An Algorithmic Approach to String Phenomenology

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Motivation

- String Pheno is hard. We want

  (Higher dimensional geometry) → [Black Box] → ($4d$ physics)

  which will satisfy many things (i.e. $N = 1$ SUSY (broken), SM-like particle spectrum, moduli stabilization, detailed particle pheno, . . .)
- Any given model is likely to fail
- E.g. Only three heterotic models with exact MSSM spectrum in the literature.
- Rather than attempting to engineer/tune a single model, we take a different approach. Can we efficiently and systematically create billions of consistent, global models and then scan for the desired properties?
- Thanks to new advances in computer speed and computational algebraic geometry, this is a reality . . .

A heterotic model

We begin with the $E_8 \times E_8$ Heterotic string in 10-dimensions:

- The geometric ingredients include:
  - A Calabi-Yau 3-fold, X
  - A holomorphic vector bundle, V, on X (with structure group $G \subset E_8$)
- Compactifying on X leads to $\mathcal{N} = 1$ SUSY in $4D$, while V breaks $E_8 \to G \times H$, where H is the Low Energy GUT group
  - $G = SU(n)$, $n = 3, 4, 5$ leads to $H = E_6, SO(10), SU(5)$
- Matter and Moduli
  - H-charged matter, $H^1(X, V)$, $H^1(X, V^\vee)$, $H^1(X, \wedge^2 V)$, ...
  - $X \Rightarrow h^{1,1}(X)$ - Kähler moduli and $h^{2,1}(X)$ - Complex structure moduli
  - $V \Rightarrow h^1(X, V \times V^\vee)$ Bundle moduli
- Historically, V chosen to be non-Abelian (Standard embedding, Heterotic SMs, etc). Here we'll take a different approach.

"Split" Gauge configurations – Sums of line bundles

- We will consider CY 3-folds defined as Complete Intersections in products of projective spaces (CICYs) (i.e. $\mathbb{P}^4[5]$, $\left[\begin{array}{c|c} \mathbb{P}^2 & 3 \\ \mathbb{P}^2 & 3 \end{array}\right]$, ...)
- Line bundles on a CY are classified by their first Chern class, $c_1 = \frac{1}{2\pi}[trF]$
- Since $F^{1,1}$ is the only non-vanishing component, can expand

$$c_1(\mathcal{L}) = \sum_{i=1}^{h^{1,1}} c_1^i(\mathcal{L}) J_i \quad (1)$$

where c_1^i are integers and J_i are a basis of $h^{1,1}$.

- Denote: $\mathcal{L} = \mathcal{O}(c_1^i(\mathcal{L}))$
- We will consider a sum of five line bundles

$$V = \bigoplus_a \mathcal{L}_a \quad (2)$$

Conditions for a good heterotic vacuum

- Condition for spinors, $c_1(V) = 0 \mod 2$.
- Anomaly Cancellation, $c_2(TX) - c_2(V) = [\mathrm{Curve}_{\mathrm{eff}}]$
- V must solve the Hermitian Yang Mills Equations

$$g^{a\bar{b}} F_{a\bar{b}} = 0, \qquad F_{ab} = F_{\bar{a}\bar{b}} = 0$$

  - V is holomorphic (automatic)
  - The sum of line bundles should be poly-stable with slope zero (not automatic) The slope, $\mu(V)$, of a vector bundle is

    $$\mu(V) \equiv \frac{1}{\mathrm{rk}(V)} \int_X c_1(V) \wedge J \wedge J$$

    where $J = t^k J_k$ is the Kähler form on X (J_k a basis for $H^{1,1}(X)$).
  - Each individual line bundle is trivial stable (i.e satisfies $g^{a\bar{b}} F_{a\bar{b}} = 0$
  - But....each part of the sum must satisfy $\mu(\mathcal{L}) = d_{ijk} c_1^i(\mathcal{L}_a) t^j t^k = 0$ simultaneously, where d_{ijk} are the triple intersection numbers and $J = t^k J_k$

$4d$ Gauge Symmetry

- What is the structure group of a fully split bundle?
- We take $c_1(V) = c_1(\bigoplus_a \mathcal{L}_a) = \bigoplus_a c_1(\mathcal{L}_a) = 0$
- For $a = 1, \ldots 5$, we have 5 abelian pieces with one "trace" condition
- Structure group: $S[U(1)^{\times 5}] \simeq U(1)^4$
- $4d$ Gauge group is commutant inside E_8, $SU(5) \times U(1)^4$
- Even though there are only four naive low energy $U(1)$ factors, we will refer to five (redundant) abelian factors for simplicity.

The Green-Schwarz Mechanism

- By dimensional reduction, the Kähler axions transform as

$$\delta\chi^i = -c_1^i(\mathcal{L}_a)\eta^a$$

- Kähler moduli kinetic terms then lead to a mass matrix for the $U(1)$ gauge bosons

$$M_{ab} = G_{ij} c_1^i(\mathcal{L}_a) c_1^j(\mathcal{L}_b)$$

- If we require all $U(1)$'s massive, must have $h^{1,1} \geq 5$
- Kähler moduli transverse to a locus Higgs the $U(1)$ symmetries. Can be done for big enough Kähler cone.
- Associated D-terms in the $4d$ theory must be satisfied. This is $1-1$ with the slope stability condition (and vanishing slope) we have already imposed.

