Flavor Violation in SUSY Grand Unified Theories of Flavor

Yukihiro Mimura (National Taiwan University)

Based on

Collaboration with B. Dutta and R.N. Mohapatra

Phys. Rev. Lett **94**, 091804 (2005); Phys. Rev. D**72**, 075009 (2005); Phys. Rev. D**80**, 095021 (2009) ; JHEP **1005**, 034 (2010).

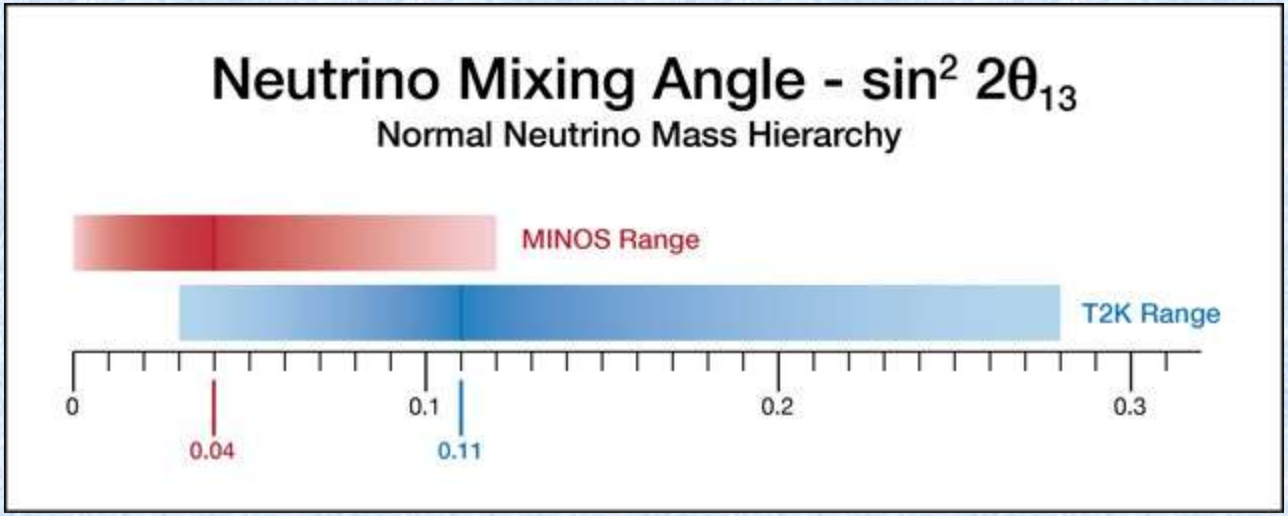
Flavor part in Collaboration with B. Dutta

Phys. Rev. Lett. **97**, 241802 (2006); Phys. Rev. D**75**, 015006 (2007).

Work in progress.

SUSY2011 @ Fermilab (2011.9.1)

Motivation



13 mixing & CP phase in the neutrino oscillation will be expected to be measured accurately.

(T2K, MINOS, WCHOOZ, NOvA, Daya Bay, …)

Naively, for the normal hierarchy case,

$$m_\nu^{\text{light}} \sim \begin{pmatrix} < O(\lambda^2) & O(\lambda) & O(\lambda) \\ O(\lambda) & O(1) & O(1) \\ O(\lambda) & O(1) & O(1) \end{pmatrix}$$

$$\Rightarrow \quad \sin^2\theta_{13} \sim \frac{\Delta m_{12}^2}{\Delta m_{23}^2} \sim \lambda^2$$

Very naive.

Can we have more accurate predictions as well as CP phase?

Flavor Violation in SUSY Grand Unified Theories of Flavor

$$\mu \to e\gamma$$

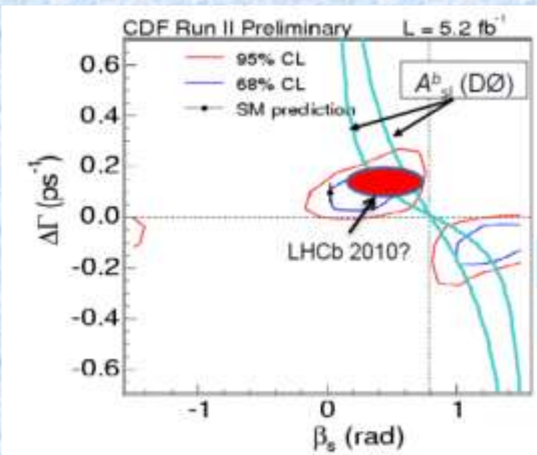
2009 data : $N_{\mathrm{sig}} \simeq 3$

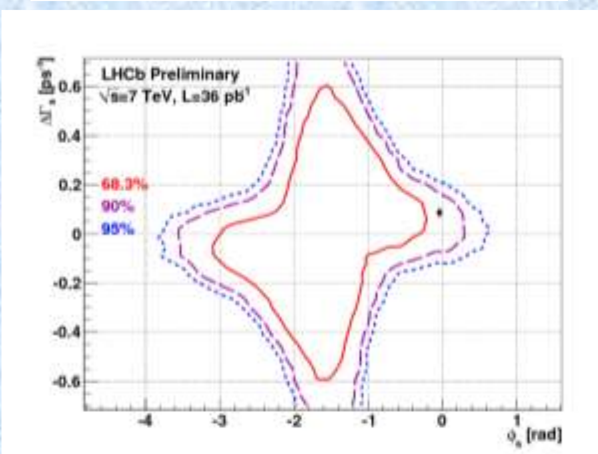
2010 data : no significant events

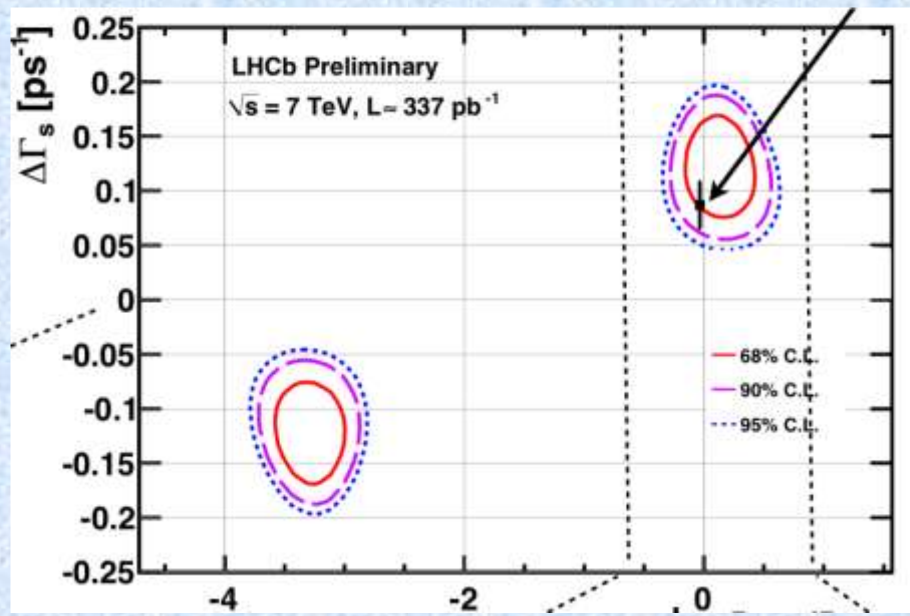
$$\mathrm{Br}(\mu \to e\gamma) < 2.4 \times 10^{-12}$$

