$\mu^- N \to e^- N$: Theory Motivation

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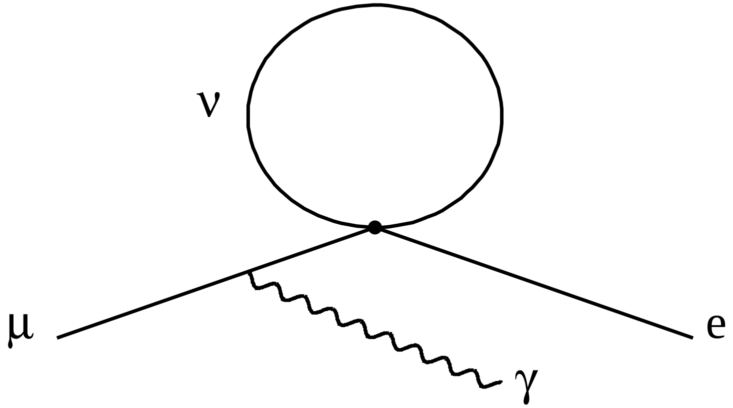
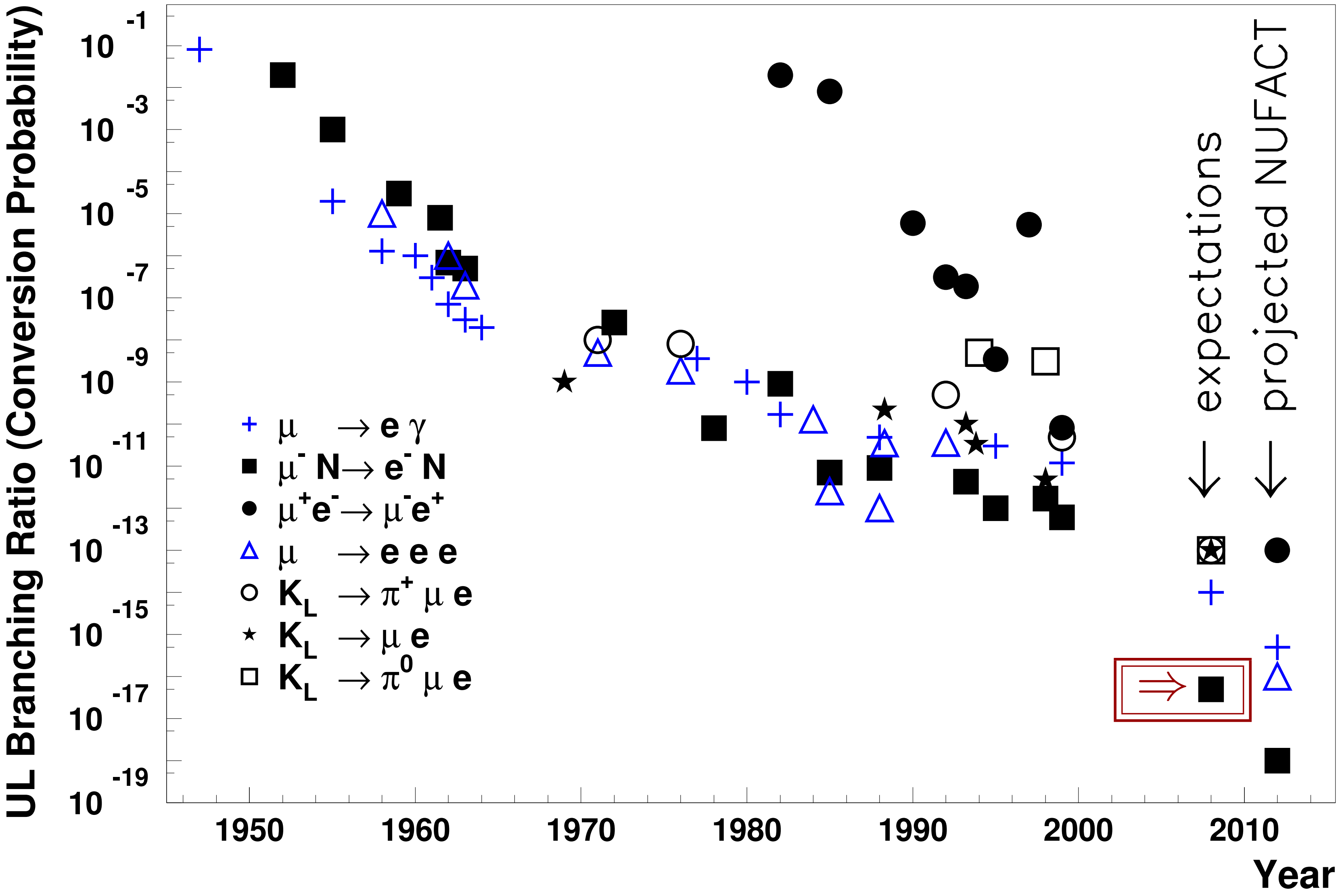
Ever since it was established that $\mu \to e\nu\bar{\nu}$, people have searched for $\mu \to e\gamma$, which was thought to arise at one-loop, like this:



The fact that $\mu \to e\gamma$ did not happen, led one to postulate that the two neutrino states produced in muon decay were distinct, and that $\mu \to e\gamma$, and other similar processes, were forbidden due to symmetries.

To this date, these so-called individual lepton-flavor numbers seem to be conserved in the case of charged lepton processes, in spite of many decades of (so far) fruitless searching for CLFV…

Searches for Lepton Number Violation



[hep-ph/0109217]

In the "New Standard Model" (νSM, equal to the old Standard Model plus operators that lead to neutrino masses) $\mu \to e\gamma$ is allowed, exactly like similar Flavor Changing Neutral Current processes which have already been observed in the quark sector (like $b \to s\gamma$).

However, just like in the quark sector, FCNC processes are GIM suppressed. In the case of charged leptons, the GIM suppression is especially efficient:

$$Br(\mu \to e\gamma) = \frac{3\alpha}{32\pi}\left|\sum_{i=2,3} U^*_{\mu i}U_{ei}\frac{\Delta m^2_{1i}}{M^2_W}\right|^2 < 10^{-54},$$

where $U_{\alpha i}$ are the elements of the leptonic mixing matrix, while Δm^2_{1i}, $i = 2, 3$ are the neutrino mass-squared differences.

This is easy to understand. Lepton mixing is unphysical if all neutrino masses were to vanish, or if all neutrino masses were identical. Hence, all charged-lepton flavor-changing processes (CLFV) are proportional to Δm^2_{ij}, which are known to be tiny...

