Muon $g-2$ and $\alpha(M_Z^2)$ re-evaluated using new precise data

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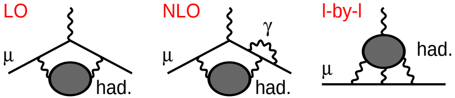
Parallel Session Talk at SUSY11@Fermilab
September 1, 2011

Based on J. Phys. **G38** (2011) 085003 (arXiv:1105.3149)
by K. Hagiwara, R. Liao, A. D. Martin, DN & T. Teubner
(HLMNT)

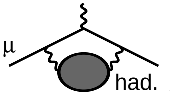
QED contribution	11 658 471.808 (0.015) $\times 10^{-10}$	Kinoshita & Nio, Aoyama et al
EW contribution	15.4 (0.2) $\times 10^{-10}$	Czarnecki et al
Hadronic contribution		
LO hadronic	**694.9 (4.3)** $\times 10^{-10}$	HLMNT11
NLO hadronic	-9.8 (0.1) $\times 10^{-10}$	HLMNT11
light-by-light	10.5 (2.6) $\times 10^{-10}$	Prades, de Rafael & Vainshtein
Theory TOTAL	**11 659 182.8 (4.9)** $\times 10^{-10}$	
Experiment	**11 659 208.9 (6.3)** $\times 10^{-10}$	world avg
Exp − Theory	**26.1 (8.0)** $\times 10^{-10}$	**3.3 σ discrepancy**

(Numbers taken from HLMNT11, arXiv:1105.3149)

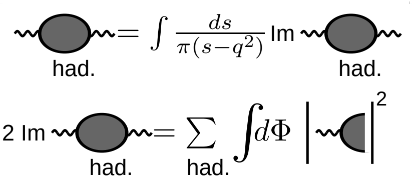
n.b.: hadronic contributions:



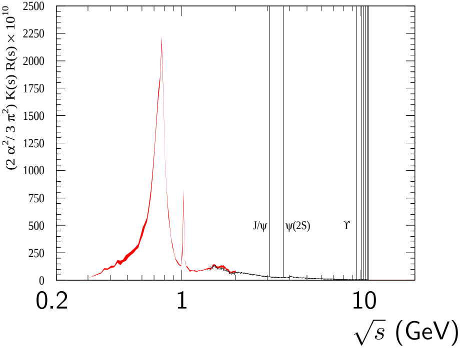
The diagram to be evaluated:



pQCD not useful. Use the dispersion relation and the optical theorem.



$$a_\mu^{\rm had,LO} = \frac{m_\mu^2}{12\pi^3}\int_{s_{\rm th}}^{\infty} ds\, \frac{1}{s}\hat{K}(s)\sigma_{\rm had}(s)$$



- Weight function $\hat{K}(s)/s = \mathcal{O}(1)/s$

$\Longrightarrow$ Lower energies more important

$\Longrightarrow$ $\pi^+\pi^-$ channel: 73% of total $a_\mu^{\rm had,LO}$

Included Hadronic Final States

Channel	Experiments with References
$\pi^+\pi^-$	OLYA [16, 17, 18], OLYA-TOF [19], NA7 [20], OLYA and CMD [21, 22], DM1 [23], DM2 [24], BCF [25, 26], MEA [27, 28], ORSAY-ACO [29], CMD-2 [10, 11, 30]
$\pi^0\gamma$	SND [31, 32]
$\eta\gamma$	SND [32, 33], CMD-2 [34, 35, 36]
$\pi^+\pi^-\pi^0$	ND [22], DM1 [37], DM2 [38], CMD-2 [10, 13, 34, 39], SND [40, 41], CMD [42]
K^+K^-	MEA [27], OLYA [43], BCF [26], DM1 [44], DM2 [45, 46], CMD [22], CMD-2 [34], SND [47]
$K_S^0K_L^0$	DM1 [48], CMD-2 [10, 14, 49], SND [47]
$\pi^+\pi^-\pi^0\pi^0$	M3N [50], DM2 [51], OLYA [52], CMD-2 [53], SND [54], ORSAY-ACO [55], $\gamma\gamma2$ [56], MEA [57]
$\omega(\to\pi^0\gamma)\pi^0$	ND and ARGUS [22], DM2 [51], CMD-2 [53, 58], SND [59, 60], ND [61]
$\pi^+\pi^-\pi^+\pi^-$	ND [22], M3N [50], CMD [62], DM1 [63, 64], DM2 [51], OLYA [65], $\gamma\gamma2$ [66], CMD-2 [53, 67, 68], SND [54], ORSAY-ACO [55]
$\pi^+\pi^-\pi^+\pi^-\pi^0$	MEA [57], M3N [50], CMD [22, 62], $\gamma\gamma2$ [56]
$\pi^+\pi^-\pi^0\pi^0\pi^0$	M3N [50]
$\omega(\to\pi^0\gamma)\pi^+\pi^-$	DM2 [38], CMD-2 [69], DM1 [70]
$\pi^+\pi^-\pi^+\pi^-\pi^+\pi^-$	M3N [50], CMD [62], DM1 [71], DM2 [72]
$\pi^+\pi^-\pi^+\pi^-\pi^0\pi^0$	M3N [50], CMD [62], DM2 [72], $\gamma\gamma2$ [56], MEA [57]
$\pi^+\pi^-\pi^0\pi^0\pi^0\pi^0$	isospin-related
$\eta\pi^+\pi^-$	DM2 [73], CMD-2 [69]
$K^+K^-\pi^0$	DM2 [74, 75]
$K_S^0\pi K$	DM1 [76], DM2 [74, 75]
K_S^0X	DM1 [77]
$\pi^+\pi^-K^+K^-$	DM2 [74]
$p\bar{p}$	FENICE [78, 79], DM2 [80, 81], DM1 [82]
$n\bar{n}$	FENICE [78, 83]
incl. (<2 GeV)	$\gamma\gamma2$ [84], MEA [85], M3N [86], BARYON-ANTIBARYON [87]
incl. (>2 GeV)	BES [88, 89], Crystal Ball [90, 91, 92], LENA [93], MD-1 [94], DASP [95], CLEO [96], CUSB [97], DHHM [98]

