Plasma Effects due to Ionization in Muon Cooling Channels

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Overview

1. Motivation
   A. Rational: Scientific Unknowns
      - Soon to be knowns?
   B. Emotional: Fear!!!
      - Is this a show-stopper?
2. Potentially Problematic Plasma Effects
   A. Plasma Formation / Ionization
   B. Plasma Beam Loading
   C. Dark Current
   D. Beam Instabilities
   E. Avalanche Formation (*incomplete*)
3. Conclusions

Motivations

- Plasma means free electrons
  - Electrons are light and move easily
  - Electron clouds have been known to cause problems
- Plasma can drive instabilities in beam
  - Beam induces wake in plasma, which in turn produces undesired fields

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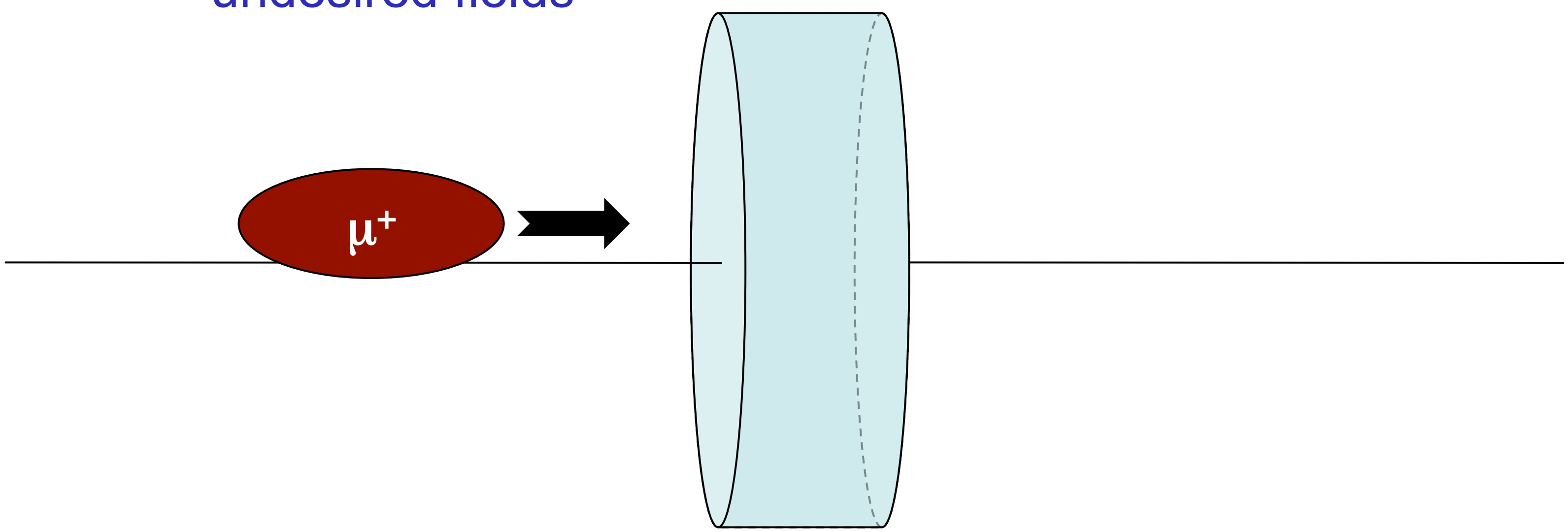