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How come we were sensitive to this in the neutrino sector?

The reason we can “see” neutrino mass effects is that the phenomenon of neutrino oscillations is an interference phenomenon.

As long as the mixing angles are large enough (which they turned out to be, for some mysterious reason) we only need to wait long enough for the tiny phase difference to build up (hence we need very long baselines to explore neutrino oscillations).

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In spite of this "technical difficulty," there are strong theoretical reasons to believe that the expected rate for flavor changing violating processes is much, much larger than νSM predictions and that, perhaps, discovery is just around the corner.

Due to the lack of SM "backgrounds," searches for rare muon processes, including $\mu\to e\gamma$, $\mu\to e^+e^-e$ and $\mu+Z\to e+Z$ (μ-e–conversion in nuclei) are considered ideal laboratories to probe effects of new physics at or slightly above the electroweak scale.

Indeed, if there is new physics at the electroweak scale (as many theorists will have you believe) and if mixing in the lepton sector is large "everywhere" the question we need to address is quite different:

Why haven't we seen charged lepton flavor violation yet?

Phenomenology of selected CLFV processes

As far as rare muon processes are concern, new physics effects can be parameterized via a handful of higher dimensional operators. Consider, for example, that the following effective Lagrangian dominates CLFV phenomena:

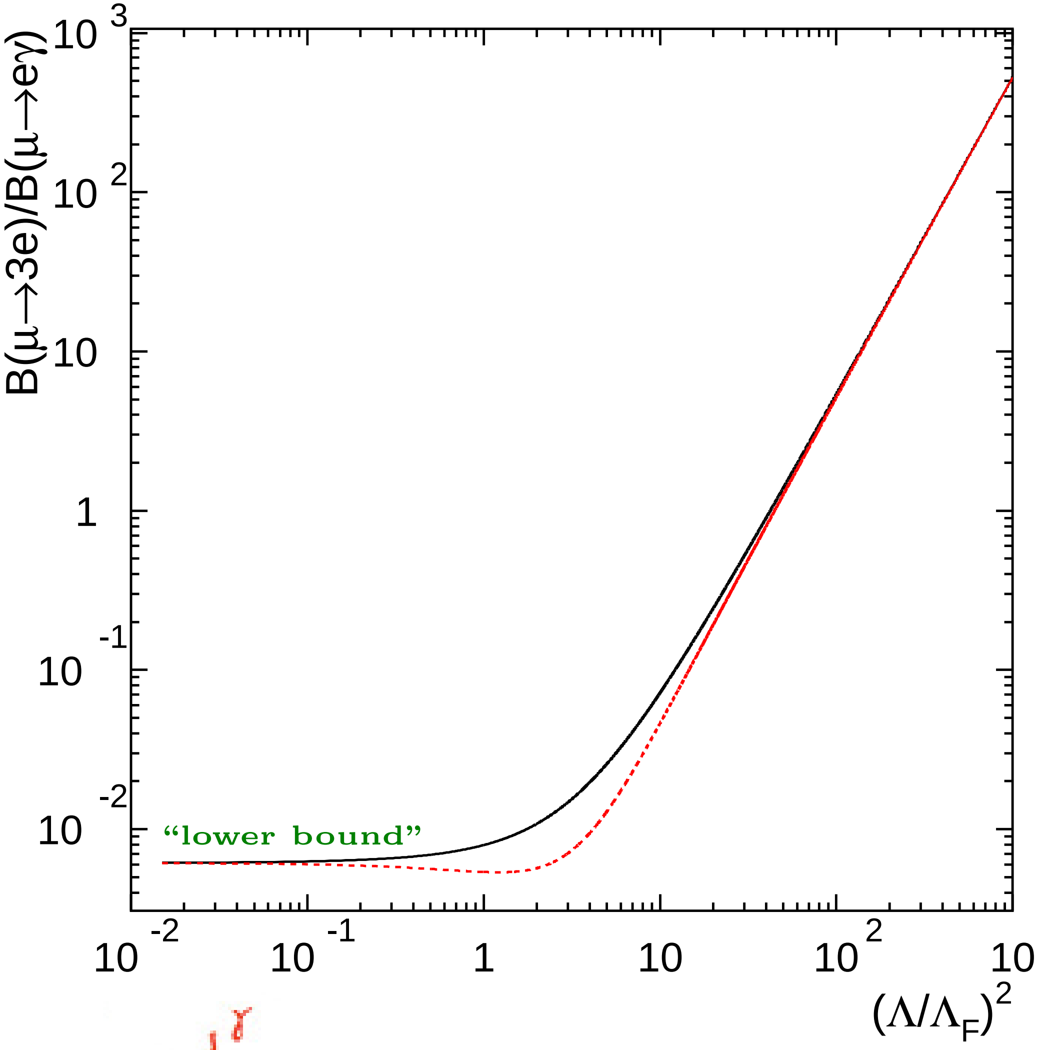
$$\mathcal{L} = \frac{m_\mu}{\Lambda^2}\bar{\mu}_R\sigma^{\mu\nu}e_L F_{\mu\nu} + \frac{1}{\Lambda_F^2}\left(\bar{\mu}_L\gamma^\mu e_L\right)\left(\bar{e}_L\gamma_\mu e_L\right) + \frac{1}{\Lambda_F^2}\left(\bar{\mu}_L\gamma^\mu e_L\right)\left(\bar{q}_L\gamma_\mu q_L\right) + \text{h.c.}$$

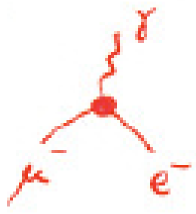
First term: mediates $\mu \rightarrow e\gamma$ and, at order α, $\mu \rightarrow eee$ and $\mu + Z \rightarrow e + Z$

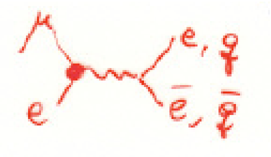
Second term: mediates $\mu \rightarrow eee$ and, at one-loop, $\mu \rightarrow e\gamma$

Third term: mediates $\mu + Z \rightarrow e + Z$ and, at one-loop, $\mu \rightarrow e\gamma$