Table 1: Experiments and references for the e^+e^- data sets for the different exclusive and the inclusive channels as used in this analysis. The recent re-analysis from CMD-2 [10] supersedes

channel	inclusive (1.43,2 GeV)		exclusive (1.43,2 GeV)	
	$a_\mu^{\rm had,LO}$	$\Delta\alpha_{\rm had}(M_Z^2)$	$a_\mu^{\rm had,LO}$	$\Delta\alpha_{\rm had}(M_Z^2)$
$\pi^0\gamma$ (ChPT)	0.13 ± 0.01	0.00 ± 0.00	0.13 ± 0.01	0.00 ± 0.00
$\pi^0\gamma$ (data)	4.50 ± 0.15	0.36 ± 0.01	4.50 ± 0.15	0.36 ± 0.01
$\pi^+\pi^-$ (ChPT)	2.36 ± 0.05	0.04 ± 0.00	2.36 ± 0.05	0.04 ± 0.00
$\pi^+\pi^-$ (data)	502.78 ± 5.02	34.39 ± 0.29	503.38 ± 5.02	34.59 ± 0.29
$\pi^+\pi^-\pi^0$ (ChPT)	0.01 ± 0.00	0.00 ± 0.00	0.01 ± 0.00	0.00 ± 0.00
$\pi^+\pi^-\pi^0$ (data)	46.43 ± 0.90	4.33 ± 0.08	47.04 ± 0.90	4.52 ± 0.08
$\eta\gamma$ (ChPT)	0.00 ± 0.00	0.00 ± 0.00	0.00 ± 0.00	0.00 ± 0.00
$\eta\gamma$ (data)	0.73 ± 0.03	0.09 ± 0.00	0.73 ± 0.03	0.09 ± 0.00
K^+K^-	21.62 ± 0.76	3.01 ± 0.11	22.35 ± 0.77	3.23 ± 0.11
$K_S^0K_L^0$	13.16 ± 0.31	1.76 ± 0.04	13.30 ± 0.32	1.80 ± 0.04
$2\pi^+2\pi^-$	6.16 ± 0.32	1.27 ± 0.07	14.77 ± 0.76	4.04 ± 0.21
$\pi^+\pi^-2\pi^0$	9.71 ± 0.63	1.86 ± 0.12	20.55 ± 1.22	5.51 ± 0.35
$2\pi^+2\pi^-\pi^0$	0.26 ± 0.04	0.06 ± 0.01	2.85 ± 0.25	0.99 ± 0.09
$\pi^+\pi^-3\pi^0$	0.09 ± 0.09	0.02 ± 0.02	1.19 ± 0.33	0.41 ± 0.10
$3\pi^+3\pi^-$	0.00 ± 0.00	0.00 ± 0.00	0.22 ± 0.02	0.09 ± 0.01
$2\pi^+2\pi^-2\pi^0$	0.12 ± 0.03	0.03 ± 0.01	3.32 ± 0.29	1.22 ± 0.11
$\pi^+\pi^-4\pi^0$ (isospin)	0.00 ± 0.00	0.00 ± 0.00	0.12 ± 0.12	0.05 ± 0.05
$K^+K^-\pi^0$	0.00 ± 0.00	0.00 ± 0.00	0.29 ± 0.07	0.10 ± 0.03
$K_S^0K_L^0\pi^0$ (isospin)	0.00 ± 0.00	0.00 ± 0.00	0.29 ± 0.07	0.10 ± 0.03
$K_S^0\pi^\mp K^\pm$	0.05 ± 0.02	0.01 ± 0.00	1.00 ± 0.11	0.33 ± 0.04
$K_L^0\pi^\mp K^\pm$ (isospin)	0.05 ± 0.02	0.01 ± 0.00	1.00 ± 0.11	0.33 ± 0.04
$K\bar{K}\pi\pi$ (isospin)	0.00 ± 0.00	0.00 ± 0.00	3.63 ± 1.34	1.33 ± 0.48
$\omega(\to\pi^0\gamma)\pi^0$	0.64 ± 0.02	0.12 ± 0.00	0.83 ± 0.03	0.17 ± 0.01
$\omega(\to\pi^0\gamma)\pi^+\pi^-$	0.01 ± 0.00	0.00 ± 0.00	0.07 ± 0.01	0.02 ± 0.00
$\eta(\to\pi^0\gamma)\pi^+\pi^-$	0.07 ± 0.01	0.02 ± 0.00	0.49 ± 0.07	0.15 ± 0.02
$\phi(\to$ unaccounted$)$	0.06 ± 0.06	0.01 ± 0.01	0.06 ± 0.06	0.01 ± 0.01
$p\bar{p}$	0.00 ± 0.00	0.00 ± 0.00	0.04 ± 0.01	0.02 ± 0.00
$n\bar{n}$	0.00 ± 0.00	0.00 ± 0.00	0.07 ± 0.02	0.03 ± 0.01
$J/\psi,\psi'$	7.30 ± 0.43	8.90 ± 0.51	7.30 ± 0.43	8.90 ± 0.51
$\Upsilon(1S-6S)$	0.10 ± 0.00	1.16 ± 0.04	0.10 ± 0.00	1.16 ± 0.04
inclusive R	73.96 ± 2.68	92.75 ± 1.74	42.05 ± 1.14	81.97 ± 1.53
pQCD	2.11 ± 0.00	125.32 ± 0.15	2.11 ± 0.00	125.32 ± 0.15
sum	692.38 ± 5.88	275.52 ± 1.85	696.15 ± 5.68	276.90 ± 1.77

