Non-Perturbative Evidence for Technicolor

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What is Technicolor?

- If EW symmetry $SU(2)_L \times U(1)_Y$ were unbroken at GeV energies, QCD would break it via strongly-coupled Higgs mechanism.
  - Pions eaten to give mass to W and Z bosons of O(100 MeV).
  - No analogue of Yukawa mechanism. Lots of very light pseudoscalar mesons due to $N_f=6$ massless flavors.
- Basic Idea: Break EW symmetry at TeV scales by adding new fermions ($\bar{Q}$,Q) with new strong interactions. [Weinberg, Susskind 1979]
- SM fermion mass: New gauge interactions broken at high scale Λ_{ETC} couple SM fermions to techniquarks. [Dimopoulis-Susskind, Eichten-Lane 1979]

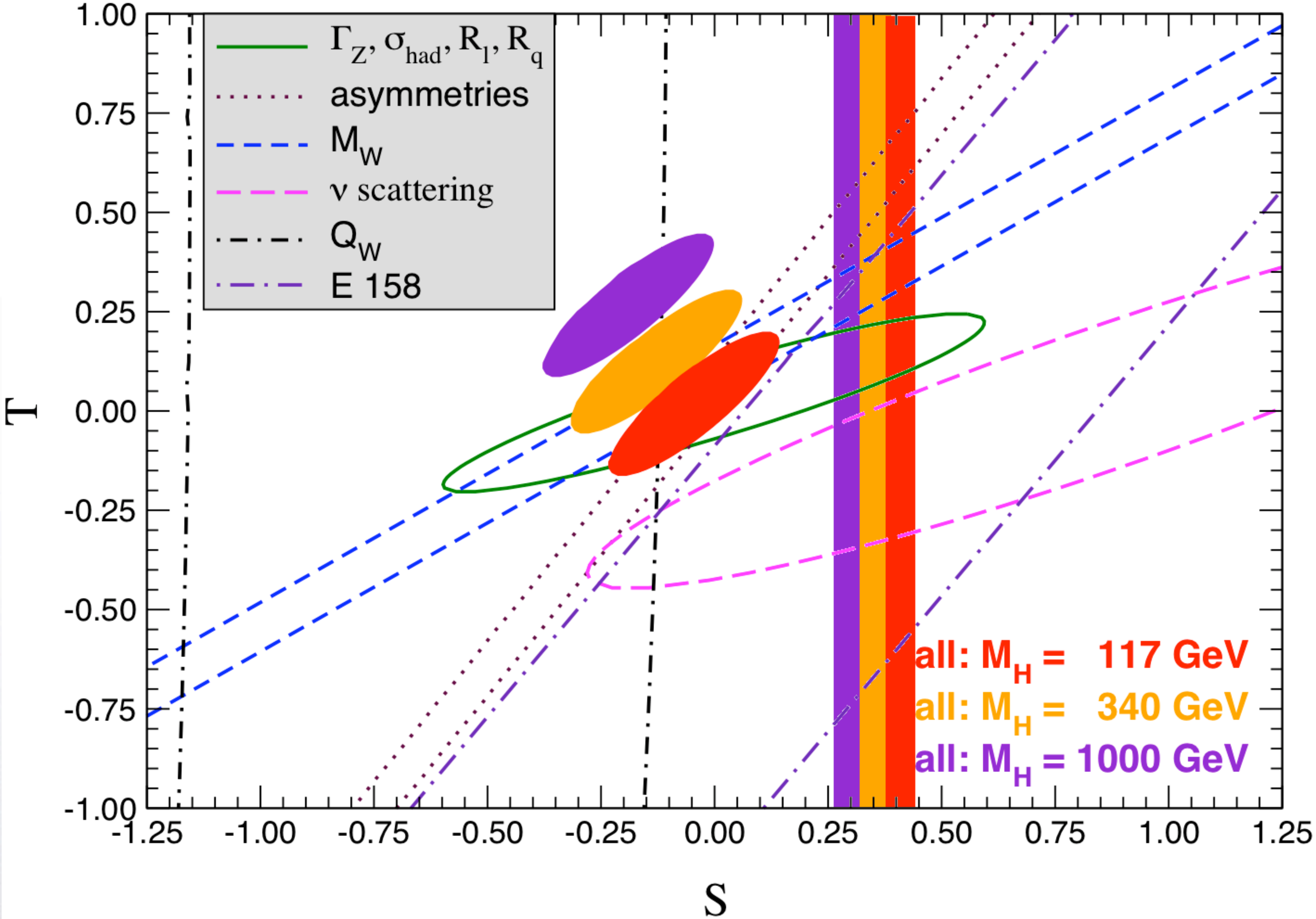
$$\text{Masses: } \frac{(\bar{Q}Q)(\bar{q}q)}{\Lambda_{\rm ETC}^2} \quad \text{FCNC's: } \frac{(\bar{q}q)(\bar{q}q)}{\Lambda_{\rm ETC}^2} \quad \Lambda_{\rm ETC} \gtrsim 1000 \text{ TeV}$$

- http://en.wikipedia.org/wiki/Technicolor_(physics)

Why did Technicolor fall out of favor?

- QCD-like strong interactions at the TeV scale can drive the Higgs mechanism, but face phenomenological challenges:
  - Either flavor changing neutral currents (FCNC) are too large or generated SM fermion masses are too small.
  - Precision EW oblique corrections (S parameter) in tension with experiment.
- A resolution: TeV strong interactions are not like QCD.
- A problem: How well do we really understand generic strongly interacting theories other than QCD?
- A solution: Lattice field theory is only now powerful enough to begin the study of strongly-coupled theories beyond QCD.

S Parameter for Scaled-Up QCD

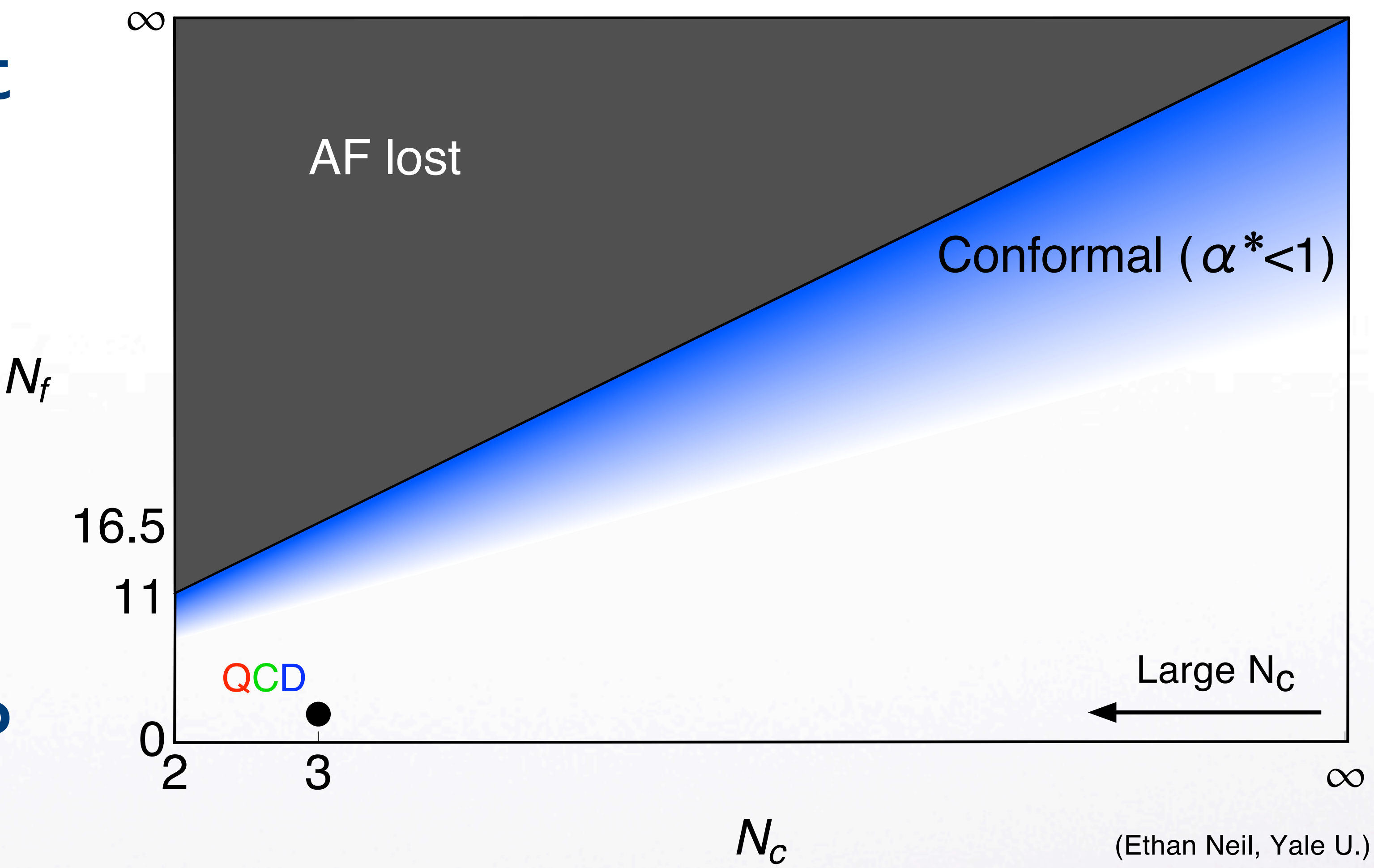


How can the lattice address Technicolor?

