Studying Neutralinos Bottom-Up at the LHC

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Work in progress....
We are all awaiting the discovery of new physics at the LHC.

**SUPER SYMMETRY**

The LHC is going great guns, but so far no sparticles.
If there is a discovery, what next?

★ Sparticle spectroscopy. Many techniques for mass measurements.

★ Although inclusive SUSY event rate (and hence SUSY reach) is mostly sensitive to gluino and squark masses only, the relative rates in various topologies will provide information about the underlying framework.

★ Information also in various energy, angle, mass, etc. distributions

Our goal here is to study the inclusive same flavour, opposite sign dilepton $(e^+e^-, \mu^+\mu^-)$ mass distribution to glean as much information as we can about neutralinos.

The mass edge of this distribution has been examined in many studies to extract $m_{\tilde{Z}_2} - m_{\tilde{Z}_1}$ which is a good starting point for SUSY mass reconstruction.

Does this distribution contain more information?
Why the $m_{\ell\ell}$ distribution?

- Reasonable rate for the signals from gluino and squark cascade decays in a wide variety of models.

- The Standard Model background can be readily suppressed with simple cuts leaving a relatively pure SUSY sample.

- The SUSY contamination to the signal from neutralino decays that we want to study can be statistically removed by flavour subtraction.

- Modulo cuts, this is a Lorentz invariant distribution that can be directly extracted from the data. Unlike energy or angular distributions, the boost of the parent neutralino does not matter.
What sort of information do we want to extract?

★ End point – old news

★ Relative sign of mass eigenvalues is physical (Kitano and Nomura discussed this as a signature of a Higgsino-like versus gaugino-like neutralinos).

★ Is the decay dominated by $Z$ or slepton exchange?

★ Can we tell more from the shape of the distribution?

★ Get really greedy and try to extract parameters of the neutralino mass matrix!
Some ground rules

In order to keep the study as bottom-up as possible do not resort to specific constrained models. mSUGRA, CMSSSM, AMSB....

Of course it is impossible to do an analysis w/o any assumptions. Will spell these out as we go along. For now, we assume an

* MSSM sparticle content

The shape of the decay distribution is fixed by neutralino decay parameters

\[(M_1, M_2, \mu, \tan \beta, m_{\tilde{\ell}_L}, m_{\tilde{\ell}_R})\]

Note that we have slipped in the flavour-degeneracy of sleptons whose violation would lead to an observable inequality of \(e^+e^-\) and \(\mu^+\mu^-\) pairs from neutralino decays.a

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aWe are tacitly assuming that neutralino polarization does not significantly affect this mass distribution.
We will use only the shape of the $m_{\ell\ell}$ distribution as the normalization depends on gluino and squark properties, decay branching fractions.

Also LHC will always mean a $pp$ collider operating at $\sqrt{s} = 14$ TeV.
Another assumption

We assume that the two body decay channels for $\tilde{Z}_2 \rightarrow \tilde{Z}_1 Z$ and $\tilde{Z}_2 \rightarrow \tilde{\ell} \ell$ are forbidden.

The first, of course leads to a mass distribution sharply peaked at $M_Z$.

Birkedal, Group and Matchev (arXiv: hep-ph/0507002) have illustrated by mSUGRA case studies that the $m_{\ell\ell}$ distribution can be used to exclude off-shell slepton decays if for the “test point” $\tilde{Z}_2 \rightarrow \tilde{\ell} \ell$ is accessible.

We do not know the details of their analysis. Without making any representation as to whether this is possible, here we simply assume that our neutralino decays via the three-body mode.

In subsequent fits that I will show you, we will make an additional technical assumption for simplicity that $m_{\tilde{\ell}L} = m_{\tilde{\ell}R}$.

Two representative case studies.
But first, it’s confession time!
We began this study long time back and selected LEP 2 and Tevatron-safe cases. Unfortunately, our analysis took longer than expected and the LHC collaborations spoiled the party.

Why show these results then?