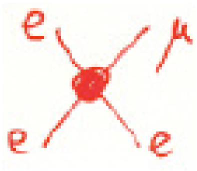
Which term wins? → Model Dependent



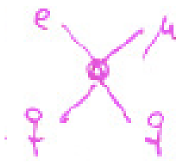


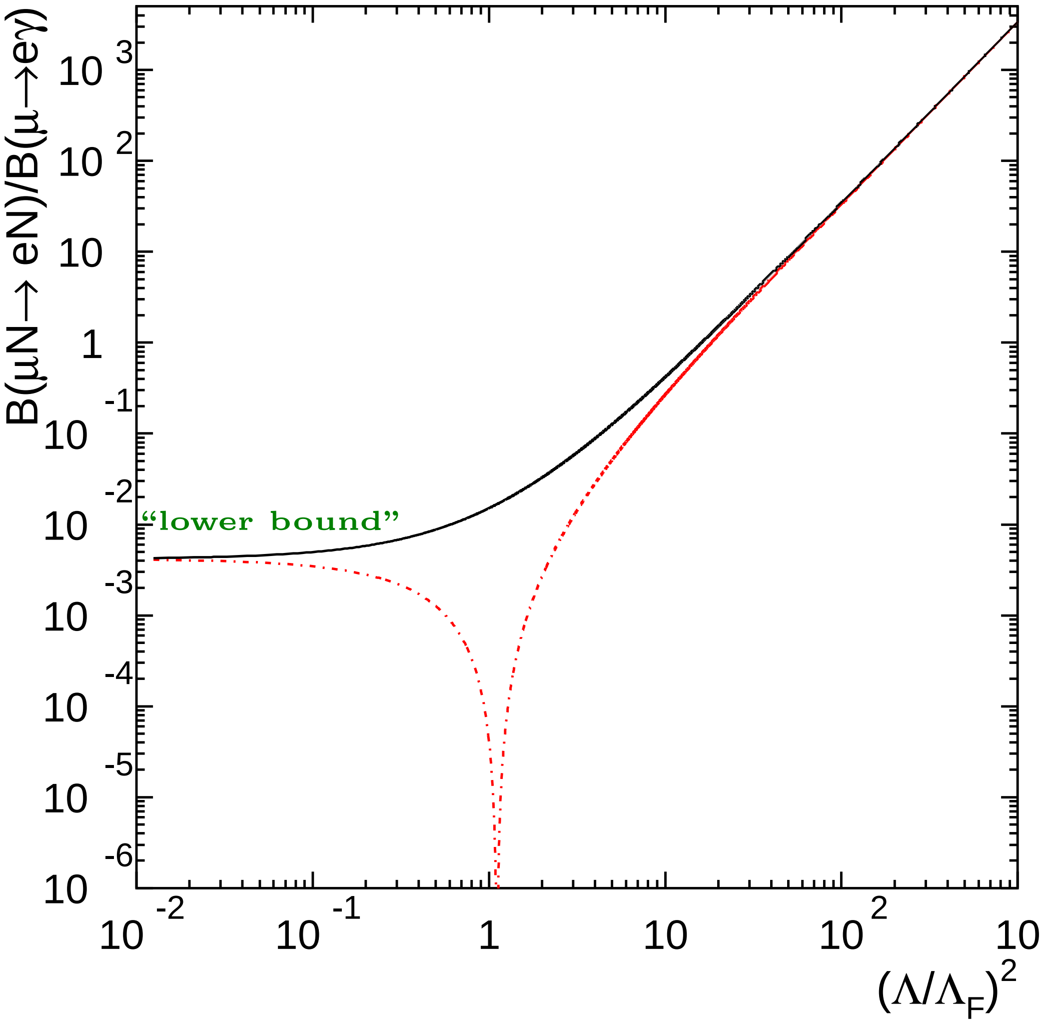


$\rightarrow \quad \Lambda \ll \Lambda_F$



$\rightarrow \quad \Lambda \gg \Lambda_F \quad \leftarrow$





On Model Dependency

Specific Models will provide estimates for the rates for CLFV processes.

On the flip side, the observation of one specific CLFV process will not determine the underlying physics mechanism.

Real strength lies in combinations of different measurements, including:

- other CLFV channels (including those involving taus);
- neutrino oscillations;
- measurements of $g-2$ and EDMs;
- collider searches for new, heavy states;
- etc.

Brief Comment on $g - 2$ and CLFV:

The effective operators that mediate $\mu \rightarrow e\gamma$ and contribute to a_μ are virtually identical:

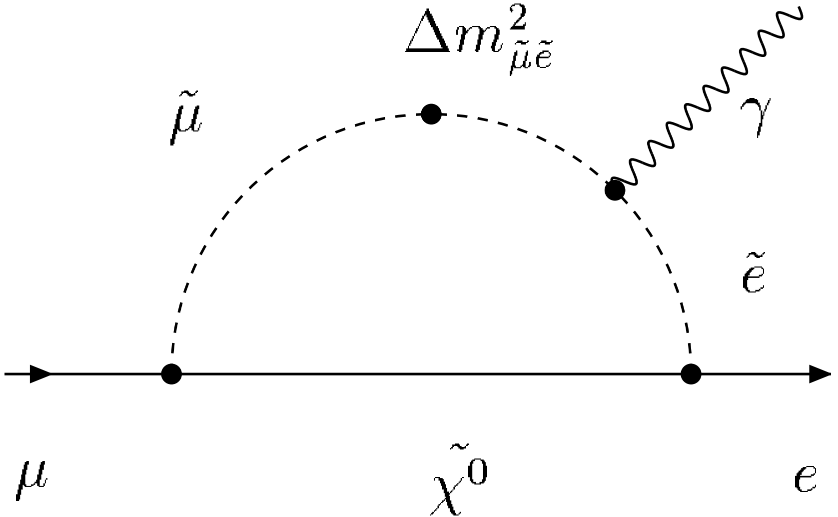
$$\frac{m_\mu}{\Lambda^2}\bar{\mu}\sigma^{\mu\nu}\mu F_{\mu\nu} \qquad \times \qquad \theta_{e\mu}\frac{m_\mu}{\Lambda^2}\bar{\mu}\sigma^{\mu\nu}eF_{\mu\nu}$$

If $\theta_{e\mu} \sim 1$, CLFV processes are a much more stringent probe of Λ.

On the other hand, if the current discrepancy ($\sim 3\sigma$ effect, of order the electroweak correction) in a_μ is due to new physics, $\theta_{e\mu} \ll 1$. This is hard to satisfy in, say, high energy SUSY breaking models...

