FLAVOR AND SCATTERING EFFECTS IN LEPTOGENESIS

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SUSY 2011

August 31, 2011
HOW TO GENERATE A BARYON ASYMMETRY?

Sakharov's conditions (1967):

- Baryon number violation
- CP violation
- Departure from equilibrium

SM:

- (B+L)!
SEE-SAW MODEL OF NEUTRINO MASSES

- Right-handed neutrinos $\psi_{Ni}$ are neutral singlets

- Can have Majorana mass term:

$$\mathcal{L} = \frac{1}{2} \bar{\psi}_{Ni} (i\not\!\partial - M_i) \psi_{Ni} + \bar{\psi}_\ell i\not\!\partial \psi_\ell - Y_i^* \bar{\psi}_\ell \phi^\dagger P_R \psi_{Ni} - Y_i \bar{\psi}_{Ni} P_L \phi \psi_\ell$$

- Mass matrix:

$$\begin{pmatrix} 0 & Y_i v \\ Y_i^* v & M_i \end{pmatrix}$$

- Eigenvalues:

$$\lambda_+ \approx M_1 \quad \lambda_- \approx |Y|^2 \frac{v^2}{M_1}$$
SEE-SA W MODEL OF NEUTRINO MASSES

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$$\mathcal{L} = \frac{1}{2} \bar{\psi}_{Ni}(i\partial - M_i)\psi_{Ni} + \bar{\psi}_\ell i\partial \psi_\ell - (Y_i)\bar{\psi}_\ell \phi \dagger P_R \psi_{Ni} - Y_i \bar{\psi}_{Ni} P_L \phi \psi_\ell$$

- Majorana mass violates lepton number

- Out of equilibrium decay of $N_1$ if couplings satisfy

$$\Gamma_{N_1} \propto \sum |Y_{1i}|^2 M_1 < H \bigg|_{T \approx M_1}$$
HOW TO GENERATE AN ASYMMETRY?

Sakharov's conditions:

- Baryon number violation
- CP violation
- Departure from equilibrium

SM + See Saw

Leptogenesis

Fukugita & Yanagida, 1986
USUAL WAY TO PREDICT ASYMMETRY:

- Calculate CP asymmetry in decays

\[ \partial_\eta f_{\ell-\bar{\ell}} = C_D [f_{\ell-\bar{\ell}}] + C_S [f_{\ell-\bar{\ell}}] \]

- Plug into Boltzmann equation

- Solve (with approximations)

  e.g. Pedestrian: Buchmuller, di Bari, Plumacher, 2000
VALID APPROACH?

- Calculate CP asymmetry in decays
  \[ N_1 \ell h \times (N_1 \rightarrow N_j \ell h) \]
  \( \equiv \) Quantum Effect

- Plug into Boltzmann equation
  \[ \partial_\eta f_{\ell-\bar{\ell}} = C_D[f_{\ell-\bar{\ell}}] + C_S[f_{\ell-\bar{\ell}}] \]
  \( \equiv \) Classical Equation

- Solve (with approximations)
  e.g. Pedestrian: Buchmuller, di Bari, Plumacher, 2000

something missed?
ONGOING EFFORT TO IMPROVE:

Canonical Framework: Nonequilibrium QFT

Applied to Leptogenesis

Buchmuller, Fredenhagen, 2000;
de Simone, Riotto, 2007;
Garny, Hohenegger, Karavtsev, Lindner, 2009, 2009;
Anisimov, Buchmuller, Drewes, Mendizabal, 2010, 2010;
Garny, Hohenegger, Karavtsev, 2010;
Beneke, Garbrecht, Herranen, PS, 2010;
Beneke, Fidler, Garbrecht, Herranen, PS 2010;
Garbrecht, 2010;
Beneke, Garbrecht, PS,... in progress

Related

Drewes, 2010;
Gagnon, Shaposhnikov, 2010;
Anisimov, Besak, Bodeker, 2010;
Herranen et al, 2011;
Fidler et al, 2011;
Garbrecht, Garny, 2011;
ONGOING EFFORT TO IMPROVE

