# Fun with F-theory GUTs and $U(1)$ Symmetries

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- Physics scenarios that extend the MSSM typically require some mechanism that controls the superpotential
- Standard Model has nice accidental Baryon number and Lepton number symmetries

$$\mathcal{L} \sim qhu^c + qh^\dagger d^c + \ell h^\dagger e^c$$

$$\frac{1}{M^2} qqq\ell + \frac{1}{M^2} u^c u^c d^c e^c \text{ forbidden}$$

| | $B$ | $L$ |
|---|---|---|
| $q$ | $\frac{1}{3}$ | 0 |
| $u^c$ | $-\frac{1}{3}$ | 0 |
| $d^c$ | $-\frac{1}{3}$ | 0 |
| $\ell$ | 0 | 1 |
| $e^c$ | 0 | $-1$ |
| $h$ | 0 | 0 |

... but supersymmetric extensions can add new couplings and interactions at the renormalizable level that violate them

... additional UV physics can introduce new violations

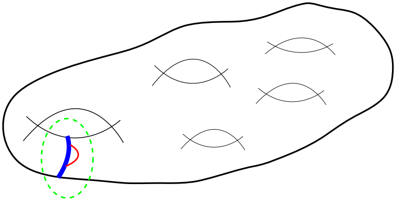
# Challenges for String Models

- This is particularly challenging for string models because they come with lots of new UV physics
  - KK modes on the brane
  - Bulk fields (SUGRA + . . .)
  - . . .
- Most robust approach is to gain control by **engineering symmetries**
  - This need not be the only option, though – internal structure of the model can help with some suppression

Our approach to studying these issues is very much in line with the paradigm of 'local-to-global model building'

[Aldazabal, Ibanez, Quevedo, Uranga], [Gray, He, Jejjala, Nelson] [Verlinde, Wijnholt]

Local physics near stacks of branes has a 'universal' description in terms of brane worldvolume physics



1. Can we understand the rules for model-building in this setting?
2. How are those rules modified/constrained when we insist on the existence of UV completions (global embeddings)?
   - 'Single-stack' GUT models→this talk
   - 'Multi-stack' quiver models→Jim Halverson's talk

# Objectives

- F-theory model-building utilizes a number of tools in addition to symmetry structure

  [Donagi, Wijnholt] [Beasley, Heckman, Vafa]

  1. Promising mechanisms for breaking the GUT group

     [Beasley, Heckman, Vafa], [Donagi, Wijnholt]
  2. Ideas for generating flavor hierarchies

     [Heckman, Vafa] [Ibanez, Font], [KIng, Leontaris, Ross]

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- But when we build models. . .

  [JM, Saulina, Schäfer-Nameki], [Blumenhagen, Grimm, Jurke, Weigand]

  [Grimm,Krause,Weigand], [Cvetic, Garcia-Etxebarria, Halverson]

  [Chen, Knapp, Kreuzer, Mayrhofer], [Knapp, Kreuzer, Mayhrofer, Walliser]

  1. These ingredients to not play nicely with $U(1)$ symmetries

     [JM, Saulina, Schäfer-Nameki]
  2. This leads to claims of unwanted features like charged exotics

The goal of this talk is to get some

**physical understanding**

for why F-theory models with $U(1)$ symmetries

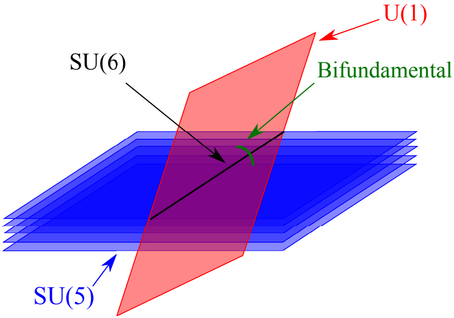
seem so constrained

# Outline

- Basic Structures of F-theory GUTs
- A Closer Look at "Hypercharge Flux"
- Implications of the "Dudas-Palti Relations"

# F-theory and Intersecting Branes

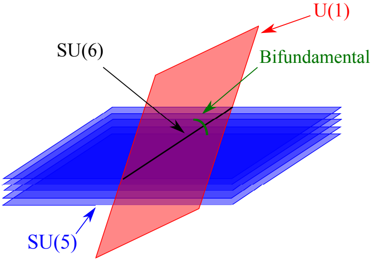
- The basic structure of F-theory models can be described in the language of intersecting branes.

