Linearized supergravity and superconformal formulation

Yutaka Sakamura (KEK)

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⇒ We have to work in the context of SUGRA

It is convenient to express higher-dimensional SUGRA in terms of N=1 superfields by using formulae of 4D off-shell SUGRA.

Superconformal formulation

[Kaku, Townsend, Nieuwenhuizen, PRD17 (1978) 3179;
Ferrara, Grisaru, Nieuwenhuizen, NPB138 (1978) 430,
Kugo, Uehara NPB226 (1983) 49, …]

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[Kugo & Ohashi, PTP108 (2002) 203, …]

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- Most of the known off-shell actions are realized by this formulation.
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  [Kugo & Ohashi, PTP108 (2002) 203, …]
- An explicit expression of the action is lengthy and complicated.

Linearized SUGRA

[Ferrara, Zumino, NPB134 (1978) 301; Siegel, Gates Jr., NPB147 (1979) 77, …]

SUGRA action is constructed
up to linear order in SUGRA fields.

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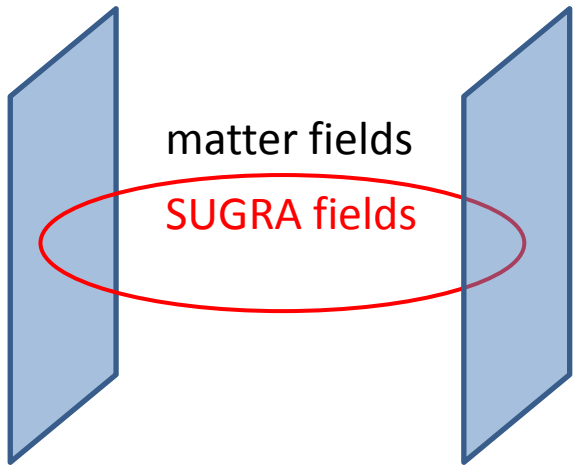
5D linearized SUGRA is constructed in terms of
N=1 superfields **for the minimal field contents**.

[Linch III, Luty, Phillips, PRD68 (2003) 025008]

Linearized SUGRA

For example, in the brane-world scenario,



Visible sector SUSY breaking

we can calculate brane-to-brane contributions,
keeping N=1 superfield structure.

[Buchbinder, et.al, PRD70 (2004) 025008]

Superconformal formulation

- We can treat general SUGRA in full order.
- 5D extension has been done.
- Explicit expressions are complicated.

Linearized SUGRA

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Purpose

Clarify the direct relation between the two formulations.

Strategy

We modify the known linearized SUGRA, and

Fields in the modified linearized SUGRA

Identify

Fields in the superconformal formulation of

[Kugo & Uehara, NBP226 (1983) 49]

Modified linearized SUGRA

Superconformal multiplets

Each supercomformal multiplet is characterized by the Weyl weight w and the chiral weight n .
(dilatation) (R-charge)

We consider the following three types of multiplets.

- Weyl multiplet (SUGRA multiplet)
- (Anti-)Chiral multiplet (matter, compensator)
- Real general multiplet (gauge multiplet, …)

Superconformal transformations

For a chiral superfield $(w = n)$,

modified part

$$\delta\Phi = \left\{ -\frac{1}{4}\bar{D}^2 L^\alpha D_\alpha - i\sigma^\mu_{\alpha\dot{\alpha}}\bar{D}^{\dot{\alpha}} L^\alpha \partial_\mu + 2w\Lambda \right\} \Phi.$$

For an anti-chiral superfield $(w = -n)$,

$$\delta\bar{\Phi} = \left\{ -\frac{1}{4}D^2 \bar{L}_{\dot{\alpha}}\bar{D}^{\dot{\alpha}} - i\sigma^\mu_{\alpha\dot{\alpha}} D^\alpha \bar{L}^{\dot{\alpha}} \partial_\mu + 2w\bar{\Lambda} \right\} \bar{\Phi}.$$