Canonical Framework: Nonequilibrium QFT

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- de Simone, Riotto, 2007;
- Garny, Hohenegger, Karavtsev, Lindner, 2009, 2009;
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- Beneke, Garbrecht, Herranen, PS, 2010;
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Related

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- Herranen et al, 2011;
- Fidler et al, 2011;
- Garbrecht, Garny, 2011;
NEQFT FOR LEPTOGENESIS

Dyson-Schwinger eqn. on CTP

\[ i \partial_u S^{ab}(u, v) = a \delta_{ab} \delta^4(u - v) + \sum_c \int d^4 w \Sigma^{ac}(u, w) S^{cb}(w, v) \]

\[ i S(u, v) = \langle \psi(u) \bar{\psi}(v) \rangle \]

lepton two point function

\[ \propto f_\ell(t, k) \]

lepton + antilepton densities
NEQFT FOR LEPTOGENESIS

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lepton two point function

lepton + antilepton densities

\[ n_\ell(t) = \int d^3k f_\ell(t, k) \]

\[ \partial_\eta(n_\ell - n_\bar{\ell}) = W + S \]
FINITE NUMBER DENSITY CORRECTIONS

Source term in hierarchical limit \((M_2 \gg M_1)\):

\[
S = 3 \text{Im}[Y_1^2 Y_2^* Y_2^* Y_2^*] \left( -\frac{M_1}{M_2} \right) \int \frac{d^3 k'}{(2\pi)^3 2 \omega_{k'}} \delta f_N(k') \Sigma_N \mu(k') \Sigma_N^\mu(k')
\]

\[
f_N - f_N^{eq}
\]

no asymmetry in equilibrium

\[
\Sigma_N^\mu(k) = \int_{p,q} \delta^4(k - p - q) p^\mu \left( 1 - f_{\ell}^{eq}(p) + f_{\phi}^{eq}(q) \right)
\]

finite density corrections

In vacuum QFT:

\[
\Sigma_N^\mu(k) = \frac{k^\mu}{16\pi}
\]
THERMAL EFFECTS: STRONG WASHOUT

blue: thermal initial $N_1$ density
red: zero initial $N_1$ density
THERMAL EFFECTS: WEAK WASHOUT

blue: thermal initial $N_1$ density
red: zero initial $N_1$ density

can be sizable!
LEPTON FLAVORS

- Neutral and charged Lepton Yukawa couplings in general not aligned

\[ \mathcal{L} = Y_{ia} \bar{\psi}_{Ni} \phi \psi_{la} + h_{ab} \bar{\psi}_{Ra} \phi^\dagger \tau \psi_{lb} + \text{h.c.} \]

- Leptogenesis usually dominated by \( N_1 \) decays

- Decay into linear combination of \( e, \mu, \tau \)

\[ N_1 \rightarrow \phi l, \quad l \sim \alpha_e l_e + \alpha_\mu l_\mu + \alpha_\tau l_\tau \]
MODIFICATION OF WASHOUT RATES

- Assume tau Yukawa in thermal equilibrium

- Projected onto states $\ell_\tau$ and $\ell_\perp$ by flavor sensitive interactions (denote as $\ell_{1,2}$)

- Boltzmann E:
  \[
  \frac{d}{d\eta} \Delta n_{\ell_i} = W_i + S_\ell
  \]

- Small washout in one flavor can largely increase the asymmetry (over 100%)
FLAVORED EVOLUTION EQUATIONS

\[
\frac{\partial q_\ell}{\partial \eta} = -[\Xi, q_\ell] - \{W, q_\ell\} + 2S - \Gamma^{fl}_\ell
\]

