Superconformal Operator Product Expansion and General Gauge Mediation

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based on
work in progress (JFF, K. Intriligator, A. Stergiou)
Outline

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   - (S)CFTs and real world
   - Applications

2 SOPE in SCFTs
   - Operator product expansion: Review
   - Superconformal operator product expansion
   - Current-current SOPE
   - Superconformal blocks

3 SOPE and GGM
   - General gauge mediation: Overview
   - Cross sections
   - Visible sector SUSY breaking masses

4 Conclusion
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Why (S)CFTs ⇒ (S)CFTs are generic

- $\mathcal{N} = 4$ SYM
- $\mathcal{N} = 1$ SQCD in conformal window Seiberg (1994)
- Non-Lagrangian candidates e.g. Benini, Tachikawa, Wecht (2009)

Why (S)OPE ⇒ (S)CFT observables

- Spectrum of operators and dimensions
- (S)OPE coefficients
- Non-local Wilson loops

⇒ Highly constrained
(S)CFT model-building applications

- Large anomalous dimensions ⇒ Suppress or enhance otherwise finely-tuned quantities
  - Conformal sequestering  \textit{Luty, Sundrum} (2001)
  - $\mu/B_\mu$ problem of gauge mediation  \textit{Roy, Schmaltz} (2007)
  - Flavor hierarchy  \textit{Poland, Simmons-Duffin} (2009)

- RG flows near (S)CFTs
  - Walking technicolor  \textit{Holdom} (1985)
  - Unparticle physics  \textit{Georgi} (2007)
(S)OPE UV/IR applications

- Non-CFTs \(\Rightarrow\) Conformally covariant OPEs
- Softly broken symmetries as spontaneously broken symmetries
  - Symmetry breaking seen as IR effect (via field or spurion vev)
  - Symmetry restored in UV theory
  \(\Rightarrow\) (S)OPE selection rules

\(\Rightarrow\) Strongly-coupled IR physics described by weakly-coupled UV physics through (S)OPE

- Example: QCD
  - Not conformal (non-trivial RG flow)
  - IR physics \(\Rightarrow\) Theory with chiral symmetry breaking \((\langle \bar{Q}Q \rangle \neq 0)\) and confinement \((\langle G^A_{\mu\nu} G^{A\mu\nu} \rangle \neq 0)\)
  - UV physics \(\Rightarrow\) Asymptotically free CFT
  \(\Rightarrow\) QCD sum rules Shifman, Vainshtein, Zakharov (1979)
Probe hadron sector from lepton sector through gauge interactions (UV/IR physics separated using OPE)

\[ \Pi_{\text{hadron}, \mu \nu}(p) = ie^2 \int d^4x \ e^{-ip \cdot x} \langle j_\mu(x) j_\nu(0) \rangle \]

\[ \mathcal{O}_i(x) \mathcal{O}_j(0) = \sum_k c_{ij}^k(x) \mathcal{O}_k(0) \]
OPE, analyticity and optical theorem ⇒ Relations between total cross section and OPE coefficients

\[ \Pi_{\text{hadron}}(s) = \frac{1}{2\pi i} \int_{s_0}^{\infty} ds' \frac{\text{Disc} \Pi_{\text{hadron}}(s')}{s' - s} \]

\[ \sigma_{e^+e^- \rightarrow \text{hadrons}}(s) = \frac{2\pi i \alpha}{s} \text{Disc} \Pi_{\text{hadron}}(s) \]
Operator product expansion expansion

\[ \mathcal{O}_i(x)\mathcal{O}_j(0) = \sum_k \frac{c_{ij}^k}{x^{\Delta_i + \Delta_j - \Delta_k}} \mathcal{O}_k(0) \]

\[ = \sum_{\text{primary } k} \frac{c_{ij}^k}{x^{\Delta_i + \Delta_j - \Delta_k}} F_{\Delta_i \Delta_j}^{\Delta_k}(x, P) \mathcal{O}_k(0) \]

- Short distance physics expressed in terms of local operators
- Wilson coefficients \(\Rightarrow\) UV physics
- Vacuum expectation values \(\Rightarrow\) IR physics
- OPE constrained by conformal symmetry \(\Rightarrow\) Wilson coefficients of descendants determined by Wilson coefficients of primaries

Ferrara, Gatto, Grillo (1971)
CFT correlation functions

2-point and 3-point correlation functions

\[ \langle O_i(x_1) O_j(x_2) \rangle = \frac{g_{ij}}{\Delta_i + \Delta_j} \]

\[ \langle O_i(x_1) O_j(x_2) O_k(x_3) \rangle = \frac{c_{ijk}}{\Delta_i + \Delta_j - \Delta_k \Delta_k + \Delta_i - \Delta_j \Delta_j + \Delta_k - \Delta_i} \]