Flavor Violation in SUSY
Grand Unified Theories of Flavor

$$\mathrm{CP}(B_s \to J/\psi\phi)$$







$$\phi_s = -2\beta_s$$

Structure
Flavor ~~Violation~~ in SUSY
Grand Unified Theories of Flavor

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Menu

1. Introduction
2. An explanation of flavor puzzle
   in GUTs
3. Proton Decay Suppression
   Flavor Structure
4. Predictions of mixing parameters
5. Conclusion

- Unification of Strong & Electroweak interactions
  $\longrightarrow$ Gauge coupling unification (SUSY GUT)

- Unification of Quarks & Leptons
  $\longrightarrow$ Yukawa unification (bottom-tau)

- Flavor puzzle
  Quark mixings are small & Lepton mixings are large.
  Charged fermion masses are hierarchical.

SO(10) Model with renormalizable Yukawa couplings

(Babu-Mohapatra,1992)

$\mathbf{10} + \overline{\mathbf{126}}$

$$\mathbf{16} \times \mathbf{16} = \mathbf{10} + \mathbf{126} + \mathbf{120}$$

Higgs fields which couple to fermions $\psi(\mathbf{16})$:

$H(\mathbf{10})$, $\bar{\Delta}(\overline{\mathbf{126}})$ and $D(\mathbf{120})$

Renormalizable Yukawa terms:

$$\frac{1}{2}h_{ij}\psi_i\psi_j H + \frac{1}{2}f_{ij}\psi_i\psi_j\bar{\Delta} + \frac{1}{2}h'_{ij}\psi_i\psi_j D$$

h, f : symmetric

h' : anti-symmetric

Neutrino Mass

$$m_\nu^{\text{light}} = M_L - M_\nu^D M_R^{-1} (M_\nu^D)^T$$

Type II Type I

$$M_L = 2\sqrt{2} f \langle \bar{\Delta}_L \rangle \qquad M_R = 2\sqrt{2} f \langle \bar{\Delta}_R \rangle$$

$$\bar{\Delta}_L : (\mathbf{1}, \mathbf{3}, 1) \qquad \bar{\Delta}_R : (\mathbf{1}, \mathbf{1}, 0)$$

$\mathrm{SU}(2)_L$ triplet

$$W_Y = \frac{1}{2} h_{ij} \psi_i \psi_j H + \frac{1}{2} f_{ij} \psi_i \psi_j \bar{\Delta} + \frac{1}{2} h'_{ij} \psi_i \psi_j D \qquad [H(\mathbf{10}),\ \bar{\Delta}(\overline{\mathbf{126}}),\ D(\mathbf{120})]$$

$$\psi\psi\bar{\Delta} \supset \ell\ell\bar{\Delta}_L + \bar{\nu}\bar{\nu}\bar{\Delta}_R$$

Simple explanation of successful fermion mass and mixings

Fermion mass matrices are rank 1 (h) + corrections (f, h')

$$Y_u = \bar{h} + r_2 \bar{f} + r_3 \bar{h}' \qquad Y_d = r_1(\bar{h} + \bar{f} + \bar{h}')$$

$$Y_e = r_1(\bar{h} - 3\bar{f} + c_e \bar{h}') \qquad m_\nu^{\text{light}} \propto \bar{f} \quad \text{(type II)}$$

2 large, 1 small neutrino mixings are obtained.

Quark mixings are small.

Dutta-YM-Mohapatra, PRD**80**, 095021 (2009)

Simple explanation of successful fermion mass and mixings

Fermion mass matrices are rank 1 (h) + corrections (f, h')

f-diagonal basis. $\bar{f}_{33}$ small $\longrightarrow$ CKM small.

$$\bar{h} = \bar{h}_{33} \begin{pmatrix} c^2 & bc & ac \\ bc & b^2 & ab \\ ac & ab & a^2 \end{pmatrix}, \quad \bar{f} = \bar{f}_{33} \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

In the limit $\lambda_1, \lambda_2 \to 0$,

$$\tan^2\theta_{\text{atm}} = \frac{b^2+c^2}{a^2}, \qquad \tan^2\theta_{\text{sol}} = \frac{c}{b}, \qquad \sin\theta_{13} = 0.$$

2 large, 1 small neutrino mixings are automatic.

$$Y_u = \bar{h} + r_2\bar{f} + r_3\bar{h}' \qquad Y_d = r_1(\bar{h} + \bar{f} + \bar{h}')$$

$$Y_e = r_1(\bar{h} - 3\bar{f} + c_e\bar{h}') \qquad m_\nu^{\text{light}} \propto \bar{f}$$ (type II)

If fine-tuning is absent to fit electron mass, we obtain

$$\sqrt{\frac{m_e}{m_\mu}} \simeq \frac{1}{3} V_{us}$$

1. $$U_{e3} \simeq \frac{1}{3} V_{us} \sin\theta_{\rm atm} \simeq 0.05$$

or

2. $$|U_{e3}| \simeq \left| \underline{\frac{m_{\nu 2}}{m_{\nu 3}} \tan\theta_s \cot\theta_a} + e^{i\gamma} c_e \frac{1}{3} V_{us} \sin\theta_a \right|$$

$$\simeq 0.11$$

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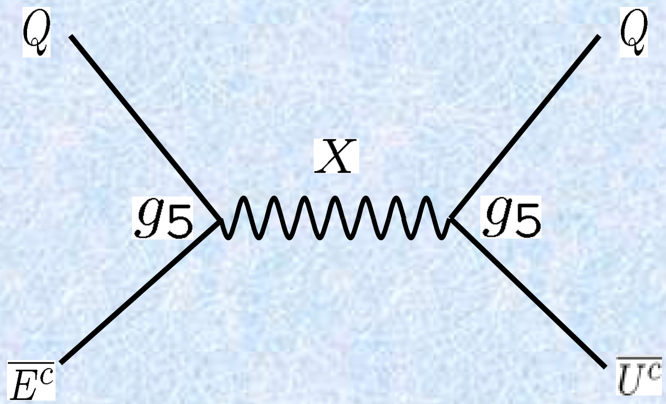
$$\simeq 0.11$$

Statement:
To suppress proton decay by a flavor structure,
solution 2 is preferable.

