Linearized supergravity and superconformal formulation

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[arXiv:1107.4247]
Introduction

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Moduli originate from the gravitational multiplet in higher-dimensional SUGRA.

We have to work in the context of SUGRA

It is convenient to express higher-dimensional SUGRA in terms of N=1 superfields by using formulae of 4D off-shell SUGRA.
Superconformal formulation

[Kaku, Townsend, Nieuwenhuizen, PRD17 (1978) 3179;
Ferrara, Grisaru, Nieuwenhuizen, NPB138 (1978) 430,
Kugo, Uehara NPB226 (1983) 49, …]

• Systematic method to construct SUGRA action
Superconformal formulation


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• Most of the known off-shell actions are realized by this formulation.
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**Superconformal formulation**


- Systematic method to construct SUGRA action
- Most of the known off-shell actions are realized by this formulation.
- We can treat general SUGRA.
- Extension to 5D has been done.
- An explicit expression of the action is lengthy and complicated.
Linearized SUGRA

[Sugraa, Zumino, NPB134 (1978) 301; Siegel, Gates Jr., NPB147 (1979) 77, ...]

SUGRA action is constructed up to linear order in SUGRA fields.

(SUGRA fields: vierbein $e_\mu^\nu$, gravitino $\psi_\mu\alpha$, ...)

28 August 2011

SUSY11@Fermilab
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Advantage

It is described in terms of superfields on the ordinary superspace ($x^\mu, \theta_\alpha$).
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**Advantage**

It is described in terms of superfields on the ordinary superspace $(x^\mu, \theta_\alpha)$.

5D linearized SUGRA is constructed in terms of N=1 superfields for the minimal field contents.

Linearized SUGRA

For example, in the brane-world scenario,

we can calculate brane-to-brane contributions,
keeping N=1 superfield structure.

**Superconformal formulation**

- We can treat general SUGRA in full order.
- 5D extension has been done.
- Explicit expressions are complicated.

**Linearized SUGRA**

- It is described by superfields.
- We cannot discuss beyond linearized order in SUGRA fields.
- 5D extension is done only for the minimal case.
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It is useful to use both formulations in a complementary manner.
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Purpose
Clarify the direct relation between the two formulations.
Strategy

We modify the known linearized SUGRA, and

Fields in the modified linearized SUGRA

Identify

Fields in the superconformal formulation of

[Kugo & Uehara, NBP226 (1983) 49]
Modified linearized SUGRA
Superconformal multiplets

Each superconformal multiplet is characterized by the Weyl weight $w$ and the chiral weight $n$. (dilatation) (R-charge)

We consider the following three types of multiplets.

• Weyl multiplet (SUGRA multiplet)
• (Anti-)Chiral multiplet (matter, compensator)
• Real general multiplet (gauge multiplet, ...)

Superconformal transformations

For a chiral superfield \((w = n)\),
\[
\delta \Phi = \left\{ -\frac{1}{4} \bar{D}^2 L^\alpha D_\alpha - i \sigma^\mu_{\alpha \dot{\alpha}} \bar{D}^\dot{\alpha} L^\alpha \partial_\mu + 2w \Lambda \right\} \Phi.
\]

For an anti-chiral superfield \((w = -n)\),
\[
\delta \bar{\Phi} = \left\{ -\frac{1}{4} D^2 \bar{L}_\dot{\alpha} \bar{D}^\dot{\alpha} - i \sigma^\mu_{\dot{\alpha} \alpha} D^\alpha \bar{L}^\dot{\alpha} \partial_\mu + 2w \bar{\Lambda} \right\} \bar{\Phi}.
\]

For a real general superfield \((n = 0)\),
\[
\delta V = \left\{ -\frac{1}{4} \bar{D}^2 L^\alpha D_\alpha - \frac{i}{2} \sigma^\mu_{\alpha \dot{\alpha}} \bar{D}^\dot{\alpha} L^\alpha \partial_\mu + w \Lambda + \text{hc} \right\} V,
\]
where
\[
\Lambda = -\frac{1}{24} \left( \bar{D}^2 D^\alpha L_\alpha + 4 \Xi \right).
\]
Superconformal transformations