- 2-point function coefficients \( g_{ij} = c_{ij}^0 \) (Zamolodchikov metric)
- 3-point function coefficients \( c_{ijk} = c_{ij}^\ell g_{\ell k} \)
Superconformal operator product expansion

Superconformal modules
Superconformal operator product expansion

\[ O_i(x)O_j(0) \overset{?}{=} \sum_{\text{superprimary } k} \frac{c_{ij}^k}{x^{\Delta_i + \Delta_j - \Delta_k}} F_{ij}^k(x, P, Q, \bar{Q}) O_k(0) \]

\[ T_\mu(z) = j_\mu^R(x) + \theta^\alpha S_{\alpha\mu}(x) + \bar{\theta}^{\dot{\alpha}} \bar{S}_{\dot{\alpha}\mu}(x) + 2\theta^{\nu\dot{\alpha}} \bar{\theta} T_{\nu\mu}(x) + \cdots \]

\[ J(z) = J(x) + i\theta j(x) - i\bar{\theta} \bar{j}(x) - \theta^{\mu\dot{\nu}} \bar{\theta} j_\mu(x) + \cdots \]

- OPE constrained by superconformal symmetry
- Sum over superprimaries instead of primaries

⇒ Wilson coefficients of superdescendants NOT fully determined by Wilson coefficients of superprimaries (existence of superconformal 3-point invariants)! Osborn (1998)
Current-current SOPE

Superconformal 3-point correlation functions for supercurrents

\[ \langle \mathcal{J}(z_1)\mathcal{J}(z_2)\mathcal{O}_{\mu_1\ldots\mu_\ell}(z_3) \rangle = \frac{1}{x_{13}^2 x_{31}^2 x_{23}^2 x_{32}^2} t_{\mathcal{J}\mathcal{J}\mathcal{O}_\ell}(X_3, \Theta_3, \bar{\Theta}_3) \]

- \( \mathcal{O}_{\mu_1\ldots\mu_\ell} \) real spin-\( \ell \) superfield with vanishing R-charge

- \( t_{\mathcal{J}\mathcal{J}\mathcal{O}_\ell}(X, \Theta, \bar{\Theta}) \) fully determined by superconformal symmetry and supercurrent conservation

JFF, Intriligator, Stergiou (2011)
Superconformal 3-point correlation functions for supercurrents

\[ t_{\mathcal{J}\mathcal{J}\mathcal{O}_{\ell}=\text{even}}^{\mu_1\ldots\mu_\ell} (X, \Theta, \bar{\Theta}) = c_{\mathcal{J}\mathcal{J}\mathcal{O}_\ell} \frac{X_+^{(\mu_1} \ldots X_+^{\mu_\ell)}}{(X \cdot \bar{X})^{2-\frac{1}{2}(\Delta-\ell)}} \]

\[ \times \left[ 1 - \frac{1}{4} (\Delta - \ell - 4)(\Delta + \ell - 6) \frac{\Theta^2 \bar{\Theta}^2}{X \cdot \bar{X}} \right] \]

\[ t_{\mathcal{J}\mathcal{J}\mathcal{O}_{\ell}=\text{odd}}^{\mu_1\ldots\mu_\ell} (X, \Theta, \bar{\Theta}) = c_{\mathcal{J}\mathcal{J}\mathcal{O}_\ell} \frac{X_+^{(\mu_1} \ldots X_+^{\mu_{\ell-1}}}{(X \cdot \bar{X})^{2-\frac{1}{2}(\Delta-\ell)}} \]

\[ \times \left[ X_\mu^{\mu_\ell)} - \frac{\ell(\Delta - \ell - 4)}{\Delta - 2} \frac{(X_- \cdot X_+)}{X \cdot \bar{X}} X_+^{\mu_\ell)} \right] \]

⇒ Relations in current-current SOPE from superconformal symmetry and supercurrent conservation
From superprimaries to superdescendants (applications to GGM)

\[
j_{\alpha}(x)j_{\dot{\alpha}}(0) = \frac{1}{x^4} \left[ (S ix \cdot \sigma)_{\dot{\alpha}}(ix \cdot \sigma \bar{S})_\alpha - x^2 \bar{Q}_{\dot{\alpha}}(ix \cdot \sigma \bar{S})_\alpha \right. \\
\left. + 2\Delta_J x^2 (ix \cdot \sigma)_{\alpha \dot{\alpha}} \right] (J(x)J(0))
\]

\[
j_\mu(x)j_\nu(0) = \frac{1}{16x^8} \left[ (x^2 \eta_{\mu\rho} - 2x_\mu x_\rho)(S\sigma^\rho \bar{S} - \bar{S}\sigma^\rho S) \right. \\
\left. \times x^4 (\bar{Q}\bar{\sigma}_\nu Q - Q\sigma_\nu \bar{Q}) + \cdots \right] (J(x)J(0))
\]