For a real general superfield $(n = 0)$,

$$\delta V = \left\{ -\frac{1}{4}\bar{D}^2 L^\alpha D_\alpha - \frac{i}{2}\sigma^\mu_{\alpha\dot{\alpha}}\bar{D}^{\dot{\alpha}} L^\alpha \partial_\mu + w\Lambda + \text{hc} \right\} V,$$

transformation parameter

where $\Lambda = -\frac{1}{24}\left(\bar{D}^2 D^\alpha L_\alpha + 4\Xi\right).$

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where $\Lambda = -\frac{1}{24}\left(\bar{D}^2 D^\alpha L_\alpha + 4\Xi\right).$

We also introduce U^μ, which transforms inhomogeneously,

$$\delta U^\mu = \frac{1}{2}\sigma^\mu_{\alpha\dot{\alpha}}\left(\bar{D}^{\dot{\alpha}} L^\alpha - D^\alpha \bar{L}^{\dot{\alpha}}\right).$$

Transformation parameters

Only the following components of L^α appear in the component transformation laws.

$$
\begin{aligned}
\xi &= \mathrm{Im}\left(\sigma^\mu_{\alpha\dot\alpha}\bar D^{\dot\alpha}L^\alpha\right)\Big|, \\
\epsilon^\alpha &= -\frac{1}{4}\bar D^2 L^\alpha\Big|, \\
\varphi &= -\frac{1}{4}\mathrm{Re}\left(D_\alpha\bar D^2 L^\alpha\right), \\
\vartheta &= \frac{1}{6}\mathrm{Im}\left(D_\alpha\bar D^2 L^\alpha\right)\Big|, \\
\lambda_{\mu\nu} &= -\frac{1}{2}\mathrm{Re}\left\{(\sigma_{\mu\nu})^\beta{}_\alpha D_\beta\bar D^2 L^\alpha\right\}\Big|, \\
\rho^\alpha &= -\frac{1}{32}D^2\bar D^2 L^\alpha\Big|.
\end{aligned}
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\xi &= \mathrm{Im}\left(\sigma^\mu_{\alpha\dot\alpha}\bar{D}^{\dot\alpha}L^\alpha\right)\Big|, && (\boldsymbol{P}\ :\ \text{translation})\\
\epsilon^\alpha &= -\frac{1}{4}\bar{D}^2L^\alpha\Big|, && (\boldsymbol{Q}\ :\ \text{SUSY})\\
\varphi &= -\frac{1}{4}\mathrm{Re}\left(D_\alpha\bar{D}^2L^\alpha\right), && (\boldsymbol{D}\ :\ \text{dilatation})\\
\vartheta &= \frac{1}{6}\mathrm{Im}\left(D_\alpha\bar{D}^2L^\alpha\right)\Big|, && (U(1)_A\ :\ \text{R-symmetry})\\
\lambda_{\mu\nu} &= -\frac{1}{2}\mathrm{Re}\left\{(\sigma_{\mu\nu})^\beta{}_\alpha D_\beta\bar{D}^2L^\alpha\right\}\Big|, && (\boldsymbol{M}\ :\ \text{local Lorentz})\\
\rho^\alpha &= -\frac{1}{32}D^2\bar{D}^2L^\alpha\Big|. && (\boldsymbol{S}\ :\ \text{conformal SUSY})
\end{aligned}
$$

Field identification

Weyl multiplet (SUGRA multiplet)

$$U_\mu = \left(\theta\sigma^\nu\bar{\theta}\right)\tilde{e}_{\nu\mu} + \bar{\theta}^2(\theta\tilde{\psi}_\mu) + \theta^2(\bar{\theta}\bar{\tilde{\psi}}_\mu) + \theta^2\bar{\theta}^2 d_\mu$$

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$$\left\{\begin{array}{rcl} \delta\tilde{e}_\nu{}^\mu & = & -\delta_\nu{}^\mu\varphi + \lambda^\mu{}_\nu + \partial_\nu\xi^\mu, \\ \delta\tilde{\psi}^\mu_\alpha & = & \left(2\sigma_\mu\bar{\tilde{\rho}} + i\sigma^\nu\bar{\sigma}^\mu\partial_\nu\epsilon\right)_\alpha, \\ \delta d^\mu & = & \frac{3}{4}\partial^\mu\vartheta + \frac{1}{4}\epsilon^{\mu\nu\rho\tau}\partial_\nu\lambda_{\rho\tau}. \end{array}\right.$$