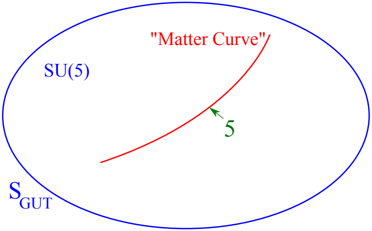



- Charged matter from open strings with one end on the stack
- Other end on some other D-brane, orientifold plane,
  $\longrightarrow$ "Matter branes"

- In F-theory models, the branes are 7-branes wrapping

$$\mathbb{R}^{3,1} \times S_2 \qquad \text{for some } \mathbb{C} \text{ surface } S_2$$




**Charged matter is effectively 6-dimensional**

- Spectrum of 4d multiplets requires further dimensional reduction
- # of 4d multiplets can be adjusted with fluxes

# Chiral Spectra from Fluxes

- Spectrum on matter curves determined by
  - "Bulk Flux"
  - "Brane Flux" ("Hypercharge Flux")

[Beasley, Heckman, Vafa] [Donagi, Wijnholt]

## $\mathbf{10}$ Matter Curve

$$\mathbf{10} \rightarrow (\mathbf{1},\mathbf{1})_{+1} \oplus (\mathbf{3},\mathbf{2})_{+1/6} \oplus (\overline{\mathbf{3}},\mathbf{1})_{-2/3}$$

| MSSM Multiplet | Chirality |
|---|---|
| $(\mathbf{1},\mathbf{1})_{+1}$ | $M^{(10)} + N^{(10)}$ |
| $(\mathbf{3},\mathbf{2})_{+1/6}$ | $M^{(10)}$ |
| $(\overline{\mathbf{3}},\mathbf{1})_{-2/3}$ | $M^{(10)} - N^{(10)}$ |

## $\overline{\mathbf{5}}$ Matter Curve

$$\overline{\mathbf{5}} \rightarrow (\overline{\mathbf{3}},\mathbf{1})_{+1/3} \oplus (\mathbf{1},\mathbf{2})_{-1/2}$$

| MSSM Multiplet | Chirality |
|---|---|
| $(\overline{\mathbf{3}},\mathbf{1})_{+1/3}$ | $M^{(\bar{5})}$ |
| $(\mathbf{1},\mathbf{2})_{-1/2}$ | $M^{(\bar{5})} - N^{(\bar{5})}$ |

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- Can engineer doublets without triplets by setting

$$M^{(\overline{5})} = 0 \qquad N^{(\overline{5})} = \pm 1$$

# "Hypercharge Flux" vs $U(1)$ Symmetries

- Model building with F-theory GUTS relies on both
  - "Hypercharge Flux"
  - $U(1)$ Symmetries

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[JM, Saulina, Schäfer-Nameki]

- Explicit constructions based on spectral covers have shown that these two are interrelated
- Distributions of "hypercharge flux" along matter curves are highly constrained
  - Lose control over 'non-GUT'ness of the spectrum
  - Resulting models exhibit charged 'quasi-chiral' exotic fields

# Understanding the Constraints

- Is there a sharp way to describe the relationship between $U(1)$ symmetries and the distribution of "hypercharge flux"?

- Is there an intrinsic physical meaning to this relationship?

# 2. A Closer Look at "Hypercharge Flux"

# Mixed Gauge Anomalies

- Hypercharge flux induces chirality so its ‘distribution’ should be limited by anomaly considerations
- Consider adding pure $U(1)_Y$ flux to a geometry that has both:

$$SU(5)_{\mathrm{GUT}} \text{ and some extra } U(1)\text{'s}$$

- By construction we should not have any 4-dimensional gauge anomalies
  - Especially interesting anomalies – $G_{MSSM} \times G_{MSSM} \times U(1)$
    - These get contributions only from chiral fields on matter curves
    - Cancellation will imply correlation between $\omega_Y$ and matter curves