- Technicolor scenario has Higgs mechanism driven by TeV-scale strong interactions with spontaneous symmetry breaking (SSB) and Nambu-Goldstone (NG) bosons.
- QCD has these features and been studied on the lattice for decades, recently with much success.
- Other strongly-coupled gauge theories likely have these features, *i.e.* other flavors (N_f), colors (N_c), *etc.*
- Lattice studies can search for the right combination that enables Technicolor to satisfy phenomenological constraints.
- Unfortunately, other theories are usually computationally more expensive than QCD for calculation: $\propto N_f^{3/2}, N_c^3, d(R)^3$

Where to look for non-QCD theories?

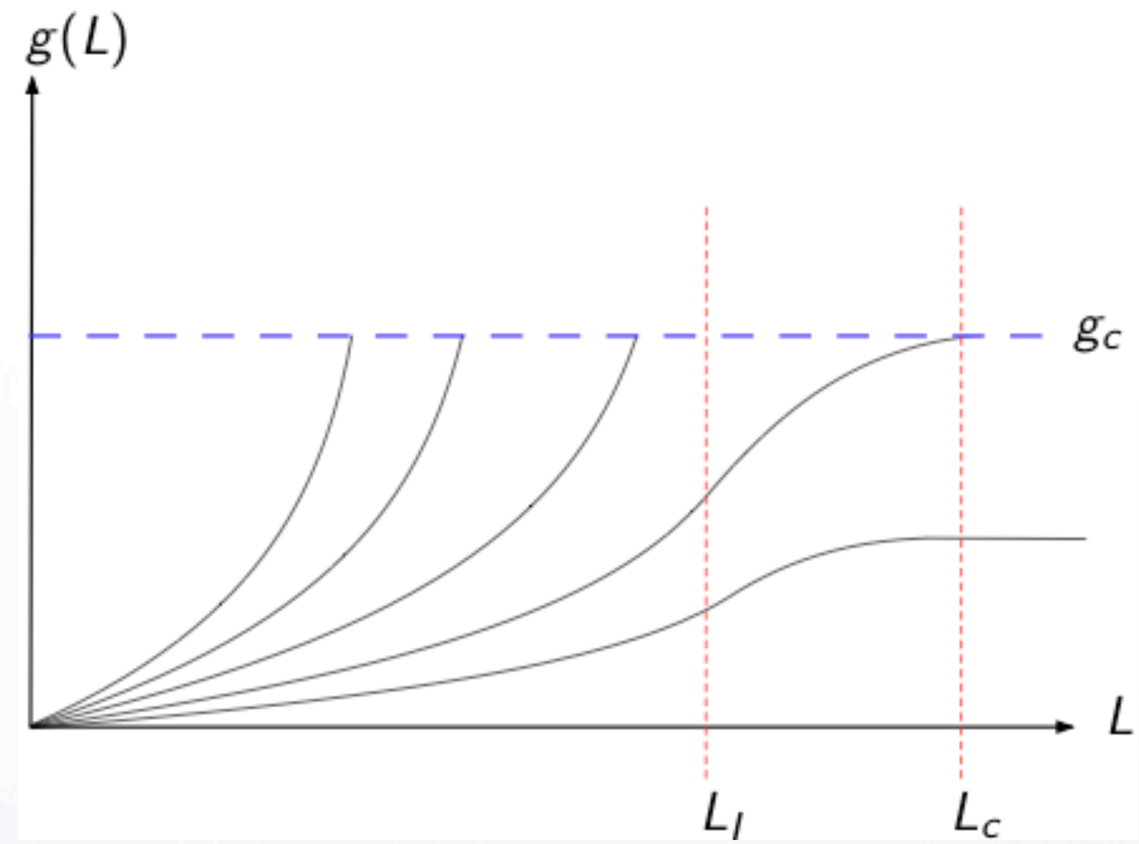
- For N_f = 0–1, confinement but no NG bosons.
- For N_c = 2, enhanced chiral symmetry means special case: Pattern of symmetry breaking yet to be determined.



- Pert. theory indicates IRFP for $N_f \lesssim 5.5 \cdot N_c$.
- Phenomenological success of large N_c calculations suggest QCD-like theories for N_f = 2–3 and $N_c \geq 3$.
- Simplest search strategy: start from QCD and increase N_f.

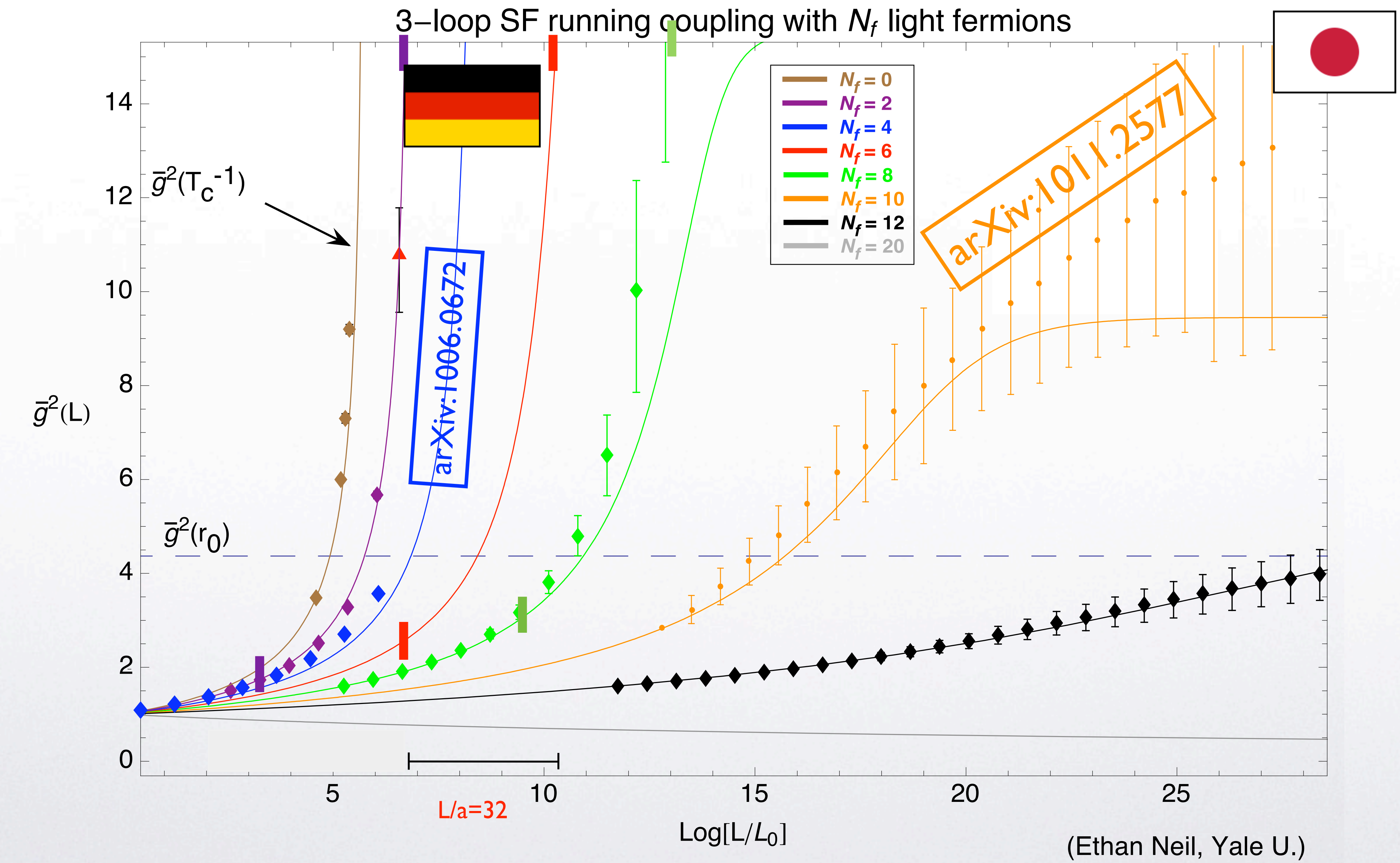
Can the running coupling be our guide?

- In QCD, $g(L)$ is asympotically free and runs rapidly until SSB and confinement: $g(L_c)=g_c$.
- As N_f increases, the running slows down.
- For large N_f, $g(L)$ flows to g_* at IR fixed point (IRFP). No SSB, no Technicolor.
- Walking theories may exist nearby theories with strongly-coupled IRFP: $g_* \lesssim g_c$.



- Unlike QCD, walking theories would have two dynamically generated scales: L_I and L_c, and in *rare* cases $L_I \ll L_c$.
- In Walking Technicolor, $L_I^{-1} = \Lambda_{ETC} \sim 1000$ TeV and $L_c^{-1} = \Lambda_{TC} \sim 1$ TeV.
- How does walking help Technicolor's FCNC problem?

