Composite Scalars with a Compact Extra Dimension

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Outline

• Review of ‘traditional’ NJL model
• 5D NJL
• Propagators in a compact dimension
• Loops with a compact dimension
• Results and conclusion
NJL Model

Integrate out massive gluon:

\[
\frac{g^2}{2} \left( \bar{\psi} \gamma^\mu \frac{\lambda^A}{2} \psi \right) \frac{\eta^{\mu\nu}}{q^2 - \Lambda^2} \left( \bar{\psi} \gamma^\nu \frac{\lambda^A}{2} \psi \right) \bigg|_{q=0}
\]

So we have an effective lagrangian at scale \( \Lambda \):

\[
\mathcal{L} = i\bar{\psi}_L^a \slashed{D} \psi_L^a + i\bar{\psi}_R^a \slashed{D} \psi_R^a + G\bar{\psi}_L^a \psi_R^a \bar{\psi}_R^b \psi_L^b + O \left( \frac{1}{N} \right)
\]

Which has a SU(N) Chiral symmetry.
NJL Model

Perform a field redefinition with an auxiliary field $H$:

$$\mathcal{L} = \bar{\psi}_L \partial_\mu \psi_L + \bar{\psi}_R \partial_\mu \psi_R + g \bar{\psi}_L \psi_R H + \text{h.c.} - \Lambda^2 |H|^2$$

and work with fermion bubble approximation:

Which produces an effective Lagrangian

$$\mathcal{L}_\mu = \bar{\psi}_L \partial_\mu \psi_L + \bar{\psi}_R \partial_\mu \psi_R + \tilde{g} H \bar{\psi}_L \psi_R + \text{h.c.} + \partial_\mu H \partial^\mu H^\dagger - \tilde{m}^2 |H|^2 - \tilde{\lambda} |H|^4$$
Top Condensation

At the low scale $\mu$ the higgs mass is:

$$m^2 = \frac{2}{\log \frac{\Lambda^2}{\mu^2}} \left( \frac{8\pi^2}{2g^2 N_c} \Lambda^2 - (\Lambda^2 - \mu^2) \right) \approx \frac{2}{\log \frac{\Lambda^2}{\mu^2}} \left( \frac{8\pi^2}{g^2 N_c} - 1 \right) \Lambda^2$$

For $\mu \ll \Lambda$ $H$ develops a vev for critical values of the gauge coupling:

$$1 - \frac{g^2 N_c}{8\pi^2} < 0 \quad \text{Chiral Symmetry spontaneously broken.}$$

Up to RG corrections $m_H = 2m_f$ so for $m_H \sim \mathcal{O}(100\text{GeV})$ the top at $m_t = 170\text{GeV}$ works best.

NJL model: Nambu, Jona-Lasino
Top Condensate model: Miransky, Tanabashi, Yamawaki, Bardden, Hill and Linder
Flat 5D

Goal: Study 4 fermion operator in bulk of compactified 5th dimension and possible bound states

\[ \frac{g^2}{\Lambda^3} \overline{t_l t_r t_r t_l} \]

\[ [\psi] = M^2 \]
\[ [g^2] = \frac{1}{M} \]
Fermions in 5d

- In 5D dimensions \( \{ \gamma^M, \gamma^N \} = 2\eta^{MN} \)
- To get a chiral spectrum on an interval choose B.C.

For example: \( \psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \) with

\[
\begin{align*}
\psi_R |_{z=0, L = 0} &= 0 \\
(\partial_z - m)\psi_L |_{z=0, L = 0} &= 0
\end{align*}
\]

massless Weyl fermion

\[
\Psi^{5D} = \begin{pmatrix} \chi(0) \\ 0 \end{pmatrix} + \sum_{n=1} \begin{pmatrix} \chi(n) \\ \psi(n) \end{pmatrix}
\]

Tower starting with mass

\[
\sim \frac{1}{L} \sim TeV
\]

So for \( \frac{g^2}{\Lambda^3} t_l t_r \bar{t}_r t_l \) these are 5D Dirac fermions whose (massless) zero modes are standard model tops
5D calculation

In KK picture:

\[ = \psi_{KK}^{(1)} + \psi_{KK}^{(2)} + ... \]

\[ = g^2 \sum_{n,n'} \int \frac{d^4 q}{(2\pi)^4} \text{Tr} \left( \frac{i}{\not{q} - m_n} \right) \left( \frac{i}{\not{q} - m_{n'}} \right) \]

For some case the mass spectrum might be simple:

\[ = g^2 \sum_{n,n'} \int \frac{d^4 q}{(2\pi)^4} \text{Tr} \left( \frac{i}{\not{q} - \frac{n\pi}{L}} \right) \left( \frac{i}{\not{q} - \frac{n'\pi}{L}} \right) \]

So while some sums have closed forms, not every one does and a bulk fermion mass is not easily incorporated.
5D mixed propagator loops

Alternatively work in a ‘mixed’ basis:

\[
\begin{align*}
&= g^2 \int_0^L dz \int_0^L dz' \int \frac{d^4 q}{(2\pi)^4} \text{Tr} \Delta_l(z, z'; q) \Delta_r(z', z; q) \\
&= \sum_n \psi^{(n)}_{kk} + \ldots
\end{align*}
\]
5D Mixed propagators

Need to solve: \((k + i\gamma^5 \partial_z - m)\Delta(z, z'; k) = i\delta(z - z')\)