- No oscillation term: suppressed by fast flavor-insensitive gauge interactions

- Decoherence only through flavor sensitive scatterings
IMPORTANT OF FLAVOR

- Total asymmetry as function of the Leptogenesis scale

- Unflavored: $M_1 > 10^{13}$

- Fully Flavored: $M_1 < 10^{11}$

**Scenario B**

blue: full solution
red: unflavored
green: fully flavored
YUKAWA INTERACTION RATES

- Responsible for
  - N1 production/decay $\rightarrow$ strength of washout
  - Flavor sensitive scatterings $\rightarrow$ scale where different flavor regimes are valid

- Difficulty
  - massless 1-$\rightarrow$2 processes zero at tree level $O(g^2 T)$
  - massive 1-$\rightarrow$2 affected by thermal masses $O(g^2 T)$
  - properly include all contributions at $O(g^2 T)$
LEADING CONTRIBUTIONS

- **Thermal masses/width**
  - thermal masses:
    - Giudice, Notari, Raidal, Riotto, Strumia, 2003;
    - Kiessig, Plumacher, Thoma, 2009;

- **Off-shell $2 \rightarrow 2$ scatterings**

- **Collinearly enhanced $2 \rightarrow 3$ processes**
  - Arnold, Moore, Yaffe, 2000;
  - Anisimov, Besak, Bodeker, 2010;
APPROACH

- Use 2PI vacuum diagrams to obtain 1PI self energies
- resums bubble subgraphs (not 2PI) into propagators
- avoids divergencies in t channel diagrams
- 2->3 following AMY, ABB
  - currently checking results, numerics. Soon!
CONCLUSIONS

‣ Ongoing effort to understand Leptogenesis at the quantum level

‣ Consistent framework to derive evolution equations for lepton asymmetry

‣ Solid formalism for calculating interaction rates

‣ Dynamics of important early universe process from first principles
BACKUP!
SUPPRESSION OF OSCILLATIONS

- Flavor blind interactions: \( \Gamma^{bl} \sim g_2^4 T \) (kinetic equilibrium)

- Oscillations: \( \Delta \omega \sim h_T^2 T < \Gamma^{bl} \) (from thermal masses)

- Toy Model:
  
  \[
  \frac{d(\delta^+)}{dt} = -i\omega \delta^+ - \Gamma^{bl}[\delta^+ + \delta^-] \\
  \frac{d(\delta^-)}{dt} = +i\omega \delta^- - \Gamma^{bl}[\delta^+ + \delta^-]
  \]

- Last term enforces: \( \delta^+ = -\delta^- + O(\omega/\Gamma^{bl})\delta^- \)

- Oscillations suppressed by large \( \Gamma^{bl} \)
DEPENDENCE ON LEPTOGENESIS SCALE

- Expansion of Universe:
  \[ H = 1.66\sqrt{g_*} \frac{T^2}{M_{pl}} \]

- Charged Higgs Yukawa interactions:
  \[ \Gamma^{fl} \propto h^2_{\tau} T \]

- Tau Yukawa in equilibrium below \(10^{12} \) GeV

- If Leptogenesis takes place at or below this scale, flavor is important
FULLY FLAVORED REGIME

- Two flavor regime \( T < 10^{12} \) GeV and three flavor regime \( T < 10^9 \) GeV

- Work in charged lepton mass basis, calculate separate washout and source terms for each flavor

\[
\frac{d}{d\eta} \Delta n_{\ell i} = W_i + S_i + \ldots
\]

- Flavor oscillations?
Separate treatment of flavors not sufficient

Washout, expansion, flavor decoherence with similar strength

Separate treatment of flavors not sufficient

ideally: find basis invariant formalism
OUR APPROACH

Derive evolution equations for number densities directly from Nonequilibrium Quantum Field Theory

Natural basis for treatment of flavor

Obtain finite temperature/density corrections to CP asymmetries

Automatic implementation of real intermediate state subtraction (no double counting)
NONEQUILIBRIUM QFT

- Conventional QFT: Calculate “in - out” correlators (S-matrix elements)

\[ \langle A | B \rangle_{out} = \langle A | U(-t, t) | B \rangle_{t \to \infty} = \langle A | S | B \rangle \]