Algorithmic Scanning

We can now begin scanning for such "split" heterotic models. In addition to the conditions mentioned, bundles must satisfy

- To break the GUT group, need Wilson Lines (and $\pi_1(X) \neq 0$). We begin "upstairs" with simply connected CY and quotient by a freely acting discrete automorphism, Γ to form X/Γ with $\pi_1(X/\Gamma) \neq 0$.
- The "upstairs" sums of line bundles must descend "downstairs". This demands $\rightarrow$ bundle equivariance.
- Must get the right spectrum

$$h^1(X,V) = 3|\Gamma| \qquad 3\ SU(5)\ \mathbf{10}\text{ families} \tag{3}$$

$$h^1(X,V^*) = 0 \qquad \text{No } \overline{\mathbf{10}}\text{ anti-families} \tag{4}$$

- It follows that $h^1(X, \wedge^2 V) - h^1(X, \wedge^2 V^*) = 3|\Gamma|$
- Chiral asymmetry of 3 $\bar{\mathbf{5}}$'s downstairs
- Also require $h^1(X, \wedge^2 V^*) > 0 \to$ At least one Higgs $\mathbf{5}\ \bar{\mathbf{5}}$ pair before quotienting
- One additional condition (involving choice of equivariant structure and Wilson line) which ensures that all Higgs triplets are removed by the Wilson line and at least one pair of Higgs doublets survives
- So what do we get? ...

Results

- Scanned $\sim 10^{12}$ models (Desktop only. Limited only by patience. New computer improvements and resources under way)
- Scanned over 23 CICYs with $h^{1,1} = 5$ which admit freely acting discrete symmetries

  $\longrightarrow$ 180 Models (105 of these have all $U(1)$s massive)
- Also 19 CICYs which are favorable, have $h^{1,1} = 4$ and freely acting symmetries

  $\longrightarrow$ 28 Models
- No results found for the 6 favorable CICYs with $h^{1,1} = 2$ or the 12 with $h^{1,1} = 3$.
- Note: Very limited integer range scanned. Easily expanded.

CICY 6784: $\left(\begin{array}{c\|ccc} \mathbb{P}^1 & 1 & 1 & 0 \\ \mathbb{P}^1 & 0 & 0 & 2 \\ \mathbb{P}^1 & 2 & 0 & 0 \\ \mathbb{P}^3 & 1 & 1 & 2 \end{array}\right)^{4,36}_{-64}$	$\mathbb{Z}_2 \times \mathbb{Z}_2$
(3,2,-2,-1)(1,-1,0,0)(-1,0,1,0)(-1,0,1,0)(-2,-1,0,1)	(2,2,1,-1)(1,-1,0,0)(1,-1,0,0)(-1,0,-2,1)(-3,0,1,0)
(2,1,0,-1)(0,1,-3,0)(0,-2,1,1)(-1,0,1,0)(-1,0,1,0)	(2,1,-3,0)(0,1,2,-1)(0,-2,-1,1)(-1,0,1,0)(-1,0,1,0)
(1,0,-1,0)(1,0,-1,0)(1,-2,0,1)(0,1,2,-1)(-3,1,0,0)	(1,2,2,-1)(1,0,-3,0)(0,-1,1,0)(0,-1,1,0)(-2,0,-1,1)
(1,1,0,-1)(1,1,0,-1)(0,-1,-2,1)(0,-2,1,1)(-2,1,1,0)	(1,0,-1,0)(1,0,-1,0)(0,-1,1,0)(0,-1,-2,1)(-2,2,3,-1)
CICY 7435: $\left(\begin{array}{c\|ccccccc} \mathbb{P}^1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ \mathbb{P}^1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ \mathbb{P}^1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ \mathbb{P}^7 & 1 & 1 & 1 & 1 & 1 & 1 & 2 \end{array}\right)^{4,44}_{-80}$	$\mathbb{Z}_2 \times \mathbb{Z}_2$
(2,1,1,-1)(2,1,-3,0)(-1,0,1,0)(-1,0,1,0)(-2,-2,0,1)	(2,1,1,-1)(2,-3,1,0)(-1,1,0,0)(-1,1,0,0)(-2,0,-2,1)
(1,2,1,-1)(1,-1,0,0)(1,-1,0,0)(0,-2,-2,1)(-3,2,1,0)	(1,1,2,-1)(1,0,-1,0)(1,0,-1,0)(0,-2,-2,1)(-3,1,2,0)
(1,2,1,-1)(1,2,-3,0)(0,-1,1,0)(0,-1,1,0)(-2,-2,0,1)	(1,1,2,-1)(1,-3,2,0)(0,1,-1,0)(0,1,-1,0)(-2,0,-2,1)

CICY 7862: $\left(\begin{array}{c\|c} \mathbb{P}^1 & 2 \\ \mathbb{P}^1 & 2 \\ \mathbb{P}^1 & 2 \\ \mathbb{P}^1 & 2 \end{array}\right)^{4,68}_{-128}$	$\mathbb{Z}_2 \times \mathbb{Z}_2$
(1,-3,0,2)(0,1,0,-1)(0,1,0,-1)(0,0,-1,1)(-1,1,1,-1)	(1,-1,-1,1)(1,-2,0,1)(0,0,-1,1)(-1,2,2,-3)(-1,1,0,0)
(1,-1,-1,1)(1,-1,-1,1)(0,1,2,-3)(-1,1,-1,1)(-1,0,1,0)	(1,0,-2,-1)(1,-2,1,2)(0,0,1,-1)(-1,1,0,0)(-1,1,0,0)
(1,0,-2,-1)(1,-2,2,1)(0,0,1,-1)(-1,1,0,0)(-1,1,-1,1)	(1,0,-2,1)(1,-2,0,1)(0,1,1,-2)(-1,1,1,-1)(-1,0,0,1)
(1,0,-2,1)(1,-2,1,0)(0,1,1,-2)(-1,1,0,0)(-1,0,0,1)	(1,0,-1,0)(1,-3,2,0)(0,1,0,-1)(0,1,0,-1)(-2,1,-1,2)
(1,0,-3,0)(1,-2,3,0)(0,0,1,-1)(0,0,1,-1)(-2,2,-2,2)	(1,0,-1,0)(1,-1,2,-2)(1,-2,1,0)(0,1,-1,0)(-3,2,-1,2)

