Flavor Structure of E_6 GUT Models

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Ref.) H. Kawase and N. Maekawa,
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The standard model: $G_{\mathsf{SM}} = SU(3)_C \times SU(2)_L \times U(1)_Y$

Quark	$q_L = (u_L, d_L)^{\mathsf{T}}$	$\cdots$	$(\mathbf{3}, \mathbf{2})_{1/6}$
	$\bar{u}_R$	$\cdots$	$(\bar{\mathbf{3}}, \mathbf{1})_{-2/3}$
	$\bar{d}_R$	$\cdots$	$(\bar{\mathbf{3}}, \mathbf{1})_{1/3}$
Lepton	$l_L = (\nu_L, e_L)^{\mathsf{T}}$	$\cdots$	$(\mathbf{1}, \mathbf{2})_{-1/2}$
	$\bar{e}_R$	$\cdots$	$(\mathbf{1}, \mathbf{1})_{1}$
Higgs	$H = (H^+, H^0)$	$\cdots$	$(\mathbf{1}, \mathbf{2})_{1/2}$

Higgs field: Vacuum expectation value (VEV) $\langle H^0 \rangle = v \neq 0$

$$SU(2)_L \times U(1)_Y \to U(1)_{\mathsf{EM}}$$

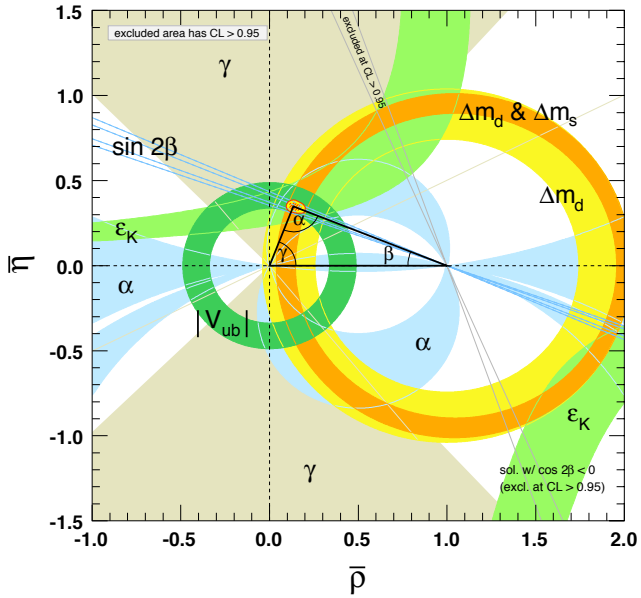
Yukawa interaction: interaction among fermions and Higgs

$$V_Y = (Y_{ff'})^{ij} \bar{f}_i f'_j H \quad (i, j = 1, 2, 3 : \text{generation})$$

The CKM matrix

Interactions of quarks are characterized by the CKM matrix.

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



Experimental value

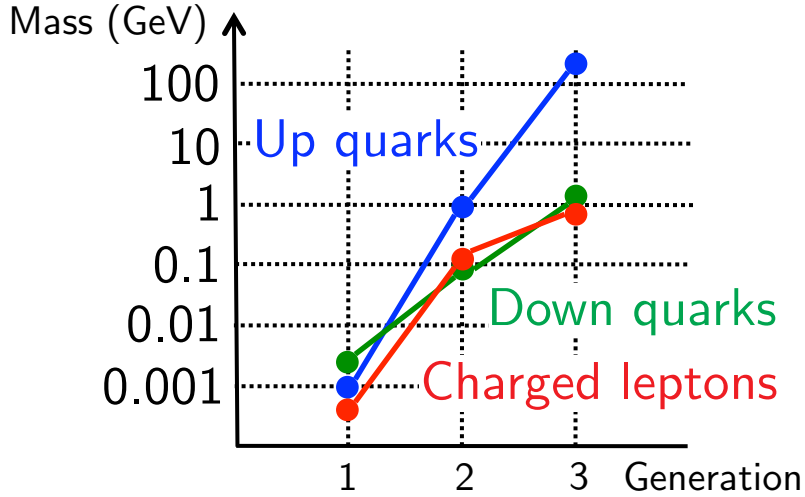
$$V_{\text{CKM}} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$$\lambda \simeq 0.23,\ A \simeq 0.81,\ \rho \simeq 0.14,\ \eta \simeq 0.35$$

$$\rho \sim \eta \sim \mathcal{O}(\lambda),\ A \sim \mathcal{O}(1) \longrightarrow V_{\text{CKM}} \sim \begin{pmatrix} 1 & \lambda & \lambda^4 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