# Mixed Gauge Anomalies

Anomalies from **10** curve with $U(1)$ charge $q_{10}$ and +1 unit of $U(1)_Y$ flux

| Mult | Chir | $SU(3)^2U(1)$ | $SU(2)^2U(1)$ | $U(1)_Y^2U(1)$ |
|---|---|---|---|---|
| $(\mathbf{1},\mathbf{1})_{+1}$ | 6 | 0 | 0 | $6q_{10}$ |
| $(\mathbf{3},\mathbf{2})_{+1/6}$ | 1 | $2q_{10}$ | $3q_{10}$ | $q_{10}/6$ |
| $(\overline{\mathbf{3}},\mathbf{1})_{-2/3}$ | $-4$ | 0 | $-16q_{10}/3$ | |
| **TOTAL:** | | $-2q_{10}$ | $3q_{10}$ | $5q_{10}/6$ |

Anomalies from $\overline{\mathbf{5}}$ curve with $U(1)$charge $q_{\overline{5}}$ and +1 unit of $U(1)_Y$ flux

| Mult | Chir | $SU(3)^2U(1)$ | $SU(2)^2U(1)$ | $U(1)_Y^2U(1)$ |
|---|---|---|---|---|
| $(\overline{\mathbf{3}},\mathbf{1})_{+1/3}$ | 2 | $2q_{\overline{5}}$ | 0 | $2q_{\overline{5}}/3$ |
| $(\mathbf{1},\mathbf{2})_{-1/2}$ | $-3$ | 0 | $-3q_{\overline{5}}$ | $-3q_{\overline{5}}/2$ |
| **TOTAL:** | | $2q_{\overline{5}}$ | $-3q_{\overline{5}}$ | $-5q_{\overline{5}}/6$ |

# Dudas-Palti Relations

- All mixed anomalies cancel provided we have

[JM]

$$\boxed{\sum_{\mathbf{10}\text{ Matter Curves, }a} q_a \int_{\Sigma_{\mathbf{10},a}} \omega_Y = \sum_{\bar{\mathbf{5}}\text{ Matter Curves, }i} q_i \int_{\Sigma_{\bar{\mathbf{5}},i}} \omega_Y}$$

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- These relations were first observed by Dudas and Palti in a set of spectral cover constructions
- They can be proven directly within the spectral cover formalism

[Dolan, JM, Saulina, Schäfer-Nameki]

- With spectral covers, it seems possible (in principle) to construct all consistent distributions of $U(1)_Y$ flux consistent with
  1. The Dudas-Palti relations
  2. The cancellation of MSSM gauge anomalies

[Dolan, JM, Saulina, Schäfer-Nameki]

$$\sum_{\mathbf{10}\text{ Matter Curves, }a} q_a \int_{\Sigma_{\mathbf{10},a}} \omega_Y = \sum_{\bar{\mathbf{5}}\text{ Matter Curves, }i} q_i \int_{\Sigma_{\bar{\mathbf{5}},i}} \omega_Y$$

All "constraints" that have been observed in F-theory GUTs with $U(1)$ symmetries are captured by these relations

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Some consequences we shall obtain from these relations can be immediately understood in terms of their underlying physics:

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Some consequences we shall obtain from these relations can be immediately understood in terms of their underlying physics:

- Generic $U(1)$'s will become anomalous once we switch on fluxes to generate a nontrivial spectrum
  - There is a nice 4d Green-Schwarz mechanism that operates to cancel anomalies

  ... but that mechanism is independent of the hypercharge flux that we use to break the GUT gauge group so

[JM]

**$U(1)$ anomalies must be the same before and after we introduce the GUT-breaking flux**

This implies that $SU(3)^2U(1)$, $SU(2)^2U(1)$, and $U(1)_Y^2U(1)$ anomalies must agree (up to rescaling by the appropriate Casimirs)

# 3. Implications of the Dudas-Palti Relations

# Simple implication

- Study $U(1)$ symmetries that commute with $SU(5)$

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- Study $U(1)$ symmetries that commute with $SU(5)$
- $G_{SM}^2 U(1)$ anomalies must all agree