Non-perturbative SF running coupling



- Results not yet confirmed in other non-pert. schemes.

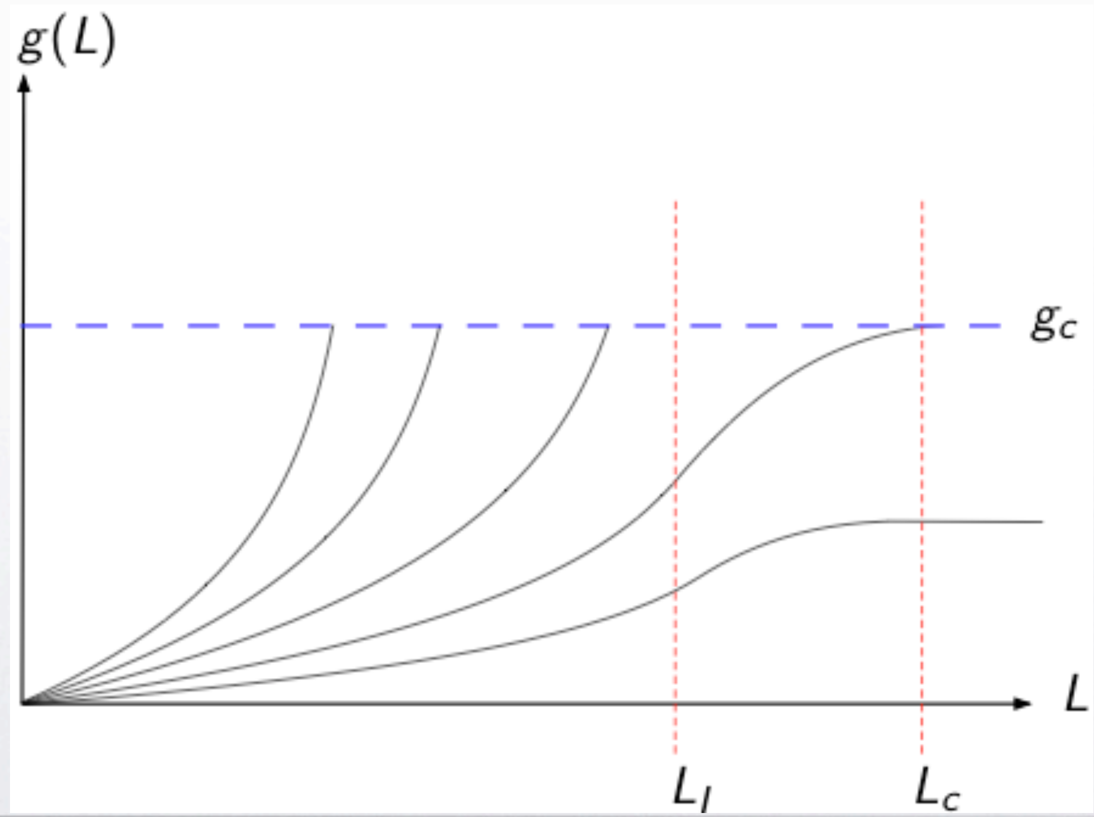
Walking Dynamics

- The relevant scale for mass generation is Λ_{ETC}, so the relevant condensate is renormalized at that scale: $\langle \overline{Q}Q \rangle$ at Λ_{ETC}.

$$\text{Masses: } \frac{(\overline{Q}Q)(\overline{q}q)}{\Lambda_{\rm ETC}^2} \quad \text{FCNC's: } \frac{(\overline{q}q)(\overline{q}q)}{\Lambda_{\rm ETC}^2} \quad \Lambda_{\rm ETC} \gtrsim 1000 \text{ TeV}$$

- The condensate is renormalized using the anomalous dimension $\gamma(\mu)$. In QCD-like theories, $\gamma(\mu) \ll 1$ for $\mu \gg \Lambda_{TC}$. Leads to $\log(\Lambda_{ETC} / \Lambda_{TC})$ enhancement.

$$\langle \overline{Q}Q \rangle_{\Lambda_{\rm ETC}} = \langle \overline{Q}Q \rangle_{\Lambda_{\rm TC}} \exp\left[\int_{\Lambda_{\rm TC}}^{\Lambda_{\rm ETC}} \frac{\gamma(\mu)}{\mu} d\mu\right]$$



- Walking dynamics ($\gamma \sim 1$) leads to power-enhanced condensates.

$$\frac{\langle \overline{Q}Q \rangle}{F_{\pi_T}^3} \sim \frac{\langle \overline{q}q \rangle}{f_\pi^3} \left(\frac{\Lambda_{\rm ETC}}{\Lambda_{\rm TC}}\right)^\gamma$$

- Now, a hierarchy of SM fermion masses can be generated while suppressing FCNC.

Lattice Strong Dynamics (LSD) Collaboration

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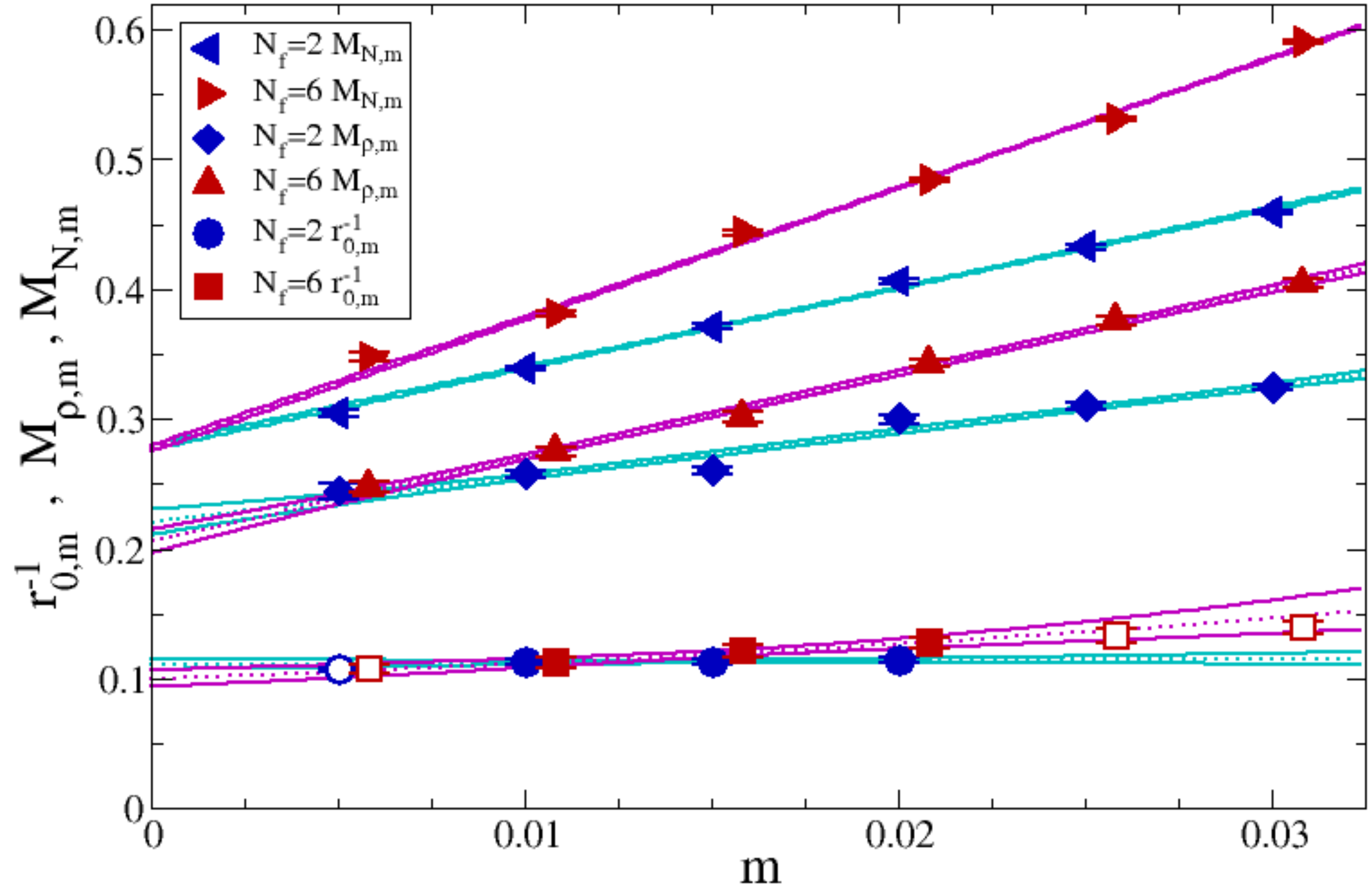
LSD: Comparing $N_f = 2$ and $N_f = 6$

- Why $N_f = 6$? It's very unlikely to walk...
- On largest computers, calculations still limited to lattices where $L / a \leq 64$.
- A walking theory should be studied on lattices where $L / a \sim 256–1024$.
- Can precursors to walking be seen in slowly running theories?