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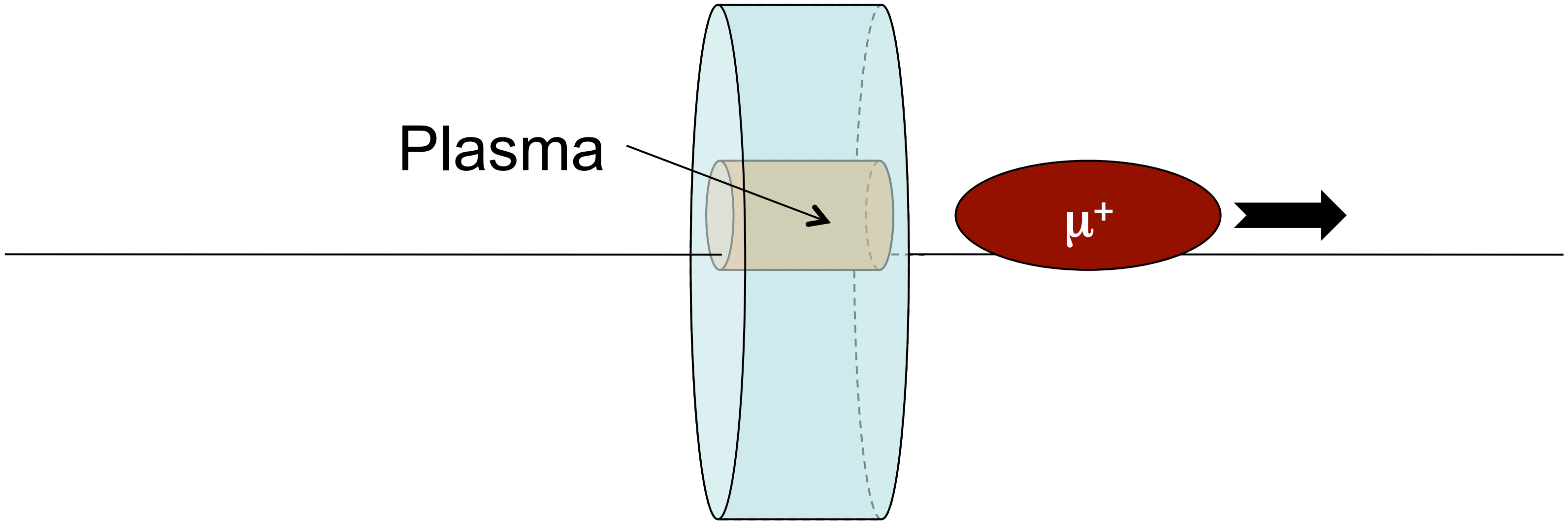

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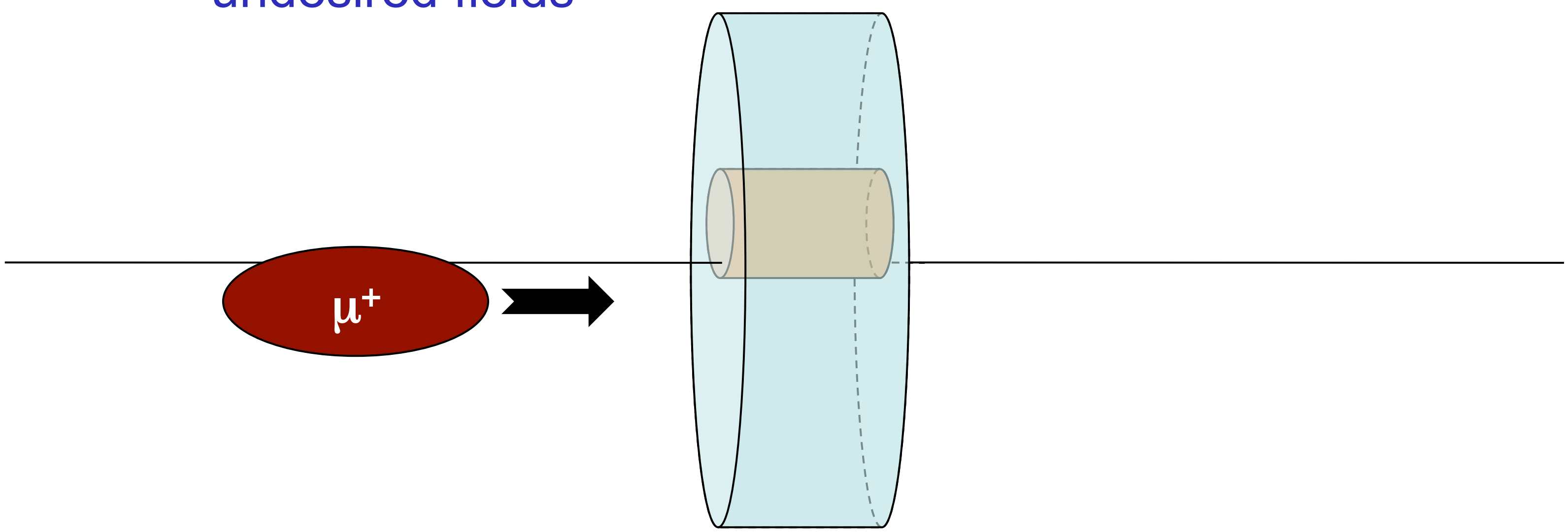