For a chiral superfield \((w = n)\),
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\]
where
\[
\Lambda = -\frac{1}{24} \left( \bar{D}^2 D^\alpha L_\alpha + 4 \Xi \right).
\]

We also introduce \(U^\mu\), which transforms inhomogeneously,
\[
\delta U^\mu = \frac{1}{2} \sigma^\mu_{\alpha \dot{\alpha}} \left( \bar{D}^{\dot{\alpha}} L^\alpha - D^\alpha \bar{L}^{\dot{\alpha}} \right).
\]
Transformation parameters

Only the following components of $L^\alpha$ appear in the component transformation laws.

\[
\begin{align*}
\xi &= \text{Im} \left( \sigma^\mu_{\alpha\alpha} \bar{D}^{\alpha} L^\alpha \right) , \\
\epsilon^\alpha &= -\frac{1}{4} \bar{D}^2 L^\alpha , \\
\varphi &= -\frac{1}{4} \text{Re} \left( D_\alpha \bar{D}^2 L^\alpha \right) , \\
\vartheta &= \frac{1}{6} \text{Im} \left( D_\alpha \bar{D}^2 L^\alpha \right) , \\
\lambda_{\mu\nu} &= -\frac{1}{2} \text{Re} \left\{ (\sigma_{\mu\nu})_{\beta\alpha} D_\beta \bar{D}^2 L^\alpha \right\} , \\
\rho^\alpha &= -\frac{1}{32} \bar{D}^2 \bar{D}^2 L^\alpha .
\end{align*}
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\end{align*}
\]

($P$ : translation)  
($Q$ : SUSY)  
($D$ : dilatation)  
($U(1)_A$ : R-symmetry)  
($M$ : local Lorentz)  
($S$ : conformal SUSY)
Field identification
Weyl multiplet (SUGRA multiplet)

\[ U_\mu = (\theta \sigma^\nu \bar{\theta}) \tilde{e}_{\nu \mu} + \bar{\theta}^2 (\theta \bar{\psi}_\mu) + \theta^2 (\bar{\theta} \bar{\psi}_\mu) + \theta^2 \bar{\theta}^2 d_\mu \]
Weyl multiplet (SUGRA multiplet)

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\[
\begin{align*}
\delta \tilde{e}_\nu^\mu &= -\delta_\nu^\mu \varphi + \lambda^\mu_\nu + \partial_\nu \xi^\mu, \\
\delta \bar{\psi}_\alpha^\mu &= \left( 2 \sigma_\mu \bar{\rho} + i \sigma^\nu \bar{\sigma}^\mu \partial_\nu \epsilon \right)_\alpha, \\
\delta d^\mu &= \frac{3}{4} \partial^\mu \vartheta + \frac{1}{4} \epsilon^{\mu \nu \rho \tau} \partial_\nu \lambda_{\rho \tau}.
\end{align*}
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\end{align*}
\]

Each component is identified as

\[
\begin{align*}
\tilde{e}_\nu^\mu &= e_\nu^\mu - \delta_\nu^\mu, \\
\bar{\psi}_\alpha^\mu &= i (\sigma^\nu \bar{\sigma}_\mu \psi_\nu)_\alpha, \\
d^\mu &= \frac{3}{4} A^\mu - \frac{1}{4} \epsilon^{\mu\nu\rho\tau} \partial_\nu \bar{\epsilon}_{\rho\tau}.
\end{align*}
\]

\[
\begin{align*}
\left( \begin{array}{l}
\begin{array}{l}
e_\nu^\mu : \text{vierbein; } \\
\psi_{\mu\alpha} : \text{gravitino; }
\end{array}
\end{array}
\right)
\left( \begin{array}{l}
A_\mu : U(1)_A \text{ gauge field}
\end{array}
\right)
\end{align*}
\]
Chiral multiplet

\[ \Phi = \phi + \theta \chi + \theta^2 F \]

\[ \delta \Phi = \left\{ \frac{-1}{4} \bar{D}^2 L^\alpha D_\alpha - i \sigma_\alpha^{\mu \alpha \dot{\alpha}} \bar{D}^\dot{\alpha} L^\alpha \partial_\mu - \frac{w}{12} (\bar{D}^2 D^\alpha L_\alpha + 4 \Xi) \right\} \Phi \]
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\]

