Fun with F-theory GUTs and $U(1)$ Symmetries

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Physics scenarios that extend the MSSM typically require some mechanism that controls the superpotential.

Standard Model has nice accidental Baryon number and Lepton number symmetries

\[ \mathcal{L} \sim qhu^c + qh^\dagger d^c + \ell h^\dagger e^c \]
\[ \frac{1}{M^2} qqq\ell + \frac{1}{M^2} u^c u^c d^c e^c \text{ forbidden} \]

\[
\begin{array}{c|cc}
 & B & L \\
\hline
q & \frac{1}{3} & 0 \\
u^c & -\frac{1}{3} & 0 \\
d^c & -\frac{1}{3} & 0 \\
\ell & 0 & 1 \\
e^c & 0 & -1 \\
h & 0 & 0 \\
\end{array}
\]

... but supersymmetric extensions can add new couplings and interactions at the renormalizable level that violate them.

... additional UV physics can introduce new violations.
Challenges for String Models

• This is particularly challenging for string models because they come with lots of new UV physics
  • KK modes on the brane
  • Bulk fields (SUGRA + . . .)
  • . . .

• Most robust approach is to gain control by engineering symmetries
  • This need not be the only option, though – internal structure of the model can help with some suppression
Our approach to studying these issues is very much in line with the paradigm of ‘local-to-global model building’

[Aldazabal, Ibanez, Quevedo, Uranga], [Gray, He, Jejjala, Nelson] [Verlinde, Wijnholt]

Local physics near stacks of branes has a ‘universal’ description in terms of brane worldvolume physics

1. Can we understand the rules for model-building in this setting?

2. How are those rules modified/constrained when we insist on the existence of UV completions (global embeddings)?
   - ‘Single-stack’ GUT models→this talk
   - ‘Multi-stack’ quiver models→Jim Halverson’s talk
Objectives

- F-theory model-building utilizes a number of tools in addition to symmetry structure
  
  [Donagi, Wijnholt] [Beasley, Heckman, Vafa]

  1. Promising mechanisms for breaking the GUT group
     [Beasley, Heckman, Vafa], [Donagi, Wijnholt]

  2. Ideas for generating flavor hierarchies
     [Heckman, Vafa] [Ibanez, Font], [KIng, Leontaris, Ross]

- But when we build models . . .
  
  [JM, Saulina, Schäfer-Nameki], [Blumenhagen, Grimm, Jurke, Weigand]

  [Grimm,Krause,Weigand], [Cvetic, Garcia-Etxebarria, Halverson]

  [Chen, Knapp, Kreuzer, Mayrhofer], [Knapp, Kreuzer, Mayrhofer, Walliser]

  1. These ingredients to not play nicely with $U(1)$ symmetries
     [JM, Saulina, Schäfer-Nameki]

  2. This leads to claims of unwanted features like charged exotics
The goal of this talk is to get some **physical understanding** for why F-theory models with $U(1)$ symmetries seem so constrained.
Outline

- Basic Structures of F-theory GUTs
- A Closer Look at "Hypercharge Flux"
- Implications of the "Dudas-Palti Relations"
F-theory and Intersecting Branes

- The basic structure of F-theory models can be described in the language of intersecting branes.

- Charged matter from open strings with one end on the stack
- Other end on some other D-brane, orientifold plane, $\rightarrow$ "Matter branes"

- In F-theory models, the branes are 7-branes wrapping $\mathbb{R}^{3,1} \times S_2$ for some $\mathbb{C}$ surface $S_2$
Charged matter is effectively 6-dimensional

- Spectrum of 4d multiplets requires further dimensional reduction
- # of 4d multiplets can be adjusted with fluxes
Chiral Spectra from Fluxes

- Spectrum on matter curves determined by
  - "Bulk Flux"
  - "Brane Flux" ("Hypercharge Flux")

[Beasley, Heckman, Vafa] [Donagi, Wijnholt]

10 Matter Curve

\[
10 \rightarrow (1, 1)_+ \oplus (3, 2)_{+1/6} \oplus (\overline{3}, 1)_{-2/3}
\]