Proton Decay

- Dimension 6 operator



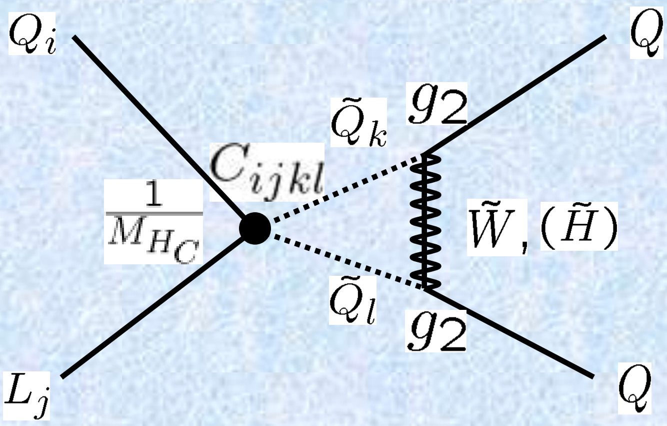
dominant mode : $p \to \pi^0 e^+$

$$A = \frac{g_5^2}{\boxed{M_X^2}}$$ (Up to Hadron matrix element)

$$M_X \sim 2 \times 10^{16} \text{ GeV}$$

$$\tau_p \sim 10^{34} - 10^{36} \text{ years}$$

- Dimension 5 operator with gaugino or Higgsino dressing



dominant mode : $p \to K^+ \bar{\nu}$

$$A = \frac{\alpha_2}{4\pi \boxed{M_{H_C} m_{\text{SUSY}}}} C_{ijkl}$$

Severe constraint to models

Dimension-5 Proton decay suppression

D5 Proton decay amplitude:

wino

$$A \simeq \frac{\alpha_2 \beta_p}{4\pi} \frac{1}{M_{H_C}} \frac{m_{\tilde{W}}}{m_{\tilde{q}}^2} C_{ijkl}$$

Lightest colored Higgs mass

squark

Function of h, f, h'

Possible solutions to suppress D5 proton decay

1. SUSY particles (squarks) are heavy.
   $\longrightarrow$ Look forward to the LHC results
2. Colored Higgs is heavy. (PRL**100** 181801)
3. Typical flavor structure of Yukawa h, f, h'.
   $\longrightarrow$ Predictive to masses and mixings

(PRL**94** 091804; PRD**72** 075009)

Dimension 5 operators : $-W_5 = \frac{1}{2}C_L^{ijkl}\underbrace{q_k q_l}_{\text{symmetric}} q_i \ell_j + C_R^{ijkl} e_k^c u_l^c u_i^c d_j^c$

Higgs triplets $(\bar{\mathbf{3}}, \mathbf{1}, 1/3) + c.c.$

$$\varphi_{\bar{T}} = (H_{\bar{T}}, D_{\bar{T}}, D'_{\bar{T}}, \bar{\Delta}_{\bar{T}}, \Delta_{\bar{T}}, \Delta'_{\bar{T}}, \Phi_{\bar{T}})$$

$$\varphi_T = (H_T, D_T, D'_T, \Delta_T, \bar{\Delta}_T, \bar{\Delta}'_T, \Phi_T)$$

mass term : $(\varphi_{\bar{T}})_a (M_T)_{ab} (\varphi_T)_b$

$$X M_T Y^\top = M_T^{\mathrm{diag}}$$

$\boxed{H_T \text{ and } \bar{\Delta}_T \text{ have opposite D-parity.}}$

$$C_L^{ijkl} = \sum_a \frac{1}{M_{T_a}} (X_{a1} h + X_{a4} f + \sqrt{2} X_{a3} h')_{ij} (Y_{a1} h + Y_{a5} f)_{kl}$$

Opposite signature

no h'

$$C_R^{ijkl} = \sum_a \frac{1}{M_{T_a}} (X_{a1} h - X_{a4} f + \sqrt{2} X_{a2} h')_{ij} (Y_{a1} h - (Y_{a5} - \sqrt{2} Y_{a6}) f + \sqrt{2}(Y_{a3} - Y_{a2}))_{kl}$$

$$\left(\begin{array}{l} C_L^{ijkl} = c h_{ij} h_{kl} + x_1 f_{ij} f_{kl} + x_2 f_{ij} f_{kl} + x_3 f_{ij} h_{kl} + x_4 h'_{ij} h_{kl} + x_5 h'_{ij} f_{kl} \\ C_R^{ijkl} = c h_{ij} h_{kl} + y_1 f_{ij} f_{kl} + y_2 f_{ij} f_{kl} + y_3 f_{ij} h_{kl} + y_4 h'_{ij} h_{kl} + y_5 h'_{ij} f_{kl} + \cdots \end{array} \right)$$

$\underline{x_3 = -y_3}$ (Dutta-YM-Mohapatra)

Proton decay amplitude $A = \frac{\alpha_2 \beta_p}{4\pi M_T m_{\mathrm{SUSY}}} \tilde{A}$

$\tau(p \to K\bar{\nu}) > 19 \times 10^{32}$ years

$\tau(n \to \pi\bar{\nu}) > 4.4 \times 10^{32}$ years

$\tau(n \to K\bar{\nu}) > 1.8 \times 10^{32}$ years

$$\tilde{A} = c\tilde{A}_{hh} + x_1 \tilde{A}_{ff} + x_2 \tilde{A}_{hf} + x_3 \tilde{A}_{fh} + x_4 \tilde{A}_{h'h} + x_5 \tilde{A}_{h'f}$$

To satisfy current bounds,

$$\tilde{A}(p \to K\bar{\nu}) < 10^{-8}, \quad \tilde{A}(n \to \pi\bar{\nu}) < 2 \cdot 10^{-8}, \quad \tilde{A}(n \to K\bar{\nu}) < 5 \cdot 10^{-8}$$

$$\left(M_T = 2 \cdot 10^{16}\ \mathrm{GeV},\ M_{\tilde{q}} = 1\ \mathrm{TeV},\ M_{\tilde{W}} = 250\ \mathrm{GeV} \right)$$

When r_2, $r_3 \sim O(1)$, $\quad \tilde{A}_{hh} \sim 10^{-4} - 10^{-5}$

(roughly, $\bar{h} \sim Y_d$)

Cancellation is unnatural.