Mystery of the standard model

Origin of the hierarchical structure of fermion masses

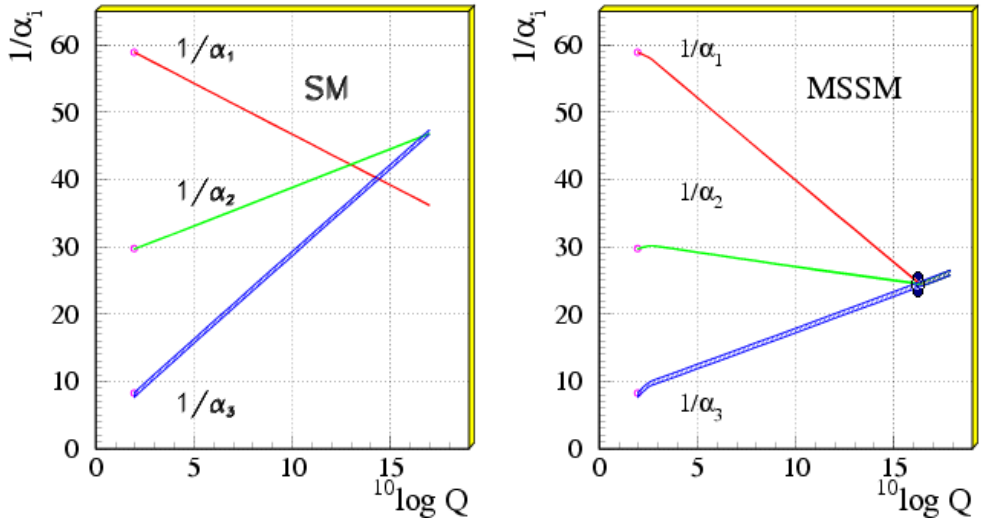


- Down quarks and charged leptons have similar structure of mass hierarchy.
- Hierarchy among up quarks is much larger.

⇒ Grand unified theory (GUT) may provide the solution.

Grand Unified Theory (GUT)

Gauge interaction of the standard model: $SU(3)_C \times SU(2)_L \times U(1)_Y$



Three gauge couplings meet at the high energy scale.

$\Rightarrow$ Unification of gauge interactions?

Minimal model: $SU(5)$ GUT

$$SU(5) \supset SU(3)_C \times SU(2)_L \times U(1)_Y$$

(SUSY) $SU(5)$ GUT

- Quarks and leptons:

  10 dimensional fields: $\mathbf{10}_1$, $\mathbf{10}_2$, $\mathbf{10}_3$ 5 dimensional fields: $\bar{\mathbf{5}}_1$, $\bar{\mathbf{5}}_2$, $\bar{\mathbf{5}}_3$

$$\mathbf{10}_i = Q_i \oplus \bar{u}_i \oplus \bar{e}_i, \quad \bar{\mathbf{5}}_i = \bar{d}_i \oplus L_i \quad (i = 1, 2, 3)$$

- Higgs fields:

  5 dimensional fields: $\mathbf{5}_H$, $\bar{\mathbf{5}}_{\bar{H}}$

$$\mathbf{5}_H = H_u \oplus H_C, \quad \bar{\mathbf{5}}_H = H_d \oplus \bar{H}_C$$

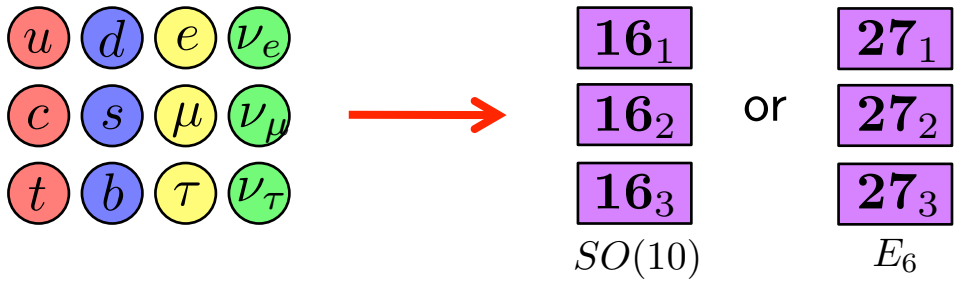
Yukawa interaction

- $(Y_{\mathbf{10}})^{ij}\mathbf{10}_i \cdot \mathbf{10}_j \cdot \mathbf{5}_H \to (Y_{\mathbf{10}})^{ij} Q_i \bar{u}_j H_u$
- $(Y_{\bar{\mathbf{5}}})^{ij}\mathbf{10}_i \cdot \bar{\mathbf{5}}_j \cdot \bar{\mathbf{5}}_{\bar{H}} \to (Y_{\bar{\mathbf{5}}})^{ij} Q_i \bar{d}_j H_d + (Y_{\bar{\mathbf{5}}})^{ij} \bar{e}_i L_j H_d$