Recently added data

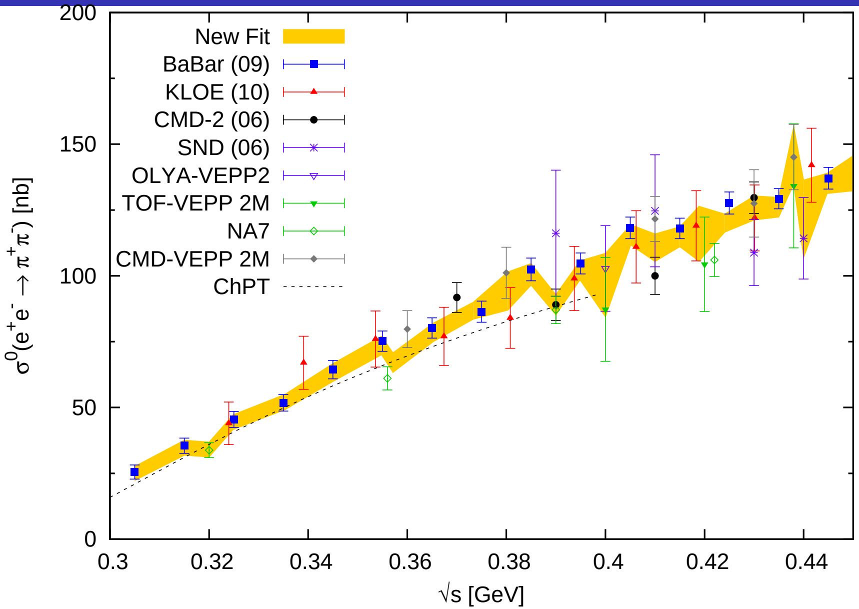
- Most important channels with changes in input data since $\sim$2006

The main exps. for 'low' energy hadronic cross sections in e^+e^-; channels

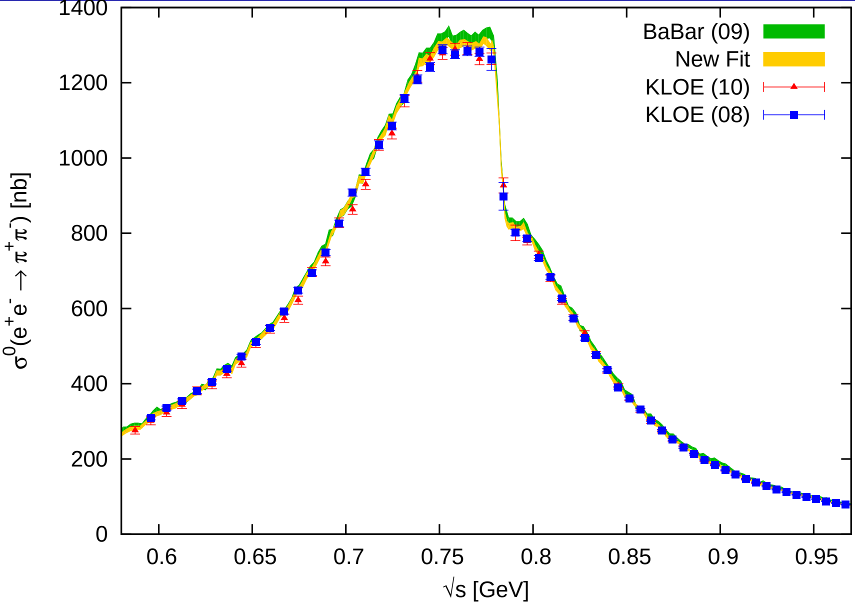
- CMD-2, [VEPP-2M], Novosibirsk (K^+K^-, $2\pi^+2\pi^-\pi^0$, $2\pi^+2\pi^-2\pi^0$)
- SND, [VEPP-2M], Novosibirsk (K^+K^-, $K_S^0K_L^0$)
- KLOE, [DAΦNE], Frascati ($\pi^+\pi^-(\gamma)$, $\omega\pi^0$)
- BaBar, [PEP-II], SLAC, Stanford ($\pi^+\pi^-(\gamma)$, $K^+K^-\pi^0$, $K_S^0\pi K$, $2\pi^+2\pi^-\pi^0$, $K^+K^-\pi^+\pi^-\pi^0$, $2\pi^+2\pi^-\eta$, $2\pi^+2\pi^-2\pi^0$)
- BELLE, [KEKB], KEK, Tsukuba
- BES, [BEPC], Beijing (inclusive $R = \sigma(e^+e^- \to hadrons)/\sigma(e^+e^- \to \mu^+\mu^-)$ data)
- CLEO, [CESR], Cornell (inclusive R)

