Dynamical Dark Matter
A New Framework for Dark-Matter Physics

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Based on work done in collaboration with Keith Dienes

The dominant paradigm in dark-matter phenomenology has been to consider scenarios in which $\Omega_{\text{DM}}$ is made up by one stable particle (or maybe two or three), but maybe nature isn't quite so simple.

It could be that many particles – maybe even a vast number – contribute nontrivially to that abundance, with each providing only a minute fraction of the total.

Some of these states may be only quasi-stable, but as long as the individual abundances are balanced against decay rates in just the right way, this can be a viable dark-matter scenario!

“Dynamical Dark Matter”
Dynamical Dark Matter: The Big Picture

Staggered onset times

Increasing Mass

Log(abundance) vs. Log(time)

$m=3H(t)$

Abundances suppressed

States surviving at present time

States decayed in past

Inflation

Reheating

Radiation-dominated

Matter/radiation equality

Matter-dominated

Present time
Over the course of this talk, I'll demonstrate how such scenarios arise naturally in the context of large extra dimensions.

Moreover, I'll provide a concrete example of a viable model of dynamical dark matter, in which all applicable constraints are satisfied, and a large number of states contribute significantly toward $\Omega_{DM}$.

This example demonstrates that dynamical dark matter is a viable framework for addressing the dark-matter question.
(General) Axions in Large Extra Dimensions

- Consider a 5D theory with the extra dimension compactified on $S_1/Z_2$ with radius $R = 1/M_c$.

- Global $U(1)_X$ symmetry broken at scale $f_X$ by a bulk scalar $\to$ bulk axion is PNGB.

- SM and an additional gauge group $G$ are restricted to the brane. $G$ confines at a scale $\Lambda_G$. Instanton effects lead to a \textit{brane-mass} term $m_X$ for the axion.

**Axion mass matrix:**

$$
\begin{pmatrix}
m_X^2 & \sqrt{2}m_X^2 & \sqrt{2}m_X^2 & \cdots \\
\sqrt{2}m_X^2 & 2m_X^2 + M_c^2 & 2m_X^2 & \cdots \\
\sqrt{2}m_X^2 & 2m_X^2 & 2m_X^2 + 4M_c^2 & \cdots \\
\vdots & \vdots & \vdots & \ddots 
\end{pmatrix}
$$

When $y \equiv M_c/m_X$ is small, substantial mixing occurs:

**Mass eigenstates** \( \tilde{\lambda} \equiv \lambda/m_X \)

$$a_{\lambda} = \sum_{n=0}^{\infty} U_{\lambda n} a_n \equiv \sum_{n=0}^{\infty} \left( \frac{r_n \tilde{\lambda}^2}{\tilde{\lambda}^2 - n^2 y^2} \right) A_{\lambda} a_n$$

**“Mixing Factor”**

$$A_{\lambda} = \frac{\sqrt{2}}{\tilde{\lambda}} \left[ 1 + \tilde{\lambda}^2 + \frac{\pi^2}{y^2} \right]^{-1/2}$$
The Three Fundamental Questions:

1. "Does the relic abundance come out right?"

\[ \Omega_{\text{tot}} = \sum_{\lambda} \Omega_{\lambda} \]  

must match

\[ \Omega_{\text{DM}}^\text{WMAP} h^2 = 0.1131 \pm 0.0034 \]

[Komatsu et al.; '09]

2. "Do a large number of modes contribute to that abundance, or does the lightest one make up essentially all of \( \Omega_{\text{DM}} \) ?"

Define:

\[ \eta \equiv 1 - \frac{\Omega_{\lambda_0}}{\Omega_{\text{tot}}} \]  

“Tower Fraction”

If \( \eta \) is \( \mathcal{O}(1) \), the full tower contributes nontrivially to \( \Omega_{\text{DM}} \).

3. "Is the model consistent with all of the applicable experimental, astrophysical, and cosmological constraints?"

Thanks to the properties of the mixing factor \( A_\lambda \), the answer to all three questions can indeed (simultaneously) be in the affirmative!
Mixing and Relic Abundances:

- At temperatures $T \gg \Lambda_G$, $m_X \approx 0$. At such temperatures, mixing is negligible, and the potential for $a_0$ effectively vanishes.

- The expectation value of $a_0$ at such temperatures is therefore undetermined:

$$\langle a_0 \rangle_{\text{init}} = \theta f X$$

- However, at $T \sim \Lambda_G$, instanton effects turn on:
  - $m_X$ becomes nonzero, so KK eigenstates are no longer mass eigenstates.
  - The zero-mode potential now has a well-defined minimum.

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\text{Coherent Oscillations (}p\sim R^{-3}\text{)}
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\text{True minimum}
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• The $a_\lambda$ are initially populated (at $t_G$) according to their overlap with $a_0$:

\[
\langle a_\lambda(t_G) \rangle = \theta \hat{f}_X A_\lambda
\]

\[
\rho_\lambda(t_G) = \frac{1}{2} \theta^2 \hat{f}_X^2 \lambda^2 A_\lambda^2
\]

• Each field begins to oscillate at a time $t_\lambda$, when **two** conditions are met:

1. $\rho_\lambda$ is nonzero (so $t \gtrsim t_G$).
2. Mass has become comparable to Hubble Parameter: $\lambda \sim 3H(t)$.

