$

$$T = \int dx \partial_1 (\phi_a{}^i M_i{}^j \bar{\phi}_j{}^a)$$

with constraints $\phi_a{}^i \bar{\phi}_i{}^b - \delta_a{}^b = 0, \quad \phi_a{}^i J_{ij} \phi^{\mathrm{T}j}{}_b = 0$

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- BPS equation

$$\left.\begin{aligned} &(D_1 \phi)_a{}^i \mp (\phi_a{}^j M_j{}^i - \Sigma_a{}^b \phi_b{}^i) = 0 \\ &(\phi_0)^{ab} \phi_b{}^i = 0 \end{aligned}\right\} \Longrightarrow \phi_0 = 0, \quad \phi \neq 0$$

Solving the BPS equation

- Rewriting the equations

$$(D_1\phi)_a{}^i - (\phi_a{}^j M_j{}^i - \Sigma_a{}^b \phi_b{}^i) = 0 \quad \Longrightarrow \quad \partial_1 f_a{}^i = f_a{}^j M_j{}^i$$

$$\boxed{\phi_a{}^i = (S^{-1})_a{}^b f_b{}^i, \quad \Sigma - iv_1 = S^{-1}\partial_1 S} \quad S \in \mathbf{C}$$

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- Solution

$$\phi_a{}^i = (S^{-1})_a{}^b H_{0b}{}^j (e^{Mx})_j{}^i$$

$H_{0a}{}^i$: moduli matrix

- Constraints

$$\phi_a{}^i \bar{\phi}_i{}^b - \delta_a{}^b = 0, \quad \phi_a{}^i J_{ij} \phi^{\mathrm{T}j}{}_b = 0 \quad \Longrightarrow \quad \begin{cases} H_0 e^{2Mx} H_0^\dagger = S S^\dagger \\ H_0 J H_0^{\mathrm{T}} = 0 \end{cases}$$

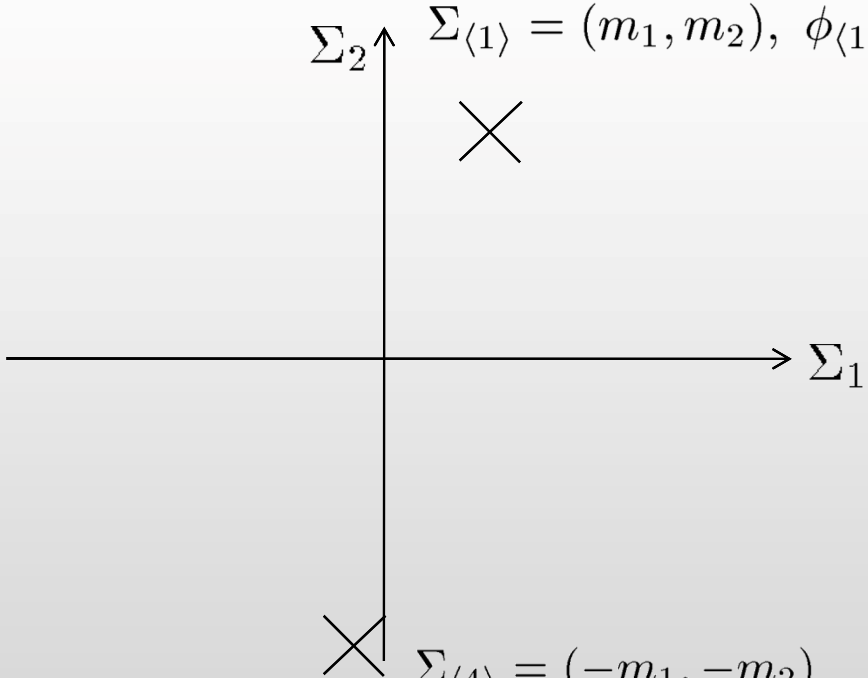
H_0 includes info of vacua, boundary conditions and positions of walls.

Solution - N=2 case

- SO(4)/U(2) case
  - 2 discrete vacua

$$(m_k - \Sigma_a){\phi_a}^{2k-1} = 0, \;\; (m_k + \Sigma_a){\phi_a}^{2k} = 0$$

$$\Sigma_{\langle 1 \rangle} = (m_1, m_2), \;\; \phi_{\langle 1 \rangle} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$



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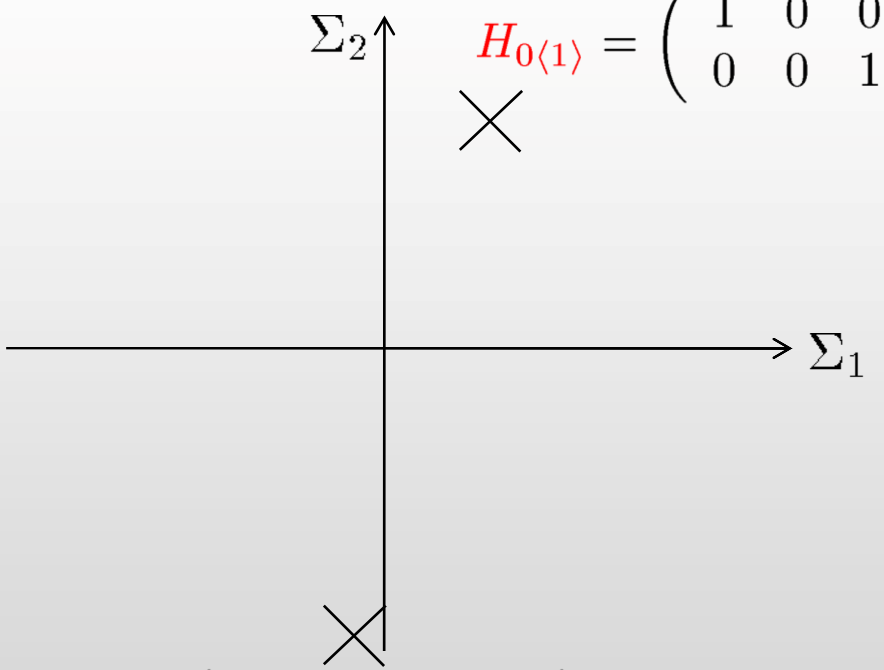
- SO(4)/U(2) case
  - 4 discrete vacua