CICY 5256: $\left(\begin{array}{c\|cccc} \mathbb{P}^1 & 1 & 1 & 0 & 0 \\ \mathbb{P}^1 & 2 & 0 & 0 & 0 \\ \mathbb{P}^1 & 0 & 0 & 1 & 1 \\ \mathbb{P}^1 & 0 & 0 & 1 & 1 \\ \mathbb{P}^3 & 1 & 1 & 1 & 1 \end{array}\right)^{5,29}_{-48}$	$\mathbb{Z}_2$
(1,-2,0,1,0)(0,1,1,1,-1)(0,1,-1,0,0)(0,0,1,-2,0)(-1,0,-1,0,1)	(1,1,0,1,-1)(1,-2,0,0,0)(0,1,1,-2,0)(-1,1,0,1,0)(-1,-1,-1,0,1)
(1,1,0,1,-1)(1,0,1,-2,0)(0,-1,0,1,0)(0,-1,-1,0,1)(-2,1,0,0,0)	
CICY 5256: $\left(\begin{array}{c\|cccc} \mathbb{P}^1 & 1 & 1 & 0 & 0 \\ \mathbb{P}^1 & 2 & 0 & 0 & 0 \\ \mathbb{P}^1 & 0 & 0 & 1 & 1 \\ \mathbb{P}^1 & 0 & 0 & 1 & 1 \\ \mathbb{P}^3 & 1 & 1 & 1 & 1 \end{array}\right)^{5,29}_{-48}$	$\mathbb{Z}_2 \times \mathbb{Z}_2$
(1,1,0,1,-1)(0,1,-2,-2,1)(0,0,1,-1,0)(0,-2,1,1,0)(-1,0,0,1,0)	(1,0,-2,1,0)(1,-2,1,0,0)(0,1,1,-2,0)(-1,1,0,0,0)(-1,0,0,1,0)
(1,1,-2,0,0)(1,-2,0,1,0)(0,1,1,-2,0)(-1,0,1,0,0)(-1,0,0,1,0)	(1,1,0,1,-1)(1,-2,0,1,0)(0,1,-2,-2,1)(-1,0,1,0,0)(-1,0,1,0,0)
(1,1,0,1,-1)(1,-2,1,0,0)(0,1,-2,-2,1)(-1,0,1,0,0)(-1,0,0,1,0)	(1,0,-2,1,0)(1,-2,1,0,0)(0,1,0,-1,0)(0,0,1,-1,0)(-2,1,0,1,0)
(1,0,-1,0,0)(1,-2,1,0,0)(0,1,1,-2,0)(0,0,-1,1,0)(-2,1,0,1,0)	(1,0,-1,0,0)(1,-2,0,1,0)(0,1,1,-2,0)(0,1,-1,0,0)(-2,0,1,1,0)
(1,0,-2,1,0)(1,-1,0,0,0)(0,1,1,-2,0)(0,-1,1,0,0)(-2,1,0,1,0)	(1,0,0,-1,0)(1,0,-2,1,0)(0,1,0,-1,0)(0,-2,1,1,0)(-2,1,1,0,0)
(1,0,-1,0,0)(1,0,-1,0,0)(0,1,1,1,-1)(0,1,1,-2,0)(-2,-2,0,1,1)	(1,0,0,-1,0)(1,0,-2,1,0)(0,1,1,1,-1)(0,1,0,-1,0)(-2,-2,1,0,1)
(1,0,1,1,-1)(1,0,-2,1,0)(0,1,0,-1,0)(0,1,0,-1,0)(-2,-2,1,0,1)	(1,0,1,1,-1)(1,0,-1,0,0)(0,1,1,-2,0)(0,1,-1,0,0)(-2,-2,0,1,1)
(1,1,-2,0,0)(1,-1,0,0,0)(0,1,1,1,-1)(0,1,0,-1,0)(-2,-2,1,0,1)	(1,1,-2,0,0)(1,0,1,-2,0)(0,-1,1,0,0)(0,-1,0,1,0)(-2,1,0,1,0)
(1,1,-2,0,0)(1,0,1,1,-1)(1,0,0,-1,0)(-1,1,0,0,0)(-2,-2,1,0,1)	
CICY 5452: $\left(\begin{array}{c\|cccc} \mathbb{P}^1 & 1 & 1 & 0 & 0 \\ \mathbb{P}^1 & 0 & 0 & 1 & 1 \\ \mathbb{P}^1 & 2 & 0 & 0 & 0 \\ \mathbb{P}^1 & 0 & 0 & 2 & 0 \\ \mathbb{P}^3 & 1 & 1 & 1 & 1 \end{array}\right)^{5,29}_{-48}$	$\mathbb{Z}_2$
(1,1,0,-2,0)(1,0,1,1,-1)(0,0,-1,1,0)(0,-1,-1,0,1)(-2,0,1,0,0)	(1,0,1,1,-1)(1,0,-2,0,0)(0,1,1,-2,0)(-1,0,0,1,0)(-1,-1,0,0,1)
(1,1,-2,0,0)(0,1,1,1,-1)(0,0,1,-1,0)(0,-2,0,1,0)(-1,0,0,-1,1)	(1,0,-2,1,0)(0,1,1,1,-1)(0,1,0,-2,0)(0,-1,1,0,0)(-1,-1,0,0,1)