In components,

\[
\begin{align*}
\delta \phi &= \xi^\mu \partial_\mu \phi + \epsilon \chi + w \varphi \phi + \frac{i w}{2} \partial \phi, \\
\delta \chi_\alpha &= \xi^\mu \partial_\mu \chi_\alpha + \frac{1}{2} \lambda_{\mu \nu} \left( \sigma^{\mu \nu} \chi \right)_\alpha + 2 \epsilon_\alpha F - 2 i \left( \sigma^{\mu \dot{\epsilon}} \right)_\alpha \partial \mu \phi \\
&\quad + \left( w + \frac{1}{2} \right) \varphi \chi_\alpha + \frac{i}{2} \left( w - \frac{3}{2} \right) \partial \chi_\alpha - 4 w \rho_\alpha \phi, \\
\delta F &= \xi^\mu \partial_\mu F - i \bar{\epsilon} \sigma^{\mu \mu} \partial_\mu \chi + \left( w + 1 \right) \varphi F + \frac{i}{2} \left( w - 3 \right) \partial F \\
&\quad + 2 \left( w - 1 \right) \rho \chi.
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&\quad + \left( w + \frac{1}{2} \right) \varphi \chi_\alpha + \frac{i}{2} \left( w - \frac{3}{2} \right) \vartheta \chi_\alpha - 4 w \rho \chi_\alpha, \\
\delta F &= \xi^\mu \partial_\mu F - i \bar{\epsilon} \sigma^{\mu \nu} \partial_\nu \chi + \left( w + 1 \right) \varphi F + \frac{i}{2} \left( w - 3 \right) \vartheta F \\
&\quad + 2(w - 1) \rho \chi.
\end{align*} \]

We can identify \( [\phi, \chi_\alpha, F] \) with a chiral multiplet.
Embedding into a general multiplet

In the global SUSY case, a chiral superfield is embedded into a general superfield by

\[ y^{\mu} \equiv x^{\mu} - i \theta \sigma^{\mu \bar{\sigma}} \bar{\theta} \rightarrow x^{\mu}. \]
Embedding into a general multiplet

In the global SUSY case, a chiral superfield is embedded into a general superfield by $y^\mu \equiv x^\mu - i\theta\sigma^\mu\bar{\theta} \rightarrow x^\mu$.

In SUGRA, this is not enough.

$\mathcal{V}(\Phi) = (1 + iU^\mu\partial_\mu)\Phi$
Embedding into a general multiplet

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In SUGRA, this is not enough.

\[
\mathcal{V}(\Phi) = (1 + iU^\mu \partial_\mu)\Phi
\]

\[
= \phi + \theta \chi + \theta^2 F - i(\theta \sigma^\mu \bar{\theta}) \left( e^{-1} \right)^\nu_\mu \partial_\nu \phi
\]

\[
- \frac{i}{2} \theta^2 \left\{ \bar{\theta} \sigma^\mu \left( e^{-1} \right)^\nu_\mu \partial_\nu \chi - 2 (\bar{\theta} \bar{\psi}_\mu) \partial_\mu \phi \right\} + i\bar{\theta}^2 (\theta \bar{\psi}^\mu) \partial_\mu \phi
\]

\[
- \frac{1}{4} \theta^2 \bar{\theta}^2 \left\{ g^{\mu \nu} \partial_\mu \partial_\nu \phi + 2i\bar{\psi}_\mu \partial_\mu \chi - 4i d^\mu \partial_\mu \phi \right\} ,
\]