[Hisano, Tobe, hep-ph/0102315]

“Bread and Butter” SUSY plus High Energy Seesaw



$$\rightarrow \quad \theta_{\tilde e\tilde\mu} \sim \frac{\Delta m^2_{\tilde e\tilde\mu}}{\tilde m}$$

$Br(\mu \rightarrow e\gamma) \simeq \frac{\alpha^3\pi}{G_F^2\tilde m^4}\theta^2_{\tilde e\tilde\mu}$, $\tilde m^2$ is a typical supersymmetric mass.

$\theta_{\tilde e\tilde\mu}$ measures the “amount” of flavor violation.

For $\tilde m$ around 1 TeV, $\theta_{\tilde e\tilde\mu}$ is severely constrained. Very big problem.

“Natural” solution: $\boxed{\boxed{\theta_{\tilde e\tilde\mu} = 0}}$ → modified by quantum corrections.

The Seesaw Mechanism

$\mathcal{L} \supset -y_{i\alpha} L^i H N^\alpha - \frac{M_N^{\alpha\beta}}{2} N_\alpha N_\beta + H.c.,$ $\Rightarrow$ N^α gauge singlet fermions, $y_{i\alpha}$ dimensionless Yukawa couplings, $M_N^{\alpha\beta}$ (very large) mass parameters.

At low energies, integrate out the "right-handed neutrinos" N_α:

$$\mathcal{L} \supset \left(y M_N^{-1} y^t\right)_{ij} L^i H L^j H + \mathcal{O}\left(\frac{1}{M_N^2}\right) + H.c.$$

y are not diagonal → right-handed neutrino loops generate non-zero $\Delta m^2_{\tilde{e}\tilde{\mu}}$

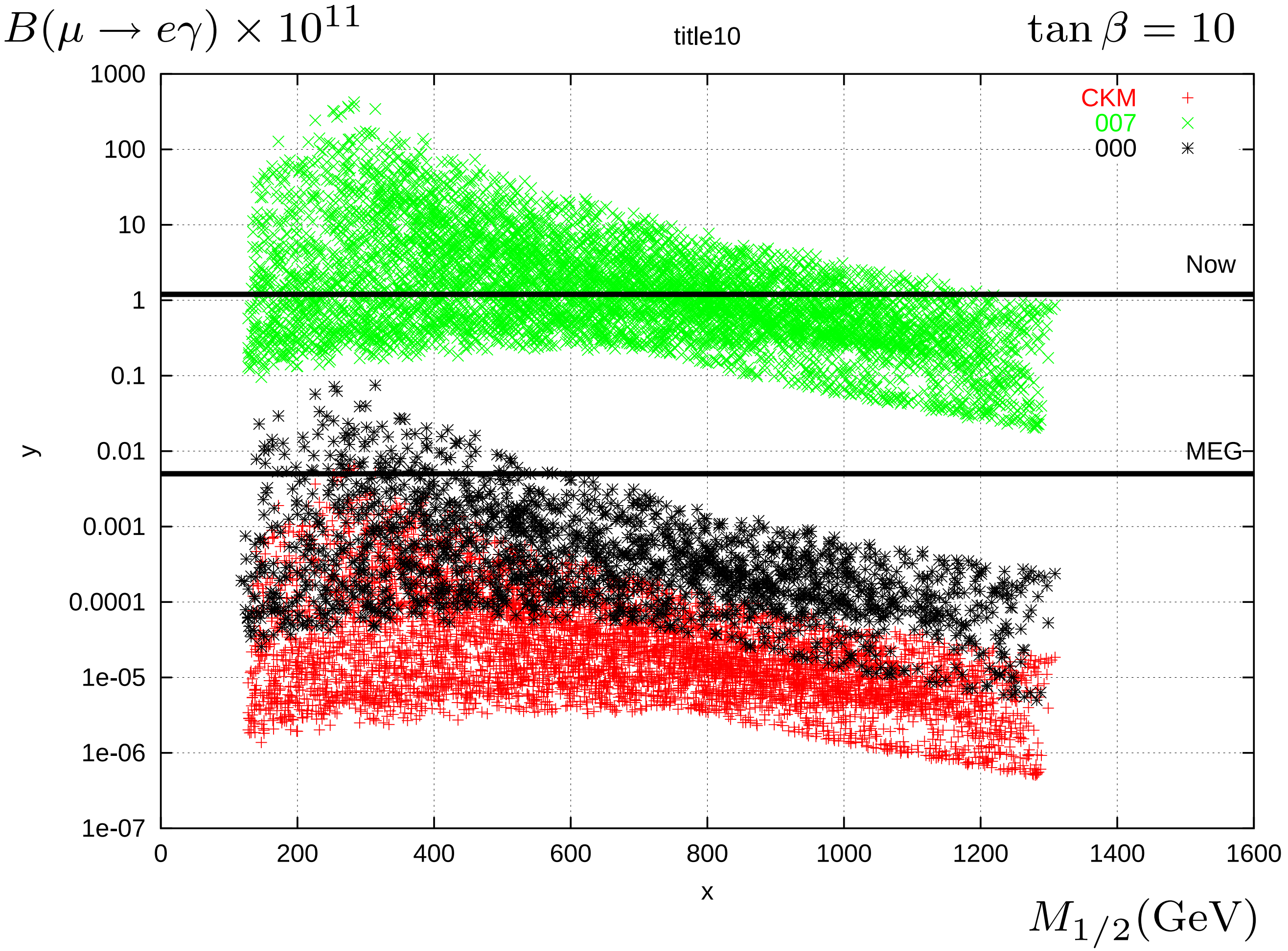
$$\left(m^2_{\tilde{\ell}_L}\right)_{ij} \simeq -\frac{3m_0^2 + A_0^2}{8\pi^2} \sum_k (y)^*_{ki} (y)_{kj} \ln \frac{M_X}{M_{N_k}} \,, \qquad X = \text{Planck}, GUT, \text{etc}$$

If this is indeed the case, CLFV would serve as another channel to probe neutrino Yukawa couplings, which are not directly accessible experimentally.

Fundamentally important for "testing" the seesaw, leptogenesis, GUTs, etc

What are the neutrino Yukawa couplings → ansatz needed!

$B(\mu \to e\gamma) \times 10^{11}$ $\tan\beta = 10$



$M_{1/2}$(GeV)

$SO(10)$ inspired model.

