Simulation of low γ_T of SPS with crabs and space-charge

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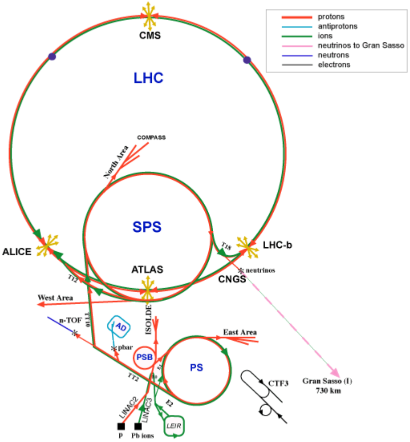
Outline

SPS (Super Proton Synchrotron)

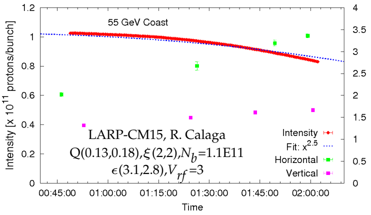
- Circumference: 6.9 km
- injection energy at 26 GeV/c
- protons for LHC at 450GeV/c
- protons for fixed target physics at 400 GeV/c
- protons for CNGS experiment at 400GeV/c



LHC: Large Hadron Collider
SPS: Super Proton Synchrotron
AD: Antiproton Decelerator
ISOLDE: Isotope Seperator OnLine DEvice
PSB: Proton Synchrotron Booster
PS: Proton Synchrotron
LINAC: LINear ACcelerator
LEIR: Low Energy Ion Ring
CNGS: Cern Neutrinos to Gran Sasso

Motivation

- During 2010 MD in the SPS, large transverse emittance growth rate (ϵ_x=80%/h, ϵ_y=42%/h) has been observed.
  - Dipole voltage ripple, space-charge, RF phase noise, chromaticity, IBS, ... contribute to the emittance growth.



- Extensive MD studies on low transition energy in the SPS are scheduled in 2011.
- SPS is one of promising candidates for testing crab cavity for the HL-LHC.
- We investigate the emittance growth in both nominal transition gamma (22.90) and low γ_T (18.01).

Low transition gamma (γ_T)

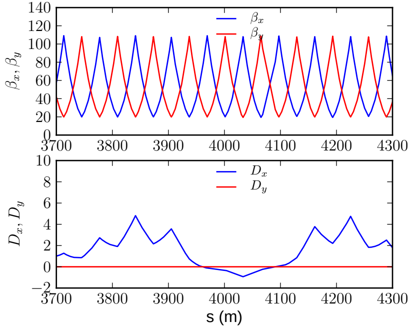
parameter	symbol	nominal	low γ_T
transition energy		22.90	18.01
transverse tune	(ν_x, ν_y)	(26.13, 26.18)	(20.13, 20.18)
natural chromaticity		(-32.68, -32.74)	(-22.79, -22.83)
sextupole strength	m^{-2}	(0.063, -0.150)	(0.045, -0.041)
max. beta	(β_x, β_y)	(111, 109)	(109, 109)
max. dispersion	(η_x, η_y)	(4.9, 0)	(8.1, 0.0)
beam energy	GeV	55	
beam intensity		1×10^{11}	
chromaticity	(ξ_x, ξ_y)	(0, 0)	
transverse emittance	mm-mrad	3.5	
long. emittance, 4σ	eV s	0.24	
rf voltage	MV	3	
particle distribution		Gaussian in (x, y, z)	

(Courtesy of I. Papaphilippou)

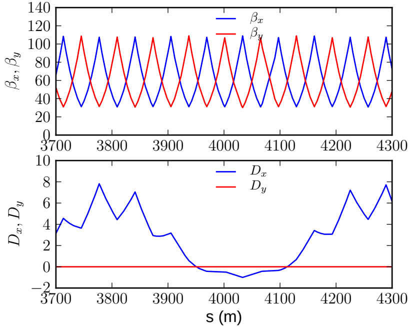
- Note) natural chromaticity and focusing sextupole strength of nominal optics are 40% larger than those of low γ_T.

Optics in nominal and low γ_T

(Nominal, γ_T=22.90)



(low γ_T, γ_T=18.01)



- Weaker focusing has the consequence of increasing beta functions and dispersions - both of which increase the beam size.
- β_{max} does not change much, but β_{min}=20m (nominal), β_{min}=34 (low γ_T).
- maximum dispersions 4.9 (nominal), 8.1 (low γ_T), but mimimum dispersion -0.91 (nominal), -0.98 (low γ_T).