$$\phi_a{}^i = (S^{-1})_a{}^b H_{0b}{}^j (e^{Mx})_j{}^i$$

$$H_0 e^{2Mx} H_0^\dagger = SS^\dagger$$

$$H_{0\langle 1\rangle} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$



$$H_{0\langle 4\rangle} = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

Solution - N=2 case

- SO(4)/U(2) case
  - Single wall

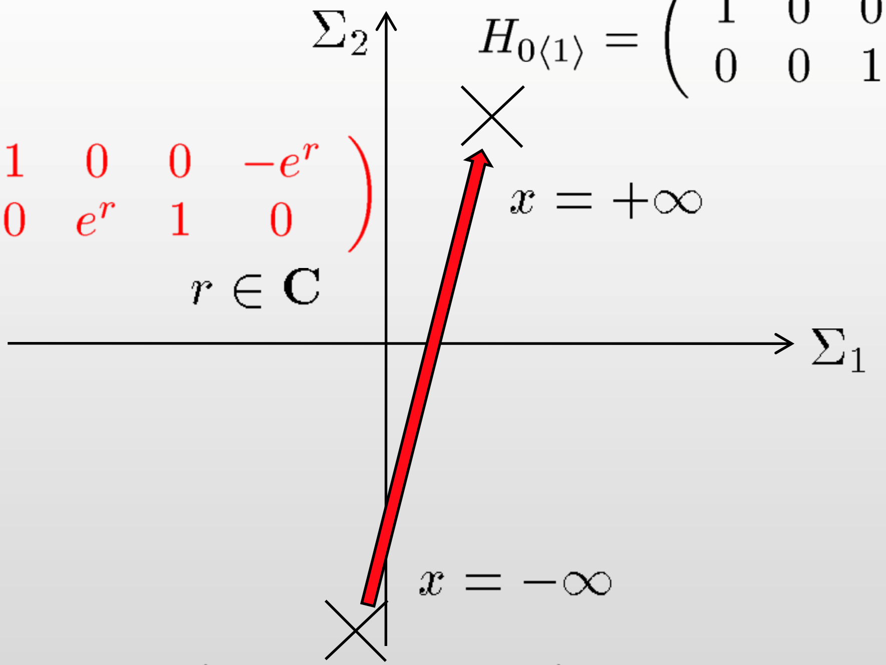
$$\phi_a{}^i = (S^{-1})_a{}^b H_{0b}{}^j (e^{Mx})_j{}^i$$

$$H_0 e^{2Mx} H_0^\dagger = S S^\dagger$$

$$H_{0\langle 1\rangle} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$H_{0\langle 1\leftarrow 4\rangle} = \begin{pmatrix} 1 & 0 & 0 & -e^r \\ 0 & e^r & 1 & 0 \end{pmatrix}$$

$$r \in \mathbf{C}$$



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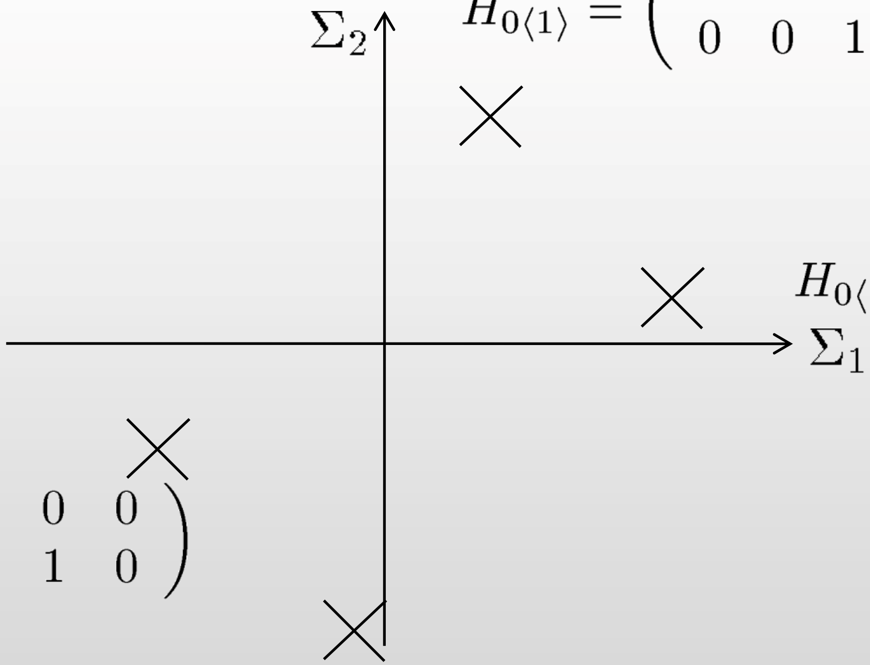
Solution - N=2 case

- Sp(2)/U(2) case
  - 4 discrete vacua



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Solution - N=2 case

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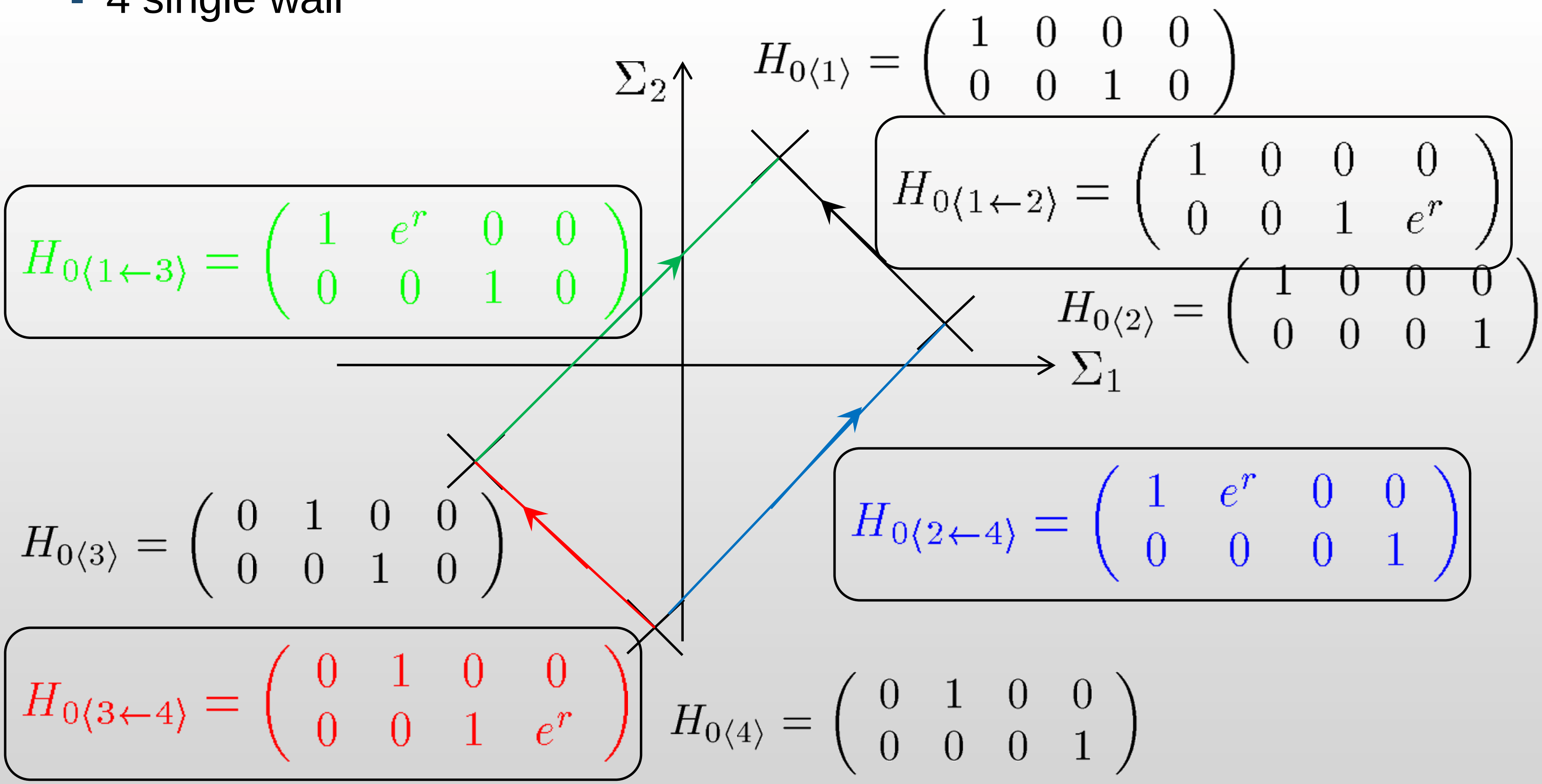