where \( (e^{-1})^\nu_\mu \equiv \delta^\nu_\mu - \bar{e}_\mu^\nu \) and \( g^{\mu \nu} \equiv \eta^{\mu \nu} - \bar{e}^{\mu \nu} - \bar{e}^\nu_\mu \).
Real general multiplet

\[ V = C' + i\theta \zeta' - i\bar{\theta}\bar{\zeta'} - \theta^2 H' - \bar{\theta}^2 \bar{H}' - (\theta \sigma^{\mu} \bar{\theta}) B'_\mu + i\theta^2 (\bar{\theta} \chi') - i\bar{\theta}^2 (\theta \chi') + \frac{1}{2} \theta^2 \bar{\theta}^2 D', \]

\[ \delta V = \left\{-\frac{1}{4} \bar{D}^2 L^\alpha D_\alpha - \frac{i}{2} \sigma^\mu_{\alpha\dot{\alpha}} \bar{D}^{\dot{\alpha}} L^\alpha \partial_\mu + w\Lambda + h.c\right\} V. \]

The transformations of \([C', \zeta', H', B'_\mu, \chi'_\alpha, D']\)
do not agree with the superconformal trf. of a real general multiplet as is.
Real general multiplet

\[ V = C' + i \theta \zeta' - i \bar{\theta} \bar{\zeta}' - \theta^2 \mathcal{H}' - \bar{\theta}^2 \bar{\mathcal{H}'} - (\theta \sigma^\mu \bar{\theta}) B'_\mu \]
\[ + i \theta^2 (\bar{\theta} \lambda') - i \bar{\theta}^2 (\theta \lambda') + \frac{1}{2} \theta^2 \bar{\theta}^2 D', \]

We need to redefine the fields.

\[
C \equiv C, \quad \zeta_\alpha \equiv \zeta'_\alpha, \quad \mathcal{H} \equiv \mathcal{H}', \\
B_\mu \equiv B'_\mu + \zeta'_\mu \psi_\mu + \bar{\zeta}'_\mu \bar{\psi}_\mu + \frac{w}{2} C' A_\mu, \\
\lambda_\alpha \equiv \lambda'_\alpha + \frac{i}{2} \left\{ \sigma^\mu \left( e^{-1} \right)_\mu^\nu \partial_\nu \bar{\zeta}' \right\} \alpha + (\sigma^\mu \bar{\sigma}^\nu \psi_\mu)_\alpha B'_\nu + \frac{w}{4} (\sigma^\mu \bar{\sigma}' \lambda')_\alpha A_\mu, \\
D \equiv D' + \frac{1}{2} g^{\mu \nu} \partial_\mu \partial_\nu C' - \left( 2 d_\mu - \frac{w}{2} A^\mu \right) B'_\mu \\
\quad + \left( \bar{\lambda}' \sigma^\mu \psi_\mu - \frac{i}{2} \partial_\nu \zeta' \sigma^\mu \bar{\sigma}^\nu \psi_\mu - i \partial_\mu \zeta' \psi_\mu + \frac{2iw}{3} \zeta' \sigma_\mu \nu \partial_\nu \psi_\mu + \text{h.c.} \right). \]
Real general multiplet

\[ V = C' + i\theta \zeta' - i\bar{\theta}\bar{\zeta}' - \theta^2 \mathcal{H}' - \bar{\theta}^2 \bar{\mathcal{H}}' - (\theta \sigma^\mu \bar{\sigma}) B'_\mu + i\theta^2 (\bar{\theta} \bar{\chi}') - i\bar{\theta}^2 (\theta \chi') + \frac{1}{2} \theta^2 \bar{\theta}^2 D', \]