T. Teubner, talk at Tau2010

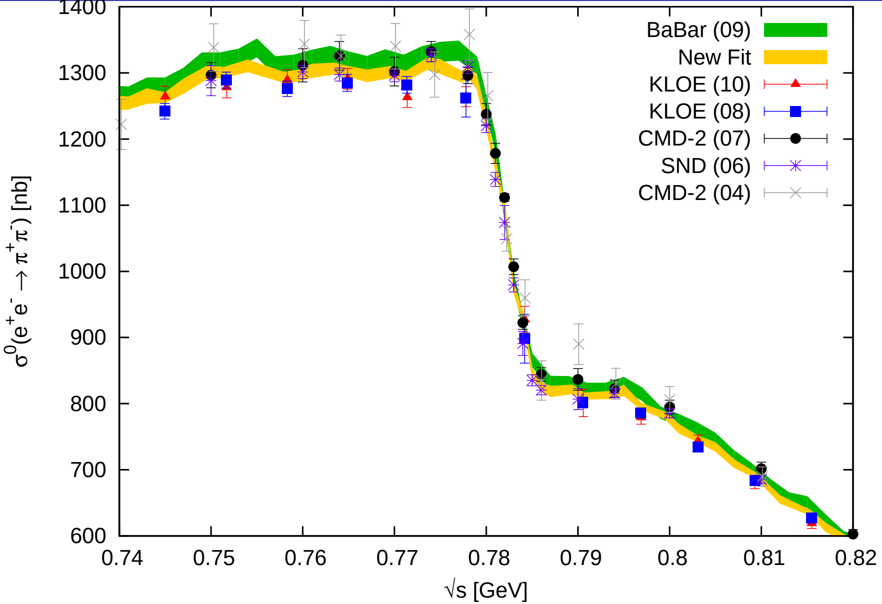
$\pi^+\pi^-$ channel: Low Energy Tail



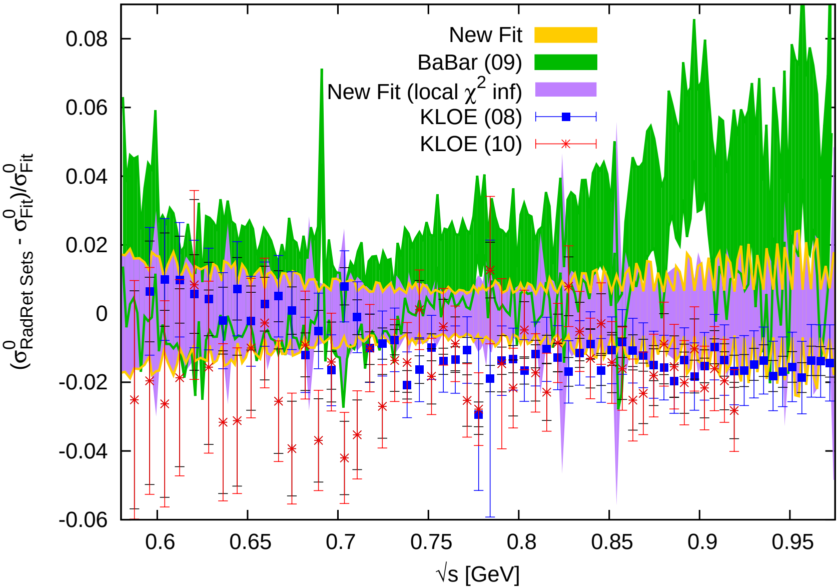
$\pi^+\pi^-$ channel: New Radiative Return Data



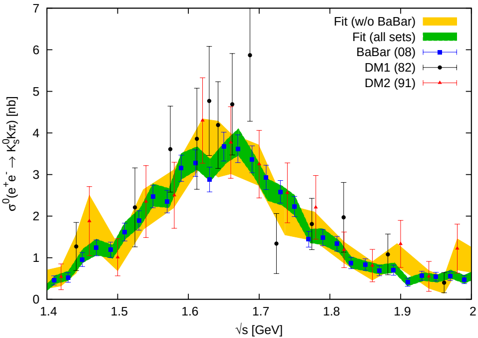
$\pi^+\pi^-$ channel: Zoom-In at ρ-ω Region

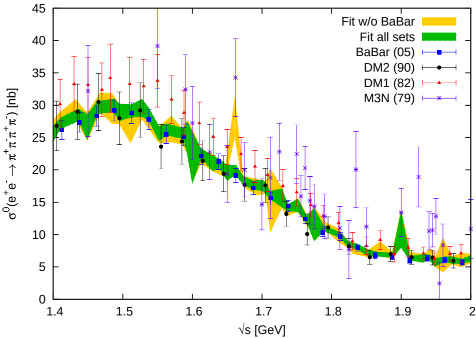


Rad. Rtn. Data (for $\pi^+\pi^-$) and Our Combined Result



Impacts from New BaBar Data in subleading channels: (1)