When r_2, $r_3 \ll 1$ and $\bar{h} \sim Y_u$, $\quad \tilde{A}_{hh} \sim 10^{-7}$

One still needs to care about cancellation in each decay mode, $p \to K\bar{\nu}_{e,\mu,\tau}$

A special choice of texture (in $\bar{h}$-diagonal basis)

$$\bar{h} \simeq \mathrm{diag}(\ll \lambda_u, \ll \lambda_c, \ \lambda_t) \qquad \lambda \sim 0.2$$

$$\bar{f} \simeq \begin{pmatrix} \sim 0 & \sim 0 & \lambda^3 \\ \sim 0 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda^2 \end{pmatrix}, \qquad \bar{h}' \simeq i \begin{pmatrix} 0 & \lambda^3 & \lambda^3 \\ -\lambda^3 & 0 & \lambda^2 \\ -\lambda^3 & -\lambda^2 & 0 \end{pmatrix}$$

Smallness of 11, 12 element of *f* is also important to suppress proton decay.

Relations are clear under this texture!

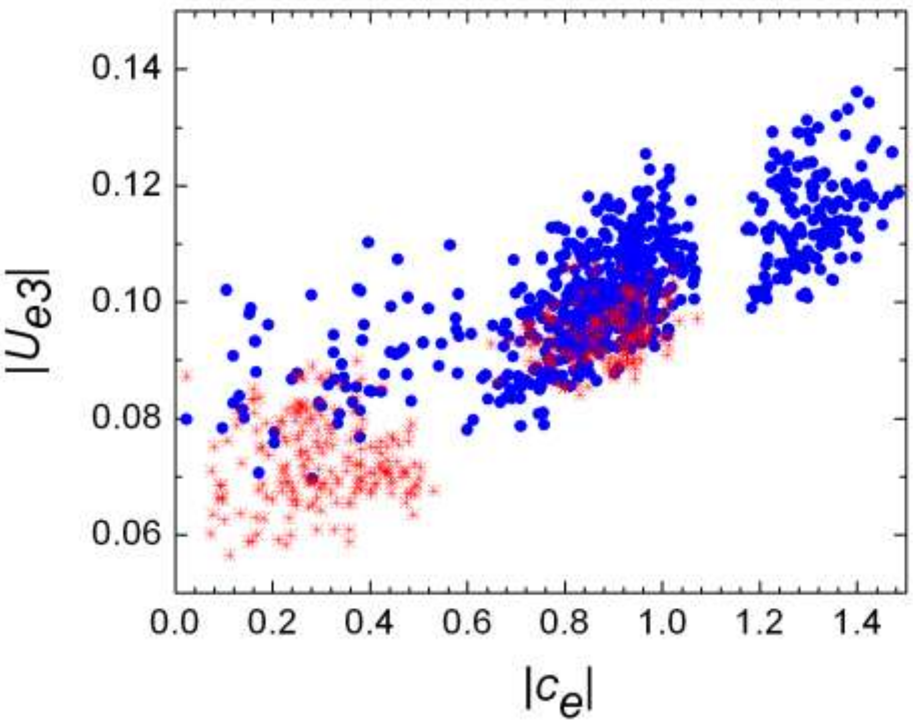
e.g. Charm Yukawa: $\lambda_c \simeq r_2\, m_s/m_b \Rightarrow |r_2| \simeq 0.1 - 0.15$

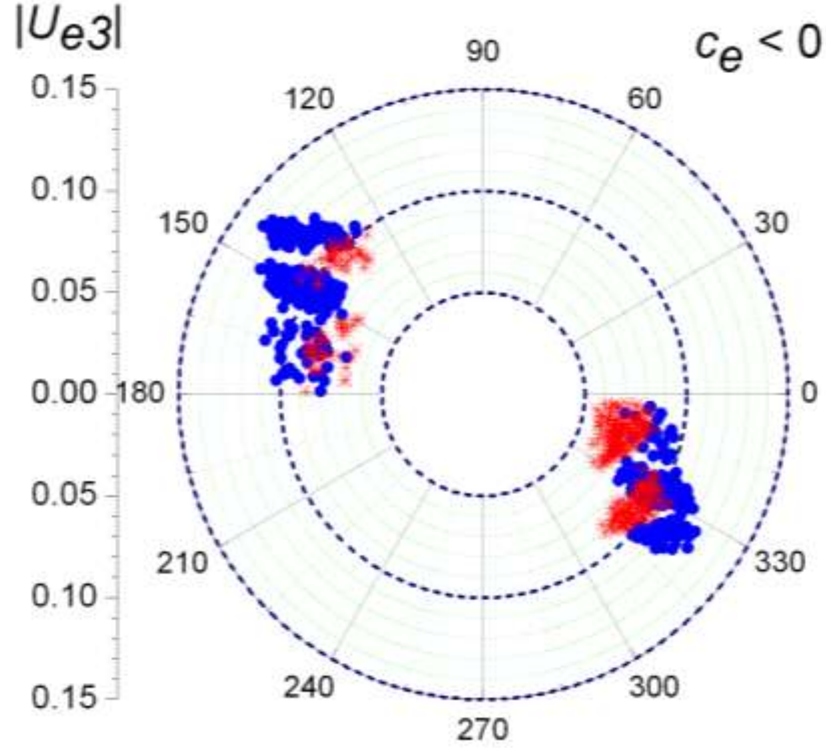
$$m_d m_s m_b \simeq c_e^2 m_e m_\mu m_\tau \Rightarrow |c_e| \sim 1.$$

$$Y_u = \bar{h} + r_2 \bar{f} + r_3 \bar{h}' \qquad Y_d = r_1(\bar{h} + \bar{f} + \bar{h}')$$

$$Y_e = r_1(\bar{h} - 3\bar{f} + c_e \bar{h}') \qquad m_\nu^{\text{light}} \propto \bar{f}$$ (type II)

Numerical fit in SO(10)