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$$U_\mu = \left(\theta\sigma^\nu\bar{\theta}\right)\tilde{e}_{\nu\mu} + \bar{\theta}^2(\theta\tilde{\psi}_\mu) + \theta^2(\bar{\theta}\bar{\tilde{\psi}}_\mu) + \theta^2\bar{\theta}^2 d_\mu$$

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Each component is identified as

$$\left\{\begin{array}{rcl} \tilde{e}_\nu{}^\mu & = & e_\nu{}^{\underline{\mu}} - \delta_\nu{}^\mu, \\ \tilde{\psi}^\mu_\alpha & = & i\left(\sigma^\nu\bar{\sigma}^\mu\psi_\nu\right)_\alpha, \\ d^\mu & = & \frac{3}{4}A^\mu - \frac{1}{4}\epsilon^{\mu\nu\rho\tau}\partial_\nu\tilde{e}_{\rho\tau}. \end{array}\right.$$

$$\left(\begin{array}{l} e_\nu{}^{\underline{\mu}} : \text{ vierbein; } \psi_{\mu\alpha} : \text{ gravitino;} \\ A_\mu\text{: } U(1)_A \text{ gauge field} \end{array}\right)$$

Chiral multiplet

$$\Phi = \phi + \theta\chi + \theta^2 F$$

$$\delta\Phi = \left\{ -\frac{1}{4}\bar{D}^2 L^\alpha D_\alpha - i\sigma^\mu_{\alpha\dot{\alpha}}\bar{D}^{\dot{\alpha}} L^\alpha \partial_\mu - \frac{w}{12}\left(\bar{D}^2 D^\alpha L_\alpha + 4\Xi\right)\right\}\Phi$$

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In components,

$$\begin{cases} \delta\phi = \xi^\mu\partial_\mu\phi + \epsilon\chi + w\varphi\phi + \frac{iw}{2}\vartheta\phi, \\ \delta\chi_\alpha = \xi^\mu\partial_\mu\chi_\alpha + \frac{1}{2}\lambda_{\mu\nu}\left(\sigma^{\mu\nu}\chi\right)_\alpha + 2\epsilon_\alpha F - 2i\left(\sigma^\mu\bar{\epsilon}\right)_\alpha\partial_\mu\phi \\ \qquad + \left(w + \frac{1}{2}\right)\varphi\chi_\alpha + \frac{i}{2}\left(w - \frac{3}{2}\right)\vartheta\chi_\alpha - 4w\rho_\alpha\phi, \\ \delta F = \xi^\mu\partial_\mu F - i\bar{\epsilon}\bar{\sigma}^\mu\partial_\mu\chi + (w+1)\varphi F + \frac{i}{2}(w-3)\vartheta F \\ \qquad + 2(w-1)\rho\chi. \end{cases}$$

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We can identify $[\phi,\ \chi_\alpha,\ F]$ with a chiral multiplet.

Embedding into a general multiplet

In the global SUSY case, a chiral superfield is embedded into a general superfield by $y^\mu \equiv x^\mu - i\theta\sigma^\mu\bar{\theta} \to x^\mu$.

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$$
\begin{aligned}
\mathcal{V}(\Phi) &= (1 + iU^\mu\partial_\mu)\Phi \\
&= \phi + \theta\chi + \theta^2 F - i(\theta\sigma^\mu\bar{\theta})\left(e^{-1}\right)_\mu{}^\nu \partial_\nu\phi \\
&\quad -\frac{i}{2}\theta^2\left\{\bar{\theta}\bar{\sigma}^\mu\left(e^{-1}\right)_\mu{}^\nu\partial_\nu\chi - 2\left(\bar{\theta}\bar{\tilde{\psi}}_\mu\right)\partial_\mu\phi\right\} + i\bar{\theta}^2\left(\theta\tilde{\psi}^\mu\right)\partial_\mu\phi \\
&\quad -\frac{1}{4}\theta^2\bar{\theta}^2\left\{g^{\mu\nu}\partial_\mu\partial_\nu\phi + 2i\tilde{\psi}^\mu\partial_\mu\chi - 4id^\mu\partial_\mu\phi\right\},
\end{aligned}
$$

where $\left(e^{-1}\right)_\mu{}^\nu \equiv \delta_\mu{}^\nu - \tilde{e}_\mu{}^\nu$ and $g^{\mu\nu} \equiv \eta^{\mu\nu} - \tilde{e}^{\mu\nu} - \tilde{e}^{\nu\mu}$.