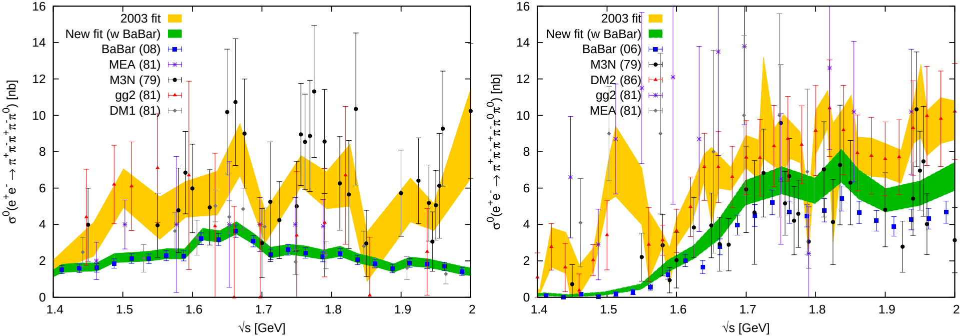




$e^+e^- \to K_S^0 K\pi$ (left) and $e^+e^- \to 2\pi^+2\pi^-$ (right)

In these cases, the new BaBar data agree well with the old data, and improve them in a peaceful way.

Impacts from New BaBar Data in subleading channels: (2)

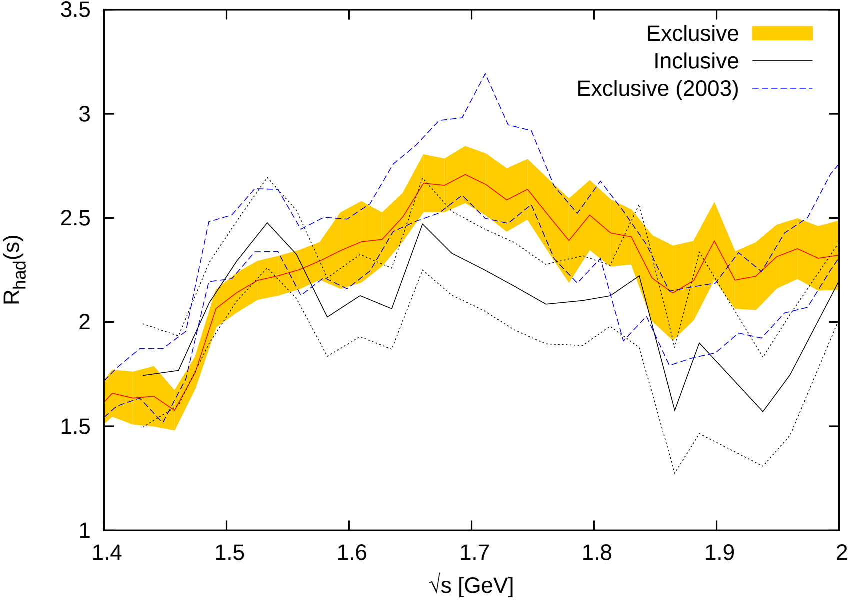


$e^+e^- \to 2\pi^+2\pi^-\pi^0$ (left) and $2\pi^+2\pi^-2\pi^0$ (right)

In these cases, the new BaBar data do not agree very well with the old data, and improve them 'radically'.

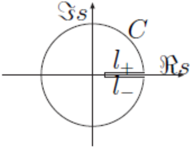
Note that the old data are really old (those from the '80s or older...)

Region between 1.4 – 2.0 GeV



Check against perturbative QCD: QCD $\sum$-rule analysis

- Evaluate QCD $\sum$-rules of the form:



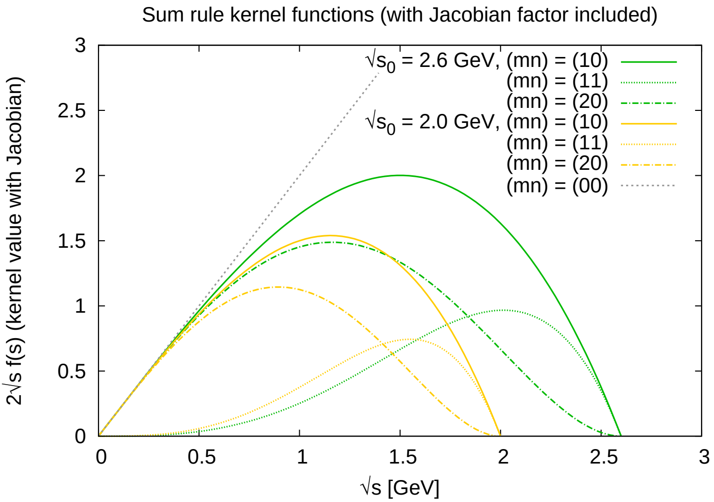
$$\int_{s_{\rm th}}^{s_0} {\rm d}s\, R(s) f(s) = \int_C {\rm d}s\, D(s) g(s)\,, \qquad \text{with} \;\; D(s) \equiv -12\pi^2 s \frac{\rm d}{{\rm d}s}\left(\frac{\Pi(s)}{s}\right)$$

- The Adler D function is calculable in pQCD: $D(s) = D_0(s) + D_{\rm m}(s) + D_{\rm np}(s)\,.$
- Take $f(s) = (1 - s/s_0)^m (s/s_0)^n$ to maximise sensitivity to the required region, $g(s)$ follows.
- Choose s_0 below the open charm threshold ($n_f = 3$ for pQCD).
- For $m = 1$, $n = 0$ one gets e.g.