$$Y_u = Y_{\mathbf{10}}, \qquad Y_d = Y_e^{\mathsf{T}} = Y_{\bar{\mathbf{5}}}$$

GUT based on larger gauge group

Matter fields are unified in $SO(10)$ GUT and E_6 GUT



$SO(10)$ GUT $\quad SO(10) \supset SU(5) \times U(1)_V$

$$\mathbf{16}_i = \mathbf{10}_i \oplus \bar{\mathbf{5}}_i \oplus \mathbf{1}_i \qquad (SU(5) \text{ decomposition})$$

E_6 GUT $\quad E_6 \supset SO(10) \times U(1)_{V'}$

$$\mathbf{27}_i = \mathbf{16}_i \oplus \mathbf{10}_i \oplus \mathbf{1}_i \quad \Rightarrow \quad [\mathbf{10}_i \oplus \bar{\mathbf{5}}_i \oplus \mathbf{1}_i] \oplus [\mathbf{5}_i \oplus \bar{\mathbf{5}}'_i] \oplus [\mathbf{1}'_i]$$

($SO(10)$ decomposition) ($SU(5)$ decomposition)

Additional $\mathbf{5}$ and $\bar{\mathbf{5}}$ fields are present for each generation.

In E_6 GUT, the mass hierarchy of fermions can be explained naturally.

$$\mathbf{27} = \mathbf{16} \oplus \mathbf{10} \oplus \mathbf{1} \quad \Rightarrow \quad [\mathbf{10} \oplus \bar{\mathbf{5}} \oplus \mathbf{1}] \oplus [\mathbf{5} \oplus \bar{\mathbf{5}}'] \oplus [\mathbf{1}']$$

Extra $\mathbf{5}$, $\bar{\mathbf{5}}$ pairs become massive due to the VEV of Higgs fields Φ and C:

- $\Phi = \mathbf{16}_\Phi \oplus \mathbf{10}_\Phi \oplus \mathbf{1}_\Phi$ $\qquad \langle \Phi \rangle = \langle \mathbf{1}_\Phi \rangle \neq 0 \quad \Rightarrow \quad E_6 \to SO(10)$
- $C = \mathbf{16}_C \oplus \mathbf{10}_C \oplus \mathbf{1}_C$ $\qquad \langle C \rangle = \langle \mathbf{16}_C \rangle \neq 0 \quad \Rightarrow \quad SO(10) \to SU(5)$

Yukawa interaction $(\mathbf{27} \cdot \mathbf{27} \cdot \mathbf{27})$

$$\Psi_i \cdot \Psi_j \cdot \langle \Phi \rangle \to \mathbf{10}_i \cdot \mathbf{10}_j \cdot \langle \mathbf{1}_\Phi \rangle \to \mathbf{5}_i \cdot \bar{\mathbf{5}}'_j \cdot \langle \mathbf{1}_\Phi \rangle$$

$$\Psi_i \cdot \Psi_j \cdot \langle C \rangle \to \mathbf{10}_i \cdot \mathbf{16}_j \cdot \langle \mathbf{16}_C \rangle \to \mathbf{5}_i \cdot \bar{\mathbf{5}}_j \cdot \langle \mathbf{16}_C \rangle$$

$\Psi_i = \mathbf{16}_i \oplus \mathbf{10}_i \oplus \mathbf{1}_i$: Matter fields $(i = 1, 2, 3)$

$$\begin{pmatrix} \bar{\mathbf{5}}_1 \\ \bar{\mathbf{5}}_2 \\ \bar{\mathbf{5}}_3 \end{pmatrix} \begin{pmatrix} \bar{\mathbf{5}}'_1 \\ \bar{\mathbf{5}}'_2 \\ \bar{\mathbf{5}}'_3 \end{pmatrix} \xleftrightarrow{\text{heavy}} \begin{pmatrix} \mathbf{5}_1 \\ \mathbf{5}_2 \\ \mathbf{5}_3 \end{pmatrix}$$