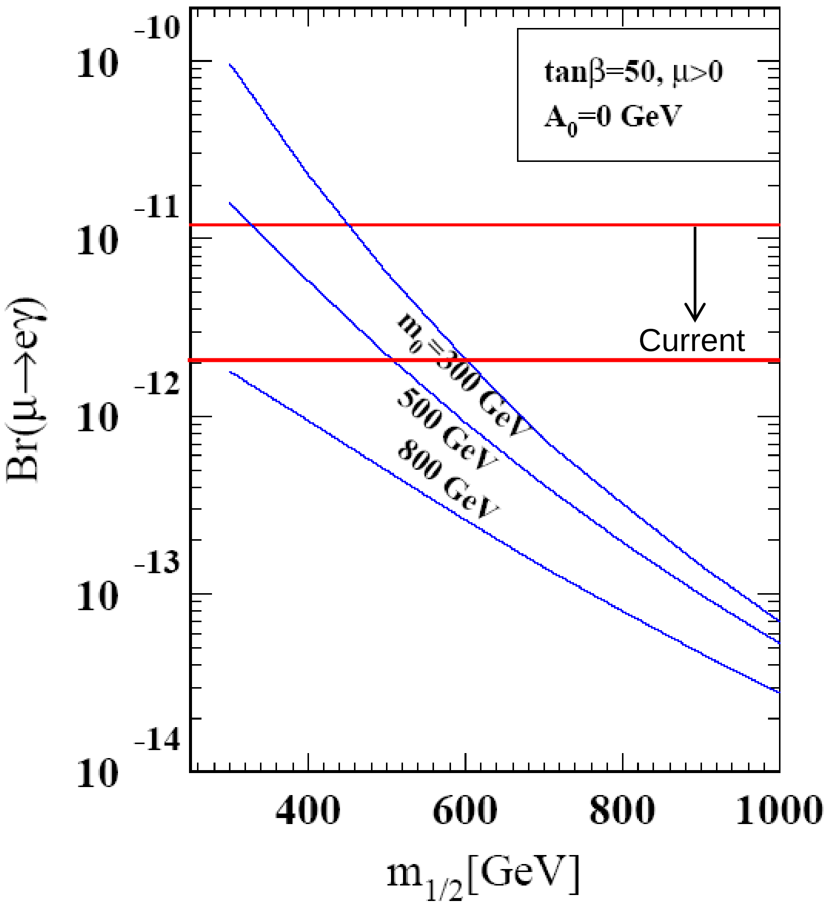
$\delta_{MNSP} \sim 150^{\circ}$ or 330°

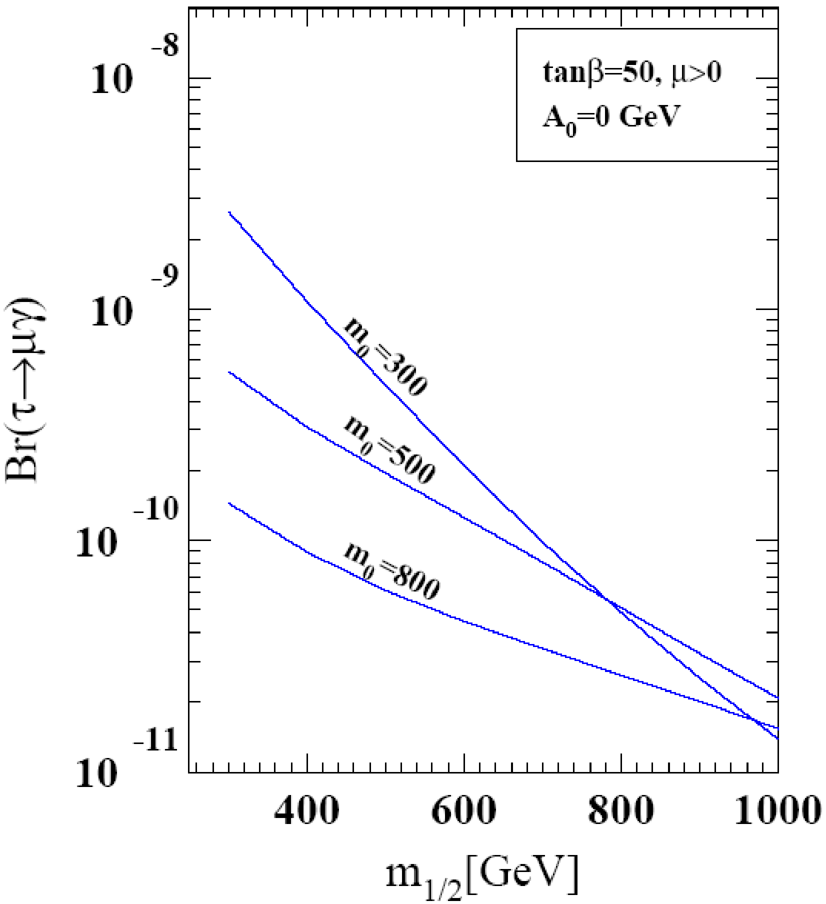
(Hermiticity of fermion mass matrix is assumed.)

Dutta-YM-Mohapatra, PRD**72**, 075009 (2005)

Measurable and testable in near-future experiments.

Lepton flavor violation





$\mu \to e\gamma$ bound : $\mathrm{BR} < 2.4 \times 10^{-12}$

$$m^2_{\tilde{D}^c} = m^2_{\tilde{L}} \simeq m_0^2 \left(\mathbf{1} - \kappa\, U^*_{\mathrm{MNSP}} \begin{pmatrix} k_1 & & \\ & k_2 & \\ & & 1 \end{pmatrix} U^{\mathrm{T}}_{\mathrm{MNSP}} \right)$$

(By RGE, in simplified situation)

$\delta^{\tilde{\ell}}_{12}$: small ⇒ $\underline{\sin\theta_{13} \sim k_2 \sin 2\theta_{12}\ \text{and}\ \delta_{\mathrm{MNSP}} \sim \pi}$

Cancellation can happen if 13 mixing is chosen.

type I

$$k_2 \simeq \sqrt{\frac{\Delta m^2_{\mathrm{sol}}}{\Delta m^2_{\mathrm{atm}}} \frac{M_2}{M_3}}$$

⇒ $\sin\theta_{13} \lesssim 0.18$

type II

$$k_2 \simeq \frac{\Delta m^2_{\mathrm{sol}}}{\Delta m^2_{\mathrm{atm}}}$$

⇒ $\sin\theta_{13} \sim 0.02$

$$(m^2_{\bar{5}})_{12} = m_0^2 \kappa \left(-\tfrac{1}{2} k_2 \sin 2\theta_{12} \cos\theta_{23} - e^{i\delta} \sin\theta_{13} \sin\theta_{23}\right) e^{i(\beta-\alpha)}$$

Type I seesaw

$$Y_\nu \bar{f}^{-1} Y_\nu^{\mathsf{T}}$$

$$= (\bar{h} - 3r_2\bar{f} + c_\nu \bar{h}')\bar{f}^{-1}(\bar{h} - 3r_2\bar{f} - c_\nu \bar{h}')$$

$$= \underline{(\bar{h} + c_\nu \bar{h}')\bar{f}^{-1}(\bar{h} - c_\nu \bar{h}') - 6r_2\bar{h}} + 9r_2^2\bar{f}$$

Cancellation is needed.