$$\int_{s_{\rm th}}^{s_0} {\rm d}s\, R(s)\left(1 - \frac{s}{s_0}\right) = \frac{i}{2\pi}\int_C {\rm d}s\,\left(-\frac{s}{2s_0} + 1 - \frac{s_0}{2s}\right) D(s)\,.$$

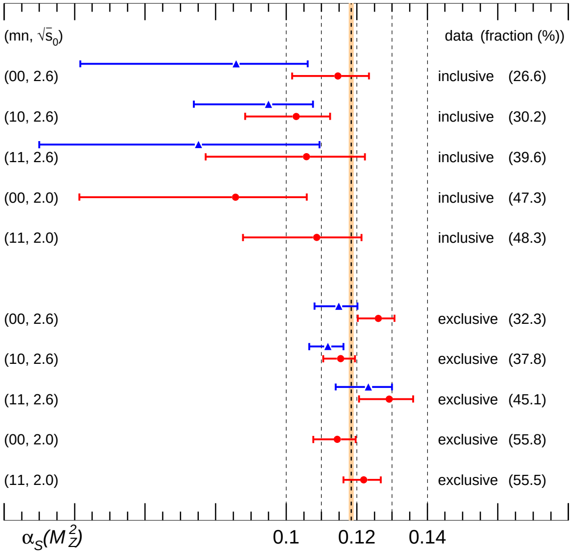
T. Teubner, talk at Tau2010

Sum Rules: choice of weight functions



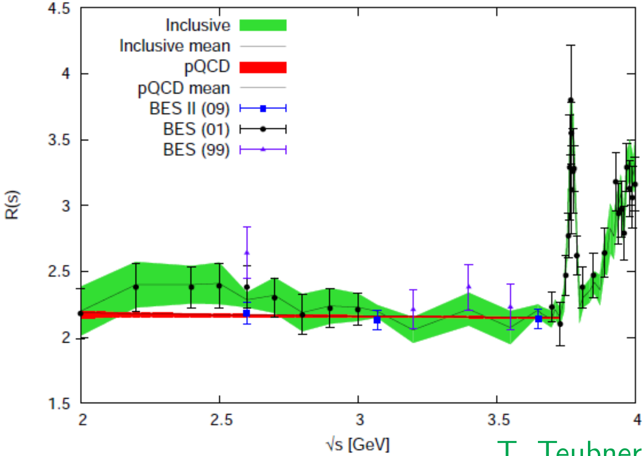
We choose the weight functions in such a way that they emphasize the region in question, 1.4–2.0 GeV.

Sum Rules for 1.4 – 2.0 GeV: Results



Sum of exclusive data: now more consistent with pQCD.

Perturbative QCD vs. inclusive data above 2 GeV (below charm threshold)

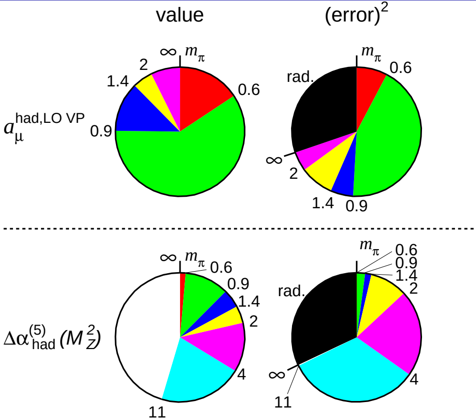


T. Teubner, talk at Tau2010

- R_{uds} from pQCD mostly below data fit in region above 2 GeV
- Latest BES data agree very well with pQCD

At 2.6–3.73 GeV we use pQCD (with the BESII (09) uncertainty)

Results: $a_\mu^{\mathrm{had,LO}}$, $a_\mu^{\mathrm{had,NLO}}$ and $\Delta\alpha_{\mathrm{had}}^{(5)}(M_Z^2)$

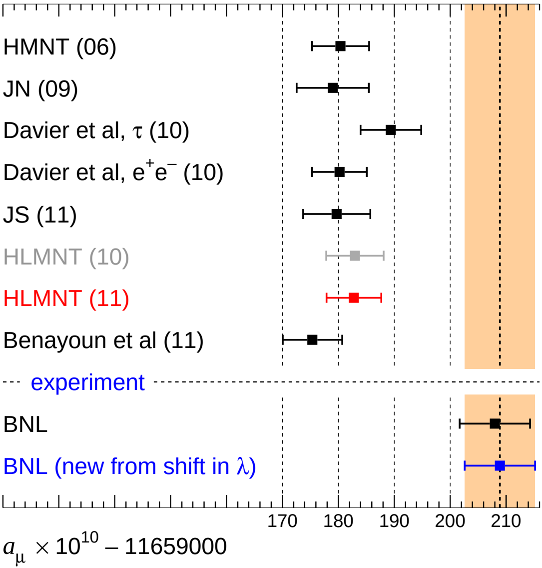


$$a_\mu^{\mathrm{had,LO}} = (694.91 \pm 3.72_{\mathrm{exp}} \pm 2.10_{\mathrm{rad}}) \times 10^{-10}$$

$$a_\mu^{\mathrm{had,NLO}} = (-9.84 \pm 0.06_{\mathrm{exp}} \pm 0.04_{\mathrm{rad}}) \times 10^{-10}$$

$$\Delta\alpha_{\mathrm{had}}^{(5)}(M_Z^2) = (276.26 \pm 1.16_{\mathrm{exp}} \pm 0.74_{\mathrm{rad}}) \times 10^{-4}$$

Full SM Result and Comparison with Other Groups



Byproducts (1): QED coupling at the Z-boson mass

★ $\alpha(M_Z^2)$: the least well known among $\{G_\mu, M_Z, \alpha(M_Z^2)\}$, which are used as **input** to precision electroweak fits.