\[
j_{\alpha}(x)j_\beta(0) = \frac{1}{x^2} Q_\beta(ix \cdot \sigma \bar{S})_\alpha (J(x)J(0))
\]

\[
j_\mu(x)J(0) = \frac{x^2 \eta_{\mu\nu} - 2x_\mu x_\nu}{4x^4} \left[ S\sigma^\nu \bar{S} - \bar{S}\bar{\sigma}^\nu S \right] (J(x)J(0))
\]

\[
S^\alpha S^\beta (J(x)J(0)) = 0
\]

\[
[x^2 Q_\alpha Q_\beta + Q_\alpha(ix \cdot \sigma \bar{S})_\beta - Q_\beta(ix \cdot \sigma \bar{S})_\alpha] (J(x)J(0)) = 0
\]
Superconformal blocks

Decomposition into primaries \text{Poland, Simmons-Duffin (2010)}

\[ \mathcal{O}^{\mu_1 \cdots \mu_\ell}(x, \theta, \bar{\theta}) = A^{\mu_1 \cdots \mu_\ell}(x) + \theta \sigma_\mu \bar{\theta} B^{\mu \mu_1 \cdots \mu_\ell}(x) + (\theta \sigma_\mu \bar{\theta})^2 D^{\mu_1 \cdots \mu_\ell}(x) + \cdots \]

- $A$ and $D$ irreducible spin-$\ell$ representations
- $B \sim M + N + L$ where $M$ spin-$(\ell + 1)$ representation and $N$ spin-$(\ell - 1)$ representation

\[ \implies A_{\text{primary}} , M_{\text{primary}} , N_{\text{primary}} , L_{\text{primary}} \text{ and } D_{\text{primary}} \]
Superconformal 3-point function and SOPE coefficients

\begin{align*}
C_{JJM}^{\ell+1;\ell=\text{even}} &= C_{JJN}^{\ell-1;\ell=\text{even}} = C_{JLL}_{\text{primary}} = 0 \\
C_{JJD}^{\ell;\ell=\text{even}} &= -\frac{\Delta(\Delta + \ell)(\Delta - \ell - 2)}{8(\Delta - 1)} C_{JJ\Lambda}^{\ell;\ell=\text{even}} \\
C_{JJA}^{\ell;\ell=\text{odd}} &= C_{JLL}_{\text{primary}} = C_{JJD}^{\ell;\ell=\text{odd}} = 0 \\
C_{JJN}^{\ell-1;\ell=\text{odd}} &= -\frac{(\ell + 2)(\Delta - \ell - 2)}{\ell(\Delta + \ell)} C_{JJM}^{\ell+1;\ell=\text{odd}}
\end{align*}

- Consistent with Lorentz symmetry

- Consistent with unitary bound \( \Rightarrow D_{\text{primary}} \) and \( N_{\text{primary}} \) are null states for \( \Delta = \ell + 2 \) (short representation)
4-point superconformal blocks

\[
\langle J(x_1) J(x_2) J(x_3) J(x_4) \rangle = \frac{1}{x_{12}^4 x_{34}^4} \sum_{\mathcal{O}_{\Delta, \ell} \in J \times J} \frac{(c_{JJ\alpha\ell})^2}{c_{\alpha\ell}} G_{\Delta, \ell}^{JJ; JJ}(u, v)
\]

\[
G_{\Delta, \ell=\text{even}}^{JJ; JJ} = g_{\Delta, \ell} + \frac{(\Delta + \ell)(\Delta - \ell - 2)}{16(\Delta + \ell + 1)(\Delta - \ell - 1)} g_{\Delta+2, \ell}
\]

\[
G_{\Delta, \ell=\text{odd}}^{JJ; JJ} = \frac{(\ell + 1)^2(\Delta + \ell)}{4(\Delta + \ell + 1)} g_{\Delta+1, \ell+1}
\]

\[
+ \frac{(\ell + 2)^2(\Delta - \ell - 2)}{\Delta - \ell - 1} g_{\Delta+1, \ell-1}
\]

- \( g_{\Delta, \ell}(u, v) \) universal 4-point conformal blocks (accounting for descendants) with \( u, v \) usual conformal cross-ratios

- \( G_{\Delta, \ell}^{JJ; JJ}(u, v) \) non-universal 4-point superconformal blocks (accounting for superdescendants)
General gauge mediation: Overview

- SUSY breaking hidden sector connected to visible sector through gauge interactions Buican, Meade, Seiberg, Shih (2008)
  - Decoupled hidden sector in $g \to 0$ limit
  - Universal visible sector SUSY breaking effects introduced via loops