Solution - N=2 case

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  - 2 double walls

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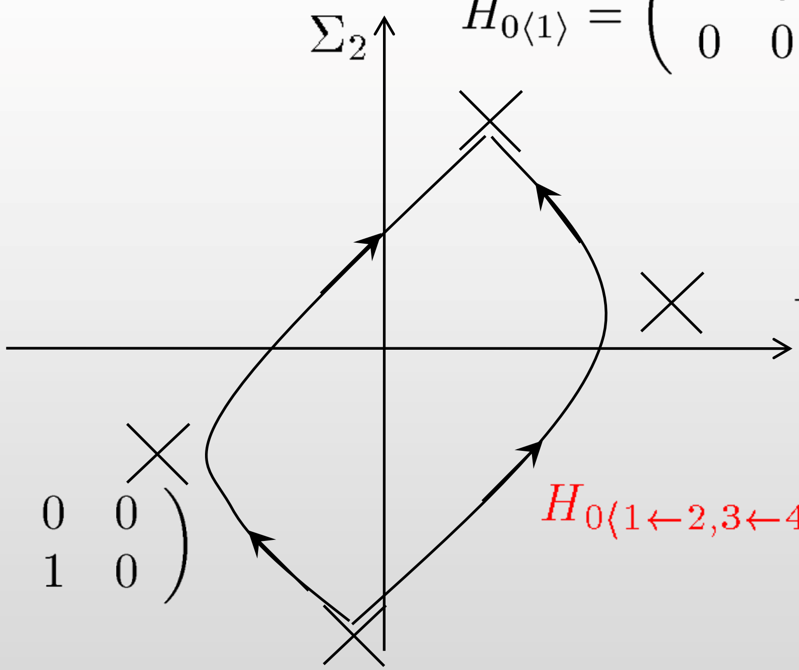

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$$H_{0\langle 1\leftarrow 2,3\leftarrow 4\rangle} = \begin{pmatrix} 1 & e^{r_2} & 0 & 0 \\ 0 & 0 & 1 & e^{r_1} \end{pmatrix}$$

$$H_{0\langle 3\rangle} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

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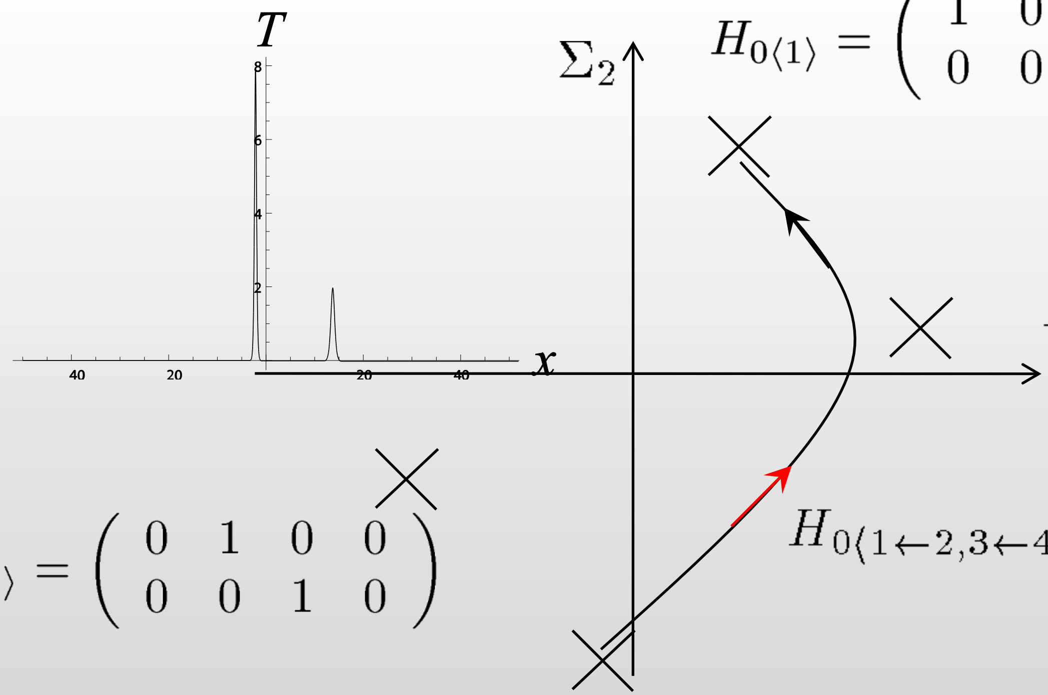
- Sp(2)/U(2) case
  - Double walls



$$H_{0\langle 1\rangle} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$H_{0\langle 2\rangle} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

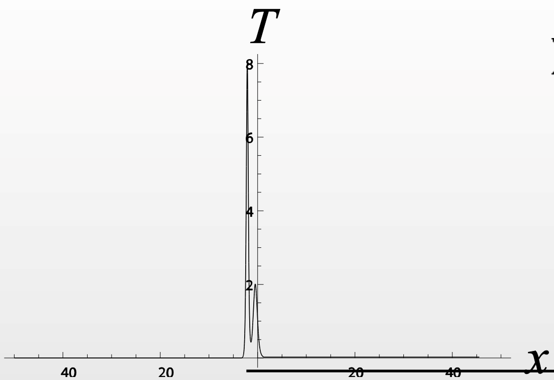
$$H_{0\langle 1\leftarrow 2,3\leftarrow 4\rangle} = \begin{pmatrix} 1 & e^{r_2} & 0 & 0 \\ 0 & 0 & 1 & e^{r_1} \end{pmatrix}$$