<table>
<thead>
<tr>
<th>MSSM Multiplet</th>
<th>Chirality</th>
</tr>
</thead>
<tbody>
<tr>
<td>((1, 1)_+)</td>
<td>(M^{(10)} + N^{(10)})</td>
</tr>
<tr>
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<td>(M^{(10)})</td>
</tr>
<tr>
<td>((\overline{3}, 1)_{-2/3})</td>
<td>(M^{(10)} - N^{(10)})</td>
</tr>
</tbody>
</table>

\[
\overline{5} \text{ Matter Curve}
\]

\[
\overline{5} \rightarrow (\overline{3}, 1)_{+1/3} \oplus (1, 2)_{-1/2}
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Chiral Spectra from Fluxes

- Spectrum on matter curves determined by
  - "Bulk Flux"
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[Beasley, Heckman, Vafa] [Donagi, Wijnholt]

### 10 Matter Curve

$$10 \rightarrow (1, 1)_{+1} \oplus (3, 2)_{+1/6} \oplus (\bar{3}, 1)_{-2/3}$$

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### $\bar{5}$ Matter Curve

$$\bar{5} \rightarrow (\bar{3}, 1)_{+1/3} \oplus (1, 2)_{-1/2}$$

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<td>$(\bar{3}, 1)_{+1/3}$</td>
<td>$M^{(5)}$</td>
</tr>
<tr>
<td>$(1, 2)_{-1/2}$</td>
<td>$M^{(5)} - N^{(\bar{5})}$</td>
</tr>
</tbody>
</table>

- Can engineer doublets without triplets by setting

$$M^{(\bar{5})} = 0 \quad N^{(\bar{5})} = \pm 1$$
"Hypercharge Flux" vs $U(1)$ Symmetries

- Model building with F-theory GUTS relies on both
  - "Hypercharge Flux"
  - $U(1)$ Symmetries

Explicit constructions based on spectral covers have shown that these two are interrelated

- Distributions of "hypercharge flux" along matter curves are highly constrained
  - Lose control over 'non-GUT'ness of the spectrum
  - Resulting models exhibit charged 'quasi-chiral' exotic fields
Understanding the Constraints

- Is there a sharp way to describe the relationship between $U(1)$ symmetries and the distribution of "hypercharge flux"?

- Is there an intrinsic physical meaning to this relationship?
2. A Closer Look at "Hypercharge Flux"
Mixed Gauge Anomalies

- Hypercharge flux induces chirality so its ‘distribution’ should be limited by anomaly considerations.
- Consider adding pure $U(1)_Y$ flux to a geometry that has both $SU(5)_{GUT}$ and some extra $U(1)$’s:
  
  $SU(5)_{GUT}$ and some extra $U(1)$’s

- By construction we should not have any 4-dimensional gauge anomalies:
  
  - Especially interesting anomalies – $G_{MSSM} \times G_{MSSM} \times U(1)$
    - These get contributions only from chiral fields on matter curves
    - Cancellation will imply correlation between $\omega_Y$ and matter curves
### Mixed Gauge Anomalies

Anomalies from \(10\) curve with \(U(1)\) charge \(q_{10}\) and +1 unit of \(U(1)_Y\) flux

<table>
<thead>
<tr>
<th>Mult</th>
<th>Chir</th>
<th>(SU(3)^2 U(1))</th>
<th>(SU(2)^2 U(1))</th>
<th>(U(1)_Y^2 U(1))</th>
</tr>
</thead>
<tbody>
<tr>
<td>((1, 1)_{+1})</td>
<td>6</td>
<td>0</td>
<td>0</td>
<td>(6q_{10})</td>
</tr>
<tr>
<td>((3, 2)_{+1/6})</td>
<td>1</td>
<td>(2q_{10})</td>
<td>(3q_{10})</td>
<td>(q_{10}/6)</td>
</tr>
<tr>
<td>((3, 1)_{-2/3})</td>
<td>-4</td>
<td>0</td>
<td>-16(q_{10}/3)</td>
<td></td>
</tr>
<tr>
<td>TOTAL:</td>
<td></td>
<td>-2(q_{10})</td>
<td>3(q_{10})</td>
<td>5(q_{10}/6)</td>
</tr>
</tbody>
</table>