$$\bar{h} \simeq \mathrm{diag}(0,0,1), \quad \bar{f} \simeq \begin{pmatrix} \sim 0 & 0 & \lambda^3 \\ 0 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda^2 \end{pmatrix}, \quad \bar{h}' \simeq i \begin{pmatrix} 0 & \lambda^3 & \lambda^3 \\ -\lambda^3 & 0 & \lambda^2 \\ -\lambda^3 & -\lambda^2 & 0 \end{pmatrix}$$

Compatible to $\bar{f}_{33}^{-1}$ suppression

Proposition:

If both the Dirac and Majorana mass matrices are in the form :

$$M_D = \begin{pmatrix} 0 & 0 & \mathrm{x} \\ 0 & \mathrm{x} & \mathrm{x} \\ \mathrm{x} & \mathrm{x} & \mathrm{x} \end{pmatrix} \qquad M_R = \begin{pmatrix} 0 & 0 & \mathrm{x} \\ 0 & \mathrm{x} & \mathrm{x} \\ \mathrm{x} & \mathrm{x} & \mathrm{x} \end{pmatrix}$$

(x denotes non-zero entry.)

the seesaw mass matrix is also in the form of

$$\mathcal{M}_\nu = -M_D M_R^{-1} M_D^T = \begin{pmatrix} 0 & 0 & \mathrm{x} \\ 0 & \mathrm{x} & \mathrm{x} \\ \mathrm{x} & \mathrm{x} & \mathrm{x} \end{pmatrix}$$

$$\mathcal{M}_\nu \propto U \begin{pmatrix} \lambda & & \\ & \epsilon & \\ & & 1 \end{pmatrix} U^T$$

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \begin{pmatrix} \cos\theta_{13} & 0 & \sin\theta_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -\sin\theta_{13} e^{i\delta} & 0 & \cos\theta_{13} \end{pmatrix} \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Suppose that the mixings from the charged lepton are small, the Unitary matrix U is the MSN matrix.

From the condition: $(\mathcal{M}_\nu)_{11} = (\mathcal{M}_\nu)_{12} = 0$

we obtain ….

Next page

$$\epsilon = -e^{-2i\delta} \tan\theta_{13}(\tan\theta_{13} + e^{i\delta}\cot\theta_{12}\sec\theta_{13}\tan\theta_{23})$$

$$\lambda = -\epsilon\tan^2\theta_{12} - \sec^2\theta_{12}\tan^2\theta_{13}e^{-2i\delta}$$

$$\frac{\Delta m^2_{12}}{\Delta m^2_{23}} = \frac{|\epsilon|^2 - |\lambda|^2}{1-|\epsilon|^2} = \frac{4\sin^2\theta_{13}\csc 2\theta_{12}\tan\theta_{23}(\sin\theta_{13}\cos\delta + \cot 2\theta_{12}\tan\theta_{23})}{1-\sin^2\theta_{13}(2+\cot^2\theta_{12}\tan^2\theta_{23}) - 2\sin^3\theta_{13}\cos\delta\cot\theta_{12}\tan\theta_{23}}$$

θ_{12} : solar neutrino mixing

θ_{23} : atmospeheric neutrino mixing

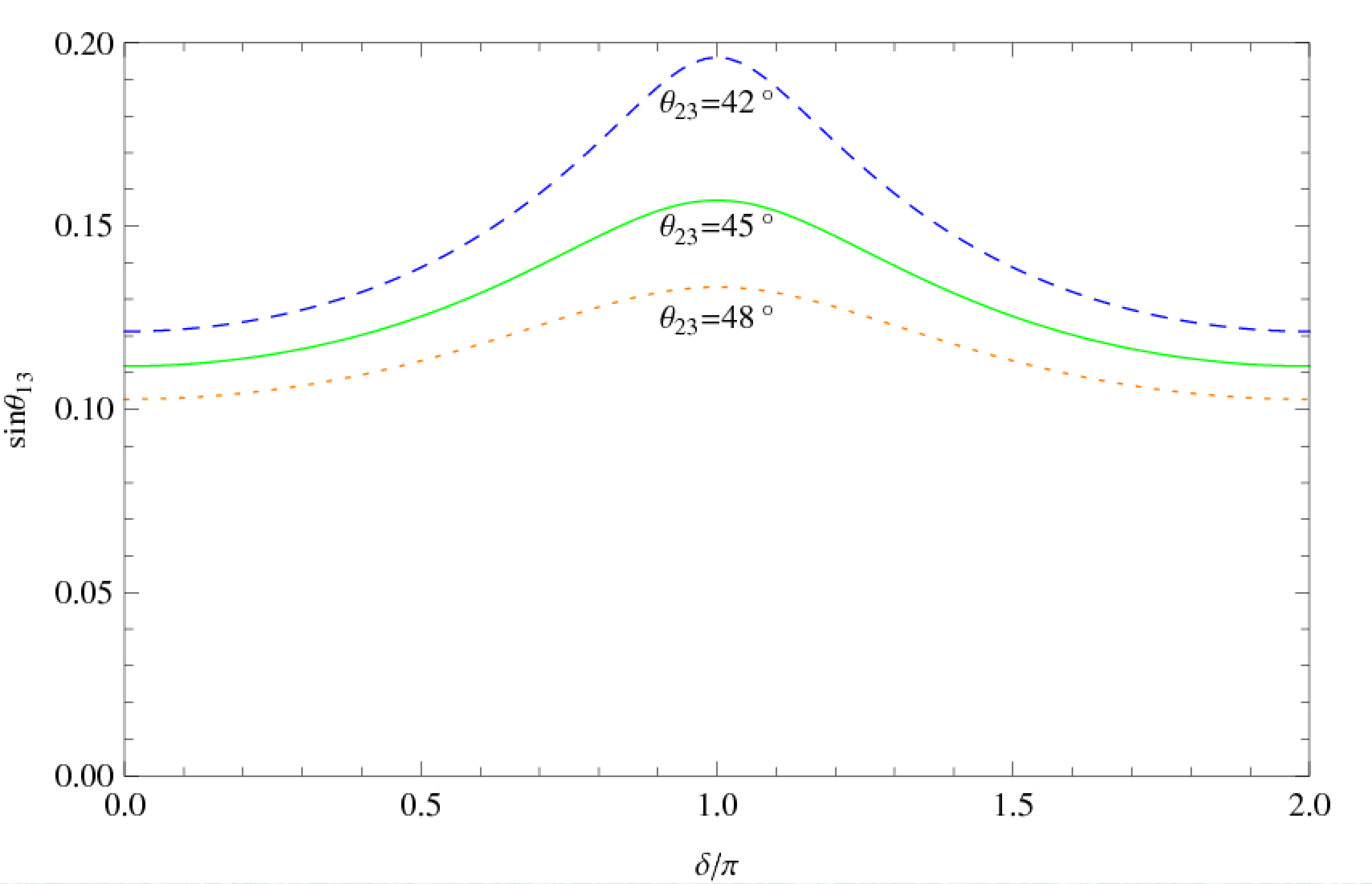
(Only the case of Normal hierarchy gives solutions in the setup.)