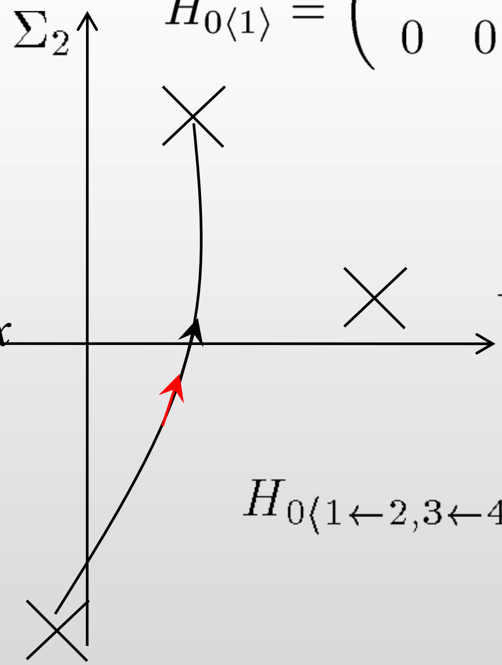
$$H_{0\langle 3\rangle} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$H_{0\langle 4\rangle} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Solution - N=2 case

- Sp(2)/U(2) case
  - Double walls





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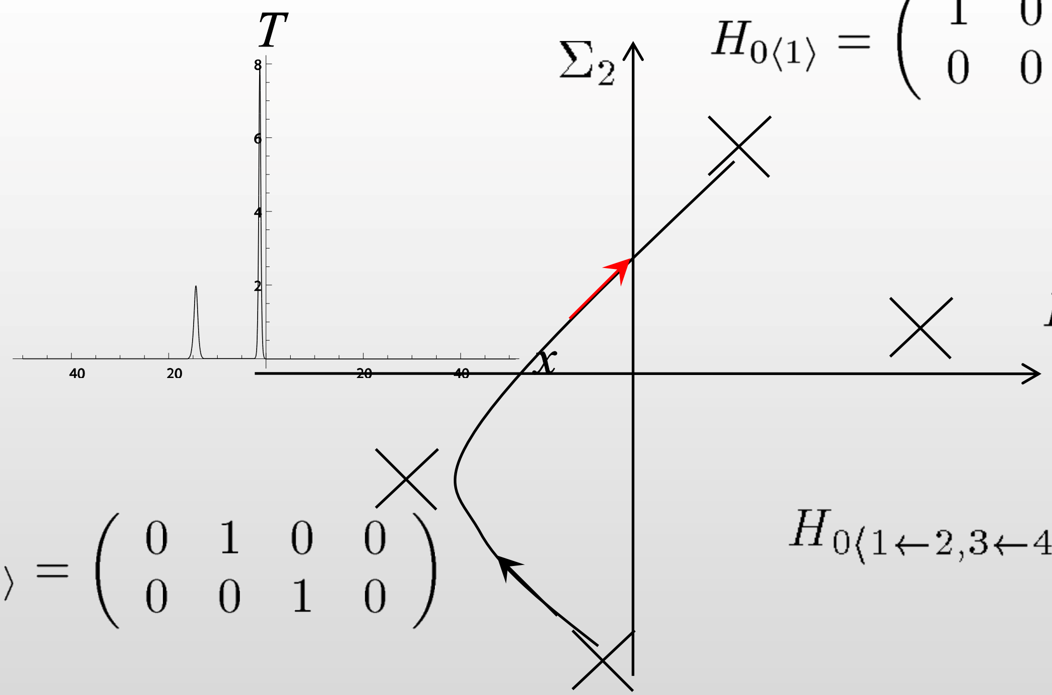
Solution - N=2 case

- Sp(2)/U(2) case
  - Double walls



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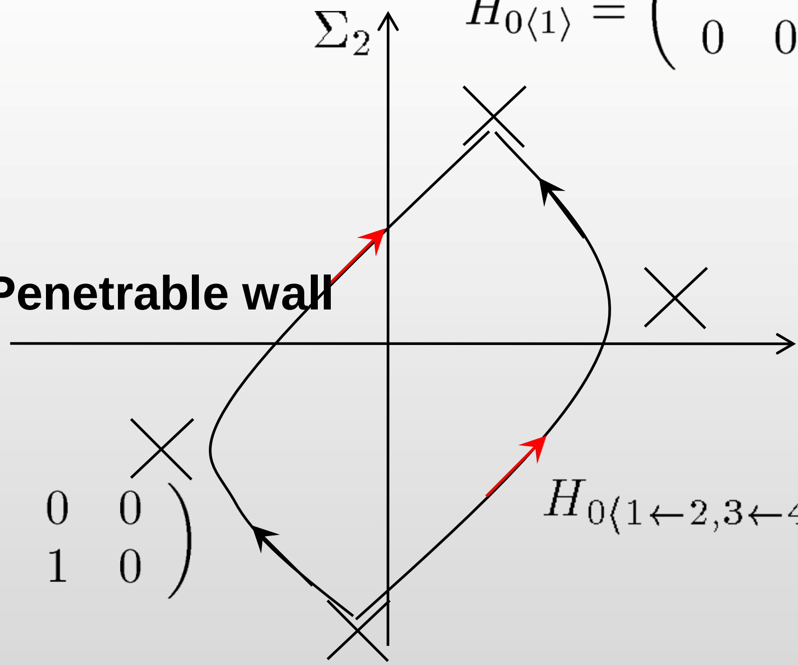
Solution - N=2 case