★ Running of α

$$\alpha(M_Z^2) = \frac{\alpha}{1 - \Delta\alpha_{\rm lep}(M_Z^2) - \Delta\alpha_{\rm had}^{(5)}(M_Z^2) - \Delta\alpha^{\rm top}(M_Z^2)}$$

where $\Delta\alpha_{\rm lep}(M_Z^2) = 0.03149769$ (Steinhauser), $\Delta\alpha^{\rm top}(M_Z^2) = -\ 0.0000728(14)$ and $\alpha = 1/137.035999679(94)$ (PDG10).

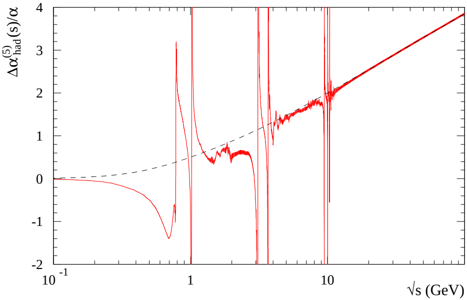
★ Similar dispersion relation: ($\Longrightarrow$ **byproduct** of $a_\mu^{\rm had,LO}$)

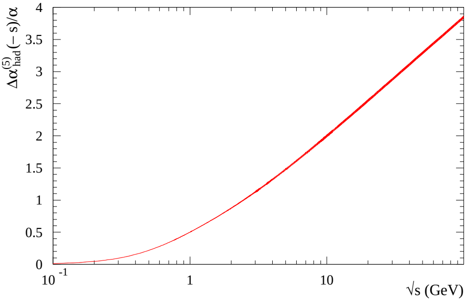
$$\Delta\alpha_{\rm had}^{(5)}(s) = -\frac{\alpha s}{3\pi} {\rm P} \int \frac{R(s')ds'}{s'(s'-s)}$$

★ Our results: $\Delta\alpha_{\rm had}^{(5)}(M_Z^2) = (276.3 \pm 1.4) \times 10^{-4}$,

$$\alpha(M_Z^2)^{-1} = 128.944 \pm 0.019.$$

The hadronic contribution $\Delta\alpha^{(5)}_{\rm had}(q^2)$ to the running QED coupling for $q^2 > 0$ (left) and $q^2 < 0$ (right)





Fortran subroutine to compute the above is available from us upon request — new version will become available soon

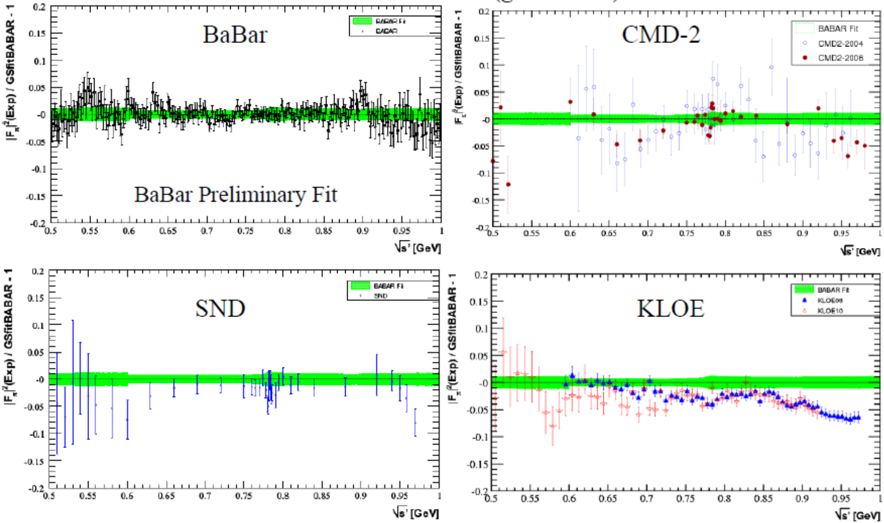
Summary

- We (HLMNT) have updated our analysis with (lots of) new data including those from KLOE and BaBar
- We find 3.3 σ discrepancy between experiment and theory $\Longrightarrow$ **New Physics?**
- Two new experiments to measure the muon $g - 2$ planned at J-PARC and Fermilab
- To establish this discrepancy more firmly, it is very important to resolve the disagreement in the $\pi^+\pi^-$ channel between the KLOE and BaBar data $\Longrightarrow$ new precise data from VEPP-2000 and SuperKEKB very welcome

Backup Slides

BaBar vs.other ee data (0.5-1.0 GeV)

direct relative comparison of cross sections with BaBar fit (stat + syst errors included)
(green band)