Anomalies from \(\overline{5}\) curve with \(U(1)\) charge \(q_{\overline{5}}\) and +1 unit of \(U(1)_Y\) flux

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<tr>
<th>Mult</th>
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<th>(SU(2)^2 U(1))</th>
<th>(U(1)_Y^2 U(1))</th>
</tr>
</thead>
<tbody>
<tr>
<td>((3, 1)_{+1/3})</td>
<td>2</td>
<td>(2q_{\overline{5}})</td>
<td>0</td>
<td>(2q_{\overline{5}}/3)</td>
</tr>
<tr>
<td>((1, 2)_{-1/2})</td>
<td>-3</td>
<td>0</td>
<td>-3(q_{\overline{5}})</td>
<td>-3(q_{\overline{5}}/2)</td>
</tr>
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<td>TOTAL:</td>
<td></td>
<td>(2q_{\overline{5}})</td>
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<td>-5(q_{\overline{5}}/6)</td>
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</table>
All mixed anomalies cancel provided we have

$$\sum_{10 \text{ Matter Curves}, a} q_a \int_{\Sigma_{10,a}} \omega_Y = \sum_{5 \text{ Matter Curves}, i} q_i \int_{\Sigma_{5,i}} \omega_Y$$
Dudas-Palti Relations

- All mixed anomalies cancel provided we have:

\[ \sum_{10 \text{ Matter Curves}, a} q_a \int_{\Sigma_{10,a}} \omega_Y = \sum_{\bar{5} \text{ Matter Curves}, i} q_i \int_{\Sigma_{\bar{5},i}} \omega_Y \]

- These relations were first observed by Dudas and Palti in a set of spectral cover constructions.
Dudas-Palti Relations

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\[ \sum_{10 \text{ Matter Curves, } a} q_a \int_{\Sigma_{10,a}} \omega_Y = \sum_{\bar{5} \text{ Matter Curves, } i} q_i \int_{\Sigma_{\bar{5},i}} \omega_Y \]

- These relations were first observed by Dudas and Palti in a set of spectral cover constructions.

- They can be proven directly within the spectral cover formalism

[JM, Dolan, JM, Saulina, Schäfer-Nameki]

- With spectral covers, it seems possible (in principle) to construct all consistent distributions of \( U(1)_Y \) flux consistent with

1. The Dudas-Palti relations
2. The cancellation of MSSM gauge anomalies

[Dolan, JM, Saulina, Schäfer-Nameki]
All "constraints" that have been observed in F-theory GUTs with $U(1)$ symmetries are captured by these relations.
Some consequences we shall obtain from these relations can be immediately understood in terms of their underlying physics:
\[ \sum_{\text{10 Matter Curves, } a} q_a \int_{\Sigma_{10,a}} \omega_Y = \sum_{\text{5 Matter Curves, } i} q_i \int_{\Sigma_{5,i}} \omega_Y \]

Some consequences we shall obtain from these relations can be immediately understood in terms of their underlying physics:

- Generic $U(1)$’s will become anomalous once we switch on fluxes to generate a nontrivial spectrum
- There is a nice 4d Green-Schwarz mechanism that operates to cancel anomalies
  
  \[ \ldots \] but that mechanism is independent of the hypercharge flux that we use to break the GUT gauge group so
$U(1)$ anomalies must be the same before and after we introduce the GUT-breaking flux

This implies that $SU(3)^2 U(1)$, $SU(2)^2 U(1)$, and $U(1)^2_{\gamma} U(1)$ anomalies must agree (up to rescaling by the appropriate Casimirs)
3. Implications of the Dudas-Palti Relations
Simple implication

- Study $U(1)$ symmetries that commute with $SU(5)$
Simple implication

- Study $U(1)$ symmetries that commute with $SU(5)$
- $G_{SM}^2 U(1)$ anomalies must all agree
  - ... but $H_u$ and $H_d$ do not contribute to the $SU(3)^2 U(1)$ anomaly