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\lambda_\alpha & \equiv \lambda'_\alpha + \frac{i}{2} \left\{ \sigma^\mu \left( e^{-1} \right)^\nu_\mu \partial_\nu \bar{\zeta}' \right\}_\alpha + (\sigma^\mu \bar{\sigma}^\nu \psi_\mu)_\alpha B'_\nu + \frac{w}{4} (\sigma^\mu \bar{\zeta}')_\alpha A_\nu, \\
D & \equiv D' + \frac{1}{2} g^{\mu\nu} \partial_\mu \partial_\nu C' - \left( 2 d^\mu - \frac{w}{2} A^\mu \right) B'_\mu \\
& \quad + \left( \bar{\zeta}' \sigma^\mu \psi_\mu - \frac{i}{2} \partial_\nu \zeta' \sigma^\mu \bar{\sigma}^\nu \psi_\mu - i \partial_\mu \zeta' \psi_\mu + \frac{2iw}{3} \zeta' \sigma^\mu \partial_\nu \psi_\mu + \text{h.c.} \right). 
\end{align*}
\]

Then, \([C, \zeta_\alpha, \mathcal{H}, B_\mu, \lambda_\alpha, D]\) is identified with a real general multiplet.
Gauge multiplet (a real general multiplet with $w = 0$)

Gauge transformation

$$V \rightarrow V + V(\Sigma) + V(\bar{\Sigma}), \quad (\Sigma : \text{chiral multiplet})$$

We can move to the Wess-Zumino gauge.

$$V_{\text{WZ}} = -(\theta \sigma^\mu \bar{\theta})(e^{-1})^\nu_\mu \hat{B}'_\nu + i\theta^2 \bar{\theta} (\bar{\lambda} - i\bar{\psi}^\mu \hat{B}'_\mu) - i\bar{\theta}^2 \theta (\lambda + i\psi^\mu \hat{B}'_\mu)$$

$$+ \frac{1}{2} \theta^2 \bar{\theta}^2 \left\{ D + \left( -\frac{i}{2} \bar{\lambda} \sigma^\mu \bar{\psi}_\mu + \text{h.c.} \right) + 2d^\mu \hat{B}'_\mu \right\}.$$}

This is possible only when $w = 0$. 
**Gauge multiplet** (a real general multiplet with \( w = 0 \))

**Gauge transformation**

\[ V \to V + \mathcal{V}(\Sigma) + \mathcal{V}(\bar{\Sigma}), \quad (\Sigma : \text{chiral multiplet}) \]

We can move to the Wess-Zumino gauge.

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V_{\text{WZ}} = - (\theta \sigma^\mu \bar{\theta}) (e^{-1})^\nu_\mu \hat{B}'_\nu + i \theta^2 \bar{\theta} (\bar{\lambda} - i \bar{\psi}^\mu \hat{B}'_\mu) - i \bar{\theta}^2 \theta (\lambda + i \psi^\mu \hat{B}'_\mu)
\]

\[
+ \frac{1}{2} \theta^2 \bar{\theta}^2 \left\{ D + \left( - \frac{i}{2} \bar{\lambda} \sigma^\mu \psi_\mu + \text{h.c.} \right) + 2 d^\mu \hat{B}'_\mu \right\}.
\]

This is possible only when \( w = 0 \).

**Field strength superfield**

\[
\mathcal{W}_\alpha^{\text{naive}} = - \frac{1}{4} \bar{D}^2 D_\alpha V
\]
Gauge multiplet (a real general multiplet with $w = 0$)

Gauge transformation

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We can move to the Wess-Zumino gauge.

$$V_{\text{WZ}} = - (\theta \sigma^\mu \bar{\theta}) \left( e^{-1} \right)_\mu^\nu \partial_\nu \bar{B}_\mu + i \theta^2 \bar{\theta} \left( \lambda - i \psi^\mu \bar{B}_\mu \right) - i \bar{\theta}^2 \theta \left( \lambda + i \bar{\psi}^\mu \bar{B}_\mu \right)$$

$$+ \frac{1}{2} \theta^2 \bar{\theta}^2 \left\{ D + \left( - \frac{i}{2} \lambda \bar{\sigma}^\mu \bar{\psi}_\mu + \text{h.c.} \right) + 2 d^\mu \bar{B}_\mu \right\}.$$

This is possible only when $w = 0$.