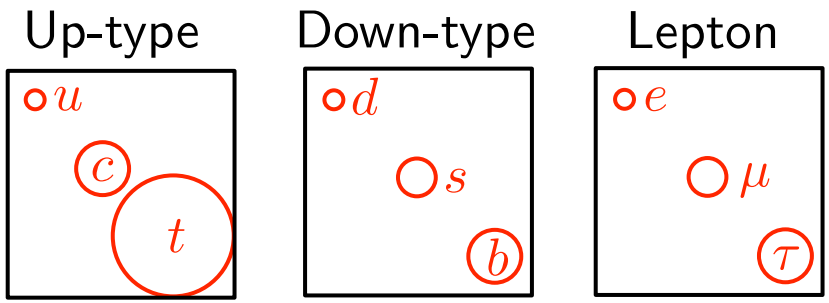
Assume that $(\bar{\mathbf{5}}_3, \bar{\mathbf{5}}'_2, \bar{\mathbf{5}}'_3)$ and $(\mathbf{5}_1, \mathbf{5}_2, \mathbf{5}_3)$ obtain large mass terms.

Massless modes: $\bar{\mathbf{5}}^0_i$

$$\begin{aligned} \bar{\mathbf{5}}^0_1 &\sim \bar{\mathbf{5}}_1 + \cdots \\ \bar{\mathbf{5}}^0_2 &\sim \bar{\mathbf{5}}'_1 + \cdots \\ \bar{\mathbf{5}}^0_3 &\sim \bar{\mathbf{5}}_2 + \cdots \end{aligned} \quad \Longrightarrow \quad \begin{pmatrix} \bar{\mathbf{5}}_1 \\ \bar{\mathbf{5}}_2 \\ \bar{\mathbf{5}}_3 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ \star & \star & \star \end{pmatrix} \begin{pmatrix} \bar{\mathbf{5}}^0_1 \\ \bar{\mathbf{5}}^0_2 \\ \bar{\mathbf{5}}^0_3 \end{pmatrix}$$

$$(Y)^{ij} \Psi_i \cdot \Psi_j \cdot H \;\Rightarrow\; (Y)^{ij} \mathbf{10}_i \cdot \mathbf{10}_j \cdot \mathbf{5}_H + (Y)^{ij} \mathbf{10}_i \cdot \bar{\mathbf{5}}_j \cdot \bar{\mathbf{5}}_{\bar{H}}$$

$$Y_u \sim Y \sim \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \qquad Y_d, Y_e^{\mathsf{T}} \sim Y \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ \star & \star & \star \end{pmatrix} \sim \begin{pmatrix} \lambda^6 & \star & \lambda^5 \\ \lambda^5 & \star & \lambda^4 \\ \lambda^3 & \star & \lambda^2 \end{pmatrix}$$



hierarchy among up quarks $\gg$ hierarchy among down quarks and leptons

Doublet-triplet splitting (DTS) problem

New fields in $SU(5)$ GUT: triplet Higgs H_C, $\bar{H}_C$

MSSM

H_u H_d

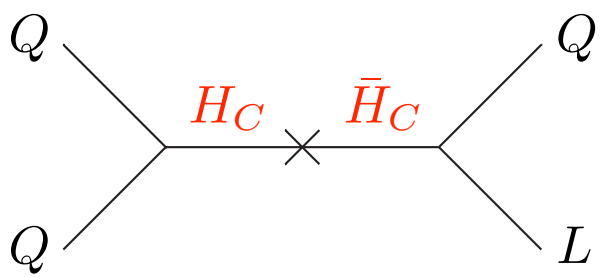
→

GUT

$\mathbf{5}_H = \boxed{H_u \; H_C}$ $\bar{\mathbf{5}}_{\bar{H}} = \boxed{H_d \; \bar{H}_C}$

Interaction involving triplet Higgs H_C and $\bar{H}_C$ violate the baryon number.

Proton decay



Mass of triplet Higgs: $M_C H_C \bar{H}_C$

$$\mathcal{M}_{p\to K^+\bar{\nu}} \sim \frac{1}{M_C}$$

$M_C > 3 \times 10^{17}$ GeV is needed.