Real general multiplet

$$
\begin{aligned}
V &= C' + i\theta\zeta' - i\bar{\theta}\bar{\zeta}' - \theta^2\mathcal{H}' - \bar{\theta}^2\bar{\mathcal{H}}' - (\theta\sigma^\mu\bar{\theta})B'_\mu \\
&\quad + i\theta^2(\bar{\theta}\bar{\lambda}') - i\bar{\theta}^2(\theta\lambda') + \frac{1}{2}\theta^2\bar{\theta}^2 D',
\end{aligned}
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$$
\delta V = \left\{ -\frac{1}{4}\bar{D}^2 L^\alpha D_\alpha - \frac{i}{2}\sigma^\mu_{\alpha\dot{\alpha}}\bar{D}^{\dot{\alpha}} L^\alpha \partial_\mu + w\Lambda + \text{hc} \right\} V.
$$

The transformations of $[C',\ \zeta'_\alpha,\ \mathcal{H}',\ B'_\mu,\ \lambda'_\alpha,\ D']$ do not agree with the superconformal trf. of a real general multiplet as is.

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\end{aligned}
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We need to redefine the fields.

$$
\begin{aligned}
C &\equiv C, \quad \zeta_\alpha \equiv \zeta'_\alpha, \quad \mathcal{H} \equiv \mathcal{H}', \\
B_\mu &\equiv B'_\mu + \zeta'\psi_\mu + \bar{\zeta}'\bar{\psi}_\mu + \frac{w}{2}C'A_\mu, \\
\lambda_\alpha &\equiv \lambda'_\alpha + \frac{i}{2}\left\{\sigma^\mu \left(e^{-1}\right)_\mu{}^\nu \partial_\nu\bar{\zeta}'\right\}_\alpha + (\sigma^\mu\bar{\sigma}^\nu\psi_\mu)_\alpha B'_\nu + \frac{w}{4}(\sigma^\mu\bar{\zeta}')_\alpha A_\mu, \\
D &\equiv D' + \frac{1}{2}g^{\mu\nu}\partial_\mu\partial_\nu C' - \left(2d^\mu - \frac{w}{2}A^\mu\right)B'_\mu \\
&\quad + \left(\bar{\lambda}'\bar{\sigma}^\mu\psi_\mu - \frac{i}{2}\partial_\nu\zeta'\sigma^\mu\bar{\sigma}^\nu\psi_\mu - i\partial_\mu\zeta'\psi^\mu + \frac{2iw}{3}\zeta'\sigma^{\mu\nu}\partial_\nu\psi_\mu + \text{h.c.}\right).
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\end{aligned}
$$

Then, $[C,\ \zeta_\alpha,\ \mathcal{H},\ B_\mu,\ \lambda_\alpha,\ D]$ is identified with a real general multiplet.

Gauge multiplet (a real general multiplet with w =0)

Gauge transformation

$$V \to V + \mathcal{V}(\Sigma) + \mathcal{V}(\bar{\Sigma}), \quad (\Sigma : \text{ chiral multiplet})$$

We can move to the Wess-Zumino gauge.

$$\begin{aligned} V_{\mathrm{WZ}} &= -(\theta\sigma^\mu\bar{\theta})\left(e^{-1}\right)_\mu{}^\nu \hat{B}'_\nu + i\theta^2\bar{\theta}\left(\bar{\lambda} - i\bar{\tilde{\psi}}^\mu \hat{B}'_\mu\right) - i\bar{\theta}^2\theta\left(\lambda + i\tilde{\psi}^\mu \hat{B}'_\mu\right) \\ &\quad + \frac{1}{2}\theta^2\bar{\theta}^2\left\{D + \left(-\frac{i}{2}\bar{\lambda}\bar{\sigma}^\mu\tilde{\psi}_\mu + \text{h.c.}\right) + 2d^\mu \hat{B}'_\mu\right\}. \end{aligned}$$

This is possible only when $w = 0$.