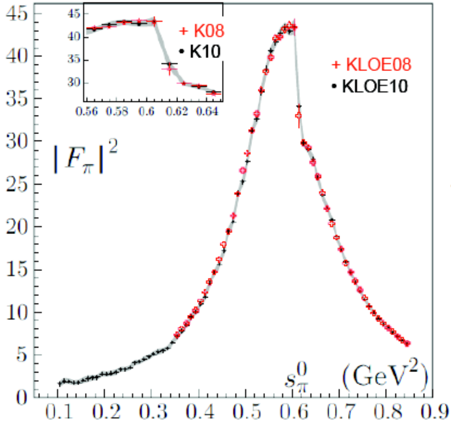


B.Malaescu ISR e+e- /g-2 TAU 2010 17

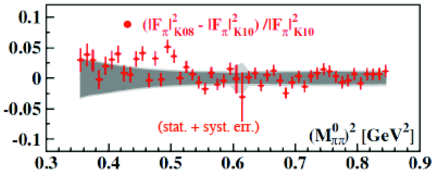
B. Malaescu, talk at Tau2010

Comparison of results: KLOE10 vs KLOE08

KLOE08 result compared to KLOE10:



Fractional difference:



band: KLOE10 error

Excellent agreement with KLOE08, expecially above 0.5 GeV^2

Combination of KLOE08 and KLOE10:

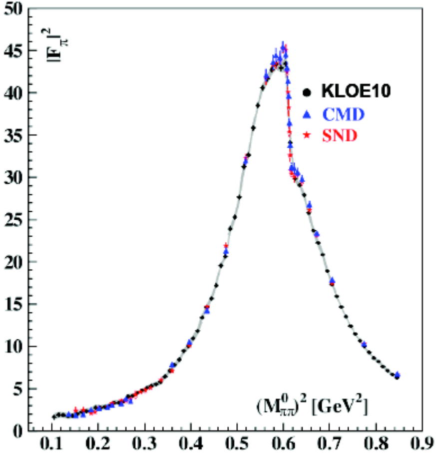
$a_\mu^{\pi\pi}(0.1\text{-}0.95\ \text{GeV}^2) = (488.6 \pm 5.0) \cdot 10^{-10}$

KLOE covers ~70% of total a_μ^{HLO} with a fractional error of 1.0%

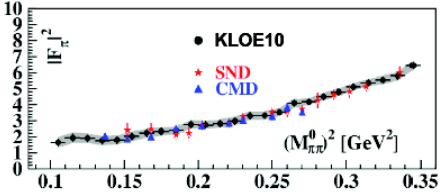
G. Venanzoni, talk at Tau2010

Comparison of results: KLOE10 vs CMD-2/SND

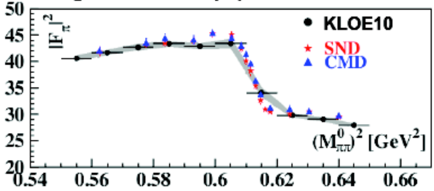
CMD and SND results compared to KLOE10:



Low $(M^0_{\pi\pi})^2$: :



Region around ρ-peak:

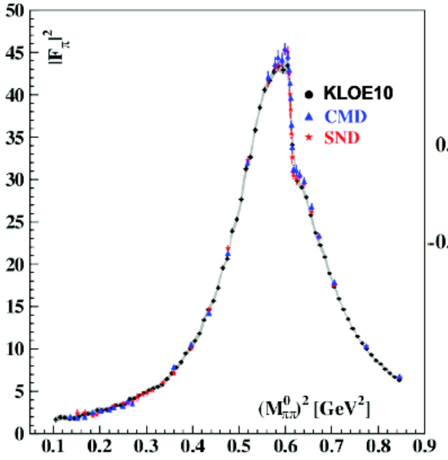


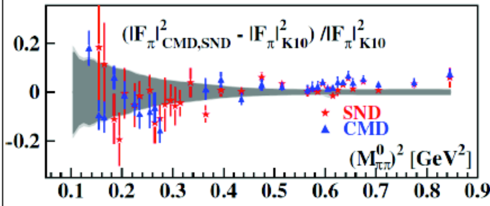
band: KLOE10 error

G. Venanzoni, talk at Tau2010

Comparison of results: KLOE10 vs CMD-2/SND

CMD and SND results compared to KLOE10: Fractional difference





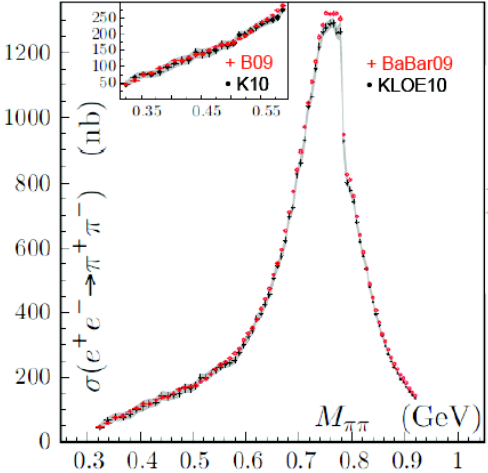
band: KLOE10 error

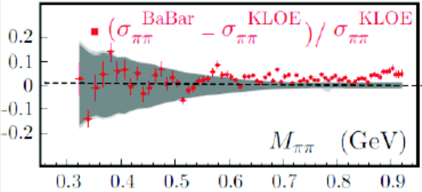
Below the ρ peak good agreement with CMD-2/SND.
Above the ρ peak KLOE10 slightly lower (as KLOE08)

G. Venanzoni, talk at Tau2010

Comparison of results: KLOE10 vs BaBar

BaBar results compared to KLOE10: Fractional difference





band: KLOE10 error

Agreement within errors below 0.6 GeV; BaBar higher by 2-3% above

G. Venanzoni, talk at Tau2010