H_u and H_d must be $\sim 10^2$ GeV ⟷ H_C and $\bar{H}_C$ must be $> 10^{17}$ GeV

In $SO(10)$ GUT (and in E_6 GUT), there is a solution for the DTS problem:

Dimopoulos-Wilczek mechanism

S. Dimopoulos and F. Wilczek, Report No. NSF-ITP-82-07
M. Srednicki, Nucl. Phys. B202 (1982), 327
S. M. Barr and S. Raby, Phys. Rev. Lett. 79 (1997), 4748

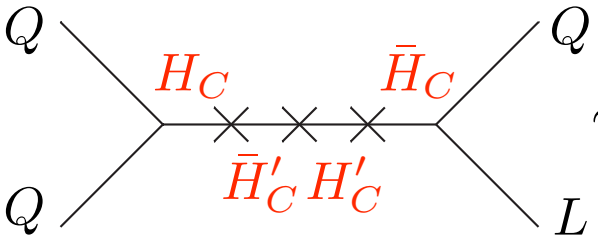
- Adjoint Higgs: $\mathbf{45}_A$

$$\langle \mathbf{45}_A \rangle = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \otimes \left(\begin{array}{c|c} v_A \cdot \mathbf{1}_{3\times 3} & \mathbf{0}_{3\times 2} \\ \hline \mathbf{0}_{2\times 3} & \mathbf{0}_{2\times 2} \end{array}\right) \propto Q_{B-L}$$

$$SO(10) \to SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

$$W_H = mH'H' + H\langle A \rangle H'$$

$$\Rightarrow (H_C \;\; H'_C) \begin{pmatrix} 0 & v_A \\ v_A & m \end{pmatrix} \begin{pmatrix} \bar{H}_C \\ \bar{H}'_C \end{pmatrix} + (H_D \;\; H'_D) \begin{pmatrix} 0 & 0 \\ 0 & m \end{pmatrix} \begin{pmatrix} \bar{H}_D \\ \bar{H}'_D \end{pmatrix}$$



$$\sim \frac{1}{M_{\text{eff}}} = \frac{m}{v_A^2}$$

- MSSM Higgs: H_D, $\bar{H}_D$
- Triplet Higgs: H_C, $\bar{H}_C$

If $m \ll v_A$, proton decay can be avoided.

Adjoint VEV $\langle A \rangle$ in E_6 GUT

$E_6 \supset SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_V \times U(1)_{V'}$

$\langle A \rangle$ can be written as a linear combination of the $U(1)$ charge:

$$\langle A \rangle = xQ_{V'} + yQ_V + zQ_Y$$

The contribution to the Yukawa coupling from the adjoint Higgs $\langle A \rangle$ is determined by the $U(1)$ charge of quarks:

$$\Psi(\langle A \rangle \Psi)H \quad \Rightarrow \quad Q\bar{u}H_u - \frac{24\epsilon + 1}{5} Q\bar{d}H_d$$

Up-type Yukawa Down-type Yukawa

$$\epsilon \equiv y/z$$

	V'	V	Y
Q	1	-1	$1/6$
$\bar{u}$	1	-1	$-2/3$
$\bar{d}$	1	3	$1/3$

$\Rightarrow$ The choice of adjoint VEV affect the structure of fermion mixing.

Realistic model of flavor

$E_6 \times SU(2)_{\mathsf{H}}$ GUT Matter: $\Psi = (\Psi_1, \Psi_2), \Psi_3$

	Ψ	Ψ_3	F	$\bar{F}$	A	Φ	C
E_6	$\mathbf{27}$	$\mathbf{27}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{78}$	$\mathbf{27}$	$\mathbf{27}$
$SU(2)_{\mathsf{H}}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\bar{\mathbf{2}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
Z_3	0	0	1	0	0	0	2

- $\langle \Phi \rangle = \langle \mathbf{1}_\Phi \rangle \neq 0 \;\Rightarrow\; E_6 \to SO(10)$
- $\langle C \rangle = \langle \mathbf{16}_C \rangle \neq 0 \;\Rightarrow\; SO(10) \to SU(5)$
- $\langle A \rangle = v_A(Q_Y + \epsilon Q_V) \neq 0 \;\Rightarrow\; SU(5) \to G_{\mathsf{SM}}$
- $\langle F\bar{F} \rangle \neq 0 \;\Rightarrow\; SU(2)_{\mathsf{H}} \to \times$