CICY 5452: $\left(\begin{array}{c\|cccc} \mathbb{P}^1 & 1 & 1 & 0 & 0 \\ \mathbb{P}^1 & 0 & 0 & 1 & 1 \\ \mathbb{P}^1 & 2 & 0 & 0 & 0 \\ \mathbb{P}^1 & 0 & 0 & 2 & 0 \\ \mathbb{P}^3 & 1 & 1 & 1 & 1 \end{array}\right)^{5,29}_{-48}$	$\mathbb{Z}_2 \times \mathbb{Z}_2$
(1,1,0,0,-1)(1,1,0,0,-1)(1,-2,0,0,1)(-1,0,-1,-1,1)(-2,0,1,1,0)	(1,1,0,0,-1)(1,1,0,0,-1)(1,-2,-1,1,1)(-1,0,1,-2,0)(-2,0,0,1,1)
(1,1,0,1,-1)(1,0,1,-2,0)(1,-1,0,0,0)(-1,0,1,0,0)(-2,0,-2,1,1)	(1,1,0,0,-1)(1,1,0,0,-1)(0,0,-1,-2,1)(0,-2,1,1,0)(-2,0,0,1,1)
(1,1,0,0,-1)(1,1,0,0,-1)(0,0,-2,-1,1)(0,-2,1,0,1)(-2,0,1,1,0)	(1,1,0,1,-1)(1,1,0,-2,0)(0,-1,1,0,0)(0,-1,1,0,0)(-2,0,-2,1,1)
(1,1,0,0,-1)(1,1,0,0,-1)(0,-1,-1,-1,1)(0,-2,1,1,0)(-2,1,0,0,1)	(1,1,0,0,-1)(1,1,0,0,-1)(0,-1,-2,1,0)(0,-2,1,0,1)(-2,1,1,-1,1)
(1,1,0,-2,0)(1,0,-1,0,0)(0,0,-1,1,0)(0,-2,1,1,0)(-2,1,1,0,0)	(1,1,-2,0,0)(1,0,0,-1,0)(0,0,1,-1,0)(0,-2,1,1,0)(-2,1,0,1,0)
(1,1,0,-2,0)(1,0,-2,1,0)(0,-1,1,0,0)(0,-1,0,1,0)(-2,1,1,0,0)	(1,1,0,1,-1)(1,-1,0,0,0)(0,1,1,-2,0)(0,-1,1,0,0)(-2,0,-2,1,1)
(1,1,0,-2,0)(1,-1,0,0,0)(0,1,1,1,-1)(0,-1,1,0,0)(-2,0,-2,1,1)	(1,1,0,-2,0)(1,-1,0,0,0)(0,1,-2,1,0)(0,-1,1,0,0)(-2,0,1,1,0)
(1,1,-2,0,0)(1,-1,0,0,0)(0,1,1,-2,0)(0,-1,0,1,0)(-2,0,1,1,0)	(1,1,0,-2,0)(1,-2,1,0,0)(0,1,-1,0,0)(0,0,-1,1,0)(-2,0,1,1,0)
(1,1,-2,0,0)(1,-2,0,1,0)(0,1,0,-1,0)(0,0,1,-1,0)(-2,0,1,1,0)	(1,0,1,-2,0)(1,0,-1,0,0)(0,1,1,1,-1)(0,-1,1,0,0)(-2,0,-2,1,1)
(1,0,0,-1,0)(1,0,-2,1,0)(0,1,0,-1,0)(0,-2,1,1,0)(-2,1,1,0,0)	(1,0,-2,1,0)(1,-1,0,0,0)(0,1,1,-2,0)(0,-1,1,0,0)(-2,1,0,1,0)
(1,0,-1,0,0)(1,-2,0,1,0)(0,1,1,-2,0)(0,1,-1,0,0)(-2,0,1,1,0)	(1,0,1,-2,0)(1,-2,0,1,0)(0,1,-1,0,0)(0,0,-1,1,0)(-2,1,1,0,0)
(1,0,0,-1,0)(1,-2,0,1,0)(0,1,-2,1,0)(0,0,1,-1,0)(-2,1,1,0,0)	(1,-1,0,0,0)(1,-1,0,0,0)(0,1,1,1,-1)(0,1,1,-2,0)(-2,0,-2,1,1)
(1,1,1,0,-1)(1,1,-2,0,0)(0,-2,1,-2,1)(-1,0,0,1,0)(-1,0,0,1,0)	(1,1,1,0,-1)(1,0,-2,1,0)(0,-2,1,-2,1)(-1,1,0,0,0)(-1,0,0,1,0)
(1,1,0,-2,0)(1,0,-2,1,0)(0,-2,1,1,0)(-1,1,0,0,0)(-1,0,1,0,0)	(1,1,-2,0,0)(1,0,1,1,-1)(0,-2,1,-2,1)(-1,1,0,0,0)(-1,0,0,1,0)
(1,1,-2,0,0)(1,0,1,-2,0)(0,-2,1,1,0)(-1,1,0,0,0)(-1,0,0,1,0)	(1,1,-2,0,0)(1,-2,0,1,0)(0,1,1,-2,0)(-1,0,1,0,0)(-1,0,0,1,0)
(1,0,1,1,-1)(1,0,-2,1,0)(0,-2,1,-2,1)(-1,1,0,0,0)(-1,1,0,0,0)	(1,0,-2,1,0)(1,-2,1,0,0)(0,1,1,-2,0)(-1,1,0,0,0)(-1,0,0,1,0)
(1,1,1,0,-1)(0,1,-2,1,0)(0,-1,0,1,0)(0,-2,1,-2,1)(-1,1,0,0,0)	(1,0,1,1,-1)(0,1,0,-1,0)(0,1,-2,1,0)(0,-2,1,-2,1)(-1,0,0,1,0)