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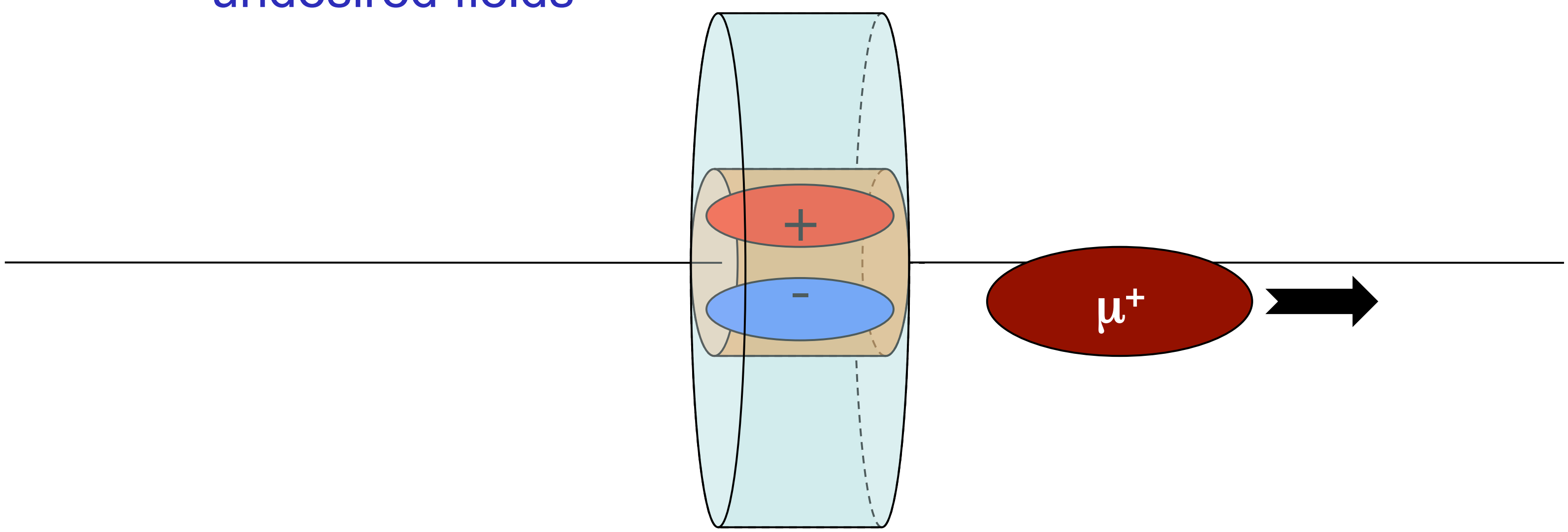