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Field strength superfield

$$\mathcal{W}_\alpha^{\text{naive}} \equiv -\frac{1}{4}\bar{D}^2 D_\alpha V$$

This is not gauge-invariant!

Gauge multiplet (a real general multiplet with w =0)

Gauge transformation

$$V \to V + \mathcal{V}(\Sigma) + \mathcal{V}(\bar{\Sigma}), \quad (\Sigma : \text{ chiral multiplet})$$

$$\begin{aligned}
\mathcal{W}_\alpha &= -\frac{1}{4} Z_\alpha{}^\beta \bar{D}^2 \left\{ D_\beta V + \frac{1}{4} D_\beta U^\mu \bar{\sigma}_\mu^{\dot{\gamma}\gamma} [D_\gamma, \bar{D}_{\dot{\gamma}}] V - i U^\mu \partial_\mu D_\beta V \right\}, \\
Z_\alpha{}^\beta &\equiv \delta_\alpha{}^\beta - \frac{1}{2} \tilde{e}_\mu{}^\nu (\sigma^\mu \bar{\sigma}_\nu)_\alpha{}^\beta - \left(\sigma^\mu \bar{\tilde{\psi}}_\mu \right)_\alpha \theta^\beta.
\end{aligned}$$

Gauge-invariant!

Gauge multiplet (a real general multiplet with w =0)

Gauge transformation

$$V \to V + \mathcal{V}(\Sigma) + \mathcal{V}(\bar{\Sigma}), \quad (\Sigma : \text{ chiral multiplet})$$

$$\begin{aligned}
\mathcal{W}_\alpha &= -\frac{1}{4} Z_\alpha{}^\beta \bar{D}^2 \left\{ D_\beta V + \frac{1}{4} D_\beta U^\mu \bar{\sigma}_\mu^{\dot{\gamma}\gamma} [D_\gamma, \bar{D}_{\dot{\gamma}}] V - i U^\mu \partial_\mu D_\beta V \right\}, \\
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\end{aligned}$$

Gauge-invariant!

In components,

$$\begin{aligned}
\mathcal{W}_\alpha &= -i\lambda_\alpha + \theta_\alpha D + i(\sigma^{\mu\nu}\theta)_\alpha \left(e^{-1}\right)_\mu{}^\rho \left(e^{-1}\right)_\nu{}^\tau \hat{F}_{\rho\tau} \\
&\quad -\theta^2 \left\{ \sigma^\mu \left(e^{-1}\right)_\mu{}^\nu \mathcal{D}_\nu \bar{\lambda} \right\}_\alpha,
\end{aligned}$$

where

$$\hat{F}_{\mu\nu} \equiv \partial_\mu \hat{B}_\nu - \partial_\nu \hat{B}_\mu + (i\psi_\mu \sigma_\nu \bar{\lambda} - i\psi_\nu \sigma_\mu \bar{\lambda} + \text{h.c.}),$$

$$\left(\mathcal{D}_\mu \bar{\lambda}\right)^{\dot{\alpha}} \equiv \left\{ \left(\partial_\mu - \frac{1}{2} \omega_\mu{}^{\nu\rho} \sigma_{\nu\rho} + \frac{3i}{4} A_\mu \right) \bar{\lambda} \right\}^{\dot{\alpha}} + (\bar{\sigma}^{\nu\rho} \bar{\psi}_\mu)^{\dot{\alpha}} \hat{F}_{\nu\rho} + i\bar{\psi}^{\dot{\alpha}} D.$$

Action formulae

F-term invariant action

$$S_F[W] = \int d^4x \int d^2\theta \, \left(1 + \tilde{\mathcal{E}}\right) W + \text{h.c.}.$$

where $\tilde{\mathcal{E}} = \tilde{e}_\mu{}^\mu - 2i\theta\sigma^\mu\bar{\psi}_\mu,$ and $\delta\tilde{\mathcal{E}} = \Xi.$

The factor $(1 + \tilde{\mathcal{E}})$ corresponds to the chiral density multiplet.