CICY 6947: $\left(\begin{array}{c\|cccccccc} \mathbb{P}^1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \mathbb{P}^1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ \mathbb{P}^1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ \mathbb{P}^1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ \mathbb{P}^7 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{array}\right)^{5,37}_{-64}$	$\mathbb{Z}_2$
(1,0,1,1,-1)(1,0,1,1,-1)(1,-2,0,-2,1)(-1,1,-2,-1,1)(-2,1,0,1,0)	(1,1,0,1,-1)(1,1,0,1,-1)(0,1,-2,-2,1)(0,-2,1,1,0)(-2,-1,1,-1,1)
(1,0,1,1,-1)(1,0,1,1,-1)(0,0,-1,-2,1)(0,-1,-1,-1,1)(-2,1,0,1,0)	(1,1,0,1,-1)(1,1,0,1,-1)(0,-2,1,1,0)(-1,0,0,-2,1)(-1,0,-1,-1,1)

CICY 6947: $\left(\begin{array}{c\|cccccccc} \mathbb{P}^1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \mathbb{P}^1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ \mathbb{P}^1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ \mathbb{P}^1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ \mathbb{P}^7 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{array}\right)^{5,37}_{-64}$	$\mathbb{Z}_2 \times \mathbb{Z}_2$
(1,1,-2,-2,1)(1,0,1,1,-1)(1,0,1,1,-1)(-1,-2,0,-1,1)(-2,1,0,1,0)	(1,1,0,1,-1)(1,1,0,1,-1)(0,-1,-2,-1,1)(0,-2,1,1,0)(-2,1,1,-2,1)
(1,1,0,-2,0)(1,0,-1,0,0)(0,0,-1,1,0)(0,-2,1,1,0)(-2,1,1,0,0)	(1,1,-2,0,0)(1,0,0,-1,0)(0,0,1,-1,0)(0,-2,1,1,0)(-2,1,0,1,0)
(1,1,0,-2,0)(1,0,-2,1,0)(0,-1,1,0,0)(0,-1,0,1,0)(-2,1,1,0,0)	(1,1,-2,0,0)(1,0,1,-2,0)(0,-1,1,0,0)(0,-1,0,1,0)(-2,1,0,1,0)
(1,1,0,-2,0)(1,-1,0,0,0)(0,1,-2,1,0)(0,-1,1,0,0)(-2,0,1,1,0)	(1,1,-2,0,0)(1,-1,0,0,0)(0,1,1,-2,0)(0,-1,0,1,0)(-2,0,1,1,0)
(1,1,0,-2,0)(1,-2,1,0,0)(0,1,-1,0,0)(0,0,-1,1,0)(-2,0,1,1,0)	(1,1,-2,0,0)(1,-2,0,1,0)(0,1,0,-1,0)(0,0,1,-1,0)(-2,0,1,1,0)
(1,0,1,-2,0)(1,0,-1,0,0)(0,1,-1,0,0)(0,-2,1,1,0)(-2,1,0,1,0)	(1,0,0,-1,0)(1,0,-2,1,0)(0,1,0,-1,0)(0,-2,1,1,0)(-2,1,1,0,0)
(1,0,1,-2,0)(1,-1,0,0,0)(0,1,-2,1,0)(0,-1,0,1,0)(-2,1,1,0,0)	(1,0,-2,1,0)(1,-1,0,0,0)(0,1,1,-2,0)(0,-1,1,0,0)(-2,1,0,1,0)
(1,0,0,-1,0)(1,-2,1,0,0)(0,1,0,-1,0)(0,1,-2,1,0)(-2,0,1,1,0)	(1,0,-1,0,0)(1,-2,0,1,0)(0,1,1,-2,0)(0,1,-1,0,0)(-2,0,1,1,0)