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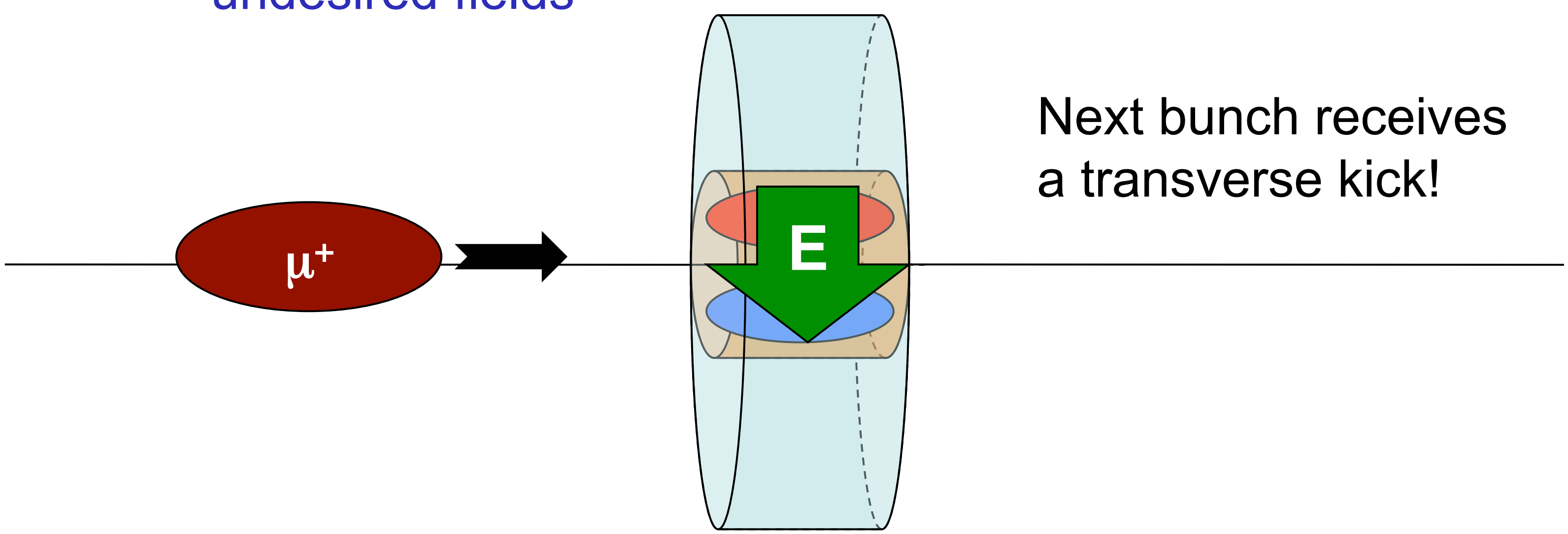

Motivations

- It's a complicated set of interdependent processes
  - Impact ionization (multiple levels of ionization)
  - Electron diffusion in the plasma
  - Electron-ion recombination (multiple ways)
- "Large" uncertainties
  - Large enough to cause concern
  - Little confidence in ionization/recombination cross sections
    - Need experiments!
  - Potentially lots of processes (coupled) to model
    - What if you forget one?
    - Difficult to model

Beam-Induced Plasma Formation

- Impact ionization leads to plasma formation

$$\frac{dn_e}{dt} = n_{beam}\, n_{mat}\, \sigma_i(v_{beam})v_{beam} \equiv F_e$$

- Plasma dissipated by other processes

$$\frac{dn_e}{dt} = -\Omega\, n_e$$

  - Recombination
  - Diffusion

$$\Big\} \; \Omega \equiv \Omega_D + \Omega_R \; \begin{cases} \Omega_D = \dfrac{D_e}{r_{beam}^2} \\ \Omega_R = n_{mat}\, \sigma_R(v_e)v_e \end{cases}$$

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$$\left.\begin{matrix} \text{Recombination} \\ \text{Diffusion} \end{matrix}\right\} \Omega \equiv \Omega_D + \Omega_R \left\{ \begin{aligned} \Omega_D &= \frac{D_e}{r_{beam}^2} \\ \Omega_R &= n_{mat}\, \sigma_R(v_e) v_e \end{aligned} \right.$$

Recombination tends to dominate!

Plasma Density Evolution

- Until the plasma density gets very large…

$$\frac{dn_e}{dt} = F_e - \Omega\, n_e$$

- Largest possible density is easy to find

$$\lim_{t\to\infty} n_e(t) \equiv n_\infty = \frac{F_e}{\Omega}$$

which is approached for an infinitely long beam (CW).

References

Ionization Cross Sections

[1] Igor D. Kaganovich, Edward Startsev and Ronald C. Davidson. *Scaling and formulary of cross sections for ion-atom impact ionization.* New Journal of Physics 8 (2006), 278.

[2] M. Reiser. *Theory and design of charged particle beams*. New York: Wiley (1994), 273-8.

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LiH Recombination

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Drift Velocities in Hydrogen

[6] United Kingdom National Physical Laboratory, *Kaye & Laby Table of Physical & Chemical Constants*, `http://www.kayelaby.npl.co.uk/atomic_and_nuclear_physics/4_4/4_4_2.html`.