- Lattice scales chosen to match confinement scale physics to ~10%.

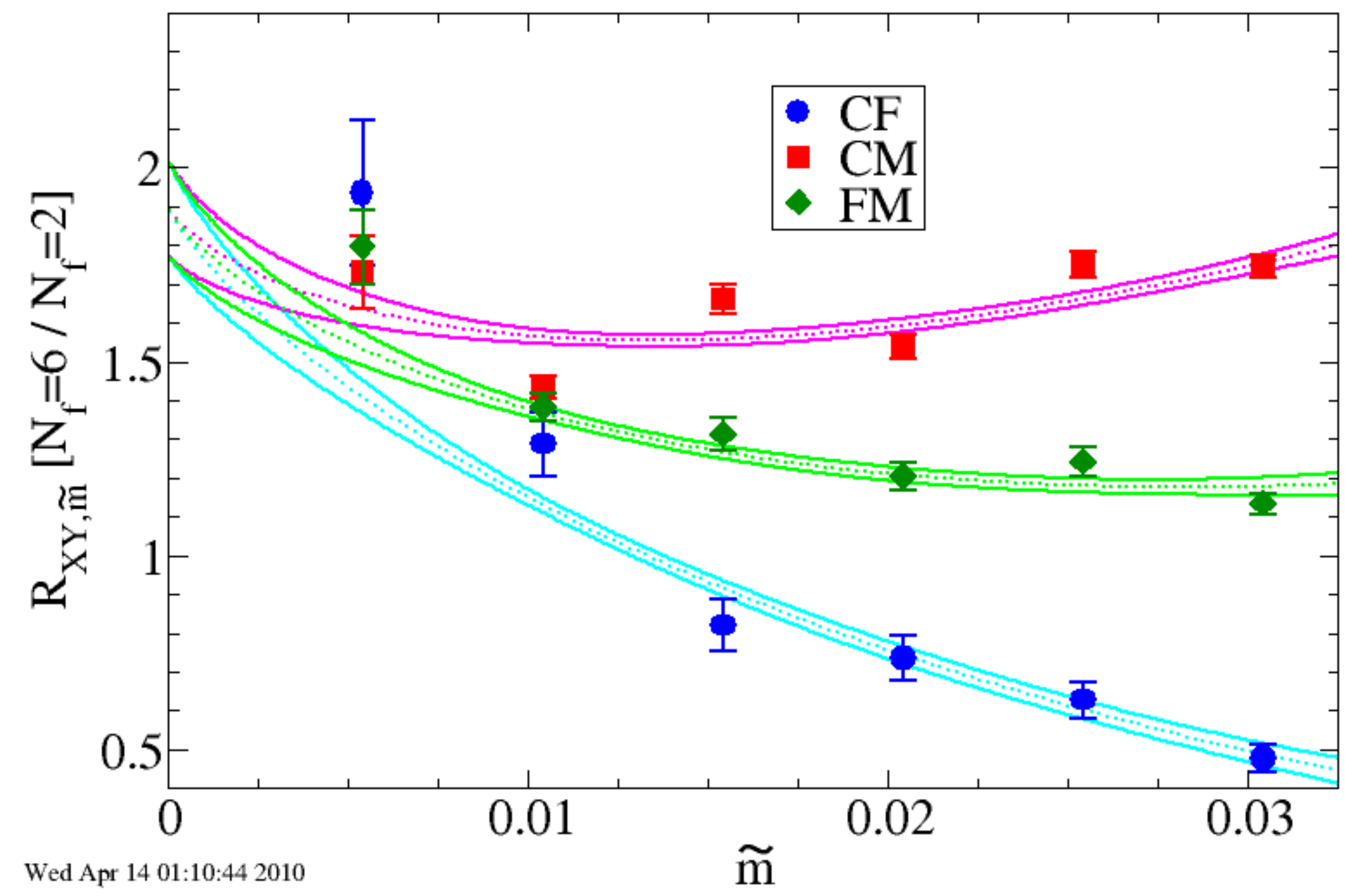
LSD: Condensate Enhancement

- Tricky to compare scale dependent quantities in two different theories.
- Definition of Enhancement:

$$\left.\frac{\langle\overline{\psi}\psi\rangle^{(N_f)}}{\langle\overline{\psi}\psi\rangle^{(2)}}\right|_{5M_\rho} \equiv \mathcal{R}(5M_\rho) \approx \frac{\exp\left(\int_{\alpha(5M_\rho)}^{\alpha(M_\rho)} \left.\frac{\gamma(\alpha)}{\pi\beta(\alpha)}\right|_{N_f} d\alpha\right)}{\exp\left(\int_{\alpha(5M_\rho)}^{\alpha(M_\rho)} \left.\frac{\gamma(\alpha)}{\pi\beta(\alpha)}\right|_{N_f=2} d\alpha\right)}$$

- GMOR Ratios

$$R = \overbrace{\frac{\langle\overline{\psi}\psi\rangle}{F_\pi^3}}^{\text{CF}} = \overbrace{\frac{M_\pi^3}{\sqrt{(2m)^3\langle\overline{\psi}\psi\rangle}}}^{\text{CM}} = \overbrace{\frac{M_\pi^2}{2mF_\pi}}^{\text{FM}} \quad \text{as} \quad m \to 0$$



- Chiral extrapolation

$$\mathcal{R}_{XY,\widetilde{m}} = \frac{R^{(N_f)}}{R^{(2)}}\left[1 + \widetilde{m}\left(\alpha_{XY10} + \alpha_{11}\log\widetilde{m}\right)\right], \quad \widetilde{m} = \sqrt{m^{(N_f)}m^{(2)}}$$

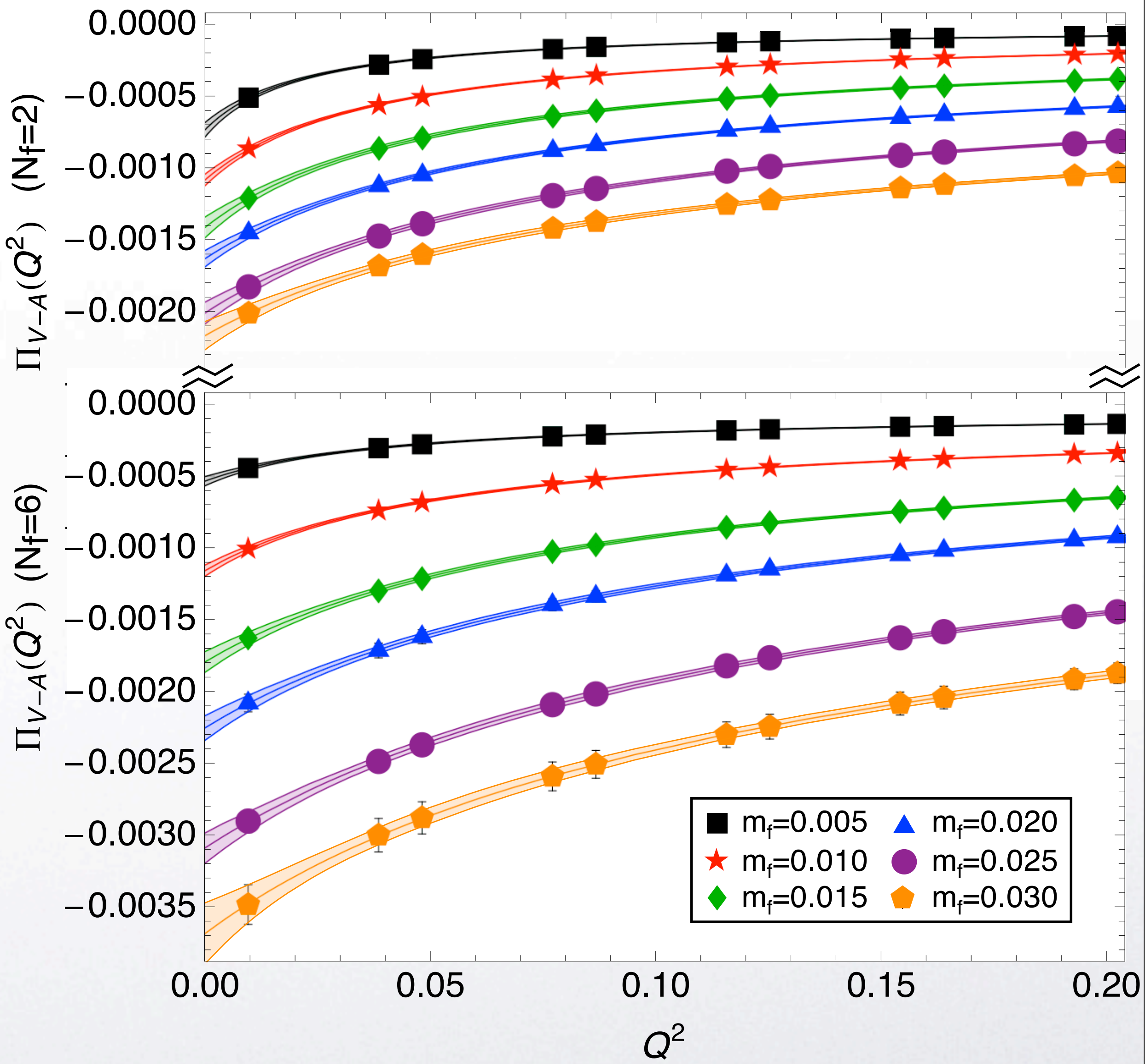
- Perturbative estimates of enhancement: $\mathcal{R}(5M_\rho) \sim 1.2\text{–}1.3$ (lat scheme)
- Enhancement bigger than expected. ***Is this a precursor to walking?***