- NEQFT: Know the “in” state \( \rho(t_0) \), want to predict the time evolution of operator:

\[ \langle t | \mathcal{O} | t \rangle = \text{Tr}[\rho(t_0)U^{\dagger}(t, t_0)\mathcal{O}U(t, t_0)] \]
CTP FORMALISM

- Instead of “in-out” correlators: Calculate “in-in” expectation values

- Possible using conventional QFT methods if we let time coordinate on Closed Time Path

- Fields get additional index $\phi^a(t, x)$ that indicates the position of the time coordinate $a = \pm$

Schwinger, 1961; Keldysh, 1964, ...

CTP $\mathcal{C}$:

$t_0$  + branch

- branch

$t$
CTP FORMALISM

- Relevant information contained in 2-point functions for bosons $\Delta(u,v)$ and fermions $S(u,v)$
- Become 2x2 matrices
- Time evolution from Dyson-Schwinger equation:

$$i\hat{\phi}_u S^{ab}(u,v) = a\delta_{ab}\delta^4(u-v) + \sum_c \int d^4w \Sigma^{ac}(u,w) S^{cb}(w,v)$$

IPI self energy
QUANTUM BOLTZMANN EQUATIONS

- Gradient & loop expansion, quasiparticle approximation (also, a Wigner transformation in between)

- Obtain evolution equations for number densities

\[
\frac{d}{d\eta} f_{N1}(k) = D(k)
\]

\[
\frac{d}{d\eta} (n_\ell - \bar{n}_\ell) = W + S.
\]

- Conformal time $\eta$ to incorporate expansion of the universe, proportional to inverse temperature
NOW WITH FLAVOR

just “add” flavor indices to field operators

\[
i S^\ell_<(u, v) = \langle \bar{\psi}_\ell(v) \psi_\ell(u) \rangle
\]

\[
i S^\ell_{ab}(u, v) = \langle \bar{\psi}_\ell^b(v) \psi_\ell^a(u) \rangle
\]

Straightforward generalization for washout and source terms

In addition: oscillations, flavor sensitive scatterings
Commutator term in kinetic equation:

\[ i \partial_\eta S^{\langle,\rangle}_\ell - \left[ k \cdot \gamma + \Sigma^H_\ell, S^{\langle,\rangle}_\ell \right] = -\frac{1}{2} \left( C_\ell + C^\dagger_\ell \right) \]

Time dependent mass basis, diagonalize self energy using \( \Sigma^H_D = U^\dagger(\eta) \Sigma^H U(\eta) \)

Additional term \( i [\Xi, S^{\langle,\rangle}_\ell] \) with \( \Xi = U^\dagger \partial_\eta U \)

\[ \partial_\eta \delta n^\pm_{\ell ab} - [\Xi, \delta n^\pm_{\ell}]_{ab} \pm i \Delta \omega_{ab} \delta n^\pm_{\ell ab} = \pm \frac{1}{2} \left( C_\ell + C^\dagger_\ell \right)_{ab} \]
FLAVOR SENSITIVE INTERACTIONS

- Main source of flavor decoherence
- Contributions from annihilation/scatterings
- All processes allowed at finite temperature
- Estimate using $\Gamma^{\text{an}} + \Gamma^{\text{sc}} \approx 0.7 \alpha_w T$
Gauge interactions enforce kinetic equilibrium also for off diagonal densities

Introduce flavored number densities $n_{\ell ab}^{\pm}$, define flavored charge density $q_{\ell ab} = n_{\ell ab}^+ - n_{\ell ab}^-$

Evolution equation:

$$\frac{\partial q_{\ell}}{\partial \eta} = -[\Xi, q_{\ell}] - \{W, q_{\ell}\} + 2S - \Gamma_{\ell}^\alpha$$
NUMERICS (CHARGED LEPTON FLAVOR BASIS)

no flavor

|Y| vs. z for different values of $h_\tau$:

- $h_\tau = 0.000$
- $h_\tau = 0.002$
- $h_\tau = 0.007$
- $h_\tau = 0.030$

full flavor
WHEN ARE FLAVOR EFFECTS IMPORTANT?