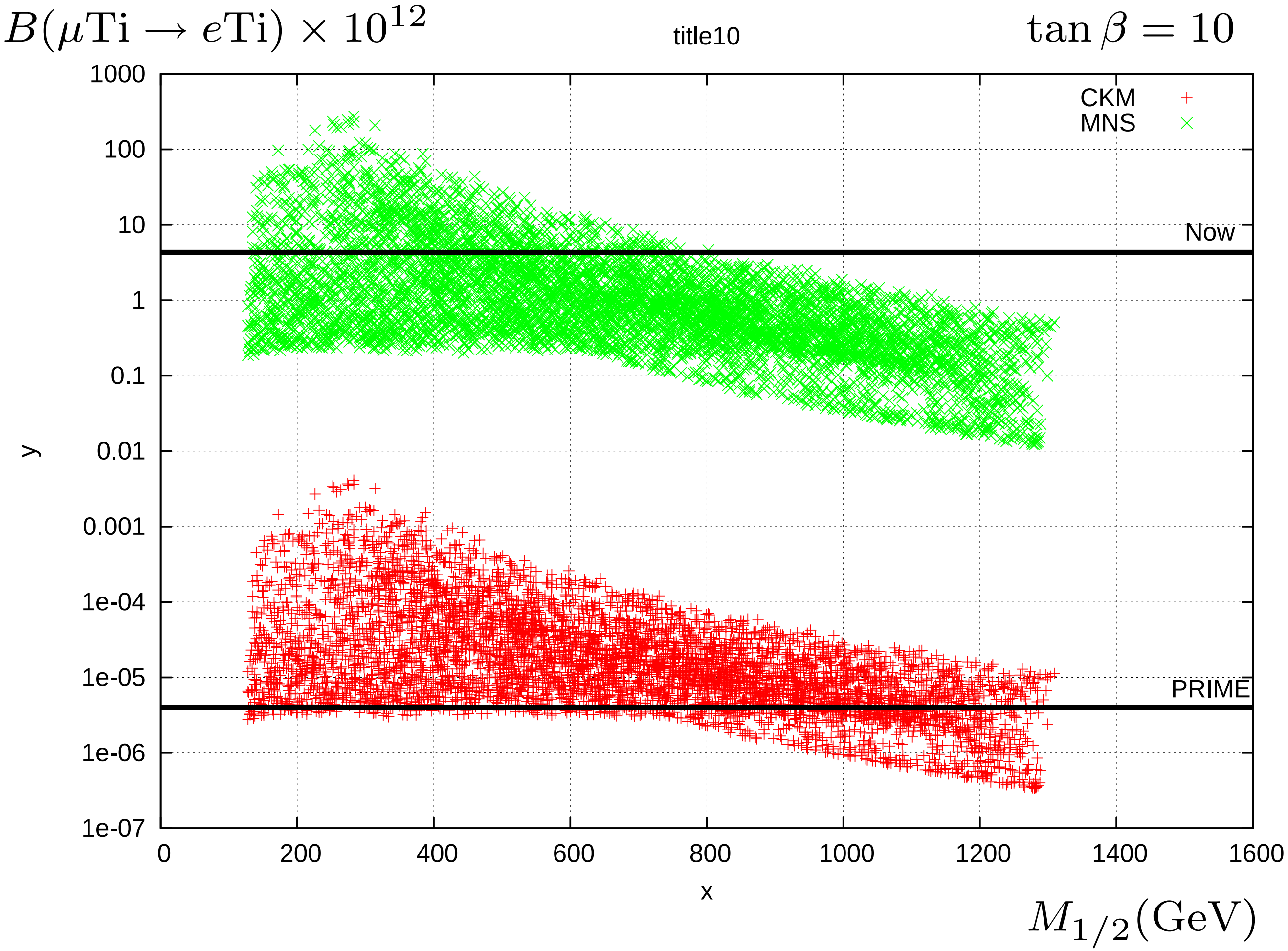
remember B scales with y^2.

$B(\mu \to e\gamma) \propto M_R^2[\ln(M_{Pl}/M_R)]^2$

[Calibbi, Faccia, Masiero, Vempati, hep-ph/0605139]

$\mu - e$ conversion is generally suppressed by a few orders of magnitude.

However, it is at least as sensitive as $\mu \to e\gamma$!



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SUSY with R-parity Violation

The MSSM Lagrangian contains several marginal operators which are allowed by all gauge interactions but violate baryon and lepton number.

A subset of these (set λ'' to zero to prevent proton decay, and ignore bi-linear terms, which do not contribute as much to CLFV) is:

$$\begin{aligned}\mathcal{L} &= \lambda_{ijk}\left(\bar{\nu}^c_{Li}e_{Lj}\tilde{e}^*_{Rk} + \bar{e}_{Rk}\nu_{Li}\tilde{e}_{Lj} + \bar{e}_{Rk}e_{Lj}\tilde{\nu}_{Li}\right) \\ &+ \lambda'_{ijk}V^{j\alpha}_{KM}\left(\bar{\nu}^c_{Li}d_{L\alpha}\tilde{d}^*_{Rk} + \bar{d}_{Rk}\nu_{Li}\tilde{d}_{L\alpha} + \bar{d}_{Rk}d_{L\alpha}\tilde{\nu}_{Li}\right) \\ &- \lambda'_{ijk}\left(\bar{u}^c_j e_{Li}\tilde{d}^*_{Rk} + \bar{d}_{Rk}e_{Li}\tilde{u}_{Lj} + \bar{d}_{Rk}u_{Lj}\tilde{e}_{Li}\right) + \text{h.c.},\end{aligned}$$

The presence of different combinations of these terms leads to very distinct patterns for CLFV. Proves to be an excellent laboratory for probing all different possibilities.

[AdG, Lola, Tobe, hep-ph/0008085]