- Sp(2)/U(2) case
  - Double walls



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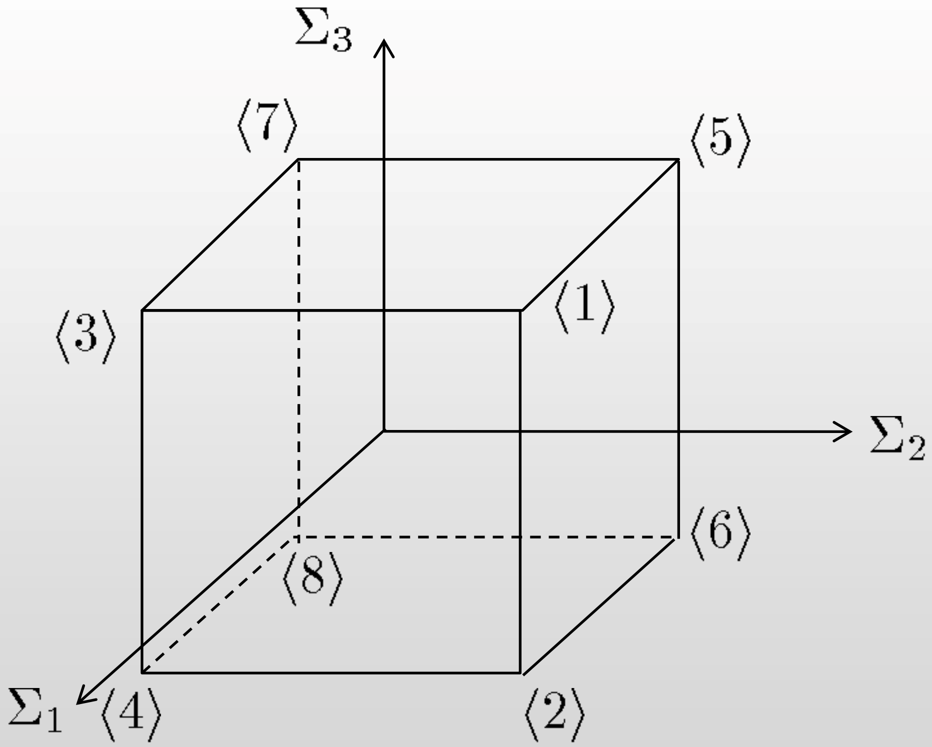
$$H_{0\langle 3\rangle} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$H_{0\langle 4\rangle} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Solution - N=3 case

- SO(6)/U(3) case

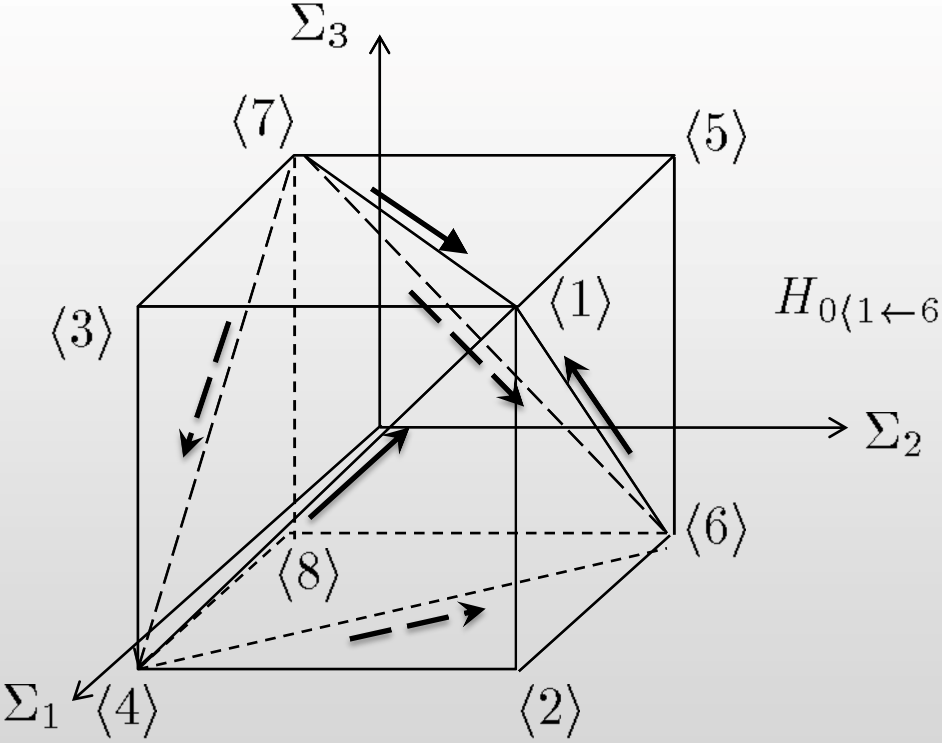


Two sets of vacua related by parity

1. <1>, <4>, <6>, <7>
2. <2>, <3>, <5>, <8>

Solution - N=3 case

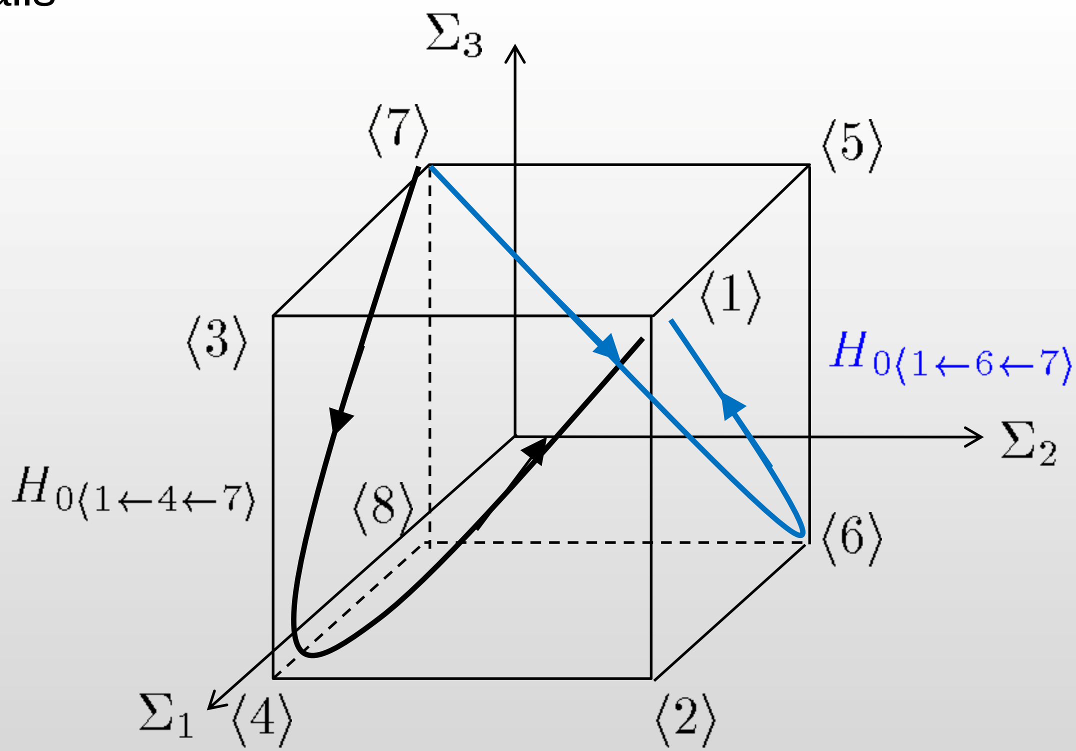
- SO(6)/U(3) case
  - single walls



$$H_{0\langle 1 \leftarrow 6 \rangle} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & -e^r \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & e^r & 0 & 0 & 1 & 0 \end{pmatrix}$$