Field strength superfield

$$\mathcal{W}_\alpha^{\text{naive}} \equiv - \frac{1}{4} \bar{D}^2 D_\alpha V$$

This is not gauge-invariant!
**Gauge multiplet** (a real general multiplet with $\mathcal{w} = 0$)

Gauge transformation

\[ V \rightarrow V + \mathcal{V}(\Sigma) + \mathcal{V}(\bar{\Sigma}), \quad (\Sigma : \text{chiral multiplet}) \]

\[
\mathcal{W}_\alpha = -\frac{1}{4} Z_{\alpha}^{\beta} \bar{D}^2 \left\{ D_\beta V + \frac{1}{4} D_\beta U^\mu \bar{\sigma}_\mu ^{\gamma \gamma} [D_\gamma, \bar{D}_\gamma] V - i U^\mu \partial_\mu D_\beta V \right\},
\]

\[
Z_{\alpha}^{\beta} \equiv \delta_{\alpha}^{\beta} - \frac{1}{2} \bar{\epsilon}_\mu ^{\nu} \left( \sigma^{\mu \nu} \bar{\sigma}_\nu \right)_{\alpha}^{\beta} - \left( \sigma^{\mu \bar{\psi}_\mu} \right)_{\alpha} \bar{\theta}^{\beta}. \quad \text{Gauge-invariant!}
\]
Gauge multiplet (a real general multiplet with u = 0)

Gauge transformation

\[ V \rightarrow V + \mathcal{V}(\Sigma) + \mathcal{V}(\bar{\Sigma}), \quad (\Sigma: \text{chiral multiplet}) \]

\[ \mathcal{W}_\alpha = -\frac{1}{4} Z_{\alpha}^\beta \bar{D}^2 \left\{ D_\beta V + \frac{1}{4} D_\beta U^\mu \bar{\sigma}^{\gamma \gamma}_\mu [D_\gamma, \bar{D}_\gamma] V - i U^\mu \partial_\mu D_\beta V \right\}, \]

\[ Z_{\alpha}^\beta \equiv \delta_{\alpha}^\beta - \frac{1}{2} \bar{\epsilon}_\mu ^\nu (\sigma^\mu \bar{\sigma}_\nu)^\beta _\alpha - (\sigma^\mu \bar{\psi}_\mu )^\beta _\alpha \theta^\beta . \quad \text{Gauge-invariant!} \]

In components,

\[ \mathcal{W}_\alpha = -i \lambda_\alpha + \theta_\alpha D + i (\sigma^{\mu \nu} \theta)_\alpha (e^{-1})^\rho _\mu (e^{-1})^\tau _\nu \bar{F}_{\rho \tau} \]

\[ -\theta^2 \left\{ \sigma^\mu (e^{-1})^\nu _\mu D_\nu \bar{\lambda} \right\} _\alpha, \]

where

\[ \bar{F}_{\mu \nu} \equiv \partial_\mu \bar{B}_\nu - \partial_\nu \bar{B}_\mu + (i \psi_\mu \sigma_\nu \bar{\lambda} - i \psi_\nu \sigma_\mu \bar{\lambda} + \text{h.c.}), \]

\[ (\mathcal{D}_\mu \bar{\lambda})^{\dot{\alpha}} \equiv \left\{ \left( \partial_\mu - \frac{1}{2} \omega^{\nu \rho}_\mu \sigma_{\nu \rho} + \frac{3i}{4} A_\mu \right) \bar{\lambda} \right\} ^{\dot{\alpha}} + (\bar{\sigma}^{\nu \rho} \bar{\psi}_\mu )^{\dot{\alpha}} \bar{F}_{\nu \rho} + i \bar{\psi}^{\dot{\alpha}} D. \]
Action formulae
F-term invariant action

\[ S_F[W] = \int d^4x \int d^2\theta \ (1 + \tilde{\mathcal{E}}) W + \text{h.c.}. \]

where \( \tilde{\mathcal{E}} = \bar{\epsilon}_\mu^\mu - 2i\theta \sigma^\mu \bar{\psi}_\mu \), and \( \delta \tilde{\mathcal{E}} = \Xi \).

The factor \( (1 + \tilde{\mathcal{E}}) \) corresponds to the chiral density multiplet.