Defining \(\Delta = \begin{pmatrix} \Delta_{LR} & \Delta_{LL} \\ \Delta_{RR} & \Delta_{RL} \end{pmatrix} = \begin{pmatrix} (-\partial_z + m)F_R & k_\mu \sigma^\mu F_L \\ k_\mu \sigma^\mu F_R & (\partial_z + m)F_L \end{pmatrix}\)

The F’s satisfy: \((\partial_{\bar{z}}^2 + (k^2 - m^2))F_{L,R} = i\delta(z - z')\)

For example, for a left-handed chiral zero mode

\[F_{l,R} = \frac{-i}{2\chi \sin \chi L} \left[\cos \chi((L - |z - z'|)) - \cos \chi((L - (z + z')))\right]\]

Where \(\chi \equiv \sqrt{k^2 - m^2}\)
5D loops

Nonlocal terms appear in the effective lagrangian:

\[
\int_0^L dz \int_0^L dz' f(q) e^{iq|z-z'|} H(z) H^\dagger(z')
\]

We are interested in the running of the parameters so we take the high energy limit to find dependence on the cutoff:

\[
\int_0^L dz \int_0^L dz' f(q) e^{iqE|z-z'|} H(z) H^\dagger(z')
\]

\[m_0(\Lambda) H H^\dagger(0) + m_L(\Lambda) H H^\dagger(L) + m(\Lambda) \int_0^L dz H H^\dagger(z)\]

- All divergences are *local*.
- All divergences are parameterized by 4D cutoff.
In bulk and on branes we naively expect divergent structure of noncompact 5 and 4 dimensions, respectively.

\[ \delta Z_{5d}^2 = g^2 (a_1 \Lambda + \text{finite}) \]
\[ \delta m_{5d}^2 = g^2 (b_1 \Lambda^3 + b_2 \Lambda + \text{finite}) \]
\[ \delta Z_{4d}^2 = g^2 (c_1 \log \Lambda + \text{finite}) \]
\[ \delta m_{4d}^2 = g^2 (d_1 \Lambda^2 + d_2 \log \Lambda + \text{finite}) \]

But in effective brane Lagrangian not all the naive corrections appear.
Effective Lagrangian

At one fermion loop we have:

\[ \mathcal{L}_{\text{brane}} = \frac{g^2}{32\pi^2} \left\{ \log(L\Lambda) \left[ (3m_l^2 + 2m_l m_r + 3m_r^2) H(0)H^\dagger(0) \right. \\
+ (m_l - m_r) \left( H(0)H^\dagger(0) \right)^\prime + H'(0)H'^\dagger(0) + 0 \leftrightarrow L \right\} + \\
\left. + 2(m_l - m_r)\Lambda \left( H(0)H^\dagger(0) - H(L)H^\dagger(L) \right) \right\} \]

All brane localized terms are related to fermion bulk mass:

- No quadratic divergences on the brane
- No 4D kinetic terms on brane
- \((\partial_z H)^2\) terms are trivial for vanishing fermion mass
If had instead taken the orbifold as a starting point we would have made an identification $\psi(+z) = \pm \gamma^5 \psi(-z)$ in order to produce a chiral spectrum.
In order to have a mass term we need
\[ \int_{-L}^{L} m(z) \bar{\psi}(z) \psi(z) \]
which is satisfied by
\[ m(z) = m(\theta(z) - \theta(-z)) \]
which violates translational invariance.

Any brane term also violates translational invariance.
Effective Lagrangian

Although \((\partial_z H)^2\) brane terms are not proportional to bulk fermion mass, they are trivial in the case of a vanishing bulk fermion mass.

Varying the action we would obtain:

\[
(\delta H \partial_z H + \delta (\partial_z H)^2)|_{z=0, L} = 0
\]

Which would be satisfied by \(\partial_z H|_{z=0, L} = 0\)
Top Condensate in 5D

2 ways to break chiral symmetry now:

• Bulk potential with critical gauge coupling $g$
  $\approx \left( m_l - m_r \right) \left( H(0)H^\dagger(0) - H(L)H^\dagger(L) \right)$

• Brane potential with fermion masses:
  $\mathcal{L}_{\text{brane}} \approx \left( m_l - m_r \right) \left( H(0)H^\dagger(0) - H(L)H^\dagger(L) \right)$
Fermion Condensate in 5D

The vev EOM: $\langle H(z) \rangle \equiv \frac{v(z)}{\sqrt{2L}} \implies v''(z) = m^2 v(z) + \frac{\lambda}{L} v(z)^3$

Can be solved exactly:

$$v(z) = \frac{2i v_0}{\kappa_-} \sqrt{\frac{\tilde{\lambda}/L}{\kappa_+}} \text{sn} \left( \sqrt{\frac{-(z-z_0)^2 \kappa_-}{2L^2}} \frac{\kappa_+}{\kappa_-} \right)$$

Where: $$\kappa_\pm = (\tilde{m}L)^2 \pm \sqrt{(\tilde{m}L)^4 - 2\tilde{\lambda}v_0}$$

For example for: $$m = \frac{2}{L}, \quad m_0 = -m_L = \frac{\sqrt{2.021}}{L}$$

We have: $$v_0 = 4.8 \cdot 10^{-4} \frac{1}{L^3}, \quad z_0 = 1.32L$$
Summary

- Analyzed scalar condensate arising from a 4 fermion operator with a compact extra dimension.
- Performed one loop approximation in mixed basis.
- Brane localized divergences are softer than expected.
- Can break chiral symmetry with bulk or brane V.
- There are power law divergences so the model is fine tuned.
- Future work: phenomenology, model in warped (RS) space.