- Some lessons from these studies may carry over to LHC-safe points that we will examine.
- These are the only results that I currently have.
Case 1: Gaugino-like neutralinos

\[ m_{\tilde{g}} = m_{\tilde{q}} = 441 \text{ GeV}, \quad (M_1, M_2, \mu, \tan \beta = 77, 127, -, 911, 10). \]

\[ m_{\tilde{Z}_1} = 77.2 \text{ GeV}, \quad m_{\tilde{Z}_2} = 127.4 \text{ GeV}, \quad B(\tilde{Z}_2 \rightarrow \ell \ell \tilde{Z}_1) = 11\% \text{ per lepton (large)} \]

\[ m_{\tilde{\ell}_L} = m_{\tilde{e}_R} = 211 \text{ GeV}. \]

Gaugino-like lighter neutralinos, with heavier -inos at the TeV scale.

\[ n_\ell = 2, \quad E_{T\ell} > 6 \text{ GeV}, \quad n_j \geq 4, \quad E_T > 50 \text{ GeV}, \quad \sum_1^4 |E_{Tj_i}| > 550 \text{ GeV} \]
The blue histogram is the ideal distribution expected for our case study from neutralinos only.

The red is the same except now the relative sign of $M_1$ and $M_2$ (and hence of $m_{\tilde{Z}_1}$ and $m_{\tilde{Z}_2}$) is flipped.

The negative relative sign of neutralino eigenvalues pushes the histogram in! Kitano and Nomura had first observed this for the gaugino-like versus higgsino-like neutralino cases. Seems as though this should be possible to tell even by eye.
\[ \frac{d\Gamma}{dm_{ll}} = \frac{d\Gamma_Z}{dm_{ll}} + \frac{d\Gamma_{\tilde{\mu}}}{dm_{ll}} + \frac{d\Gamma_{\tilde{\tau}Z}}{dm_{ll}}. \]  

Here,

\[ \frac{d\Gamma_Z}{dm_{ll}} = \frac{4\pi^2 w_{12}^2 g^2 (1 + (-1 + 4 \sin^2 \theta_W)^2) m_{ll} \sqrt{\lambda(m_{ll}^2, m_{\tilde{Z}_1}^2, m_{\tilde{Z}_2}^2)}}{12(2\pi)^5 \cos^2 \theta_W (m_{ll}^2 - m_{\tilde{Z}}^2)^2 4m_{\tilde{Z}}^3} \times \]

\[ ((m_{\tilde{Z}_2}^2 - m_{\tilde{Z}_1}^2)^2 - 2m_{ll}^4 + m_{ll}^2 (m_{\tilde{Z}_2}^2 + m_{\tilde{Z}_1}^2) + 6am_{\tilde{Z}_1} m_{\tilde{Z}_2} m_{ll}^2). \]

Here, \( a \) is the relative sign of \( \tilde{Z}_1, \tilde{Z}_2 \) mass eigenvalues. \( i.e. \ a = \pm 1. \)

BTW, it is just this “\( a \)-term” that causes the distribution to shift inward when \( a = -1 \).
The contribution from the pure slepton-mediated decay is,

$$\frac{d\Gamma_{\tilde{l}\tilde{l}}}{dm_{ll}} = \frac{1}{32\pi^3} \frac{m_{ll}}{4m_{\tilde{Z}}^3} \left( A_{Z_1}^2 A_{Z_2}^2 F_{\tilde{l}}(m_{\tilde{l}L}) + B_{Z_1}^2 B_{Z_2}^2 F_{\tilde{l}}(m_{\tilde{l}R}) \right),$$

$$F_{\tilde{l}}(m_{\tilde{l}}) = -\sqrt{\lambda(m_{ll}^2, m_{\tilde{Z}_1}^2, m_{\tilde{Z}_2}^2)} - 4\sqrt{\lambda(m_{ll}^2, m_{\tilde{Z}_1}^2, m_{\tilde{Z}_2}^2)} \frac{(m_{\tilde{Z}_2}^2 - m_{\tilde{l}}^2)(m_{\tilde{Z}_1}^2 - m_{\tilde{l}}^2)}{(\Delta(m_{\tilde{l}}, m_{\tilde{Z}_1}, m_{\tilde{Z}_2}) + m_{ll}^2)^2 - \lambda(m_{ll}^2, m_{\tilde{Z}_1}^2, m_{\tilde{Z}_2}^2)}$$