- Three regimes (neglecting muon, electron Yukawas and assuming that flavors are not aligned)
- Unflavored: Single flavor approximation is good
- Fully Flavored: Off-diagonal densities can be neglected
- Intermediate: Full evolution equation needs to be solved
Early universe has zero baryon number, but today's universe has a nonzero $B$.

Lepton number $L$ violated by Majorana mass term.

Electroweak sphalerons can convert the lepton asymmetry into a baryon asymmetry.

Generating a lepton asymmetry sufficient
CP VIOLATION

- Must be able to distinguish particles from anti-particles

- In Leptogenesis: CP violated in decays of heavy right-handed neutrinos:

\[ N_1 \rightarrow L \]
\[ N_1 \rightarrow L, N_{2,3} \]
\[ N_1 \rightarrow L, N_{2,3}, H \]
\[ N_1 \rightarrow L, N_{2,3}, H, L \]

Figure 4: Feynman diagrams contributing to SM thermal leptogenesis.
QM: Observables are expectation values of operators

\[ \Gamma(N_1 \rightarrow H \ell^+) = |\langle N_1 | \mathcal{H}_{\text{int}} | H \ell^+ \rangle|^2 = |A|^2 \]

Asymmetry:

\[ Y_L = \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}} \]

\[ A = h_1 A_0 \]

\[ \bar{A} = h_1^* A_0 \]

Simplest case:

No asymmetry since \( \bar{\Gamma} = \Gamma \)
CP VIOLATION III

- Add one loop correction

\[ \mathcal{A} = h_1 A_0 + h_1^* (h_2)^2 A_1 \]
\[ \bar{\mathcal{A}} = h_1^* A_0 + h_1 (h_2^*)^2 A_1 \]

- Asymmetry proportional to interference term

\[ Y_L \propto \Im(h_1 h_1^* h_2^*) \Im(A_0 A_1) \]

- Note: Requires complex couplings and complex \( A_1 \)
In early universe: Expansion with Hubble rate

\[ H = 1.66 \sqrt{g_*} \frac{T^2}{M_{pl}} \]

Processes with rates \( \Gamma \lesssim H \) go out of EQ

Distributions \( f(k, t) \) deviate from
\[ f^{eq}(k) = \frac{1}{e^{\beta E(k)} \mp 1} \]
Temperature:

\[ T = \frac{T_{\text{com}}}{a(\eta)} = \frac{1}{a(\eta)} \sqrt{\frac{a_R m_{\text{Pl}}}{2}} \left( \frac{45}{g_* \pi^3} \right)^{1/4} \]

Expansion rate (radiation dominated):

\[ a(\eta) = a_R \eta \]

“time variable”

\[ z = M_1/T \propto \eta \]

Time derivative becomes

\[ \frac{d}{d\eta} = a_R \frac{d}{dz} \]
THE ELECTROWEAK SPHALERON

- $B + L$ current is anomalous in the SM
- $T = 0$: Tunneling between configurations with different $B + L$ highly suppressed
- $T \gtrsim \text{TeV}$: In equilibrium
- $\Delta B = \Delta L = 3$
- have no proton decay

$\prod_i (q_i \bar{l}_i q_{i'} \bar{l}_{i'})$, (2)

$\Delta B = \Delta L = 3$, (3)
THE BARYON ASYMMETRY

The number we have to explain is

\[ Y_{\Delta B} = \frac{n_B - n_{\bar{B}}}{s} = (8.75 \pm 0.23) \times 10^{-11} \]

Entropy \( s = g_*(2\pi^2/45)T^3 \) is conserved, related to photon density: \( s = 7.04 n_\gamma \)

Measured using BBN (deuterium abundance) and CMB anisotropies (temperature fluctuations)