(1,0,1,-2,0)(1,-2,0,1,0)(0,1,-1,0,0)(0,0,-1,1,0)(-2,1,1,0,0)	(1,0,0,-1,0)(1,-2,0,1,0)(0,1,-2,1,0)(0,0,1,-1,0)(-2,1,1,0,0)
(1,0,-1,0,0)(1,-2,1,0,0)(0,1,1,-2,0)(0,0,-1,1,0)(-2,1,0,1,0)	(1,0,-2,1,0)(1,-2,1,0,0)(0,1,0,-1,0)(0,0,1,-1,0)(-2,1,0,1,0)
(1,1,0,-2,0)(1,0,-2,1,0)(0,-2,1,1,0)(-1,1,0,0,0)(-1,0,1,0,0)	(1,1,-2,0,0)(1,0,1,-2,0)(0,-2,1,1,0)(-1,1,0,0,0)(-1,0,0,1,0)
(1,1,0,-2,0)(1,-2,1,0,0)(0,1,-2,1,0)(-1,0,1,0,0)(-1,0,0,1,0)	(1,1,-2,0,0)(1,-2,0,1,0)(0,1,1,-2,0)(-1,0,1,0,0)(-1,0,0,1,0)
(1,0,1,-2,0)(1,-2,0,1,0)(0,1,-2,1,0)(-1,1,0,0,0)(-1,0,1,0,0)	(1,0,-2,1,0)(1,-2,1,0,0)(0,1,1,-2,0)(-1,1,0,0,0)(-1,0,0,1,0)
CICY 6732: $\left(\begin{array}{c\|cccccc} \mathbb{P}^1 & 1 & 1 & 0 & 0 & 0 & 0 \\ \mathbb{P}^1 & 0 & 0 & 1 & 1 & 0 & 0 \\ \mathbb{P}^1 & 0 & 0 & 0 & 0 & 1 & 1 \\ \mathbb{P}^1 & 0 & 0 & 0 & 0 & 2 & 0 \\ \mathbb{P}^5 & 1 & 1 & 1 & 1 & 1 & 1 \end{array}\right)^{5,37}_{-64}$	$\mathbb{Z}_2$
(1,1,1,0,-1)(1,0,-2,1,0)(1,-1,0,0,0)(-1,1,1,-1,0)(-2,-1,0,0,1)	(1,1,1,0,-1)(1,-1,1,-1,0)(1,-1,0,0,0)(-1,0,-2,0,1)(-2,1,0,1,0)
(1,1,1,0,-1)(1,1,-1,1,-1)(0,0,-2,1,0)(0,-2,1,-1,1)(-2,0,1,-1,1)	(1,1,0,-2,0)(1,0,1,1,-1)(0,1,-1,0,0)(0,-1,0,1,0)(-2,-1,0,0,1)
(1,1,-2,0,0)(1,0,1,1,-1)(0,1,0,-1,0)(0,-1,1,0,0)(-2,-1,0,0,1)	(1,1,1,0,-1)(1,-1,0,0,0)(0,1,0,-2,0)(0,0,-1,1,0)(-2,-1,0,1,1)
(1,1,1,0,-1)(1,-1,0,0,0)(0,1,-2,1,0)(0,0,1,-1,0)(-2,-1,0,0,1)	(1,1,0,1,-1)(1,-1,1,-1,0)(0,1,-2,1,0)(0,0,1,-1,0)(-2,-1,0,0,1)
(1,1,0,1,-1)(1,-1,0,0,0)(0,1,-2,0,0)(0,0,1,-1,0)(-2,-1,1,0,1)	(1,1,0,1,-1)(1,-1,1,-1,0)(0,0,-1,1,0)(0,-1,0,-2,1)(-2,1,0,1,0)
(1,1,0,-2,0)(1,0,-1,0,0)(0,1,1,1,-1)(-1,0,0,1,0)(-1,-2,0,0,1)	(1,1,-2,0,0)(1,0,0,-1,0)(0,1,1,1,-1)(-1,0,1,0,0)(-1,-2,0,0,1)
(1,1,1,0,-1)(1,0,0,-2,0)(0,0,-1,1,0)(-1,1,0,0,0)(-1,-2,0,1,1)	(1,1,1,0,-1)(1,0,-2,1,0)(0,0,1,-1,0)(-1,1,0,0,0)(-1,-2,0,0,1)
(1,1,0,1,-1)(1,0,-2,1,0)(0,0,1,-1,0)(-1,1,1,-1,0)(-1,-2,0,0,1)	(1,1,0,1,-1)(1,0,-2,0,0)(0,0,1,-1,0)(-1,1,0,0,0)(-1,-2,1,0,1)
(1,1,1,0,-1)(1,-1,1,-1,0)(0,1,-2,1,0)(-1,1,0,0,0)(-1,-2,0,0,1)	(1,1,1,0,-1)(1,-1,0,0,0)(0,0,1,-2,0)(-1,0,0,1,0)(-1,0,-2,1,1)
(1,1,0,1,-1)(1,-2,0,1,0)(0,0,-1,1,0)(-1,1,1,-1,0)(-1,0,0,-2,1)	(1,1,1,0,-1)(1,-2,0,1,0)(0,-1,-2,0,1)(-1,1,1,-1,0)(-1,1,0,0,0)
(1,1,1,0,-1)(0,0,1,-2,0)(0,-1,0,1,0)(0,-1,-2,1,1)(-1,1,0,0,0)	