$$+ \left( \Delta(m_{\tilde{l}}, m_{\tilde{Z}_1}, m_{\tilde{Z}_2}) + \frac{2am_{\tilde{Z}_1} m_{\tilde{Z}_2} m_{ll}^2}{\Delta(m_{\tilde{l}}, m_{\tilde{Z}_1}, m_{\tilde{Z}_2}) + m_{ll}^2} \right) \ln \frac{\Delta(m_{\tilde{l}}, m_{\tilde{Z}_1}, m_{\tilde{Z}_2}) + m_{ll}^2 + \sqrt{\lambda(m_{ll}^2, m_{\tilde{Z}_1}^2, m_{\tilde{Z}_2}^2)}}{\Delta(m_{\tilde{l}}, m_{\tilde{Z}_1}, m_{\tilde{Z}_2}) + m_{ll}^2 - \sqrt{\lambda(m_{ll}^2, m_{\tilde{Z}_1}^2, m_{\tilde{Z}_2}^2)}}.$$

Again, notice the “a-term”.

Functional form of $Z$ and slepton contributions completely different-looking.
Finally, the mixed $Z$ and slepton contribution, \( i.e. \) the cross term is,

\[
\frac{d\Gamma_{lZ}}{dm_{ll}} = -\frac{g m_{ll}}{32\pi^3 m_Z^3 (m_{l l}^2 - m_Z^2) \cos \theta_W} \times \\
(A Z_1 A Z_2 w_{12} (1 - 2 \sin^2 \theta_W) F_{lZ}(m_{R l}) + B Z_1 B Z_2 w_{12} 2 \sin^2 \theta_W F_{lZ}(m_{R l}))
\]

with

\[
F_{lZ}(m_{l l}) = \frac{1}{2} (-\Delta(m_{l l}, m_{Z 1}, m_{Z 2}) + m_{ll}^2) \sqrt{\lambda(m_{l l}^2, m_{Z 1}^2, m_{Z 2}^2)} + ((m_{\tilde{l}}^2 - m_{Z 1}^2)(m_{\tilde{l}}^2 - m_{Z 2}^2) - a m_{Z 1} m_{Z 2} m_{ll}^2) \ln \frac{\Delta(m_{l l}, m_{Z 1}, m_{Z 2}) + m_{ll}^2 + \sqrt{\lambda(m_{l l}^2, m_{Z 1}^2, m_{Z 2}^2)}}{\Delta(m_{l l}, m_{Z 1}, m_{Z 2}) - m_{ll}^2 - \sqrt{\lambda(m_{l l}^2, m_{Z 1}^2, m_{Z 2}^2)}}
\]

Again, a quite different-looking functional form, again with an "\( a \)-term". Since the functional forms look so different can we distinguish $Z$-mediated from slepton-mediated decay?
THEORETICAL EXPECTATIONS: Slepton versus $Z$-mediated decays

Mostly slepton-mediated

$Z$ and slepton shapes different

It'd seem clear that we can tell between $Z$ and slepton mediated decays.

However, if we wish to do so from this data alone, we do not know the parameters. We know the end points well, so fix the end point (within the less bin), and vary other things...
An illustration for a slightly different parameter point.

Here, \( M = \frac{m\tilde{Z}_2 + m\tilde{Z}_1}{2}, \) \( m = \frac{m\tilde{Z}_2 - m\tilde{Z}_1}{2} \)

The real thing (black curve) coincides with the pure slepton-mediated (blue) curve for slightly different parameters and almost coincides with the pure \( Z \)-mediated (red) curve for very different parameters!

Even with fixed endpoint do not have information of \( m\tilde{Z}_2 + m\tilde{Z}_1 \) from this data alone! D I S C O U R A G I N G.
WHAT'S HAPPENING? (A partial answer)

Assuming that the $Z$ and slepton propagators can be shrunk to a point so that we have a four point interaction, by Fierz transforming it is not difficult to show this takes the form:

$$
\begin{align*}
&Z\text{-exchange} & &\tilde{\ell}_{L/R}\text{-exchange} \\
\overline{Z}_1\gamma_\mu(\gamma_5)^{\theta} & \tilde{\ell}_2\bar{\ell}\gamma_\mu(a + b\gamma_5)\ell & \overline{Z}_1\gamma_\mu(\gamma_5)^{\theta} & \tilde{\ell}_2\bar{\ell}\gamma_\mu(1 \pm \gamma_5)\ell
\end{align*}
$$

Of course, $a + b\gamma_5 = (a + b)\frac{1+\gamma_5}{2} - (a - b)\frac{1-\gamma_5}{2}$.