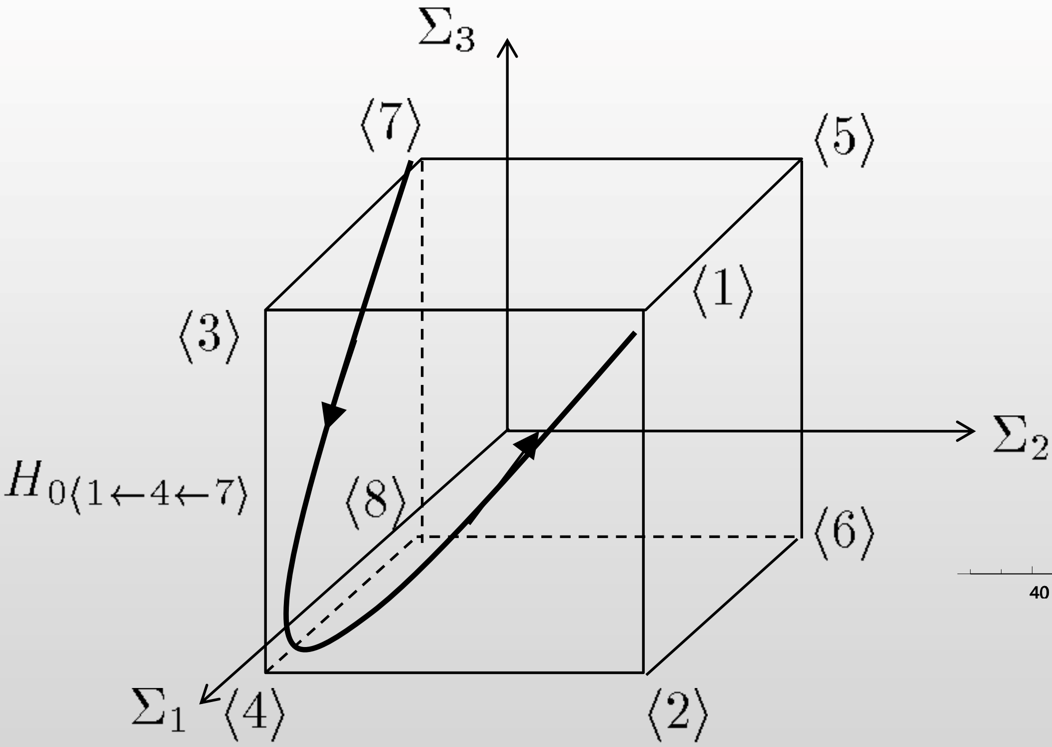
Solution - N=3 case

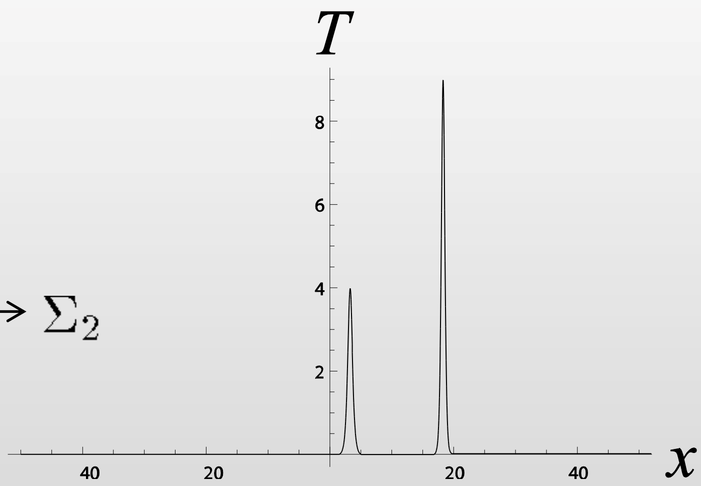
- SO(6)/U(3) case
  - Double walls



Solution - N=3 case

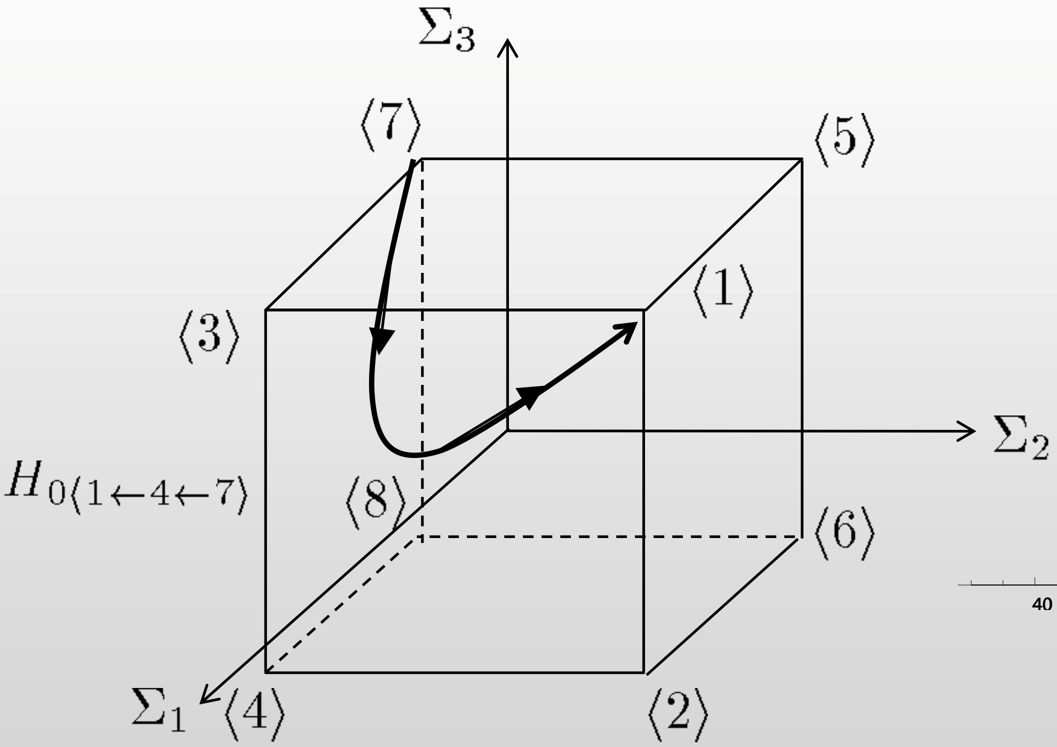
- SO(6)/U(3) case
  - Compressed walls

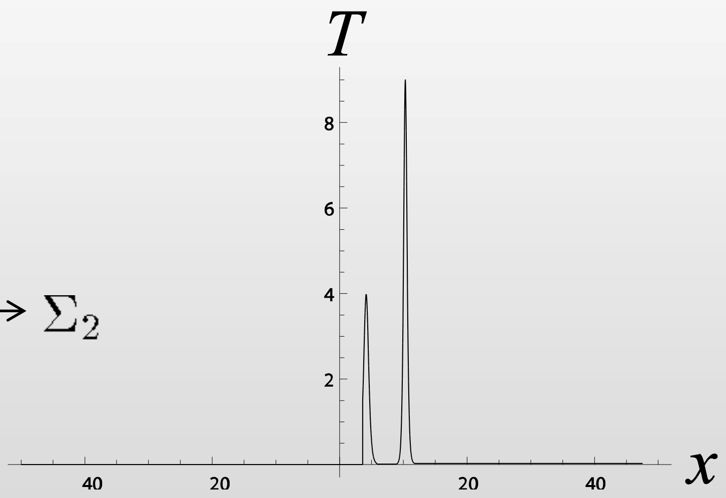




Solution - N=3 case

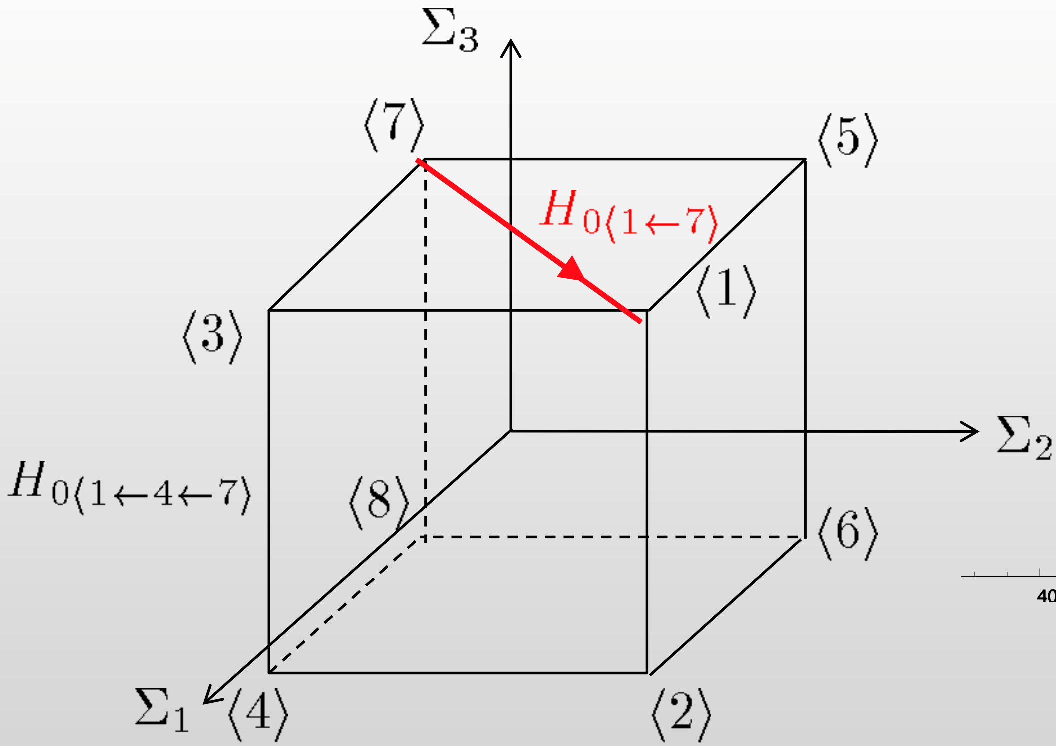
- SO(6)/U(3) case
  - Compressed walls

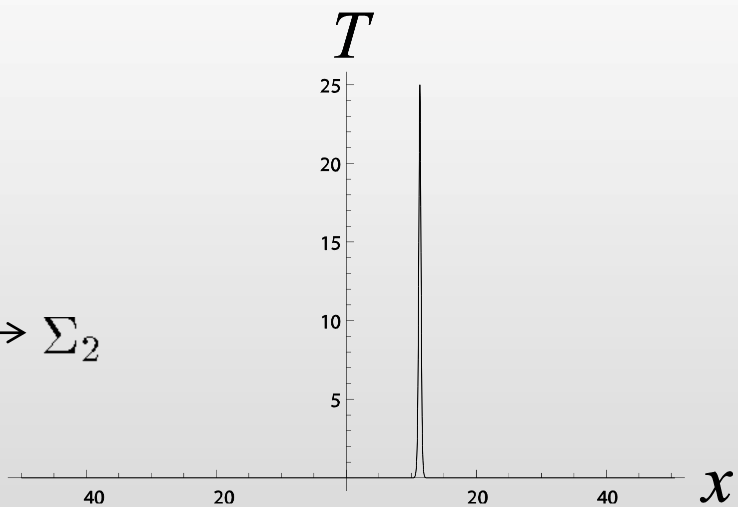