  $\implies H_u$ and $H_d$ must be vectorlike wrt $U(1)$

- Such a $U(1)$ cannot help with the $\mu$ problem
• We often like $U(1)$’s that are flavor blind as well
  • Such $U(1)$’s are compatible with a particularly nice flavor scenario but we don’t say anything about flavor structure here

[Heckman, Vafa]

(except that we do not use $U(1)$’s to manipulate flavor)
• We often like $U(1)$’s that are flavor blind as well
  • Such $U(1)$’s are compatible with a particularly nice flavor scenario but we don’t say anything about flavor structure here [Heckman, Vafa]

(except that we do not use $U(1)$’s to manipulate flavor)

• Only one $U(1)$ that
  • Gives all 10’s a common charge
  • Gives all 5 a common charge
  • Gives $H_u$ and $H_d$ opposite charges
  • Preserves the MSSM superpotential

\[
W_{\text{MSSM}} = 10_M \times 10_M \times 5_H + 10_M \times \bar{5}_M \times \bar{5}_H
\]

\[
\begin{array}{c|cccc}
U(1) & 10_M & \bar{5}_M & 5_H & \bar{5}_H \\
\hline
1 & -3 & -2 & 2
\end{array}
\]
This is $U(1)_\chi \to$ linear combination of $U(1)_Y$ and $U(1)_{B-L}$

- Only flavor-blind $U(1)$ consistent with exact MSSM spectrum
- $U(1)_\chi$ is nice because it contains a $\mathbb{Z}_2$ matter parity subgroup
  
- Issues in spectral cover models if you want to break $U(1)_\chi$ while preserving $\mathbb{Z}_2$ matter parity → see Christoph Ludeling’s talk

[Ludeling, Nilles, Stephan]
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• Inadequate for dealing with

\[ W_\mu \sim \mu H_u H_d \]

\[ W_{\text{Dim 5 proton decay}} \sim \frac{1}{\Lambda} Q^3 L \]
$U(1)_{PQ}$

**Problem**

$W_\mu \sim \mu H_u H_d$

**Dim 5 proton decay**

$W_{\text{Dim 5}} \sim \frac{1}{\Lambda} Q^3 L$

- Would like to engineer a $U(1)_{PQ}$ symmetry to deal with $\mu$ and dim 5 proton decay

  $$Q(H_u) + Q(H_d) \neq 0$$

- Anomaly analysis tells us that this is not possible without introducing quasi-chiral exotics
  - Come in non-$SU(5)$ multiplets
    - must give 'non-universal' contribution to mixed $G_{SM}^2 U(1)$ anomalies
  - Non-chiral wrt MSSM but chiral wrt $U(1)_{PQ}$
Dealing with Exotics

- In principle, exotics can lift since they will couple to MSSM singlets $X_i$ that carry $PQ$ charge

$$W \sim X_i f_{\text{exotic}} \bar{f}_{\text{exotic}} \implies M_{\text{Exotic},i} \sim \langle X_i \rangle$$

- Expectation values $\langle X_i \rangle$ can strongly break $U(1)_{PQ}$ and regenerate dangerous operators from

$$\int d^2 \theta \frac{X_i^{n_i}}{\Lambda^{\sum_i n_i - 1}} H_u H_d \quad \text{and/or} \quad \frac{X_i^{n_i}}{\Lambda} \int d^2 \theta Q^3 L$$

- Suppression of operators favors small $\langle X_i \rangle$
- Unification favors large $\langle X_i \rangle$
Dealing with Exotics

[Dolan, JM, Saulina, Schäfer-Nameki]

- Best possible scenario: exotics come in a combination that yields universal shift of MSSM $\beta$ functions
- Dudas-Palti relations have something to say about the structure of exotics, though. . .
Dealing with Exotics

[Dolan, JM, Saulina, Schäfer-Nameki]

- Best possible scenario: **exotics** come in a combination that yields universal shift of MSSM $\beta$ functions
- **Dudas-Palti** relations have something to say about the structure of **exotics**, though...