$\Rightarrow$ Current-current correlation functions (even without hidden sector Lagrangian)

$$\mathcal{J}(z) = J(x) + i\theta j(x) - i\bar{\theta}j(x) - \theta\sigma^{\mu}\bar{\theta} j_{\mu}(x) + \cdots$$

$$\langle J(x)J(0) \rangle, \langle j_\alpha(x)\bar{j}_\dot{\alpha}(0) \rangle, \langle j_\mu(x)j_\nu(0) \rangle, \langle j_\alpha(x)j_\beta(0) \rangle$$
Operator product expansion

Strongly-coupled hidden sector and OPE

\[ J(x)J(0) = \sum_k c^k_{JJ}(x)O_k(0) \]

\[ i \int d^4x \, e^{-ip \cdot x} J(x)J(0) = \sum_k \tilde{c}^k_{JJ}(p)O_k(0) \]

- Relations between Wilson coefficients in same OPE and different OPEs
  - \( \langle J(x)J(0) \rangle \) overdetermined \( \checkmark \)
  - \( \langle j_\alpha(x)\bar{j}_\alpha(0) \rangle, \langle j_\mu(x)j_\nu(0) \rangle \) and \( \langle j_\alpha(x)j_\beta(0) \rangle \) determined in terms of \( \langle J(x)J(0) \rangle \) \( \checkmark \)

\[ \implies \text{Knowledge of strongly-coupled quantities in terms of } J(x)J(0) \text{ OPE} \]
OPE constraints, analyticity and optical theorem

\[
\sigma_{\text{visible} \rightarrow D^* \rightarrow \text{hidden}} (s) = -\frac{(4\pi\alpha)^2}{s} \sum_k \text{Im}[\tilde{c}^k_J J_j(s)] \langle O_k(0) \rangle
\]

\[
\sigma_{\text{visible} \rightarrow \lambda^*_\alpha \rightarrow \text{hidden}} (s) = f_{1/2}(\text{Im}[\tilde{c}^k_J J_j(s)])
\]

\[
\sigma_{\text{visible} \rightarrow A^*_\mu \rightarrow \text{hidden}} (s) = f_1(\text{Im}[\tilde{c}^k_J J_j(s)])
\]

- Good approximation with first few terms
- Consistent with direct computation in ordinary gauge mediation
Visible sector SUSY breaking masses

OPE constraints and analyticity

\[ M_{\text{gaugino}} \approx \sum_k \alpha \frac{\text{Im}[s^{d_k/2} \tilde{c}_{JJ}(s)]}{2^{d_k-1} d_k M_M} \langle Q^2(\mathcal{O}_k(0)) \rangle \]

\[ m_{\text{sfermion}}^2 \approx 4\pi\alpha Y \langle J(x) \rangle - \sum_k \frac{\alpha^2 c_2 \text{Im}[s^{d_k/2} \tilde{c}_{JJ}(s)]}{2^{d_k+1} \pi d_k M_M} \langle \bar{Q}^2 Q^2(\mathcal{O}_k(0)) \rangle \]

- Reproduce the usual \( f(x) \) and \( g(x) \) functions of ordinary gauge mediation  
  Martin (1996)
**OPE constraints and analyticity**

\[
J^A(x)J^B(0) = \tau \frac{\delta^{AB} \mathbb{1}}{16\pi^4 x^4} + \frac{k d^{ABC} J^C(0)}{16\pi^2 x^2} + \frac{\delta^{AB} K(0)}{4\pi^2 x^2 - \gamma K} + c_i^{AB} \frac{O_i(0)}{x^{4-\Delta_i}} + \ldots
\]

\[
M_{\text{gaugino}} \approx -\frac{\alpha \pi w \gamma Ki}{8M^2} \langle Q^2(O_i(0)) \rangle
\]

\[
m_{\text{sfermion}}^2 \approx 4\pi \alpha Y \langle J(x) \rangle + \frac{\alpha^2 c_2 w \gamma Ki}{64M^2} \langle \bar{Q}^2 Q^2(O_i(0)) \rangle
\]

- Good approximation with first few terms
- \(f(x), g(x) \sim 1/2 + \cdots\) instead of \(f(x), g(x) \sim 1 + \mathcal{O}(x)\) in ordinary gauge mediation
Features and applications

Features

- Undetermined superconformal 3-point correlation functions
- Fully determined superconformal current-current SOPE
- Relations between current-current OPEs
- Well-defined current 4-point superconformal blocks

Applications to GGM

- All current-current OPEs determined by $J(x)J(0)$ OPE
- Relations between Wilson coefficients
- Cross sections and SUSY breaking masses from $J(x)J(0)$
- Incalculable strongly-coupled models made tractable