• In the approximation that the instanton potential turns on rapidly, we have two regimes:
The Contribution from Each Field

Mixing factor from $A^2_\lambda$

Decay suppression

\[ \Omega_\lambda = 3 \left( \frac{\theta f_X m_X}{M_P} \right)^2 t^2_\lambda \left[ 1 + \frac{\chi^2}{m^2_X} + \frac{\pi^2 m^2_X}{M^2_c} \right]^{-1} e^{-\Gamma_\lambda (t-t_G)} \]

\[ t^2_G \text{ (simultaneous)} \text{ or } 4/\lambda^2 \text{ (staggered)} \]

Case I: All Simultaneous

Case II: A Lot of Staggering

Time-evolution factor (for $t_\lambda$ during reheating)

\[ \begin{align*}
\frac{1}{4} & \quad 2/\lambda \leq t \leq t_{Rh} \\
\frac{4}{9} \left( \frac{t}{t_{Rh}} \right)^{1/2} & \quad t_{Rh} \leq t \leq t_{MRE} \\
\frac{1}{4} \left( \frac{t_{MRE}}{t_{Rh}} \right)^{1/2} & \quad t \geq t_{MRE}
\end{align*} \]
E Pluribus Unum: $\Omega_{\text{tot}}$ from $\Omega_\lambda$

The total relic abundance at present time is obtained by summing over these individual contributions.

The upshot: $\Omega_{\text{DM}}$ consistent with WMAP results for $\hat{f}_X \sim 10^{14} - 10^{15}$ GeV.
Tower Fractions

- When $\Lambda_G$ is small and $t_G$ occurs very late, all modes begin oscillating simultaneously at $t_G$ and contribute “democratically” to $\Omega_{DM}$.

- When $\Lambda_G$ is large and $t_G$ occurs early, $t_\chi$ for the relevant modes are staggered in time. Lighter modes contribute proportionally more to $\Omega_{DM}$.

Tower fraction $\eta$ with Small $\Lambda_G$

Tower fraction $\eta$ with Large $\Lambda_G$
Mixing and stability:

- Couplings between SM fields and the $a_\lambda$ are proportional to $\tilde{\lambda}^2 A_\lambda$.
- This results in a decay-width suppression for modes with $\lambda \lesssim m_X^2 / M_c$

\[ \Gamma_\lambda \propto \frac{\lambda^3}{f_X^2} \left( \tilde{\lambda}^2 A_\lambda \right)^2 \]

- Comparing to the relic-abundance results, above we find that the $a_\lambda$ with large $\Gamma_\lambda$ automatically have suppressed $\Omega_\lambda$!

This balance between $\Omega_\lambda$ and $\Gamma_\lambda$ rates relaxes constraints related to:

- Distortions to the CMB
- Features in the diffuse X-ray and gamma-ray background
- Disruptions of BBN
- Late entropy production
Mixing and axion production:

**Without mixing:**
(e.g. KK-graviton production)

\[ \sigma_{\text{prod}} \propto \frac{1}{M_P^2} \left( \frac{E}{M_c} \right) \]

**With mixing:**

\[ \sigma_{\text{prod}} \propto \frac{1}{f_X^2} N^2(E) \]

where

\[ N^2(E) \equiv \sum_{\lambda} \left( \tilde{\lambda}^2 A_{\lambda} \right)^2 \]

Suppression significantly relaxes limits from processes in which axions are produced, but not detected directly, including those from:

- Supernova energy-loss rates
- Stellar evolution
- Collider production (j+E_T, γ+E_T, ...)

Decoherence phenomena (also related to axion mixing) suppress detection rates from:

[Dienes, Dudas, Gherghetta; '99]

- Microwave-cavity experiments
- Helioscopes
- “Light-shining-through-walls” (LSW) experiments, etc.
Constraints on Dark Towers

- Therefore, while a great many considerations constrain scenarios involving light bulk axions, they can all be simultaneously satisfied.

- 5D Theory Inconsistent

- $\Lambda_G = 1$ GeV

- $\Lambda_G = 1$ TeV

- \[
\begin{align*}
\text{GC stars} & \quad \text{SN1987A} & \quad \text{Diffuse photon spectra} \\
\text{Eötvös experiments} & \quad \text{Helioscopes (CAST)} & \quad \text{DM overabundant} \\
\text{Collider limits} & \quad \text{Thermal production} & \quad \text{5D Theory Inconsistent}
\end{align*}
\]
Summary

• There's no reason to assume that a single, stable particle accounts for all of the non-baryonic dark matter in our universe.

• Indeed, there are simple, well-motivated BSM scenarios in which a large number of particles contribute non-trivially toward \( \Omega_{DM} \).

• Production mechanisms (e.g. misalignment production) exist which naturally generate relic abundances for the contributing fields in such a way that an inverse correlation exists between \( \Omega_\lambda \) and \( \Gamma_\lambda \).

• The same mass-mixing which gives rise to this correlation automatically suppresses the interactions between the lighter modes and the SM fields, making these particles less dangerous from a phenomenological perspective.

The Take-Home Message:

Dynamical dark matter is as viable a framework in which to address the dark matter question as any other.