Finally, in massless lepton limit, the left and right leptons are "independent particles" so no interference between left and right slepton graphs.

Hence $Z$ and slepton amplitudes same except for deviation from point propagators.

For $Z$ propagator, corrections are $O(\frac{m^{2}_{\ell\ell}}{M^{2}_{Z}})$, but still not bad; compensation with different parameters??
This plot is made for an idealized study – neutralino leptons only, no lepton cut. Realistic effects can only make this worse.

The reduction of $\chi^2$ at high slepton mass illustrates the degeneracy of the spectra that we just discussed.

Expect that the higgsino-like case with $|\mu| \ll |M_{1,2}|$ to have the same issues (even worse as ino-splitting is smaller still).
Another case study

The gaugino-like case with one mass edge and shape was discouraging because we could not tell from $m_{\ell\ell}$ distribution alone whether the decay was $Z$- or slepton-mediated.

Exceptions would be where the mass gap was larger so $Z$ was closer to its mass-shell and propagator effects changed the shape. But this may be regarded as somewhat exceptional.

In higgsino-like neutralino case, the mass gap is generally smaller, so this does not help.

Led to think of mixed bino-higgsino DM. This happens in many models when for some reason $\mu$ is not terrifically large.

Indeed, it is possible that there is a double mass edge from both $\tilde{Z}_2$ and $\tilde{Z}_3$ decays. Occurs, for instance, in so-called High $M_2$ DM models. (Baer, Mustafayev, Summy, XT)
The idea behind high $M_2$ DM: An ununified high value of $M_2$ at GUT scale.

$$\frac{dm^2_{H_u}}{dt} = \frac{2}{16\pi^2} \left( -\frac{3}{5} g_1^2 M_1^2 - 3 g_2^2 M_2^2 + \frac{3}{10} g_1^2 S + 3 f_t^2 X_t \right),$$

High $M_2(GUT)$ has two effects.

1. It initially causes $m^2_{H_u}$ to increase more rapidly as we reduce the renormalization scale $Q$.

2. It increases the $t$-squark masses (that enter via the $X_t$ term) more than in models with universal parameters.

Thus $m^2_{H_u}$ initially goes up faster than in unified gaugino mass models, but then comes down faster because of increased $X_t$.

$$\mu^2 = \frac{m^2_{H_d} - m^2_{H_u} \tan^2 \beta}{(\tan^2 \beta - 1)} - \frac{M_Z^2}{2} \approx -m^2_{H_u}$$

Depending on the balance the weak scale value of $-m^2_{H_u}$, and hence $|\mu|$ may be relatively small.
High $M_2$ can give a small value of $-m_{H_u}^2 \sim \mu^2$

$m_0 = 300 \text{GeV}, \ m_{1/2} = 300 \text{GeV}, \ \tan \beta = 10, \ A_0 = 0, \ \mu > 0, \ m_t = 171.4 \text{GeV}$

At the weak scale we have $M_1 < |\mu| \ll M_2$.

Motivates our next case study with a double mass edge.
A double mass edge case

\[ m_{\tilde{g}} = m_{\tilde{q}} = 441 \text{ GeV}; \quad m_{\tilde{\ell}_L} = m_{\tilde{e}_R} = 211 \text{ GeV}; \]

\[(M_1, M_2, \mu, \tan \beta = -70, 400, 120, 10).\]

\[ m_{\tilde{Z}_1} = 63 \text{ GeV}, \quad m_{\tilde{Z}_2} = 113 \text{ GeV}, \quad m_{\tilde{Z}_3} = 138 \text{ GeV}, \quad m_{\tilde{Z}_4} = 418 \text{ GeV}; \]

\[ B(\tilde{\nu}_L \rightarrow \ell \ell \tilde{Z}_1) = 2.2\%, \quad B(\tilde{\nu}_L \rightarrow \ell \ell \tilde{Z}_1) = 1.6\%; \quad B(\tilde{\nu}_L \rightarrow \ell \ell \tilde{Z}_1) = 1.9\% \text{ per lepton} \]

\[ B(\tilde{u}_L \rightarrow \tilde{Z}_2 u) = 20\%; \quad B(\tilde{u}_L \rightarrow \tilde{Z}_3 u) = 12\%; \quad B(\tilde{d}_L \rightarrow \tilde{Z}_2 d) = 29\%; \]