CW Beam Maximum Plasma Densities

- Consider HP GH_2, LH_2, and LiH absorbers
- Consider a CW muon beam before and after cooling

	N	Radius	KE	n_{beam}
Initial	$55. \times 10^{12}$	0.15 m	150 MeV	4.3×10^{7} m^{-3}
Final	3.3×10^{12}	0.01 m	150 MeV	5.8×10^{8} m^{-3}

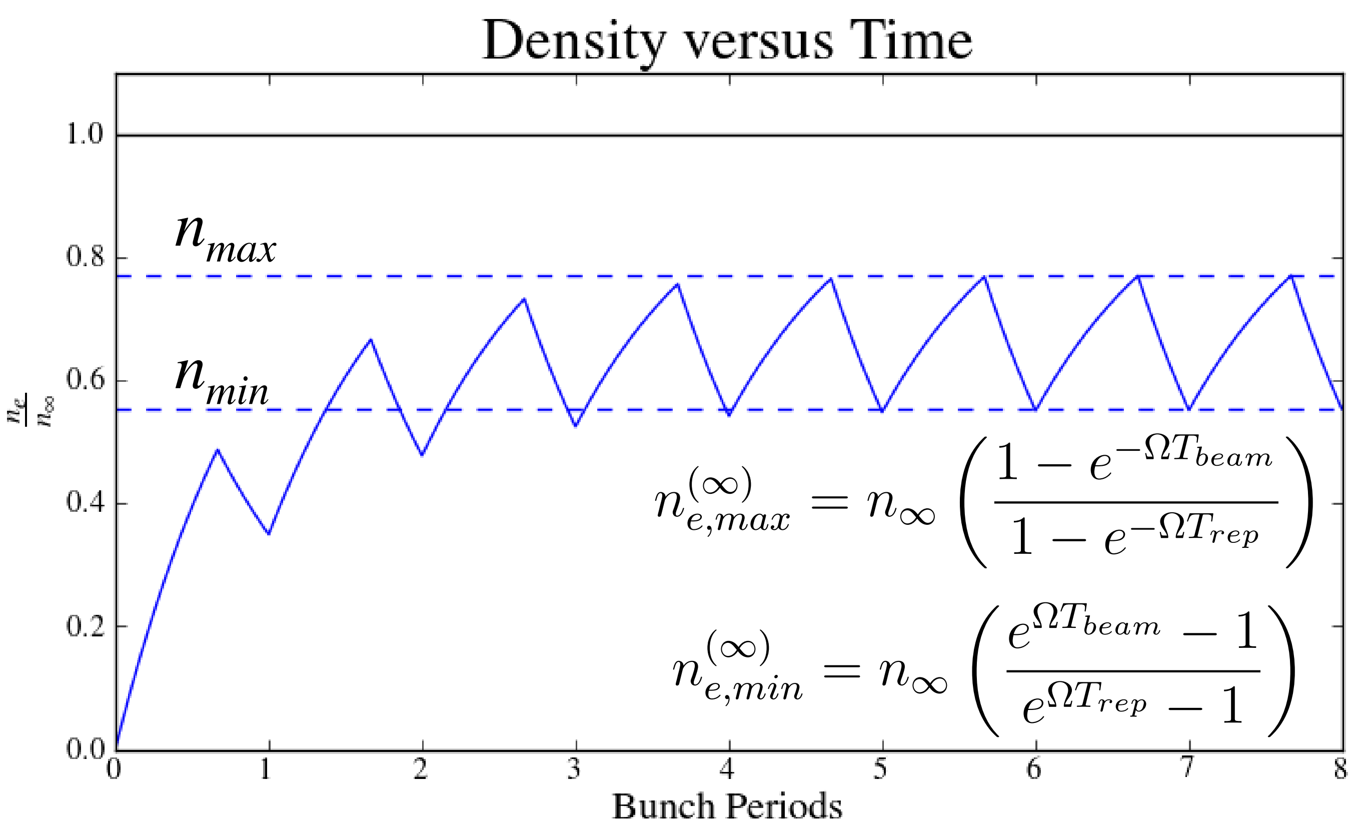
- Maximum (CW) Densities in each material

Comparable!