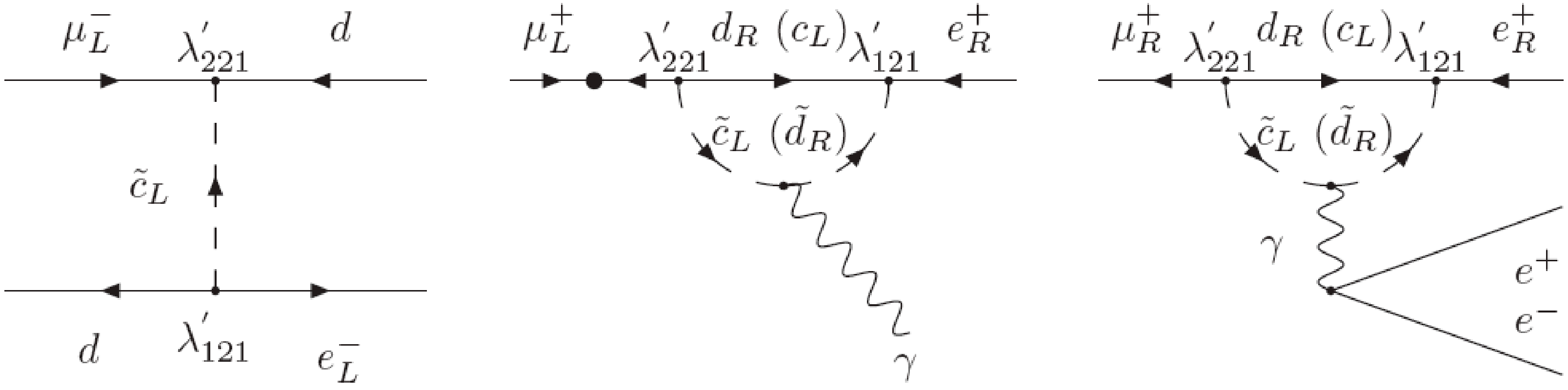


Figure 4: *Lowest order Feynman diagrams of lepton flavour violating processes induced by* $f'_{121}f'_{221}$ *couplings (see Eq. (2.1)).*

$$\frac{\mathrm{Br}(\mu^+\to e^+\gamma)}{\mathrm{Br}(\mu^+\to e^+e^-e^+)} = 1.1$$

$$(m_{\tilde{d}_R} = m_{\tilde{c}_L} = 300 \text{ GeV})$$

$$\boxed{\frac{\mathrm{R}(\mu^-\to e^- \text{ in Ti (Al)})}{\mathrm{Br}(\mu^+\to e^+e^-e^+)} = 2\ (1) \times 10^5}$$

$\mu - e$-conversion “only hope”!

[AdG, Lola, Tobe, hep-ph/0008085]

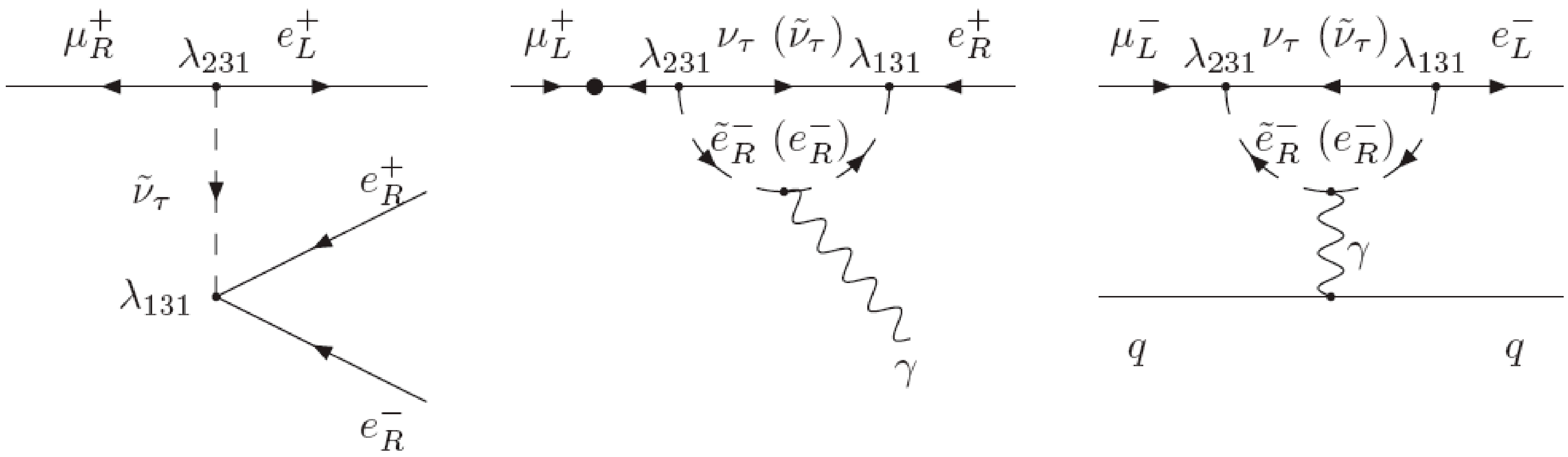


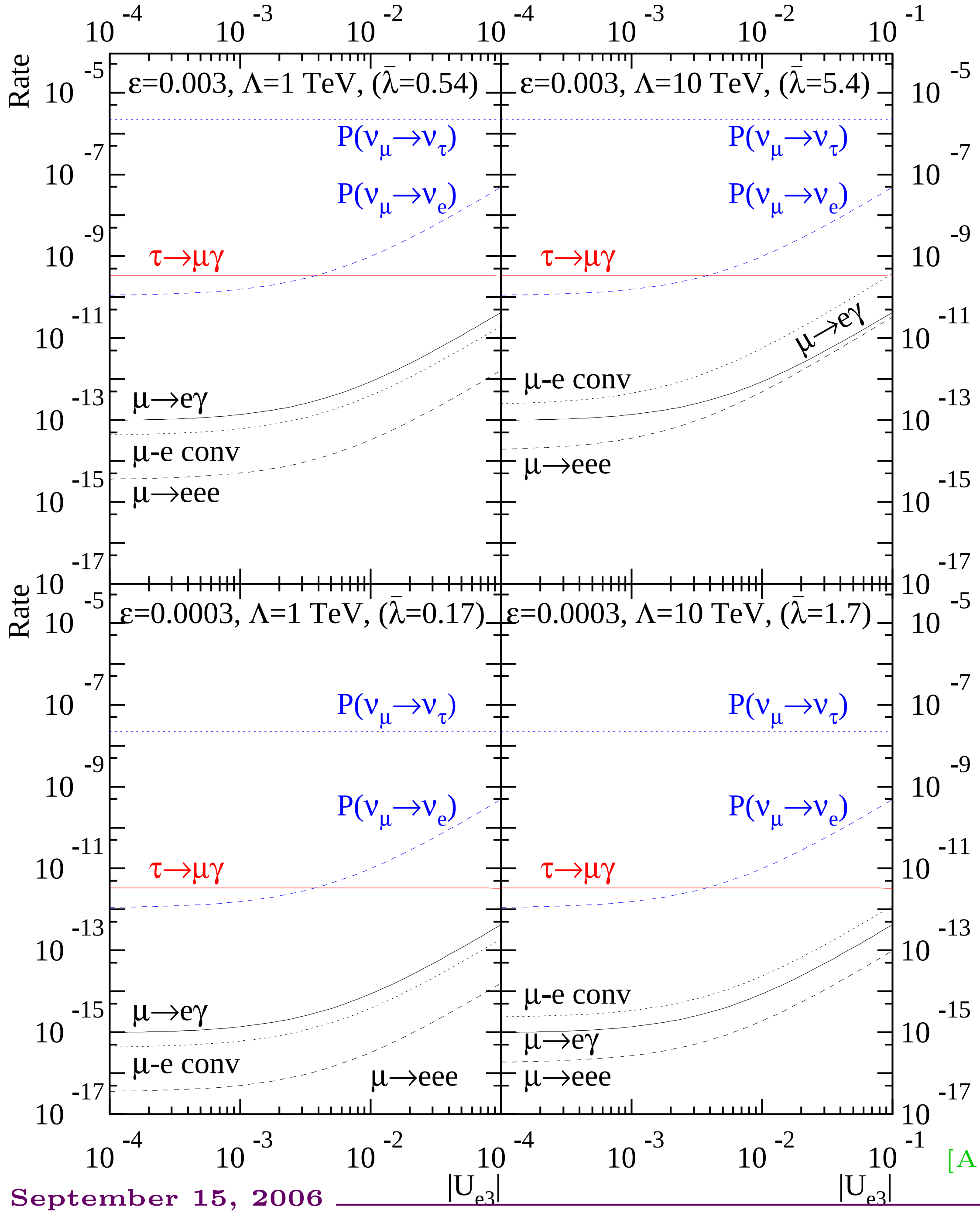
Figure 1: *Lowest order Feynman diagrams for lepton flavour violating processes induced by* $\lambda_{131}\lambda_{231}$ *couplings (see Eq. (2.1)).*