LSD: Polarization Tensor for S Parameter

- S for N_f / 2 EW doublets

$$S = 4\pi \frac{N_f}{2} \left[\Pi'_{\mathrm{VV}}(0) - \Pi'_{\mathrm{AA}}(0)\right] + \Delta S_{\mathrm{SM}}$$
$$= \frac{1}{3\pi} \int_0^\infty \frac{ds}{s} \left\{ \frac{N_f}{2} \left[R_V(s) - R_A(s)\right] - \frac{1}{4} \left[1 - \left(1 - \frac{m_h^2}{s}\right)^3 \Theta(s - m_h^2)\right] \right\}$$

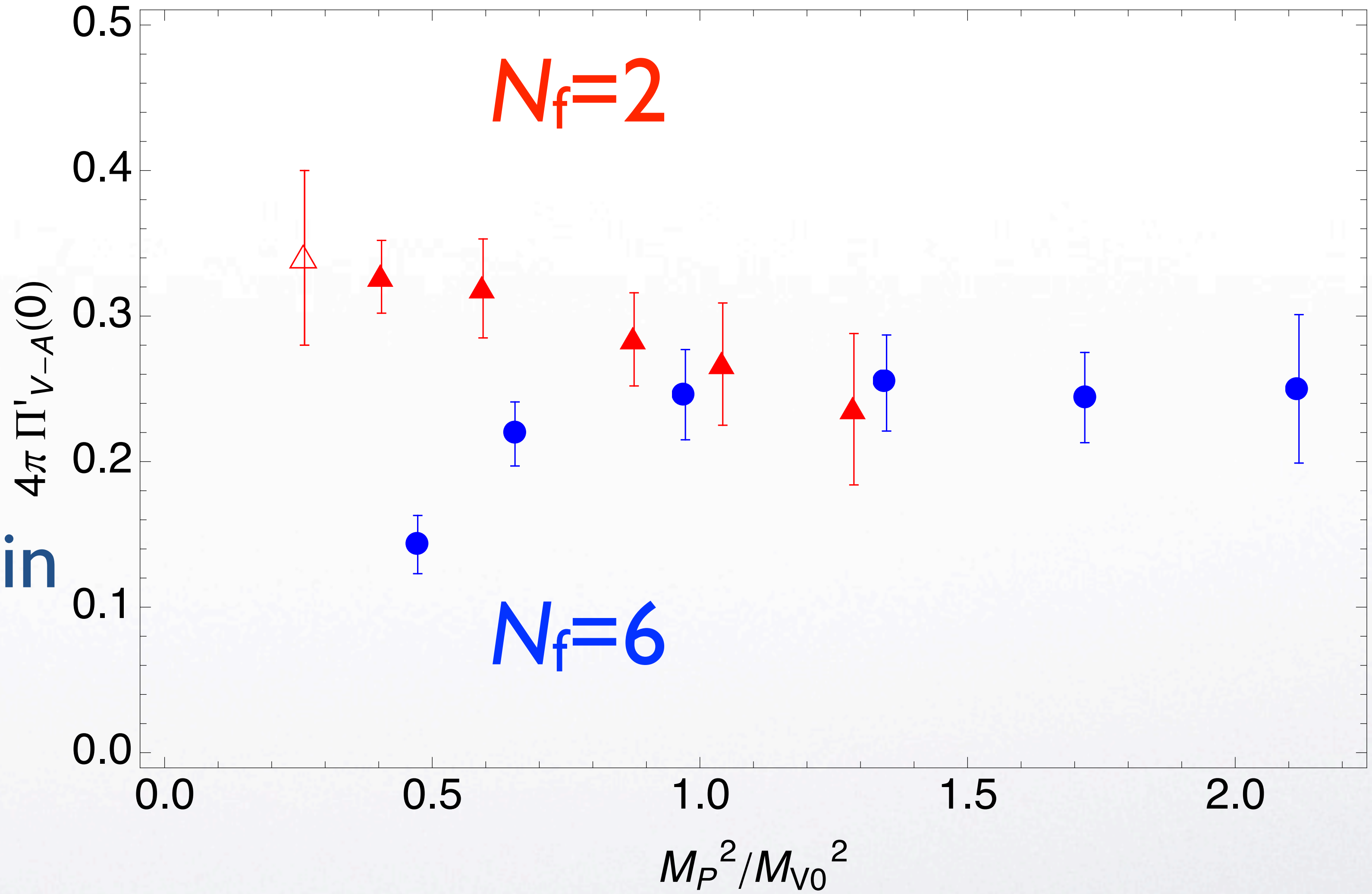
- Pade(1,2) fit of $\Pi_{V\text{-}A}(Q^2)$ assumes Q^{-2} scaling as $Q^2 \to \infty$ [1st WSR].
- Slope shows decreasing trend with decreasing mass for $N_f = 6$.
- *n.b.* smaller S for fewer EW doublets.



D. Schaich & E. Neil

LSD: Flavor dependence of $\Pi'_{V\text{-}A}(0)$

- Polarization tensor computed for one EW doublet.
- Filled symbols $M_P \cdot L \geq 4$.
- Plot *vs.* M_P^2 instead of m, in units of M_{V0}.
- $\Pi' \sim \log M_P^2$ as $M_P^2 \to 0$.



- Free field value for $\Pi' = 1/2\pi = 0.159\ldots$

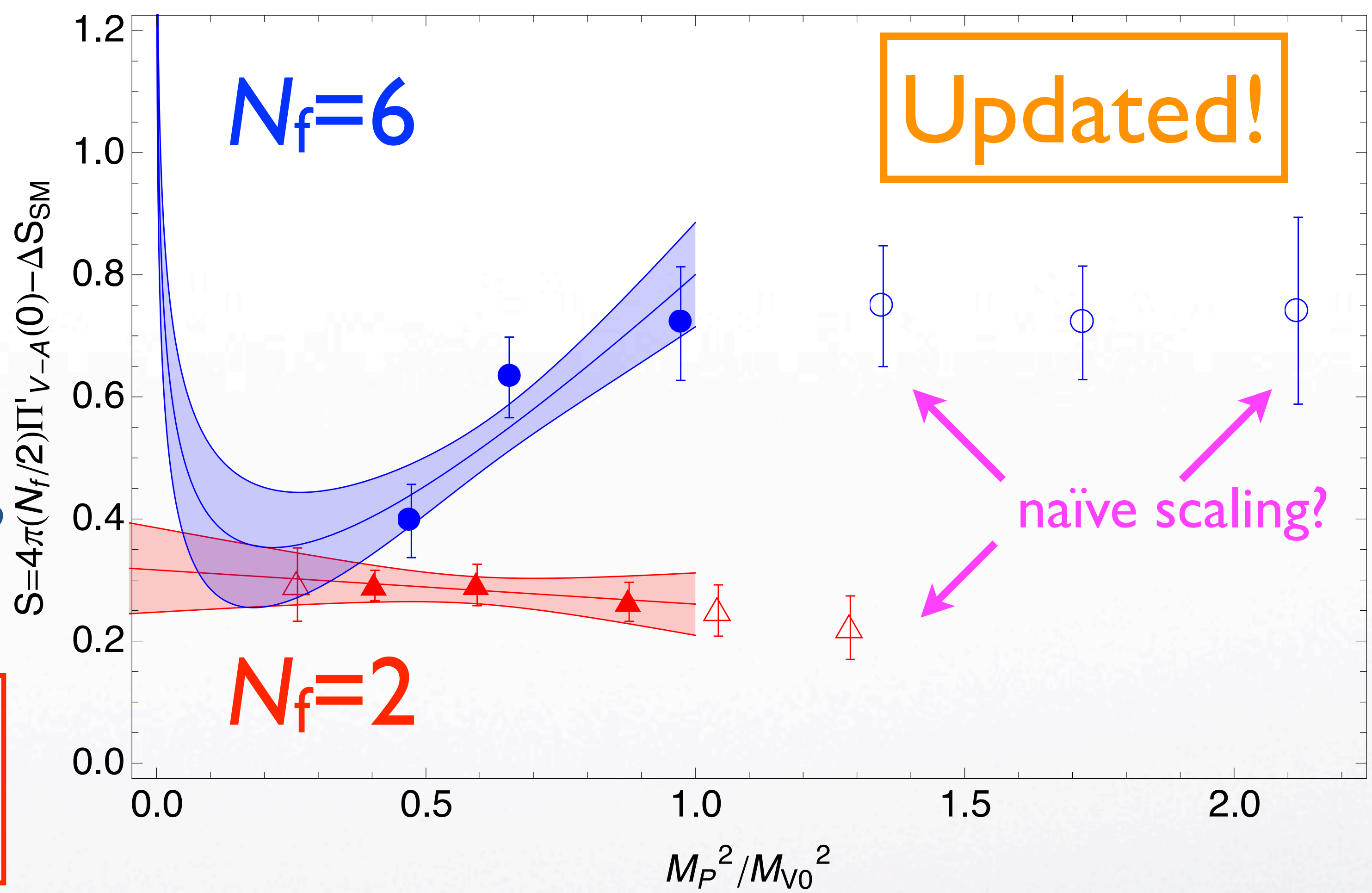
Flavor dependence of S Parameter

- Very naïve scaling for S

$$S \propto \frac{N_f}{2}\frac{N_c}{3}$$

- Walking conjectured to reduce S by parity doubling, e.g. single-pole dominance:

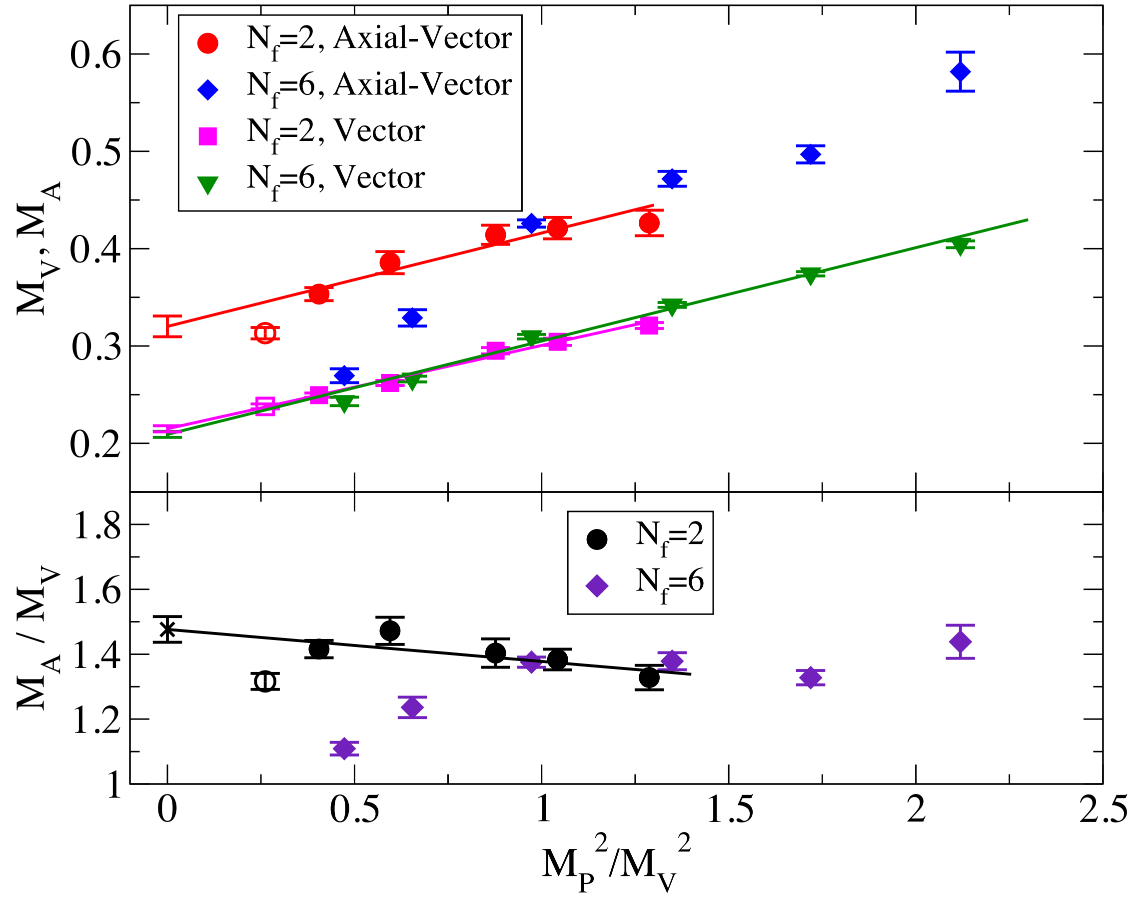
$$S \sim 4\pi\left(\frac{N_f}{2}\right)\left[\frac{F_V^2}{M_V^2} - \frac{F_A^2}{M_A^2}\right]$$

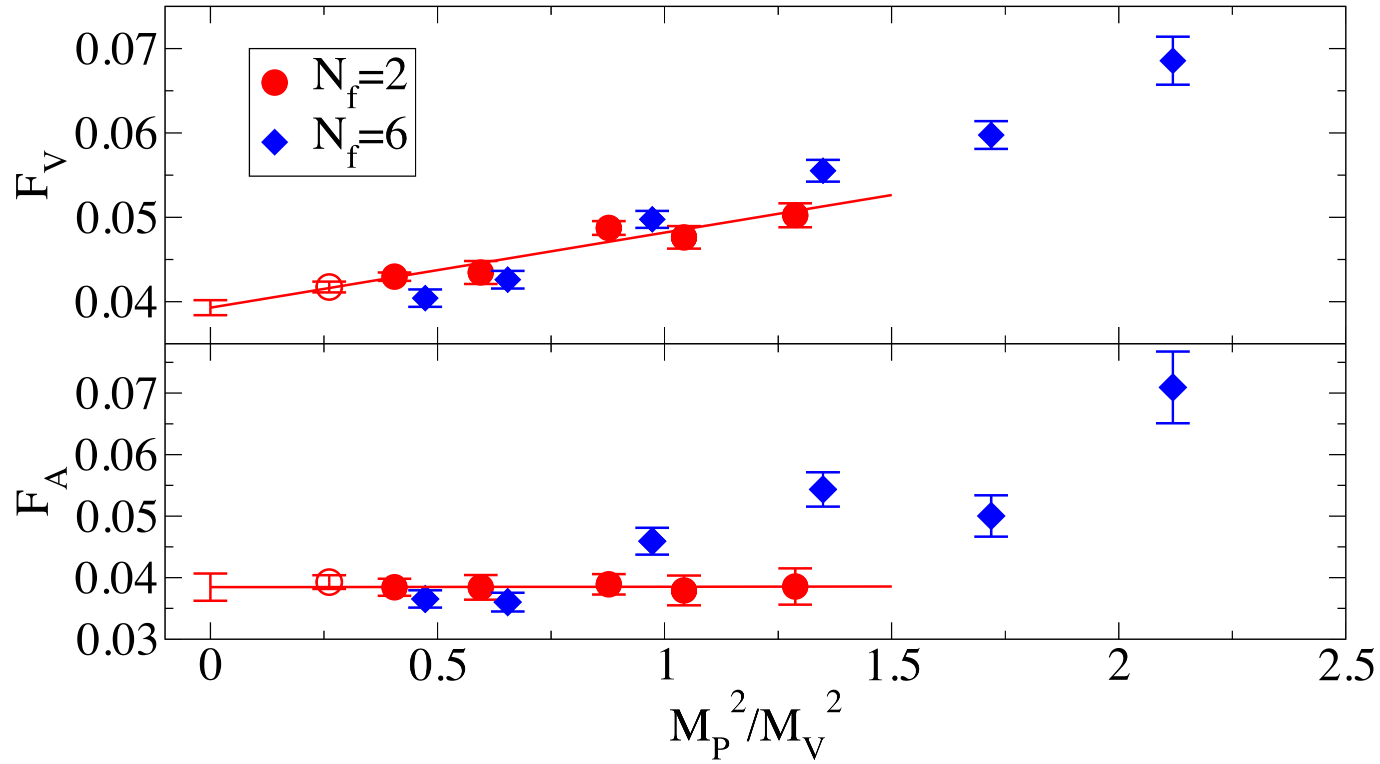


- After ΔS_{SM} subtraction, S reduced relative to naïve scaling for N_f=6. ***Is it a precursor of walking behavior?***

- *n.b.* S for N_f=6 still log divergent until spectrum of PNGB's fixed.

Flavor dependence of parity partners





- Note slope of M_V *vs.* M_P^2 roughly independent of N_f, not true for M_V *vs.* m.