Using the experimental data, we obtain 13 mixing as a prediction.

(Cubic equation of 13 mixing for given CP phase).

For $\delta = \pi/2$,

$$\sin^2\theta_{13} = \frac{\frac{\Delta m^2_{12}}{\Delta m^2_{23}}}{4\cot 2\theta_{12}\csc 2\theta_{12}\tan^2\theta_{23} + \frac{\Delta m^2_{12}}{\Delta m^2_{23}}(2+\cot^2\theta_{12}\tan^2\theta_{23})}$$



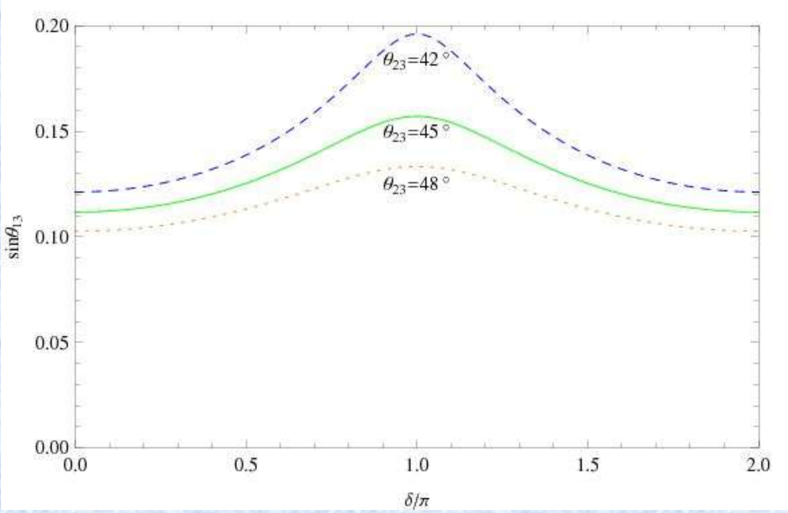
0.20
0.15
0.10
0.05
0.00
0.0
0.5
1.0
1.5
2.0
$\sin\theta_{13}$
δ/π
θ_{23}=42°
θ_{23}=45°
θ_{23}=48°

$$U_{\mathrm{MNSP}} = \begin{pmatrix} \tilde{V}_{11} & \tilde{V}_{12} & \tilde{V}_{13} \\ \tilde{V}_{21} & \tilde{V}_{22} & \tilde{V}_{23} \\ \tilde{V}_{31} & \tilde{V}_{32} & \tilde{V}_{33} \end{pmatrix} \begin{pmatrix} \cos\theta_s & \sin\theta_s & \sim 0.1 \\ -\cos\theta_a \sin\theta_s & \cos\theta_a \cos\theta_s & -\sin\theta_a \\ -\sin\theta_a \sin\theta_s & \sin\theta_a \cos\theta_s & \cos\theta_a \end{pmatrix}$$

Diagonalization matrix for charged lepton

13 neutrino mixing can be corrected by $\tilde{V}_{12} \sin\theta_a$.

Naively, $\tilde{V}_{12} \sim \sqrt{m_e/m_\mu}$



The flavor structure
to suppress proton decay
can be probed by
accurate measurement of
the 13 mixing and CP phase.

If both the Dirac and Majorana mass matrices are in the form :

$$M_D = \begin{pmatrix} 0 & 0 & \mathrm{x} \\ 0 & \mathrm{x} & \mathrm{x} \\ \mathrm{x} & \mathrm{x} & \mathrm{x} \end{pmatrix} \qquad M_R = \begin{pmatrix} 0 & 0 & \mathrm{x} \\ 0 & \mathrm{x} & \mathrm{x} \\ \mathrm{x} & \mathrm{x} & \mathrm{x} \end{pmatrix}$$

(x denotes non-zero entry.)

Note : This flavor structure can enhance left-right neutrino mixing.

$$(M_R^{-1})_{33} = 0 \qquad\qquad O(10^{-2})$$

cf. Naively, $\sin\theta_{\mathrm{LR}} \simeq \frac{m_D}{m_R} \simeq \sqrt{\frac{m_\nu^{\mathrm{light}}}{m_R}} \sim 10^{-6}\sqrt{\frac{1\ \mathrm{TeV}}{m_R}}$

If 1st (and 2nd) right-handed neutrino is at TeV scale,
LFV can be enhanced to the current exp. bounds.

(work in progress: Haba-Horita-Kaneta-YM)

Summary

- We study the predictions of 13 mixing and CP phase in neutrino oscillations in SUSY GUTs.
- The flavor structure to suppress proton decay is considered.
- The flavor structure can be probed by precise measurements of neutrino oscillation parameters as well as LFV.
- Neutrino oscillations and LFV will lead us a scheme of vast scope to investigate GUT scale physics (if the LHC finds low energy SUSY).

Back up slides

Fortunately, many of the SO(10) models predicts measurable 13 neutrino mixing.

It will be measured near future.

HyperKamiokande will be designed (hopefully).

Nucleon decay experiments may disentangle the models.

Relation of bottom-tau mass conversion and large atm. mixing

(Bajc-Senjanovic-Vissani)

$$\begin{cases} M_d = r_1(\bar{h} + \bar{f})\, v_d \\ M_e = r_1(\bar{h} - 3\bar{f})\, v_d \\ m_\nu^{\mathrm{II}} \propto \bar{f} \end{cases}$$

$$\Rightarrow \quad \boxed{M_d - M_e \propto m_\nu^{\mathrm{II}}}$$

$$\left.\begin{array}{l} M_d \simeq \begin{pmatrix} \cdot & \cdot & O(\lambda^3) \\ \cdot & O(\lambda^2) & O(\lambda^2) \\ O(\lambda^3) & O(\lambda^2) & 1 \end{pmatrix} m_b \\ \\ M_e \simeq \begin{pmatrix} \cdot & \cdot & O(\lambda^3) \\ \cdot & O(\lambda^2) & O(\lambda^2) \\ O(\lambda^3) & O(\lambda^2) & 1 \end{pmatrix} m_\tau \end{array}\right\} \quad m_\nu^{\mathrm{II}} \propto \begin{pmatrix} \cdot & \cdot & \lambda \\ \cdot & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix}$$

$$\underline{m_b - m_\tau \sim O(\lambda^2) m_b}$$

$$W_Y = \frac{1}{2} h_{ij}\psi_i\psi_j H + \frac{1}{2} f_{ij}\psi_i\psi_j \bar{\Delta} + \frac{1}{2} h'_{ij}\psi_i\psi_j D \qquad [H(\mathbf{10}),\ \bar{\Delta}(\overline{\mathbf{126}}),\ D(\mathbf{120})]$$

$$[\Delta(\mathbf{126}),\ \Phi(\mathbf{210})]$$

Higgs doublets

$$\varphi_d = (H_d^{10}, D_d^1, D_d^2, \bar{\Delta}_d, \Delta_d, \Phi_d) \qquad \varphi_u = (H_u^{10}, D_u^1, D_u^2, \Delta_u, \bar{\Delta}_u, \Phi_u)$$

mass term : $(\varphi_d)_a (M_{\text{doub.}})_{ab} (\varphi_u)_b \qquad U M_{\text{doub.}} V^{\mathsf{T}} = M_{\text{doub.}}^{\text{diag}}$