CICY 6770: $\left(\begin{array}{c\|cc} \mathbb{P}^1 & 1 & 1 \\ \mathbb{P}^1 & 1 & 1 \\ \mathbb{P}^1 & 1 & 1 \\ \mathbb{P}^1 & 2 & 0 \\ \mathbb{P}^1 & 0 & 2 \end{array}\right)^{5,37}_{-64}$	$\mathbb{Z}_2$
(1,-2,0,0,0)(0,1,1,-2,0)(0,1,0,0,-1)(0,-1,-1,1,1)(-1,1,0,1,0)	(1,-2,0,0,0)(0,1,1,0,-2)(0,1,0,-1,0)(0,-1,-1,1,1)(-1,1,0,0,1)
(1,-2,0,0,1)(0,1,0,0,-2)(0,1,-1,1,0)(0,0,0,-1,1)(-1,0,1,0,0)	(1,-2,0,1,0)(0,1,0,-2,0)(0,1,-1,0,1)(0,0,0,1,-1)(-1,0,1,0,0)
(1,-2,0,0,0)(0,1,1,-2,0)(0,1,0,0,-1)(0,0,-1,1,1)(-1,0,0,1,0)	(1,-2,0,0,0)(0,1,1,0,-2)(0,1,0,-1,0)(0,0,-1,1,1)(-1,0,0,0,1)
(1,-1,-1,1,1)(0,1,0,-2,1)(0,0,1,0,-2)(0,-1,0,1,0)(-1,1,0,0,0)	(1,-1,-1,1,1)(0,1,0,1,-2)(0,0,1,-2,0)(0,-1,0,0,1)(-1,1,0,0,0)
(1,-1,0,0,0)(0,1,0,-2,1)(0,0,1,0,-2)(0,0,-1,1,1)(-1,0,0,1,0)	(1,-1,0,0,0)(0,1,0,1,-2)(0,0,1,-2,0)(0,0,-1,1,1)(-1,0,0,0,1)
(1,-1,1,-1,1)(0,1,0,0,-2)(0,0,-1,1,0)(0,0,-1,0,1)(-1,0,1,0,0)	(1,-1,1,1,-1)(0,1,0,-2,0)(0,0,-1,1,0)(0,0,-1,0,1)(-1,0,1,0,0)
(1,0,0,-2,0)(0,1,-2,0,1)(0,-1,1,1,-1)(0,-1,1,0,0)(-1,1,0,1,0)	(1,0,0,0,-2)(0,1,-2,1,0)(0,-1,1,0,0)(0,-1,1,-1,1)(-1,1,0,0,1)
(1,0,0,-2,0)(0,1,-1,0,0)(0,0,1,0,-1)(0,-2,1,1,0)(-1,1,-1,1,1)	(1,0,0,0,-2)(0,1,-1,0,0)(0,0,1,-1,0)(0,-2,1,0,1)(-1,1,-1,1,1)
CICY 6777: $\left(\begin{array}{c\|cccc} \mathbb{P}^1 & 1 & 1 & 0 & 0 \\ \mathbb{P}^1 & 0 & 0 & 0 & 2 \\ \mathbb{P}^1 & 0 & 0 & 2 & 0 \\ \mathbb{P}^1 & 2 & 0 & 0 & 0 \\ \mathbb{P}^3 & 1 & 1 & 1 & 1 \end{array}\right)^{5,37}_{-64}$	$\mathbb{Z}_2$
(1,1,1,0,-1)(1,0,-2,-1,1)(1,-2,0,-1,1)(-1,1,1,1,-1)(-2,0,0,1,0)	(1,1,0,1,-1)(1,0,-1,0,0)(0,0,1,-1,0)(0,-2,-1,0,1)(-2,1,1,0,0)
(1,0,1,1,-1)(1,-1,0,0,0)(0,1,0,-1,0)(0,-1,-2,0,1)(-2,1,1,0,0)	(1,0,0,-1,0)(1,-1,-2,0,1)(0,1,1,1,-1)(0,-1,1,0,0)(-2,1,0,0,0)
(1,0,0,-1,0)(1,-2,-1,0,1)(0,1,1,1,-1)(0,1,-1,0,0)(-2,0,1,0,0)	(1,1,1,0,-1)(0,1,-1,0,0)(0,0,1,-2,0)(0,-2,-1,1,1)(-1,0,0,1,0)
(1,1,0,1,-1)(0,1,1,-2,0)(0,0,-1,1,0)(0,-2,-1,0,1)(-1,0,1,0,0)	(1,1,-1,-1,0)(0,1,1,1,-1)(0,0,-1,-2,1)(0,-2,1,1,0)(-1,0,0,1,0)
(1,1,1,0,-1)(0,1,0,-2,0)(0,-1,1,0,0)(0,-1,-2,1,1)(-1,0,0,1,0)	(1,0,1,1,-1)(0,1,1,-2,0)(0,-1,0,1,0)(0,-1,-2,0,1)(-1,1,0,0,0)
(1,-1,1,-1,0)(0,1,1,1,-1)(0,1,-2,1,0)(0,-1,0,-2,1)(-1,0,0,1,0)	
CICY 6890: $\left(\begin{array}{c\|ccccc} \mathbb{P}^1 & 1 & 1 & 0 & 0 & 0 \\ \mathbb{P}^1 & 0 & 0 & 1 & 1 & 0 \\ \mathbb{P}^1 & 0 & 0 & 0 & 0 & 2 \\ \mathbb{P}^1 & 0 & 0 & 2 & 0 & 0 \\ \mathbb{P}^4 & 1 & 1 & 1 & 1 & 1 \end{array}\right)^{5,37}_{-64}$	$\mathbb{Z}_2$
(1,1,1,0,-1)(1,0,-1,0,0)(1,-2,0,1,0)(-1,1,1,-1,0)(-2,0,-1,0,1)	(1,1,1,0,-1)(1,0,-1,0,0)(0,0,1,-2,0)(0,-1,0,1,0)(-2,0,-1,1,1)
(1,1,0,1,-1)(1,0,1,-2,0)(0,0,-1,1,0)(0,-1,1,0,0)(-2,0,-1,0,1)	(1,1,-1,-1,0)(1,0,1,1,-1)(0,0,-1,-2,1)(0,-1,0,1,0)(-2,0,1,1,0)