Solution - N=3 case

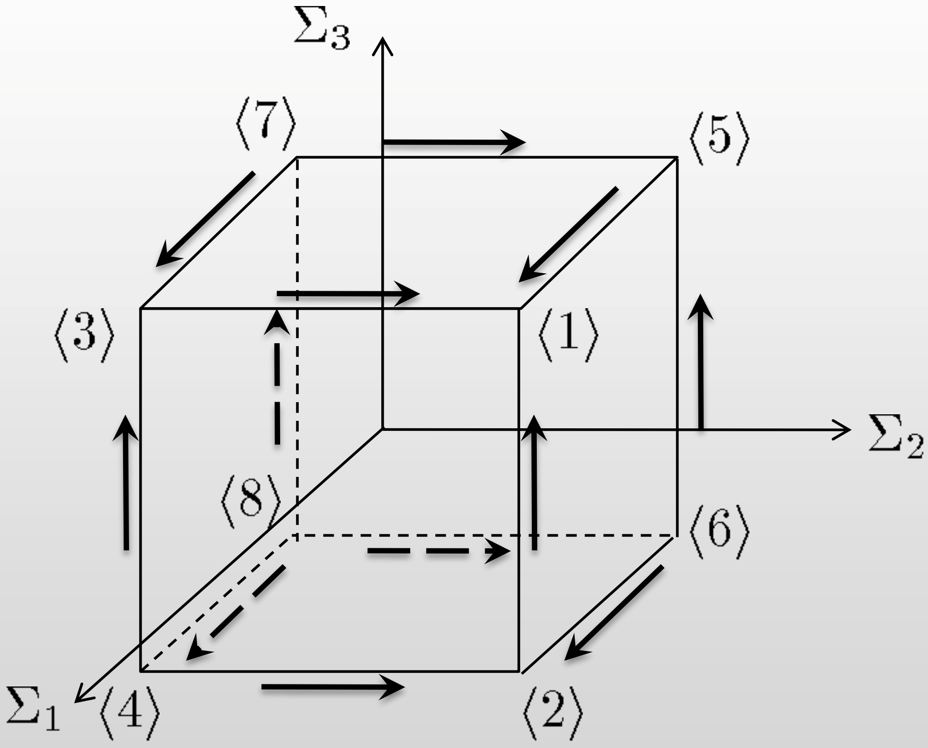
- SO(6)/U(3) case
  - Compressed walls





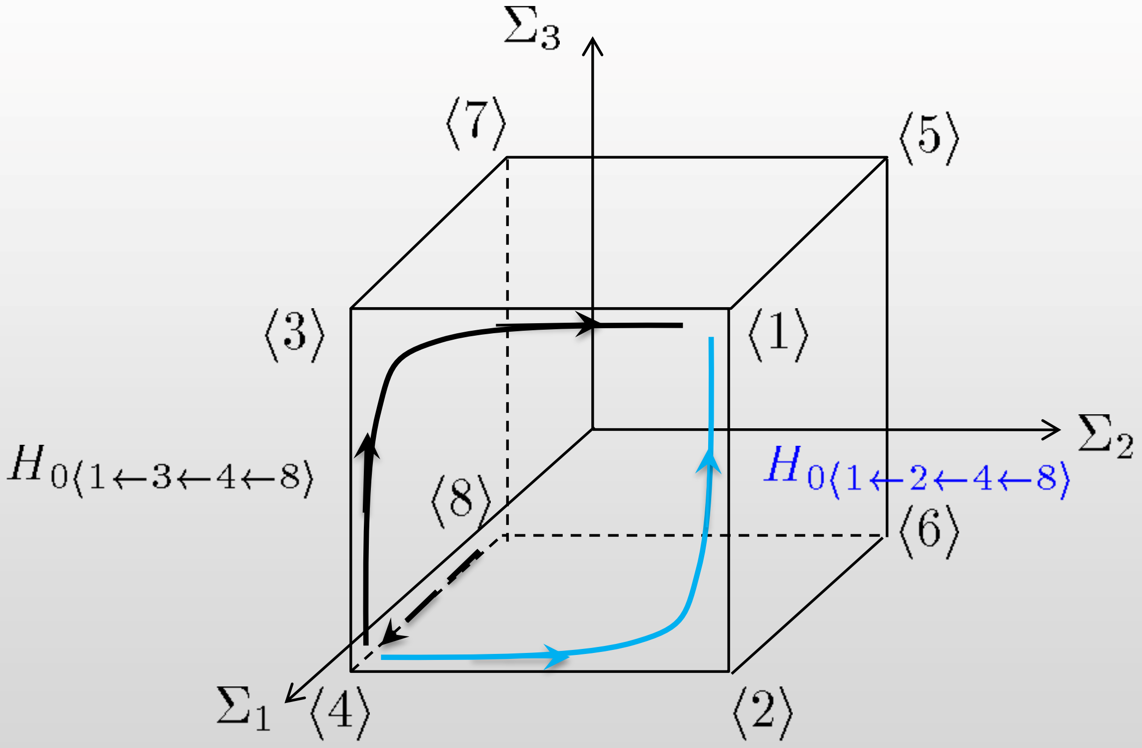
Solution - N=3 case

- Sp(3)/U(3) case
  - 8 discrete vacua, 12 single walls



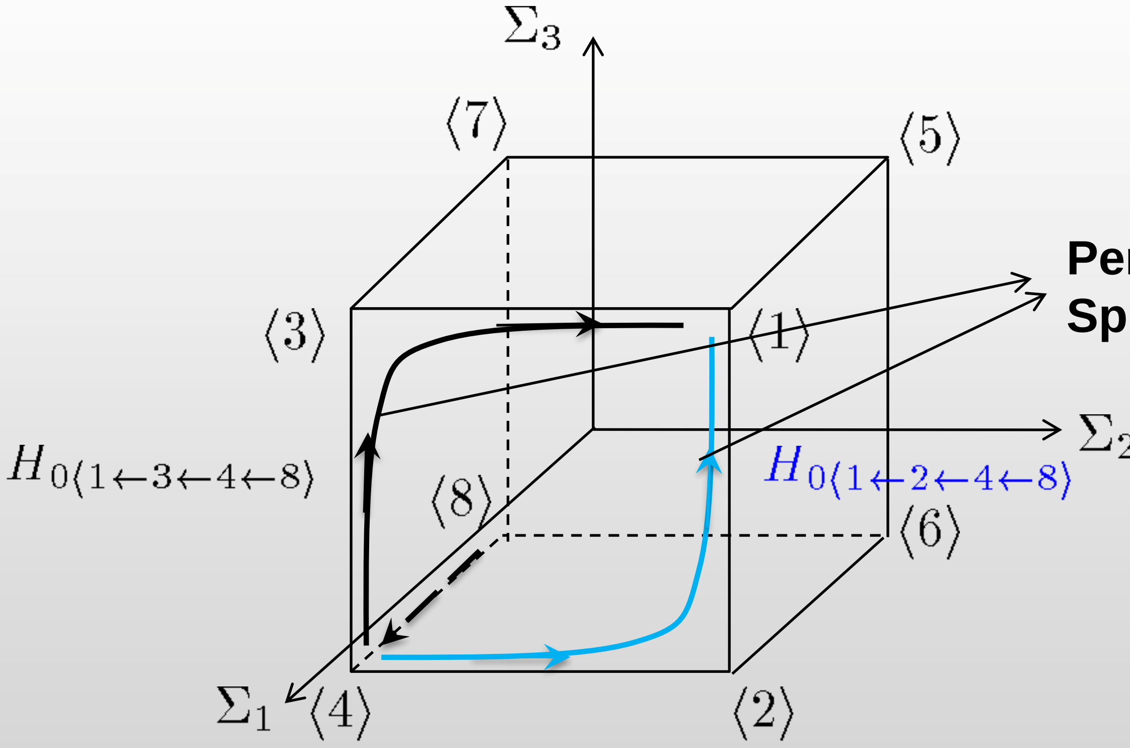
Solution - N=3 case

- Sp(3)/U(3) case
  - Double, triple walls



Solution - N=3 case

- Sp(3)/U(3) case
  - Double, triple walls



Penetrable like Sp(2)/U(2) case

Summary & Discussion

- Investigating domain walls in massive Kähler NLSM on SO(2N)/U(N) and Sp(N)/U(N) in 3-dimensional space-time.
- Found 2^{N-1} and 2^N discrete vacua in SO(2N)/U(N) and Sp(N)/U(N) models.
- BPS wall solutions
  - Deriving BPS domain wall solutions up to N=3 case.
  - Properties: Compressed wall, Penetrable walls.
- Future direction.
  - Complex solution such as wall-vortex system.