Crab cavity at COLDEX location

- Longitudinal location: 4009 m

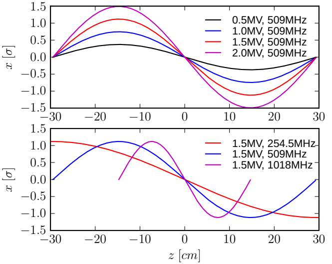
parameter	nominal	low γ_T
(β_x, β_y)	(30,77)	(42, 80)
(η_x, η_x')	(-0.5,-0.02)	(-0.6, -0.02)
(ν_x, ν_y)	(15.173, 15.176)	(11.742, 11.770)

- Crab cavity parameters
  - Voltage: 0.325MV (55GeV)
  - Frequency: 400MHz (λ=750mm)
  - Global scheme with horizontal crossing
- Crab cavity kicks:

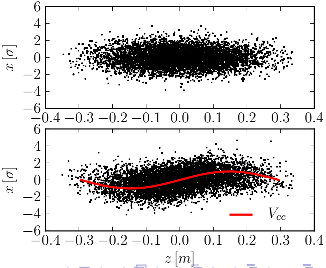
$$\Delta x' = \frac{qV_{cc}}{E_0} \sin(kz) \,, \Delta \frac{\Delta E}{E} = \frac{qV_{cc}}{E_0} \cos(kz) \cdot kx$$

- Couples the hor. and long. planes.
- Makes the closed orbit dependent on z.
- Distorts bunch shape for long bunches.

Closed orbit due to crab cavity



Large bunch length



Stability analysis model

- Consider horizontal and longitudinal phase space, i.e., (x, x', z, δ)
- Linear transfer map btwn crab cavity and main RF ($M_{RF\to CC}, M_{RF\to CC}$), RF map (M_{RF}), and CC map (M_{CC}).
- One turn map is

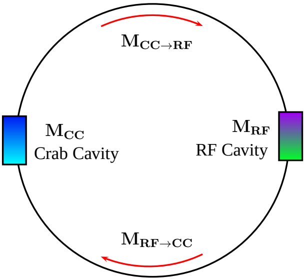
$$T = M_{RF\to CC} \cdot M_{RF} \cdot M_{RF\to CC} \cdot M_{CC}$$

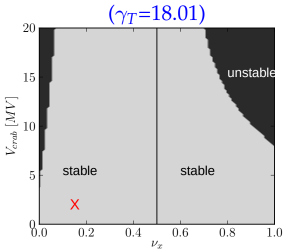
$$M_{CC} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \xi & 0 \\ 0 & 0 & 1 & 0 \\ \xi & 0 & 0 & 1 \end{pmatrix} \qquad \xi = \frac{qV_{cc}k}{E_0}$$

$$M_{RF} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \Delta & 1 \end{pmatrix} \qquad \Delta = \frac{qV_{rf}h}{\beta^2 E_0 R}$$

- Crab cavity couples the horizontal and longitudinal planes (synchro-beta coupling).
- The characteristic polynomial of the matrix T is $Q(\rho) = \rho^2 - b_1\rho + b_2$, where $\rho = \lambda + \frac{1}{\lambda}$.
- The stability conditions are

$$\boxed{\mathbf{b_1^2 - 4b_2 > 0, \quad 4 - \left| b_1 \pm \sqrt{b_1^2 - 4b_2} \right| > 0}}$$





Beam-Beam Simulation (BBSIM) code

- 6D weak-strong tacking code.
- Linear transfer matrices btwn nonlinear elements + nonlinear kicks at the nonlinear elements (thin lens approximation: dipole, quadrupole, sextupole, mulitpole, etc.).
- Space charge: (1) 2-D and 3-D Poisson solver using Conjugate Gradient and (2) 2-D solver with FFT.
- Beam-beam force: (1) Gaussian beam profile and (2) Poisson solver with FFT.
- Multiple-slice model for finite bunch length effects.
- Lorentz boost to handle crossing angle collisions.
- Modules: crab crossing, wire and electron lens compensation, etc.
- Fully parallelized with MPI.
- Simulations agree well with measurements in the Tevatron, RHIC. Also applied to wire compensation in the SPS, LHC.
- Diagnostics: Beam loss, emittance growth, beam profiles, BTFs, dynamic aperture, tune footprints.