(1,1,1,0,-1)(1,-1,1,1,-1)(0,1,-2,-1,1)(0,-2,0,1,0)(-2,1,0,-1,1)	(1,1,0,1,-1)(1,-2,1,0,0)(0,1,-1,0,0)(0,0,1,-1,0)(-2,0,-1,0,1)
(1,0,1,1,-1)(1,0,-1,0,0)(0,1,0,-1,0)(0,-2,1,0,0)(-2,1,-1,0,1)	(1,1,1,0,-1)(1,0,0,-2,0)(0,-1,0,1,0)(-1,0,1,0,0)(-1,0,-2,1,1)
(1,1,1,0,-1)(1,0,-2,1,0)(0,-2,-1,0,1)(-1,1,1,-1,0)(-1,0,1,0,0)	(1,1,1,0,-1)(1,-2,0,1,0)(0,1,0,-1,0)(-1,0,1,0,0)(-1,0,-2,0,1)
(1,0,1,1,-1)(1,0,-2,1,0)(0,-1,0,1,0)(-1,1,1,-1,0)(-1,0,0,-2,1)	(1,0,1,-2,0)(1,-1,0,0,0)(0,1,1,1,-1)(-1,0,0,1,0)(-1,0,-2,0,1)
(1,0,1,1,-1)(1,-2,0,1,0)(0,1,0,-1,0)(-1,1,1,-1,0)(-1,0,-2,0,1)	(1,0,1,1,-1)(1,-2,0,0,0)(0,1,0,-1,0)(-1,1,-2,0,1)(-1,0,1,0,0)
(1,0,0,-1,0)(1,-2,1,0,0)(0,1,1,1,-1)(-1,1,0,0,0)(-1,0,-2,0,1)	(1,1,1,0,-1)(0,1,0,-2,0)(0,0,-1,1,0)(0,-2,-1,1,1)(-1,0,1,0,0)
CICY 7447: $\left(\begin{array}{c\|cc} \mathbb{P}^1 & 1 & 1 \\ \mathbb{P}^1 & 1 & 1 \\ \mathbb{P}^1 & 1 & 1 \\ \mathbb{P}^1 & 1 & 1 \\ \mathbb{P}^1 & 1 & 1 \end{array}\right)^{5,45}_{-80}$	$\mathbb{Z}_2 \times \mathbb{Z}_2$
(0,1,0,-2,1)(0,1,-2,1,0)(0,0,1,1,-2)(0,-1,1,0,0)(0,-1,0,0,1)	(1,-2,0,0,1)(0,1,-2,0,1)(0,0,1,1,-2)(0,0,1,-1,0)(-1,1,0,0,0)
(1,-2,0,0,1)(0,1,0,1,-2)(0,0,1,-2,1)(0,0,-1,0,1)(-1,1,0,1,-1)	(1,-2,-1,1,1)(0,1,1,-2,0)(0,1,-1,0,0)(0,0,1,1,-2)(-1,0,0,0,1)
CICY 7487: $\left(\begin{array}{c\|cc} \mathbb{P}^1 & 0 & 2 \\ \mathbb{P}^1 & 1 & 1 \\ \mathbb{P}^1 & 1 & 1 \\ \mathbb{P}^1 & 1 & 1 \\ \mathbb{P}^1 & 1 & 1 \end{array}\right)^{5,45}_{-80}$	$\mathbb{Z}_2 \times \mathbb{Z}_2$
(1,-2,0,0,1)(0,1,-2,0,1)(0,0,1,1,-2)(0,0,1,-1,0)(-1,1,0,0,0)	(1,-2,0,0,1)(0,1,0,1,-2)(0,0,1,-2,1)(0,0,-1,0,1)(-1,1,0,1,-1)
(1,0,-2,0,1)(1,-2,1,0,0)(0,1,1,0,-2)(-1,1,0,0,0)(-1,0,0,0,1)	(1,-1,-1,0,1)(1,-2,0,0,1)(0,1,0,1,-2)(0,1,0,-1,0)(-2,1,1,0,0)
(1,-1,-1,1,0)(1,-1,-1,0,1)(0,1,0,-2,1)(0,0,1,1,-2)(-2,1,1,0,0)	(1,-1,0,0,0)(1,-1,-1,0,1)(0,1,0,-2,1)(0,0,1,1,-2)(-2,1,0,1,0)
(1,-1,1,-1,0)(1,-1,-2,1,1)(0,1,1,-2,0)(0,0,0,1,-1)(-2,1,0,1,0)	(1,-1,1,-1,0)(1,-1,-2,1,1)(0,1,1,0,-2)(0,0,0,-1,1)(-2,1,0,1,0)
(1,0,-2,0,1)(1,-2,0,1,0)(0,1,0,-1,0)(0,0,1,0,-1)(-2,1,1,0,0)	(1,0,-2,0,1)(1,-2,0,1,0)(0,1,0,0,-1)(0,0,1,-1,0)(-2,1,1,0,0)
(1,0,-2,0,1)(1,-2,1,0,0)(0,1,0,-1,0)(0,0,1,0,-1)(-2,1,0,1,0)	(1,0,-2,0,1)(1,-2,1,0,0)(0,1,0,0,-1)(0,0,1,-1,0)(-2,1,0,1,0)
(1,0,-2,0,1)(1,-2,1,0,0)(0,1,0,0,-1)(0,0,1,0,-1)(-2,1,0,0,1)	(1,0,-2,0,1)(1,-2,1,1,-1)(0,1,0,-1,0)(0,0,1,0,-1)(-2,1,0,0,1)
(1,0,-2,0,1)(1,-2,1,1,-1)(0,1,0,0,-1)(0,0,1,-1,0)(-2,1,0,0,1)	(1,0,-1,0,0)(1,-2,0,0,1)(0,1,1,-2,0)(0,0,0,1,-1)(-2,1,0,1,0)
(1,0,-1,0,0)(1,-2,0,0,1)(0,1,1,0,-2)(0,0,0,-1,1)(-2,1,0,1,0)	(1,0,-1,0,0)(1,-2,1,0,0)(0,1,0,-2,1)(0,0,0,1,-1)(-2,1,0,1,0)
(1,0,-1,0,0)(1,-2,1,0,0)(0,1,1,-2,0)(0,0,-1,1,0)(-2,1,0,1,0)	(1,0,-1,0,0)(1,-2,1,0,0)(0,1,1,-2,0)(0,0,0,1,-1)(-2,1,-1,1,1)
(1,0,-1,0,0)(1,-2,0,0,1)(0,1,0,1,-2)(0,1,0,-1,0)(-2,0,1,0,1)	(1,0,-1,0,0)(1,-2,0,0,1)(0,1,1,0,-2)(0,1,-1,0,0)(-2,0,1,0,1)
(1,0,-1,0,0)(1,-2,1,-1,1)(0,1,0,1,-2)(0,1,0,-1,0)(-2,0,0,1,1)	(1,0,0,-1,0)(1,0,-2,0,1)(1,-2,0,1,0)(-1,1,1,0,-1)(-2,1,1,0,0)

Constraints from $U(1)$ Symmetries

Although Green-Schwarz $U(1)$ symmetries are all broken, global remnants remain. Examples include:

- Residual effects constrain both the perturbative and non-perturbative Lagrangian in an easy to calculate way
- Important for pheno and computability (i.e. Kähler potential)
- This persists even in the veved configurations
  - Can lead to interesting hierarchies
  - Prevent proton decay operators
  - Or be dangerously restrictive?

Conclusions and future work

- It is possible to build phenomenologically relevant heterotic models using the simplest possible gauge configurations – sums of line bundles.
- Split gauge configurations give rise to Green-Schwarz anomalous $U(1)$ symmetries $\rightarrow$ can constrain lagrangian, phenomenology

The plan for the future:

- Combine this work with broader algorithmic program
  - Moduli stabilization scenarios (including using the gauge bundle itself to fix the complex structure moduli, while remaining a CY background)
  - Numerically determining the CY metric, g and gauge field, F allows us to compute Kähler metric, normalized Yukawa couplings
  - Soft susy-breaking, masses

The End