For simplicity, suppose all **exotics** lifted by 1 singlet, $X$

$$W \sim X f_{\text{exotic}} \bar{f}_{\text{exotic}}$$

**Dudas-Palti** $\implies q_{H_u} + q_{H_d} = q_X \Delta$

where $\Delta$ measures the non-universal $\beta$ function shifts

$$\Delta = \delta b_2 - \delta b_3 = \frac{1}{6} (5\delta b_1 + 3\delta b_2 - 8\delta b_3)$$
Unification Issues

$q_{H_u} + q_{H_d} = q_x \Delta \quad \Delta = \delta b_2 - \delta b_3 = \frac{1}{6} (5\delta b_1 + 3\delta b_2 - 8\delta b_3)$

- Impossible for exotics to preserve 1-loop gauge coupling unification

- We could also try to adjust the charge of $X$ in order to crank up the powers $m, n$ in

\[
\int d^2 \theta \left( \frac{X}{\Lambda} \right)^m H_u H_d \quad \frac{1}{\Lambda} \int d^2 \theta \left( \frac{X}{\Lambda} \right)^n Q^3 L
\]
Unification Issues

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Dudas-Palti \( \implies -m = n = \Delta ! \)
Unification Issues

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\[
\int d^2\theta \left( \frac{X}{\Lambda} \right)^{-\Delta} \wedge H_u H_d \quad \frac{1}{\Lambda} \int d^2\theta \left( \frac{X}{\Lambda} \right)^{\Delta} Q^3 L
\]

\[ \text{Dudas-Palti} \implies -m = n = \Delta! \]
\[
q_{H_u} + q_{H_d} = q_x \Delta \\
\Delta = \delta b_2 - \delta b_3 = \frac{1}{6} (5\delta b_1 + 3\delta b_2 - 8\delta b_3)
\]

\[
\int d^2 \theta \left( \frac{X}{\Lambda} \right)^{-\Delta} \wedge H_u H_d \\
\frac{1}{\Lambda} \int d^2 \theta \left( \frac{X}{\Lambda} \right)^\Delta Q^3 L
\]

- General tension between unification and proton decay/\(\mu\) prob
  - Dealing with exotics from \(U(1)_{PQ}\) forces us to break it so strongly that it may not address the problems it was meant to solve

- Similar story for multiple singlets \(X_i\)

- Small hope remains: \(U(1)_Y\) flux also distorts unification

[Donagi, Wijnholt] [Blumenhagen]

- Maybe we can use this to gain some wiggle room?

[Dolan, JM, Schäfer-Nameki, in progress]
Further lessons

- So far everything we have said is essentially local...
  - Engineering $U(1)$’s globally is a very subtle matter
    [Hayashi, Kawano, Tsuchiya, Watari], [Grimm, Weigand], [JM, Saulina, Schäfer-Nameki]

- From global studies, it seems that $U(1)_{PQ}$ symmetries are generically Higgs’ed by GUT singlets away from the $SU(5)$ stack
  [JM, Saulina, Schäfer-Nameki,...]

  - Not true for $U(1)_X$

- Natural suppression mechanism for PQ violating terms (like exotic masses)
  - Seems likely to be model (ie geometry) dependent
The general lesson seems to be that $Z'$s are not ubiquitous in F-theory GUTs

In fact, the only $Z'$ that $SU(5)$ F-theory GUT models like is $U(1)_\chi$; $PQ$'s will be Higgs'ed at a high scale and only approximate selection rules, which seem model-dependent (and amount to tuning certain Yukawa couplings from a low energy point of view), will remain
Summary

• “All constraints” that have appeared in spectral cover models of F-theory GUTs are encoded in the Dudas-Palti relations.

• The Dudas-Palti relations are a consequence of 4-dimensional anomaly cancellation so their physical origin is clear.

• Several model-building implications:
  • Only $U(1)_{B-L}$ is consistent with the precise MSSM spectrum.
  • Models that use $U(1)$ symmetries to address $\mu$ or dimension 5 proton decay come equipped with exotics.

  → General tension between $\mu$/proton decay and unification.