2.5-D space-charge kicks

- Transverse electric field (fast 2D Poisson solver, $\vec{E} = -\nabla\phi$)

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\phi\left(x, y\right) = -\frac{1}{\epsilon_0}\rho\left(x, y\right)$$

- longitudinal electric field (ρ_L line density)

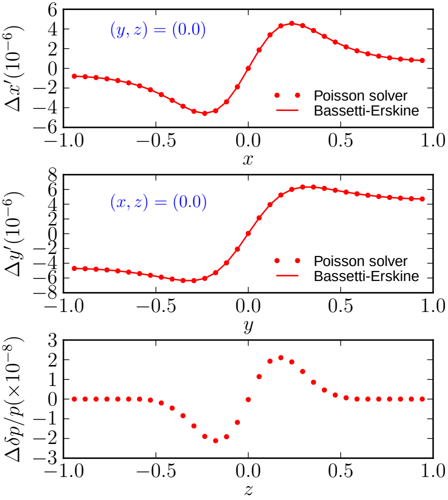
$$E_z = -\frac{g}{4\pi\epsilon_0\gamma^2}\frac{d}{dz}\rho_L\left(z\right)$$

- Space charge kicks:

$$\Delta\vec{r'} = \frac{qL}{m_0c^2\beta^2\gamma^3}\vec{E}\left(x, y\right)\frac{\rho_L(z)}{\rho_0},$$

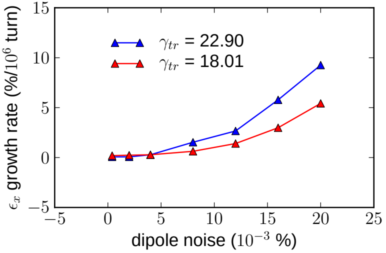
$$\Delta\frac{\delta p}{p} = \frac{qL}{m_0c^2\beta^2\gamma}E_z\left(z\right).$$

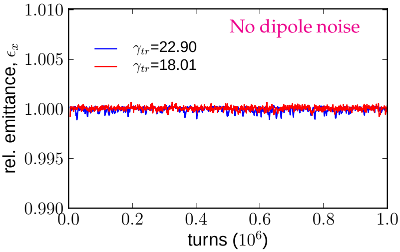
- Benchmark with 10000 particles in SPS optics. Space-charge kicks obtained by Poisson solver and Bassetti-Erskine formula are well agreed.



Emittance growth vs dipole noise

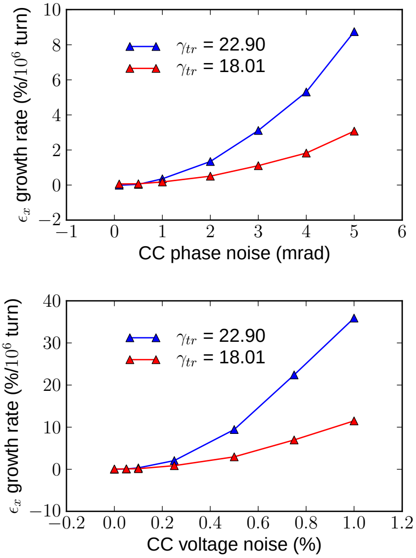
- Large emittance growth in the SPS (MD2010). Expect that dipole noise contributes to the emittance growth.
- Gaussian distribution with 10000 particles, 10^6 turns (23 seconds).
- Model: sextupole + dipole voltage ripple (white noise)
- No crab cavity is included in the model.
- Emittance growth is (2 times) less in low γ_T.
- Sextupole strength of nominal is 40% larger than low γ_T.
- Vertical and longitudinal emittance growth is insignificant.
- Voltage ripple of LHC after active filtering is 2.5×10^{-3}%.





Emittance growth vs CC phase/voltage noise

- Model: sextupole, crab cavity noises (phase/voltage)
- 55GeV beam energy.
- CC voltage: 0.325MV, CC frequency: 400MHz
- Gaussian distribution with 10000 particles, 10^6 turns (23 seconds).
- Emittance growth is (3 times) less in low γ_T.
- Vertical and longitudinal emittance growth is insignificant.
- 509MHz crab cavity gives the same result. CC frequency does not change the emittance growth rate for rms bunch length 0.18m.

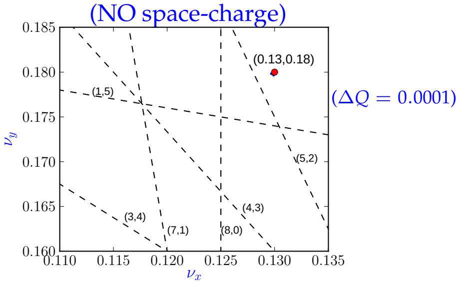


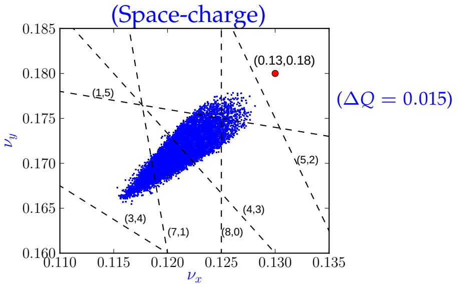
Tune footprint (space charge)