This action is invariant only when \( w=3 \).
D term invariant action

\[ S_D[K] \equiv 2 \int d^4x \int d^4\theta \left\{ 1 + \frac{1}{3} (\vec{E}_1 + \vec{\varepsilon} + \vec{\bar{\varepsilon}}) \right\} K, \]

where \( \vec{E}_1 \equiv \frac{1}{4} \vec{\sigma}_\mu \lbrack D_\alpha, \bar{D}_{\bar{\alpha}} \rbrack U^\mu. \)

This is invariant only when \( \nu = 2. \)
\[ \bar{\mathcal{E}} = \bar{\epsilon}_\mu^\mu - 2i \theta \sigma^\mu \bar{\psi}_\mu \] is a redundant superfield.
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We can absorb \( \tilde{\cal E} \) by redefinition,

\[
\begin{align*}
\hat{\Phi} & \equiv \left(1 + \frac{w}{3} \tilde{\cal E} \right) \Phi, \\
\hat{\cal V} & \equiv \left\{ 1 + \frac{w}{6} (\tilde{\cal E} + \bar{\cal E}) \right\} \cal V.
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\end{align*}
\]

Then, the action formulae become

\[
\begin{align*}
S_F[W] &= \int d^4 x \int d^2 \theta \, \tilde{\mathcal{V}}, \\
S_D[K] &= 2 \int d^4 x \int d^4 \theta \left(1 + \tilde{E}_1\right) \tilde{K}.
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\end{align*}
\]

Now the dependence of the SUGRA fields are absorbed into the component fields, except for \( \tilde{E}_1 \).
Summary

- We modify the **linearized SUGRA** in a way that each component is identified with those in the superconformal formulation.
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Summary

• We modify the linearized SUGRA in a way that each component is identified with those in the superconformal formulation.

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• This work provides a basis for an extension to higher-dimensional SUGRA.

Future work

• Construct 5D linearized SUGRA for general field content.
Gauge kinetic term

As for the gauge kinetic term, \( \bar{\epsilon} \)-dependence automatically cancels because

\[
\mathcal{W}^\alpha \mathcal{W}_\alpha = (1 - \bar{\epsilon}) \hat{\mathcal{W}}^\alpha \hat{\mathcal{W}}_\alpha,
\]

where

\[
\hat{\mathcal{W}}_\alpha \equiv -\frac{1}{4} \bar{D}^2 \left( D_\alpha \hat{V} + \frac{1}{4} D_\alpha U^\mu \sigma_\mu^{\beta \beta} [D_\beta, \bar{D}_\beta] \hat{V} - i U^\mu \partial_\mu D_\alpha \hat{V} \right).
\]

Thus, the action is rewritten as

\[
S_{\text{kin}}^{\text{gauge}}[V] = S_F \left[ -\frac{1}{4} \mathcal{W}^\alpha \mathcal{W}_\alpha \right]
\]

\[
= \int d^4x \int d^2\theta \left( 1 + \bar{\epsilon} \right) \left( -\frac{1}{4} \mathcal{W}^\alpha \mathcal{W}_\alpha \right) + \text{h.c.}
\]

\[
= \int d^4x \int d^2\theta \left( -\frac{1}{4} \hat{\mathcal{W}}^\alpha \hat{\mathcal{W}}_\alpha \right) + \text{h.c.}
\]
**SUGRA kinetic term**

The SUGRA kinetic term is expressed as

\[
S_{SG}^{\text{kin}} = \int d^4x \int d^4\theta \Omega_0 \left\{ -\frac{1}{24} U_\mu D^\alpha \bar{D}^2 D_\alpha U^\mu + \frac{1}{9} \bar{E}_1^2 - \frac{1}{3} (\partial_\mu U^\mu)^2 \right\},
\]

where \( \bar{E}_1 \equiv \frac{1}{4} \bar{\sigma}_\mu^{\dot{\alpha}\alpha} [D_\alpha, \bar{D}_{\dot{\alpha}}] U^\mu \).