Light doublets

$$H_d = U_{1a}^* (\varphi_d)_a \qquad H_u = V_{1a}^* (\varphi_u)_a$$

U, V : unitary matrices

$$Y_u = \bar{h} + r_2 \bar{f} + r_3 \bar{h}'$$
$$Y_d = r_1(\bar{h} + \bar{f} + \bar{h}')$$
$$Y_e = r_1(\bar{h} - 3\bar{f} + c_e \bar{h}')$$
$$Y_\nu = \bar{h} - 3 r_2 \bar{f} + c_\nu \bar{h}'$$

$$\bar{h} = V_{11} h \qquad \bar{f} = U_{14}/(\sqrt{3} r_1) f$$
$$\bar{h}' = (U_{12} + U_{13}/\sqrt{3})/r_1 h'$$
$$r_1 = \frac{U_{11}}{V_{11}}$$
$$r_2 = r_1 \frac{V_{15}}{U_{14}} \qquad r_3 = r_1 \frac{V_{12} - V_{13}/\sqrt{3}}{U_{12} + U_{13}/\sqrt{3}}$$
$$c_e = \frac{U_{12} - \sqrt{3} U_{13}}{U_{12} + U_{13}/\sqrt{3}} \qquad c_\nu = r_1 \frac{V_{12} + \sqrt{3} V_{13}}{U_{12} + U_{13}/\sqrt{3}}$$

$$Y_u = \bar{h} + r_2\bar{f} + r_3\bar{h}'$$
$$Y_d = r_1(\bar{h} + \bar{f} + \bar{h}')$$
$$Y_e = r_1(\bar{h} - 3\bar{f} + c_e\bar{h}')$$
$$Y_\nu = \bar{h} - 3r_2\bar{f} + c_\nu\bar{h}'$$

$$\bar{h} = V_{11}h \qquad \bar{f} = U_{14}/(\sqrt{3}r_1)f$$
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$$r_2 = r_1\frac{V_{15}}{U_{14}} \qquad r_3 = r_1\frac{V_{12} - V_{13}/\sqrt{3}}{U_{12} + U_{13}/\sqrt{3}}$$
$$c_e = \frac{U_{12} - \sqrt{3}U_{13}}{U_{12} + U_{13}/\sqrt{3}} \qquad c_\nu = r_1\frac{V_{12} + \sqrt{3}V_{13}}{U_{12} + U_{13}/\sqrt{3}}$$

If Higgs potential is symmetric under the exchange $\Delta \leftrightarrow \bar{\Delta}$

$\longrightarrow$ $r_1 = 1 \quad (\tan\beta \sim 50).$

$\longrightarrow$ top-bottom-tau Yukawa unification (approximately)

Even if $r_1 \neq 1$, we may have bottom-tau unification (approximately).

Top-bottom-tau unification:
Banks, Olechowski-Pokorski, Ananthanarayan-Lazarides-Shafi,
Dimopoulos-Hall-Raby, Murayama-Olechowski-Pokorski
Blazek-Dermisek-Raby, Baer-Ferrandis, Tobe-Wells, ….

Consider only h and f

$$h = \begin{pmatrix} c \\ b \\ a \end{pmatrix} \begin{pmatrix} c & b & a \end{pmatrix}, \quad f = \begin{pmatrix} f_1 & & \\ & f_2 & \\ & & f_3 \end{pmatrix} \qquad |f_2/f_3| \sim 0.2$$

In this parameterization, diagonalization matrix of Y_e is the neutrino mixing matrix by definition.

$$U_{\mathsf{MNSP}} = V_e^* \qquad V_e Y_e Y_e^\dagger V_e^\dagger = Y_e^{\mathsf{diag}}$$

$$Y_u = \bar{h} + r_2 \bar{f} + r_3 \bar{h}' \qquad Y_d = r_1(\bar{h} + \bar{f} + \bar{h}')$$

$$Y_e = r_1(\bar{h} - 3\bar{f} + c_e \bar{h}') \qquad m_\nu^{\mathsf{light}} \propto \bar{f}$$ (type II)

$$\begin{pmatrix} \cos\theta_s & \sin\theta_s & 0 \\ -\sin\theta_s & \cos\theta_s & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c \\ b \\ a \end{pmatrix} = \begin{pmatrix} 0 \\ \sqrt{b^2+c^2} \\ a \end{pmatrix}$$

$$\tan\theta_s = -c/b$$

$$\tan\theta_a = \sqrt{b^2+c^2}/a$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_a & -\sin\theta_a \\ 0 & \sin\theta_a & \cos\theta_a \end{pmatrix} \begin{pmatrix} \cos\theta_s & \sin\theta_s & 0 \\ -\sin\theta_s & \cos\theta_s & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c \\ b \\ a \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \sqrt{a^2+b^2+c^2} \end{pmatrix}$$

$U_0 \equiv$

$$U_0 = \begin{pmatrix} \cos\theta_s & \sin\theta_s & 0 \\ -\cos\theta_a \sin\theta_s & \cos\theta_a \cos\theta_s & -\sin\theta_a \\ -\sin\theta_a \sin\theta_s & \sin\theta_a \cos\theta_s & \cos\theta_a \end{pmatrix}$$

Assumption: $\bar{h} \gg \bar{f}, \bar{h}'$ and $\bar{h}$ is rank 1.

- CKM mixings are small.
- Approximate bottom-tau Yukawa unification.
- $3\frac{m_s}{m_b} \simeq \frac{m_\mu}{m_\tau}$
- $V_{cb} \sim \frac{m_s}{m_b}$, $V_{ub} \sim V_{cb} \frac{m_{\nu 2}}{m_{\nu 3}}$
- Atmospheric and solar neutrino mixings are generically O(1).
  But, 13 mixing is related to mass ratio $\sin\theta_{13} \sim \frac{m_{\nu 2}}{m_{\nu 3}}$