- Gaussian distribution in (x, y, z) with 10^4 particles.
- Apply space-charge kicks at quadrupole locations.
  - 18, 36, 72, and 208 kicks per turn are tested.
  - 72 kicks/turn is chosen.
- Tune shift for bunched beam due to space-charge

$$\Delta Q = -\frac{N_b r_p}{4\pi B \beta \gamma^2 \epsilon_N}$$

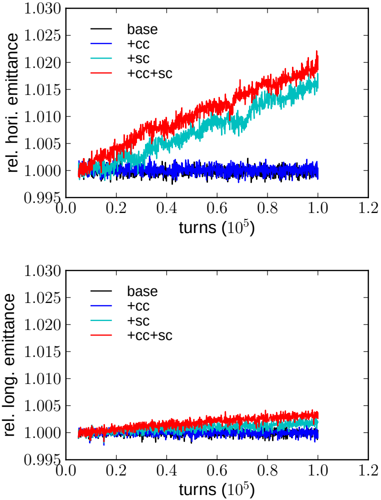
- Tune shift of particles with small betatron amplidue (55GeV, $N_b = 10^{11}$, ϵ_N=3.5μm, $\sigma_z = 0.18m$), $\Delta Q = 0.015$.
- 6-th, 7-th, and 8-th resonance lines are spanned.





Emittance growth (space-charge)

- low γ_T (18.01) lattice.
- No noises are added in the model.
- Space charge kicks at 72 locations.
- Space-charge induces 40%/hr emittance growth in both horizontal and vertical planes. 5%/hr emittance growth in longitudinal plane.
- Crab cavity + space-charge increase emittance. 7%/hr increase with crab cavity.
- γ_T (22.90) has the same growth.
- Space-charge is expected to have a significant contribution to observed emittance growth in MD2010
- Results need to be confirmed with more particles in simulation.

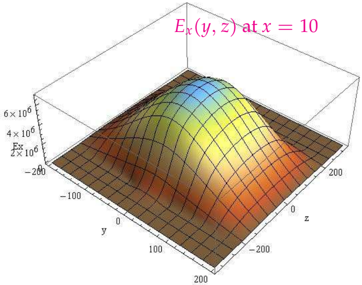


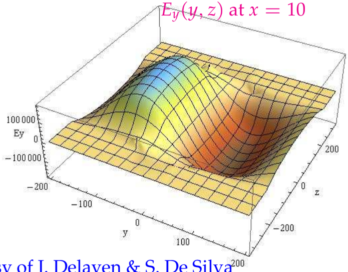
JLab crab cavity simulation model

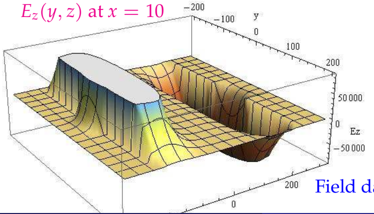
- Electromagnetic fields obtained from a model of the JLab design (J. Delayen) .
- E_y and E_z fields are smaller and much smaller respectively than the E_x fields.
- Buildup Hamiltonian from the EM fields, and calculate crab cavity kicks as follows:

$$\Delta x = 0, \Delta y = 0$$
$$\Delta p_x = \frac{1}{1+\delta}\left((p_x - A_x)\frac{\partial A_x}{\partial x} + \left(p_y - A_y\right)\frac{\partial A_y}{\partial x}\right) + \frac{\partial A_z}{\partial x}$$
$$\Delta p_y = \frac{1}{1+\delta}\left((p_x - A_x)\frac{\partial A_x}{\partial y} + \left(p_y - A_y\right)\frac{\partial A_y}{\partial y}\right) + \frac{\partial A_z}{\partial y}$$
$$\Delta z = \frac{1}{2(1+\delta)^2}\left(2p_xA_x - A_x^2 + 2p_yA_y - A_y^2\right)$$
$$\Delta \delta = \frac{1}{1+\delta}\left((p_x - A_x)\frac{\partial A_x}{\partial z} + \left(p_y - A_y\right)\frac{\partial A_y}{\partial z}\right) + \frac{\partial A_z}{\partial z}$$

- Implementation in BBSIM is in progress.







Field data courtesy of J. Delayen & S. De Silva

Summary

- Linear stability model of crab cavity has been studied for low γ_T lattice. SPS parameters are far away from unstable boundaries.
- Emittance growth rate studies for nominal and low γ_T optics. Noises affect low γ_T optics (two times) less than nominal optics.
- Space-charge increases emittance in both nominal and low γ_T optics significantly. The emittance growth is approximately 40%/hr in both horizontal and vertical planes. Needs to be confirmed with more particles.
- Combination of crab cavity and space-charge increases emittance growth further. Needs to be confirmed with more particles.
- Proposed crab cavity simulation model using electromagnetic fields of crab cavity from JLab. Implementation in BBSIM is in progress. Wakefields will be added.

Thank you for your attention!