$$\frac{\mathrm{Br}(\mu^+\to e^+\gamma)}{\mathrm{Br}(\mu^+\to e^+e^-e^+)} = \frac{4\times 10^{-4}\left(1-\frac{m^2_{\tilde{\nu}_\tau}}{2m^2_{\tilde{e}_R}}\right)^2}{\beta} \simeq 1\times 10^{-4} \qquad (\beta\sim 1)$$

$$\frac{\mathrm{R}(\mu^-\to e^-\ \text{in Ti (Al)})}{\mathrm{Br}(\mu^+\to e^+e^-e^+)} = \frac{2\ (1)\times 10^{-5}}{\beta}\left(\frac{5}{6}+\frac{m^2_{\tilde{\nu}_\tau}}{12m^2_{\tilde{e}_R}}+\log\frac{m_e^2}{m^2_{\tilde{\nu}_\tau}}+\delta\right)^2 \simeq 2\ (1)\times 10^{-3},$$

$\mu^+ \to e^+e^-e^+$ most promising channel!

[AdG, Lola, Tobe, hep-ph/0008085]



[AdG, Giudice, Strumia, Tobe, hep-ph/0107156]

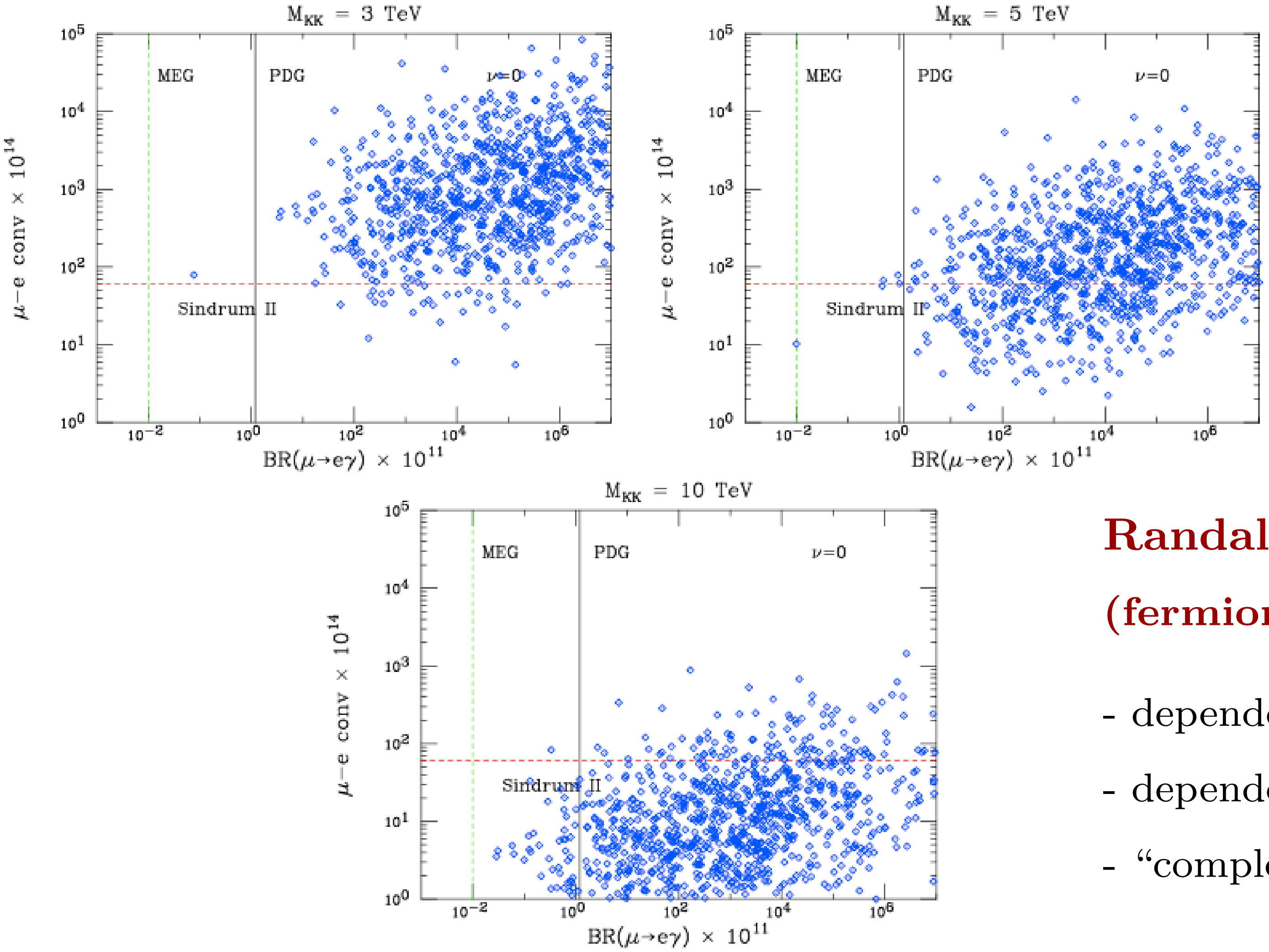
Large Extra-Dimensions

(right-handed neutrinos in the bulk)