	HP GH_2	LH_2	LiH
Initial	4.7×10^{6} m^{-3}	4.7×10^{6} m^{-3}	3.0×10^{7} m^{-3}
Final	6.3×10^{7} m^{-3}	6.3×10^{7} m^{-3}	4.1×10^{8} m^{-3}

Bunched-Beam Plasma Evolution

- Assume $F_e(t)$ is a periodic step-function…



Bunched-Beam Plasma Evolution

- Assume a 201.25 MHz micro-bunching (20 bunches) and a single-pulse approximation to initial beam

	N	T_{bunch}	n_{beam}
Micro	2.75×10^{12}	1 ns	1.4×10^{14} m^{-3}
Pulse	$55. \times 10^{13}$	100 ns	2.8×10^{13} m^{-3}

- Assume a train of 805 MHz bunches at 15 Hz for the final beam

	N	T_{bunch}	n_{beam}
Pulse	3.3×10^{12}	0.5 ns	7.7×10^{16} m^{-3}

Max/Min Bunched-Beam Plasma Densities

- Initial and final beam approximations:

n_{max}	HP GH_2	LH_2	LiH
Micro	1.6×10^{13} m^{-3}	1.6×10^{13} m^{-3}	1.0×10^{14} m^{-3}
Pulse	3.1×10^{12} m^{-3}	3.1×10^{12} m^{-3}	2.0×10^{13} m^{-3}
Final	8.4×10^{15} m^{-3}	8.4×10^{15} m^{-3}	5.5×10^{16} m^{-3}

n_{min}	HP GH_2	LH_2	LiH
Micro	0	0	0
Pulse	0	0	0
Final	0	0	0

Plasma densities much less than material densities (~10^{28} m^{-3}), comparable with beam densities!

No residual plasma between bunches!

RF Plasma Currents

- What if the plasma is driven by RF?
  - For example, in HP GH_2 filled RF cavities
  - RF cavity will accelerate free electrons easily
  - Predicted ionization levels are not "full ionization"
    - Could seed an avalanche
    - Would expect Paschen's Law to still apply
- Can predict currents in 15 MV/m fields
  - Drift velocity of electrons given by mobility in gas
  - With drift velocity, ratio of currents can be computed

$$\frac{I_e}{I_{beam}} = \frac{n_e \, v_e}{n_{beam} \, v_{beam}}$$

	HPG	LH2
v_e	7.4×10^3 m/s	5.5×10^3 m/s

Small!!!

Beam-Driven Plasma Currents

- Energetic beam fields can drive further ionization
  - Can lead to an avalanche!

$$\dot{n}_{2+} = (j_b/e)\sigma_{2i} n_{20} + n_e n_{20} \alpha_{2i} - n_e n_{2+} (\alpha_{2dr} + \alpha_{2cr}) + n_{20} n_{1+} \alpha_{cx} .$$

$$\dot{n}_{10} = -(j_b/e)\sigma_{1i} n_{10} + 2n_e n_{20} \alpha_{2d} + 2n_e n_{2+} \alpha_{2dr} - n_e n_{10} \alpha_{1i} + n_{20} n_{1+} \alpha_{cx} + n_e n_{1+} \alpha_{1cr} .$$

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Beam-Driven Plasma Currents

- Beam-induced Impact Ionization:

$$\dot{n}_{2+} = \boxed{(j_b/e)\sigma_{2i}n_{20}} + n_e n_{20}\alpha_{2i} - n_e n_{2+}(\alpha_{2dr} + \alpha_{2cr}) + n_{20}n_{1+}\alpha_{cx}\,.$$

$$\dot{n}_{10} = -\boxed{(j_b/e)\sigma_{1i}n_{10}} + 2n_e n_{20}\alpha_{2d} + 2n_e n_{2+}\alpha_{2dr} - n_e n_{10}\alpha_{1i} + n_{20}n_{1+}\alpha_{cx} + n_e n_{1+}\alpha_{1cr}\,.$$

$$\dot{n}_{1+} = \boxed{(j_b/e)\sigma_{1i}n_{10}} + n_e n_{10}\alpha_{1i} - n_{20}n_{1+}\alpha_{cx} - n_e n_{1+}\alpha_{1cr}$$