Assuming $H_u \sim \mathbf{10}_\Phi$ and $H_d \sim \mathbf{10}_\Phi \cos\theta + \mathbf{16}_C \sin\theta$,

$$(Y_\Phi)^{ij}\Psi_i\Psi_j\Phi + (Y_C)^{ij}\Psi_i\Psi_j C \;\Rightarrow\; (Y_u)^{ij}Q_i\bar{u}_j H_u - (Y_d)^{ij}Q_i\bar{d}_j H_d$$

Yukawa interactions

$$Y_\Phi : \begin{pmatrix} 0 & \Psi(A\Psi) & 0 \\ \Psi(A\Psi) & (\overline{F}\Psi)^2 & (\overline{F}\Psi)\Psi_3 \\ 0 & (\overline{F}\Psi)\Psi_3 & \Psi_3\Psi_3 \end{pmatrix} \Phi$$

$$Y_C : \begin{pmatrix} 0 & (F\Psi)^2 & (F\Psi)\Psi_3 \\ (F\Psi)^2 & 0 & 0 \\ (F\Psi)\Psi_3 & 0 & 0 \end{pmatrix} C$$

Parameters are chosen to produce the hierarchical Yukawa matrices:

$$Y_u \sim \begin{pmatrix} 0 & \lambda^5 & 0 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ 0 & \lambda^2 & 1 \end{pmatrix}, \quad Y_d \sim \begin{pmatrix} \lambda^6 & \lambda^{5.5} & -\frac{24\epsilon+1}{5}\lambda^5 \\ \lambda^5 & \lambda^{4.5} & \lambda^4 \\ \lambda^3 & \lambda^{2.5} & \lambda^2 \end{pmatrix}$$

This term is dependent on the choice of adjoint VEV $\langle A \rangle$.

The CKM matrix V_{CKM} can be obtained by diagonalizing Y_u and Y_d:

$$V_{\text{CKM}} \sim \begin{pmatrix} 1 & \lambda & (4\epsilon+1)\lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4) \quad \Leftrightarrow \quad V_{\text{CKM}}^{\text{exp}} \sim \begin{pmatrix} 1 & \lambda & \lambda^4 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

Relation with the DTS problem

If $\epsilon = -1/4$, the correct structure of CKM matrix can be obtained.

$$V_{\text{CKM}} \sim \begin{pmatrix} 1 & \lambda & \lambda^4 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

$\epsilon = -1/4$ corresponds to $\langle \mathbf{45}_A \rangle \propto Q_{B-L}$: $\quad Q_{B-L} = \frac{2}{5} Q_Y - \frac{1}{10} Q_V$

$\langle \mathbf{45}_A \rangle \propto Q_{B-L}$ is just the direction which can solve the DTS problem

$$\langle \mathbf{45}_A \rangle = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \otimes \left(\begin{array}{c|c} v_A \cdot \mathbf{1}_{3\times 3} & \mathbf{0}_{3\times 2} \\ \hline \mathbf{0}_{2\times 3} & \mathbf{0}_{2\times 2} \end{array} \right)$$

Measurement of the CKM matrix element V_{ub}: low-energy physics

$\downarrow$

Manner of the GUT symmetry breaking: high-energy physics

Summary

- In E_6 GUT, mass hierarchy of fermions can be explained naturally.

$$\begin{pmatrix} \bar{\mathbf{5}}_1 \\ \bar{\mathbf{5}}_2 \\ \mathbf{5}_3 \end{pmatrix} \quad \begin{pmatrix} \bar{\mathbf{5}}'_1 \\ \mathbf{5}'_2 \\ \bar{\mathbf{5}}'_3 \end{pmatrix} \xleftrightarrow{\text{heavy}} \begin{pmatrix} \mathbf{5}_1 \\ \mathbf{5}_2 \\ \mathbf{5}_3 \end{pmatrix}$$

- $\langle \mathbf{45}_A \rangle \propto Q_{B-L}$ is a natural solution for the DTS problem.

$$\langle \mathbf{45}_A \rangle = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \otimes \left(\begin{array}{c|c} v_A \cdot \mathbf{1}_{3\times 3} & \mathbf{0}_{3\times 2} \\ \hline \mathbf{0}_{2\times 3} & \mathbf{0}_{2\times 2} \end{array} \right)$$

- In the realistic model, the correct structure of the CKM matrix can be obtained only if $\langle \mathbf{45}_A \rangle \propto Q_{B-L}$ is satisfied.

$$V_{\text{CKM}} \sim \begin{pmatrix} 1 & \lambda & \lambda^4 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$