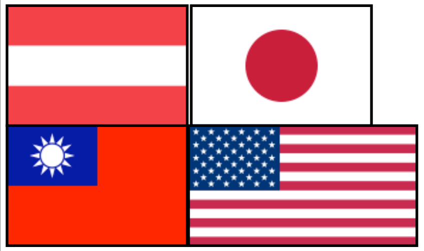
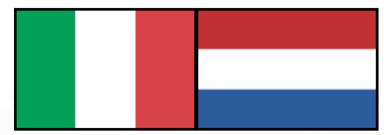
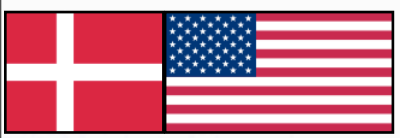
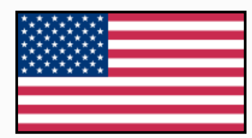
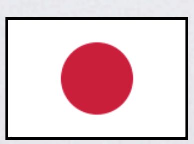
arXiv:1009.5967 [hep-ph]

Conclusions

- For SU(3) running coupling studies for various N_f suggest a walking theory may exist for $8 < N_f < 12$ flavors.
- Direct study of walking theories beyond the current capabilities of current computers, algorithms, ...
- Searches for precursors of walking behavior as the running slows with increasing N_f support the vision that a walking theory can solve Technicolor's phenomenological problems.
- For $N_f = 6$, non-perturbative condensates are enhanced and S parameter reduced relative to perturbative expectations.
- Technicolor remains a viable option for physics at the TeV scale.
- New results in two weeks: https://latt11.llnl.gov/

Backup Slides

A Dozen Lattice BSM Efforts Worldwide

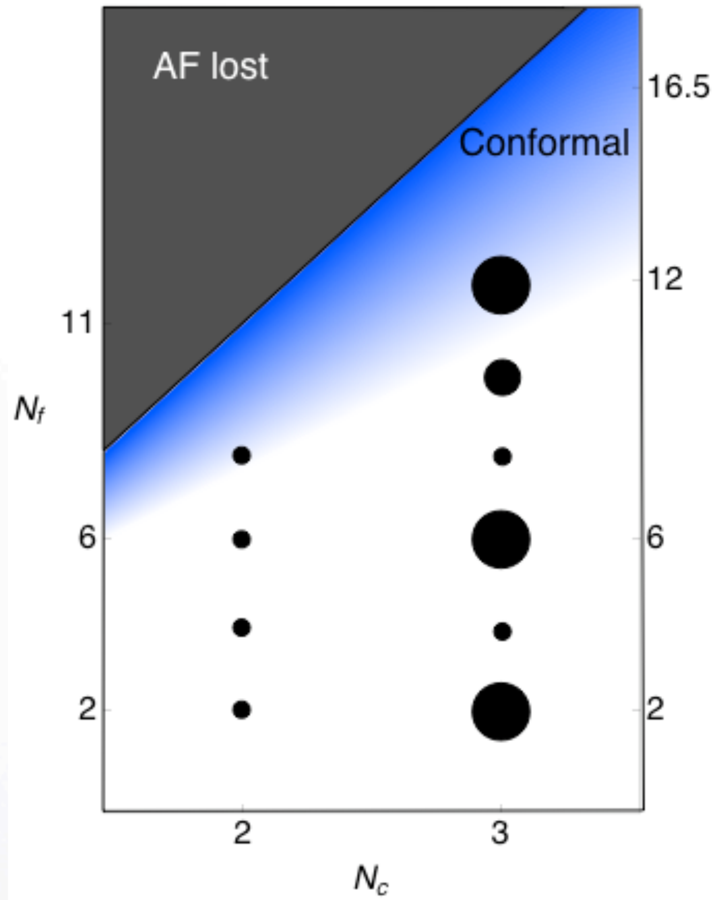
	Aoyama et al.		DeGrand et al.
	Del Debbio et al.		Deuzeman et al.
	Catteral et al.		**LSD**
	Hietanen et al.		A. Hasenfratz
	**LHC**		Jin-Mawhinney
	Yamada et al.		Kogut-Sinclair

Technicolor on the Lattice (II)

- Tools developed for study of Lattice QCD:
  - Non-perturbative Running Coupling
  - Non-perturbative Renormalization of Operators
  - Light Hadron and Glueball Spectrum
  - Chiral Observables (condensate, Dirac eigenvalues)
  - Thermodynamic Observables (T_c , EoS)
- Are tools optimized for QCD useful for non-QCD studies?
  - Exception: Monte Carlo methods using Wilsonian RG?
  - Can finite-size scaling methods be adapted from stat. mech.?

LSD Program Overview

- SU(2) and SU(3) gauge theories with N_f domain wall fundamental fermions.
- Initial focus on SU(3): code readiness and QCD experience.
- Preparing SU(2) code for production.
- Majority of flops so far spent on SU(3) with N_f=2,6,10.



- Exploration of IR: QCD-like, conformal or "walking".
- Phenomenology: S parameter, condensate enhancement.
- One PRL, recent preprint: arXiv:1009.5967 [hep-ph]

Flavor dependence of NLO ChiPT

$$M_\pi^2 = 2mB\left\{1+\frac{2mB}{(4\pi F)^2}\left[2\alpha_8-\alpha_5+N_f\left(2\alpha_6-\alpha_4\right)+\frac{1}{N_f}\log\frac{2mB}{(4\pi F)^2}\right]\right\}$$

$$F_\pi = F\left\{1+\frac{2mB}{(4\pi F)^2}\left[\frac{1}{2}\left(\alpha_5+N_f\alpha_4\right)-\frac{N_f}{2}\log\frac{2mB}{(4\pi F)^2}\right]\right\}$$

$$\langle\bar{q}q\rangle = F^2B\left\{1+\frac{2mB}{(4\pi F)^2}\left[\frac{1}{2}\left(2\alpha_8+\eta_2\right)+2N_f\alpha_6-\frac{N_f^2-1}{N_f}\log\frac{2mB}{(4\pi F)^2}\right]\right\}$$

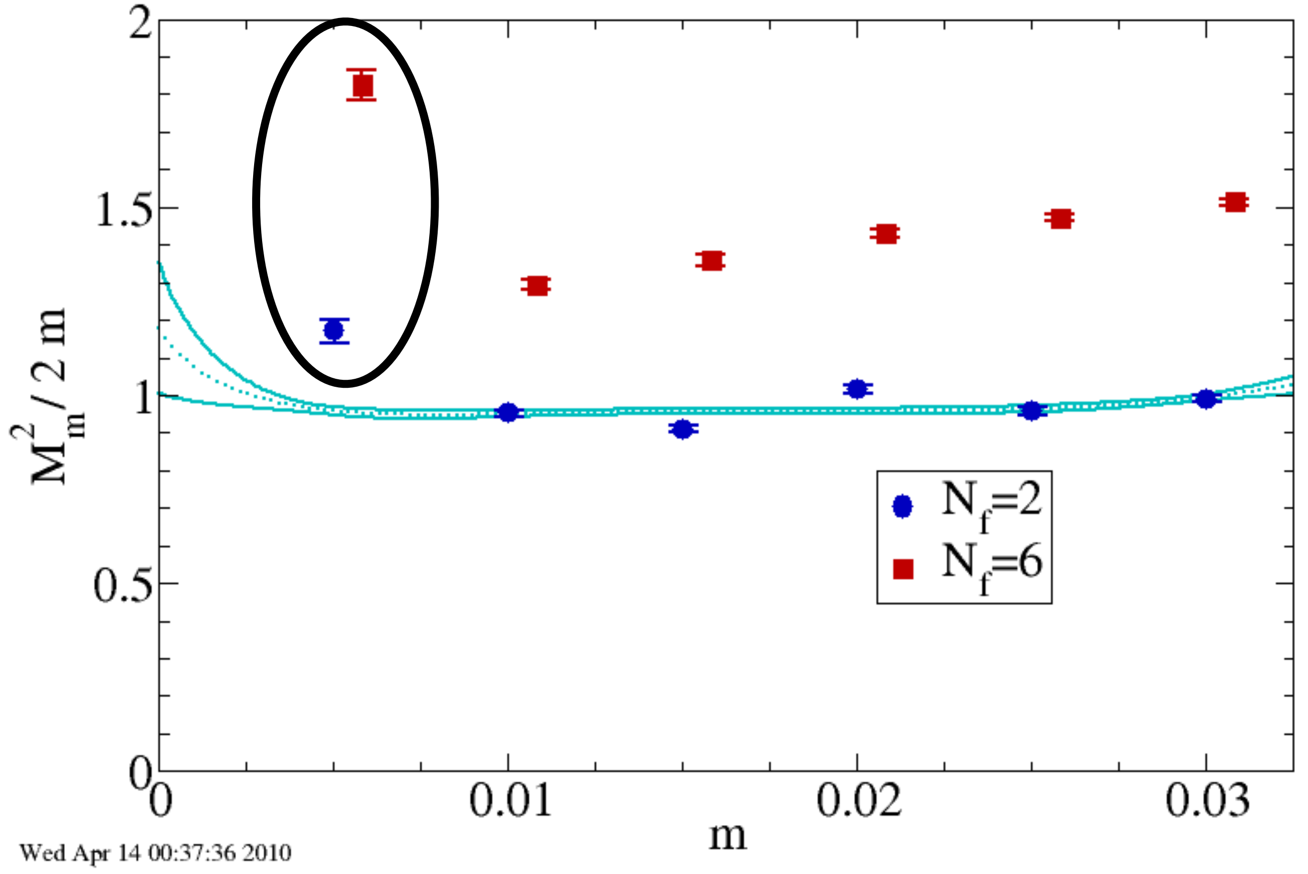
- The leading non-analytic terms are enhanced in the condensate and f_π but suppressed in $(M_\pi)^2$.
- The $\alpha_i \sim O(1)$ low energy constants.
- $\eta_2 \sim O(a^{-2})$ contact term: UV-sensitive slope for condensate.