  ... but $H_u$ and $H_d$ do not contribute to the $SU(3)^2 U(1)$ anomaly

$$\boxed{\Longrightarrow \; H_u \text{ and } H_d \text{ must be vectorlike wrt } U(1)}$$

- Such a $U(1)$ cannot help with the $\mu$ problem

- We often like $U(1)$'s that are flavor blind as well
  - Such $U(1)$'s are compatible with a particularly nice flavor scenario but we don't say anything about flavor structure here

[Heckman, Vafa]

(except that we do not use $U(1)$'s to manipulate flavor)

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- Only one $U(1)$ that
  - Gives all $\mathbf{10}$'s a common charge
  - Gives all $\overline{\mathbf{5}}$ a common charge
  - Gives $H_u$ and $H_d$ opposite charges
  - Preserves the MSSM superpotential

$$W_{MSSM} = \mathbf{10}_M \times \mathbf{10}_M \times \mathbf{5}_H + \mathbf{10}_M \times \overline{\mathbf{5}}_M \times \overline{\mathbf{5}}_H$$

| | $\mathbf{10}_M$ | $\overline{\mathbf{5}}_M$ | $\mathbf{5}_H$ | $\overline{\mathbf{5}}_H$ |
|---|---|---|---|---|
| $U(1)$ | 1 | $-3$ | $-2$ | 2 |

# $B - L$!

| | $\mathbf{10}_M$ | $\overline{\mathbf{5}}_M$ | $\mathbf{5}_H$ | $\overline{\mathbf{5}}_H$ |
|---|---|---|---|---|
| $U(1)$ | 1 | $-3$ | $-2$ | 2 |

- This is $U(1)_\chi \to$ linear combination of $U(1)_Y$ and $U(1)_{B-L}$
  - Only flavor-blind $U(1)$ consistent with exact MSSM spectrum
- $U(1)_\chi$ is nice because it contains a $\mathbb{Z}_2^{\text{matter parity}}$ subgroup
  - Issues in spectral cover models if you want to break $U(1)_\chi$ while preserving $\mathbb{Z}_2^{\text{matter parity}}$ $\to$ see Christoph Ludeling's talk

[Ludeling, Nilles, Stephan]

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[Ludeling, Nilles, Stephan]

- Inadequate for dealing with

**$\mu$ Problem**

$$W_\mu \sim \mu H_u H_d$$

**Dim 5 proton decay**

$$W_{\text{Dim 5}} \sim \frac{1}{\Lambda} Q^3 L$$

# $U(1)_{PQ}$

**$\mu$ Problem**

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**Dim 5 proton decay**

$$W_{\text{Dim 5}} \sim \frac{1}{\Lambda} Q^3 L$$

- Would like to engineer a $U(1)_{PQ}$ symmetry to deal with $\mu$ and dim 5 proton decay

$$Q(H_u) + Q(H_d) \neq 0$$

- Anomaly analysis tells us that this is not possible without introducing quasi-chiral exotics
    - Come in non-$SU(5)$ multiplets
        - must give 'non-universal' contribution to mixed $G^2_{SM} U(1)$ anomalies
    - Non-chiral wrt MSSM but chiral wrt $U(1)_{PQ}$

# Dealing with Exotics

- In principle, exotics can lift since they will couple to MSSM singlets $X_i$ that carry $PQ$ charge

$$W \sim X_i f_{\text{exotic}} \overline{f}_{\text{exotic}} \implies M_{\text{Exotic},i} \sim \langle X_i \rangle$$

- Expectation values $\langle X_i \rangle$ can strongly break $U(1)_{PQ}$ and regenerate dangerous operators from

$$\int d^2\theta \, \frac{{X_i}^{n_i}}{\Lambda^{\sum_i n_i - 1}} H_u H_d \text{ and/or } \frac{{X_i}^{n_i}}{\Lambda} \int d^2\theta Q^3 L$$

  - Suppression of operators favors small $\langle X_i \rangle$
  - Unification favors large $\langle X_i \rangle$

# Dealing with Exotics

[Dolan, JM, Saulina, Schäfer-Nameki]

- Best possible scenario: exotics come in a combination that yields universal shift of MSSM $\beta$ functions
- Dudas-Palti relations have something to say about the structure of exotics, though. . .