\[ B(\tilde{u}_L \rightarrow \tilde{Z}_3 d) = 26\%; \quad B(\tilde{q}_R \rightarrow q \tilde{Z}_1) = 83\%; \quad B(\tilde{q}_R \rightarrow q \tilde{Z}_3) = 14\%. \]
Fit the shape of the flavour-subtracted OS dilepton histogram to

\[
(M_1, M_2, \mu, \tan \beta, m_{\tilde{\ell}}) + 3 \text{ normalizations.}
\]

\[
a \times \frac{d\Gamma}{d\ell\ell} (\tilde{Z}_2 \rightarrow \tilde{Z}_1 \ell\ell) + b \times \frac{d\Gamma}{d\ell\ell} (\tilde{Z}_3 \rightarrow \tilde{Z}_1 \ell\ell) + c \times \frac{d\Gamma}{d\ell\ell} (\tilde{Z}_3 \rightarrow \tilde{Z}_2 \ell\ell)
\]
ONE MORE COMPLICATION

Our formula has no lepton cuts in it, the data do!

Expect most distortion due to this at low $m_{\ell\ell}$.

Also expect most of the effect of the lepton cut to be mostly from kinematics rather than details of matrix element.

Poor man’s correction obtained by taking ratio of flavour-subtracted OS dilepton mass distributions by running MC events without and with lepton $E_T$ cut($E_T(\ell) > 6$ GeV) for different choices of other parameters and relative signs of mass eigenvalues that give the same end point.

One such Ratio $R$ for each decay, but $R$ a universal function of the endpoint.

Since $R$ is a correction, we hope that getting it super-precisely will be a nicety.

But this is a patch until someone thinks of a cleverer way to handle the effect of the lepton cut.
Each endpoint has its own $R$-function.

We use the Red curves to obtain the theory expectation after the lepton cuts to get the fitted values of parameters.
Marginalize over other parameters.

\[ \Delta \chi^2 = 4 \]

\[ m_{\tilde{\ell}}^{\text{fit}} = (168 - 188) \text{ GeV}; \quad m_{\tilde{\ell}} = 170 \text{ GeV}. \]

Unlike in the gaugino-like case discussed above, the fact that we have to simultaneously fit three different neutralino decay shapes nails down the slepton mass for this mixed neutralino case.
Marginalize over other parameters.

\[ M_1^{\text{fit}} = (-95, -40) \text{ GeV}; \quad M_1 = -70 \text{ GeV} \quad \mu^{\text{fit}} = (72 - 122) \text{ GeV}; \quad \mu = 120 \text{ GeV} \]

\[ M_2^{\text{fit}} = (360, 465) \text{ GeV}; \quad M_2 = 400 \text{ GeV} \]

Precision not much degraded compared to pure neutralino MC study.
Attempt to extract information about neutralinos from dilepton mass distribution at the LHC w/in the MSSM.

Aside from endpoints, the shape of the mass distribution yields the relative sign of the neutralino mass eigenvalues.

If just one neutralino is accessible, extraction of neutralino parameters appears impossible without additional information because of theoretical degeneracies.

For models where two neutralinos are accessible and both decay via three body modes, we may be able to extract neutralino parameters and even slepton mass in a completely bottom-up manner, w/o resorting to various constrained models!
You are all undoubtedly thinking this is at best academic because the case studies I have shown are already excluded.

Will this have interest in light of new limits? I do not know.

In models with unified gaugino masses, this will probably not work if \( m_{\tilde{q}} \sim m_{\tilde{g}} \) since the lower limit on the gluino mass implies that \( m_{\tilde{Z}_2} - m_{\tilde{Z}_1} > M_Z \).

In models with highly compressed spectra though this may not be an issue. Will need higher integrated luminosity (\( \mathcal{O}(100) \text{ fb}^{-1} \)) to compensate for reduced production rates, making this part of the far future program. Depending on what the LHC finds, the pay-off could be high.

While such scenarios are not in vogue today, this example illustrates that more bottom-up studies than have been looked at may be possible at the LHC.
RESULTS OF THE FITS: PURE CASE (no lepton cuts)

Marginalize over other parameters.
Marginalize over other parameters.

Shifts of central values; don’t know whether the $\tan \beta$ minimum is real.