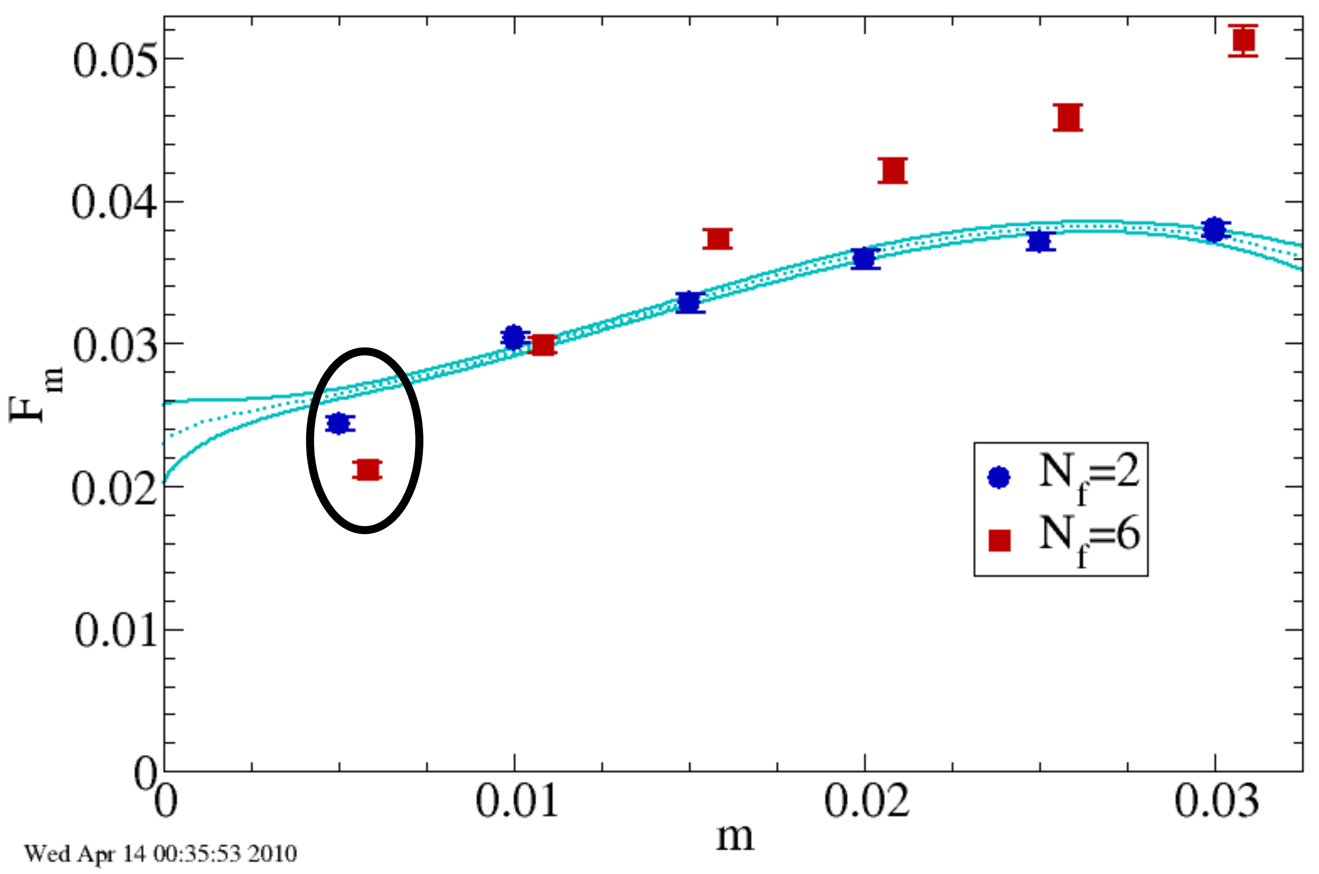
Non-analytic flavor factors in NNLO ChiPT

	$m \log(m)$	$m^2 \log^2(m)$
M_π^2	N_f^{-1}	$-3/8\, N_f^2 + 1/2 - 9/2\, N_f^{-2}$
F_π	$-1/2\, N_f$	$3/16\, N_f^2 + 1/2$
$\langle qq \rangle$	$-N_f + N_f^{-1}$	$3/2 - 3/2\, N_f^{-2}$

- J. Bijnens and J. Lu, JHEP11(2009)116 [arXiv:0910.5424]
- Small NLO coeff for M_π^2 is not generic and doesn't persist to higher orders.
- Can NNLO formulae help us extrapolate $N_f \gg 2$ results?

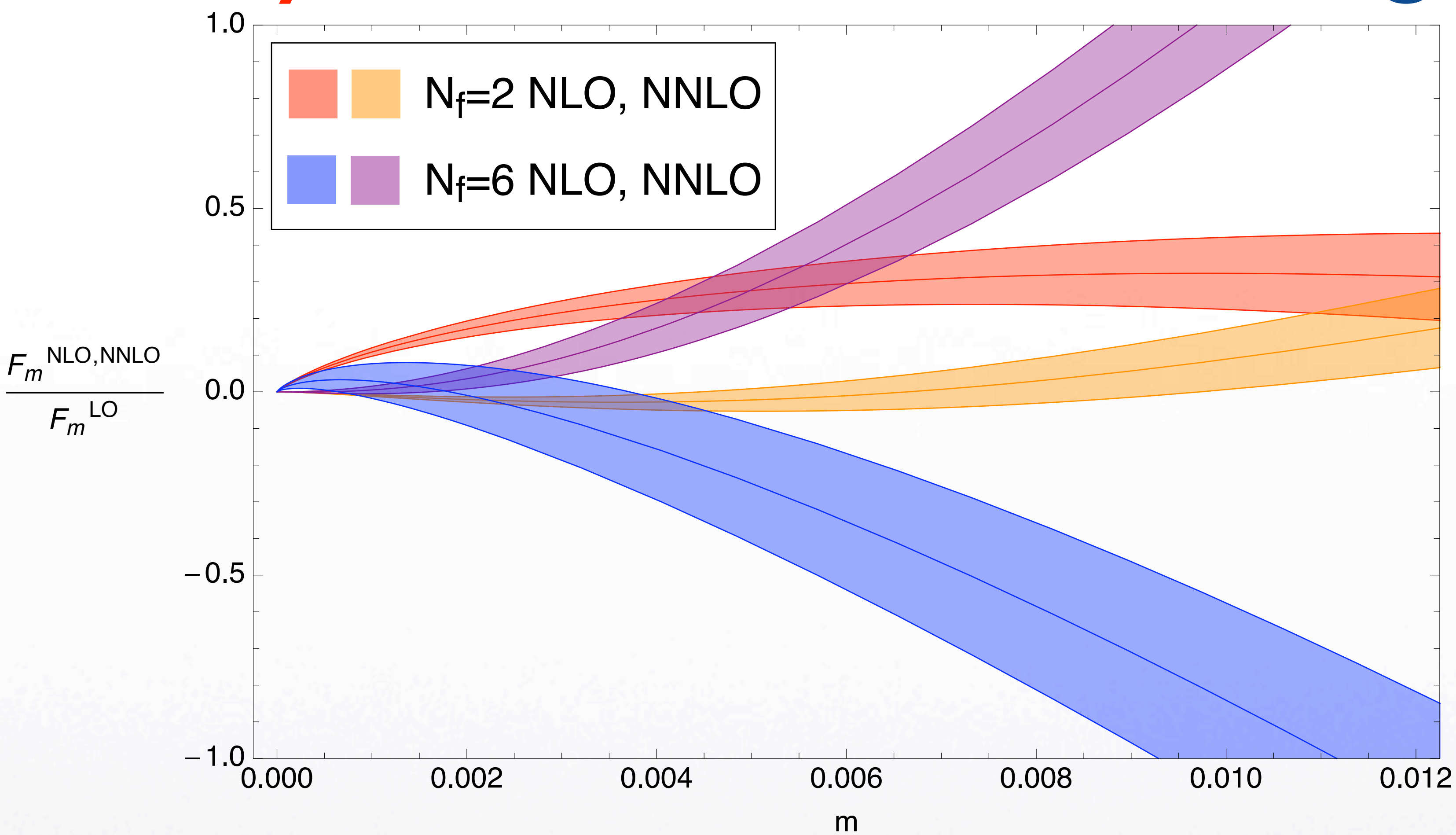
Preliminary: Basic Chiral Observables





- NNLO ChiPT fits work fine for Nf=2.
- NNLO expression for general Nf recently derived by Bijnens and Lu [JHEP11(2009)116].

Preliminary: χPT Radius of Convergence



- Smaller quark masses needed for reliable NNLO extrapolation for $N_f > 2$ [E.T. Neil *et al.*, PoS(CD09)088].
- On $32^3 \times 64$, $m \cong 0.01$: $M_\pi \cdot L \sim 4$ and $F_\pi \cdot L \sim 1$. $48^3 \times 64$ lattices needed to reach smaller quark masses.