This action is invariant only when $w=3$.

D term invariant action

$$S_D[K] \equiv 2\int d^4x \int d^4\theta \left\{1 + \frac{1}{3}\left(\tilde{E}_1 + \tilde{\mathcal{E}} + \bar{\tilde{\mathcal{E}}}\right)\right\} K,$$

where $\tilde{E}_1 \equiv \frac{1}{4}\bar{\sigma}_\mu^{\dot{\alpha}\alpha}[D_\alpha, \bar{D}_{\dot{\alpha}}]U^\mu.$

This is invariant only when $w = 2$.

$\tilde{\mathcal{E}} = \tilde{e}_\mu{}^\mu - 2i\theta\sigma^\mu\bar{\psi}_\mu$ is a redundant superfield.

already contained in U^μ

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We can absorb $\tilde{\mathcal{E}}$ by redefinition,

$$\begin{cases} \hat{\Phi} \equiv \left(1 + \frac{w}{3}\tilde{\mathcal{E}}\right)\Phi, \\ \hat{V} \equiv \left\{1 + \frac{w}{6}\left(\tilde{\mathcal{E}} + \bar{\tilde{\mathcal{E}}}\right)\right\} V. \end{cases}$$

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Then, the action formulae become

$$\begin{cases} S_F[W] = \int d^4x \int d^2\theta\, \hat{W}, \\ S_D[K] = 2\int d^4x \int d^4\theta\, \left(1 + \tilde{E}_1\right)\hat{K}. \end{cases}$$

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Now the dependence of the SUGRA fields are absorbed into the component fields, except for $\tilde{E}_1$.

Summary

- We modify the linearized SUGRA in a way that each component is identified with those in the superconformal formulation.

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Summary

- We modify the linearized SUGRA in a way that each component is identified with those in the superconformal formulation.
- This identification makes it possible to use both formulations in a complementary way.
- This work provides a basis for an extension to higher-dimensional SUGRA.

Future work

- Construct 5D linearized SUGRA for general field content.

Gauge kinetic term

As for the gauge kinetic term, $\tilde{\mathcal{E}}$-dependence automatically cancels because

$$\mathcal{W}^\alpha \mathcal{W}_\alpha = \left(1 - \tilde{\mathcal{E}}\right) \hat{\mathcal{W}}^\alpha \hat{\mathcal{W}}_\alpha,$$

where $\hat{\mathcal{W}}_\alpha \equiv -\frac{1}{4}\bar{D}^2 \left(D_\alpha \hat{V} + \frac{1}{4} D_\alpha U^\mu \bar{\sigma}_\mu^{\dot{\beta}\beta} [D_\beta, \bar{D}_{\dot{\beta}}] \hat{V} - i U^\mu \partial_\mu D_\alpha \hat{V} \right).$

Thus, the action is rewritten as

$$\begin{aligned} S_{\text{kin}}^{\text{gauge}}[V] &= S_F \left[-\frac{1}{4} \mathcal{W}^\alpha \mathcal{W}_\alpha \right] \\ &= \int d^4x \int d^2\theta \, \left(1 + \tilde{\mathcal{E}}\right) \left(-\frac{1}{4} \mathcal{W}^\alpha \mathcal{W}_\alpha \right) + \text{h.c.} \\ &= \int d^4x \int d^2\theta \, \left(-\frac{1}{4} \hat{\mathcal{W}}^\alpha \hat{\mathcal{W}}_\alpha \right) + \text{h.c.}. \end{aligned}$$

SUGRA kinetic term

The SUGRA kinetic term is expressed as

$$S_{\mathrm{SG}}^{\mathrm{kin}} = \int d^4x \int d^4\theta\, \Omega_0 \left\{ -\frac{1}{24} U_\mu D^\alpha \bar{D}^2 D_\alpha U^\mu + \frac{1}{9}\tilde{E}_1^2 - \frac{1}{3}(\partial_\mu U^\mu)^2 \right\},$$

where $\tilde{E}_1 \equiv \frac{1}{4}\bar{\sigma}_\mu^{\dot{\alpha}\alpha}[D_\alpha, \bar{D}_{\dot{\alpha}}]U^\mu$.