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- Dudas-Palti relations have something to say about the structure of exotics, though...

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- For simplicity, suppose all exotics lifted by 1 singlet, $X$

$$W \sim X f_{\text{exotic}} \bar{f}_{\text{exotic}}$$

$$\text{Dudas-Palti} \implies q_{H_u} + q_{H_d} = q_X \Delta$$

where $\Delta$ measures the non-universal $\beta$ function shifts

$$\Delta = \delta b_2 - \delta b_3 = \frac{1}{6}(5\delta b_1 + 3\delta b_2 - 8\delta b_3)$$

# Unification Issues

$$q_{H_u} + q_{H_d} = q_X \Delta \qquad \Delta = \delta b_2 - \delta b_3 = \frac{1}{6}(5\delta b_1 + 3\delta b_2 - 8\delta b_3)$$

- Impossible for exotics to preserve 1-loop gauge coupling unification

---

- We could also try to adjust the charge of $X$ in order to crank up the powers $m, n$ in

$$\int d^2\theta \left(\frac{X}{\Lambda}\right)^m \Lambda H_u H_d \qquad \frac{1}{\Lambda}\int d^2\theta \left(\frac{X}{\Lambda}\right)^n Q^3 L$$

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$$\int d^2\theta \left(\frac{X}{\Lambda}\right)^{-\Delta} \Lambda H_u H_d \qquad \frac{1}{\Lambda}\int d^2\theta \left(\frac{X}{\Lambda}\right)^{\Delta} Q^3 L$$

Dudas-Palti $\implies -m = n = \Delta!$

# $\mu$ Problem/Proton Decay and Unification

$$q_{H_u} + q_{H_d} = q_X \Delta \qquad \Delta = \delta b_2 - \delta b_3 = \frac{1}{6}(5\delta b_1 + 3\delta b_2 - 8\delta b_3)$$

$$\int d^2\theta \left(\frac{X}{\Lambda}\right)^{-\Delta} \Lambda H_u H_d \qquad \frac{1}{\Lambda}\int d^2\theta \left(\frac{X}{\Lambda}\right)^{\Delta} Q^3 L$$

- General tension between unification and proton decay/$\mu$ prob
  - Dealing with exotics from $U(1)_{PQ}$ forces us to break it so strongly that it may not address the problems it was meant to solve
- Similar story for multiple singlets $X_i$
- Small hope remains: $U(1)_Y$ flux also distorts unification

  [Donagi, Wijnholt] [Blumenhagen]
  - Maybe we can use this to gain some wiggle room?

    [Dolan, JM, Schäfer-Nameki, in progress]

# Further lessons

- So far everything we have said is essentially local...
  - Engineering $U(1)$'s globally is a very subtle matter

    [Hayashi, Kawano, Tsuchiya, Watari], [Grimm, Weigand], [JM, Saulina, Schäfer-Nameki]
- From global studies, it seems that $U(1)_{PQ}$ symmetries are generically Higgs'ed by GUT singlets away from the $SU(5)$ stack

  [JM, Saulina, Schäfer-Nameki,...]
  - Not true for $U(1)_\chi$
- Natural suppression mechanism for PQ violating terms (like exotic masses)
  - Seems likely to be model (ie geometry) dependent

**The general lesson seems to be that $Z'$s are not ubiquitous in F-theory GUTs**

In fact, the only $Z'$ that $SU(5)$ F-theory GUT models like is $U(1)_\chi$; *PQ*'s will be Higgs'ed at a high scale and only approximate selection rules, which seem model-dependent (and amount to tuning certain Yukawa couplings from a low energy point of view), will remain

# Summary

- "All constraints" that have appeared in spectral cover models of F-theory GUTs are encoded in the Dudas-Palti relations
- The Dudas-Palti relations are a consequence of 4-dimensional anomaly cancellation so their physical origin is clear
- Several model-building implications
  - Only $U(1)_{B-L}$ is consistent with the precise MSSM spectrum
  - Models that use $U(1)$ symmetries to address $\mu$ or dimension 5 proton decay come equipped with exotics

    $\longrightarrow$ General tension between $\mu$/proton decay and unification