-no ambiguity in y (neutrinos Dirac)

-dependency on UV-completion

$$\frac{B(\mu\to e\gamma)}{B(\mu-e\ \text{conv})}\in[0.1-10]$$



Randall-Sundrum Model

(fermions in the bulk)

- dependency on UV-completion(?)
- dependency on Yukawa couplings
- "complementarity" between $\mu \to e\gamma$, $\mu - e$ conv

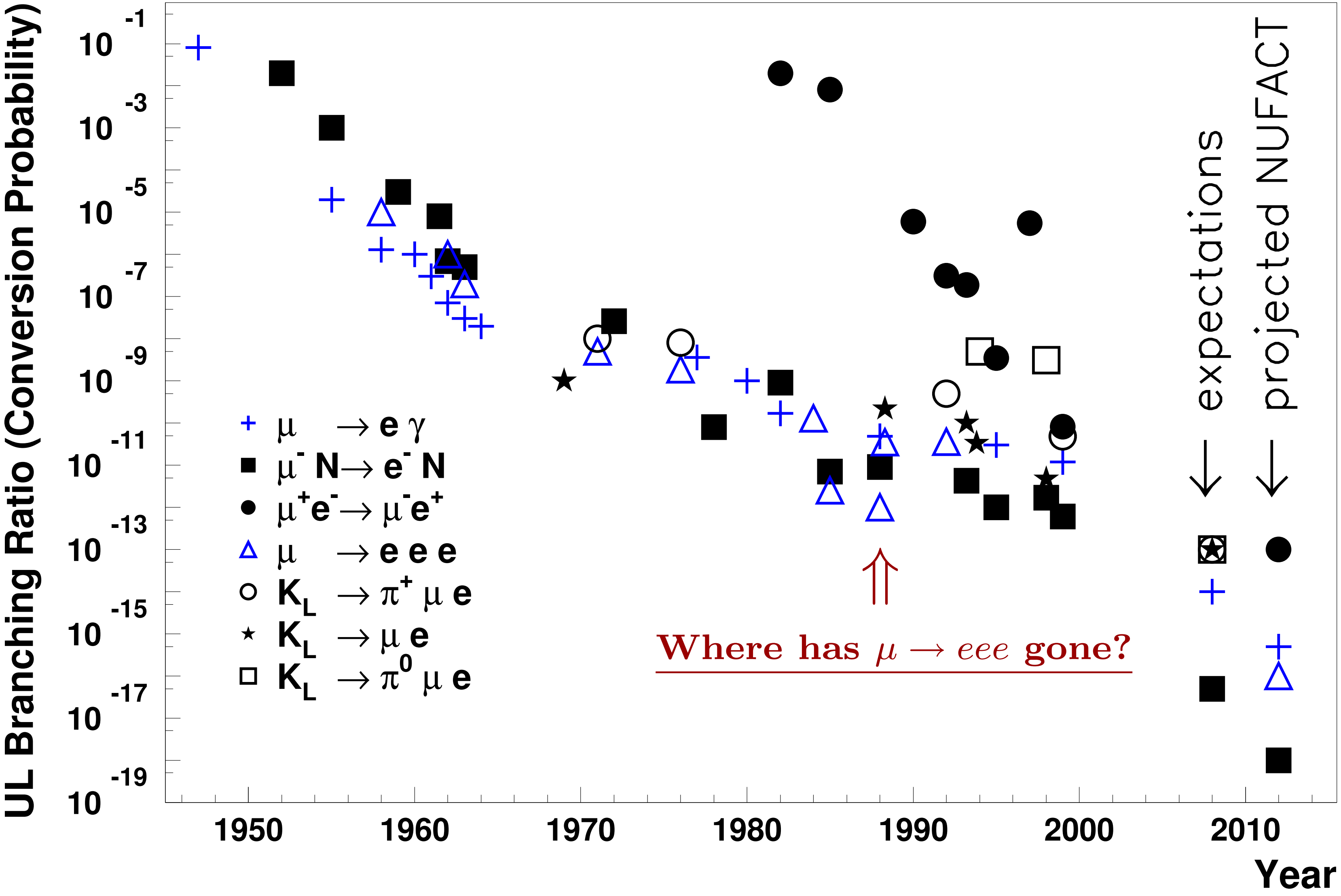
FIG. 6: Scan of the $\mu \to e\gamma$ and $\mu - e$ conversion predictions for $M_{KK} = 3, 5, 10$ TeV and $\nu = 0$. The solid line denotes the PDG bound on $BR(\mu \to e\gamma)$, while the dashed lines indicate the SINDRUM II limit on $\mu - e$ conversion and the projected MEG sensitivity to $BR(\mu \to e\gamma)$.

[Agashe, Blechman, Petriello, hep-ph/0606021]

Summary and Conclusions

- We know that charged lepton flavor violation must occur. Effects are, however, really tiny in the νSM (neutrino masses too small).
- If there is new physics at the electroweak scale, there is every reason to believe that CLFV is well within the reach of next generation experiments. Indeed, it is fair to ask: 'Why haven't we seen it yet?'
- It is fundamental to probe all CLFV channels. There is priceless information in ratios of branching ratios! While in some scenarios $\mu \to e\gamma$ is the "largest" channel, there is no theorem that guarantees this (and many exceptions).

Searches for Lepton Number Violation



Where has $\mu \to eee$ gone?

[hep-ph/0109217]

Summary and Conclusions (Cont.)

- Rate for $\mu - e$ conversion “guaranteed” to be at least few $\times 10^{-3}$ the branching ration for $\mu \to e\gamma$. Could be much, much larger!
- If we see $\mu \to e\gamma$ at MEG, we “must” see $\mu - e$ conversion. On the other hand, if we don’t see it $\mu \to e\gamma$ at the 10^{-13} level, $\mu - e$ conversion the best bet to improve sensitivity to CLFV in the future (?).
- CLFV may be intimately related to the new physics unveiled with the discovery of non-zero neutrino masses. It may play a fundamental role in our understanding of the seesaw mechanism, GUTs, the baryon-antibaryon asymmetry of the Universe. We won’t know for sure until we see it!