Beam-Driven Plasma Currents

- Free-electron-induced Impact Ionization:

$$\dot{n}_{2+} = (j_b/e)\sigma_{2i} n_{20} + \boxed{n_e n_{20} \alpha_{2i}} - n_e n_{2+}(\alpha_{2dr} + \alpha_{2cr}) + n_{20} n_{1+} \alpha_{cx} .$$

$$\dot{n}_{10} = -(j_b/e)\sigma_{1i} n_{10} + 2n_e n_{20}\alpha_{2d} + 2n_e n_{2+}\alpha_{2dr} - \boxed{n_e n_{10} \alpha_{1i}} + n_{20} n_{1+}\alpha_{cx} + n_e n_{1+}\alpha_{1cr} .$$

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Beam-Driven Plasma Currents

- Electron-ion Recombination:

$$\dot{n}_{2+} = (j_b/e)\sigma_{2i}n_{20} + n_e n_{20}\alpha_{2i} - \boxed{n_e n_{2+}(\alpha_{2dr} + \alpha_{2cr})} + n_{20}n_{1+}\alpha_{cx} .$$

$$\dot{n}_{10} = -(j_b/e)\sigma_{1i}n_{10} + 2n_e n_{20}\alpha_{2d} + \boxed{2n_e n_{2+}\alpha_{2dr}} - n_e n_{10}\alpha_{1i} + n_{20}n_{1+}\alpha_{cx} + \boxed{n_e n_{1+}\alpha_{1cr}} .$$

$$\dot{n}_{1+} = (j_b/e)\sigma_{1i}n_{10} + n_e n_{10}\alpha_{1i} - n_{20}n_{1+}\alpha_{cx} - \boxed{n_e n_{1+}\alpha_{1cr}}$$

Beam-Driven Plasma Currents

- Molecular Disassociation:

$$\dot{n}_{2+} = (j_b/e)\sigma_{2i}n_{20} + n_e n_{20}\alpha_{2i} - n_e n_{2+}(\alpha_{2dr} + \alpha_{2cr})$$
$$+ n_{20}n_{1+}\alpha_{cx} .$$

$$\dot{n}_{10} = -(j_b/e)\sigma_{1i}n_{10} + \boxed{2n_e n_{20}\alpha_{2d}} + 2n_e n_{2+}\alpha_{2dr}$$
$$- n_e n_{10}\alpha_{1i} + n_{20}n_{1+}\alpha_{cx} + n_e n_{1+}\alpha_{1cr} .$$

$$\dot{n}_{1+} = (j_b/e)\sigma_{1i}n_{10} + n_e n_{10}\alpha_{1i} - n_{20}n_{1+}\alpha_{cx} - n_e n_{1+}\alpha_{1cr}$$

Beam-Driven Plasma Currents

- Charge Exchange:

$$\dot{n}_{2+} = (j_b/e)\sigma_{2i}n_{20} + n_e n_{20}\alpha_{2i} - n_e n_{2+}(\alpha_{2dr} + \alpha_{2cr}) + \boxed{n_{20}n_{1+}\alpha_{cx}}.$$

$$\dot{n}_{10} = -(j_b/e)\sigma_{1i}n_{10} + 2n_e n_{20}\alpha_{2d} + 2n_e n_{2+}\alpha_{2dr} - n_e n_{10}\alpha_{1i} + \boxed{n_{20}n_{1+}\alpha_{cx}} + n_e n_{1+}\alpha_{1cr}.$$

$$\dot{n}_{1+} = (j_b/e)\sigma_{1i}n_{10} + n_e n_{10}\alpha_{1i} - \boxed{n_{20}n_{1+}\alpha_{cx}} - n_e n_{1+}\alpha_{1cr}$$

Conclusions

- Plasma densities comparable with beam densities
- Recombination time scales much shorter than beam time scales
  - No residual plasma left in material between pulses
  - No beam instabilities driven
- RF-driven plasma currents small compared with beam current
  - No beam loading
- Avalanche possible, but maybe avoidable
  - Requires further investigation
- Largest uncertainties are in cross sections
  - Need to check with experiment

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