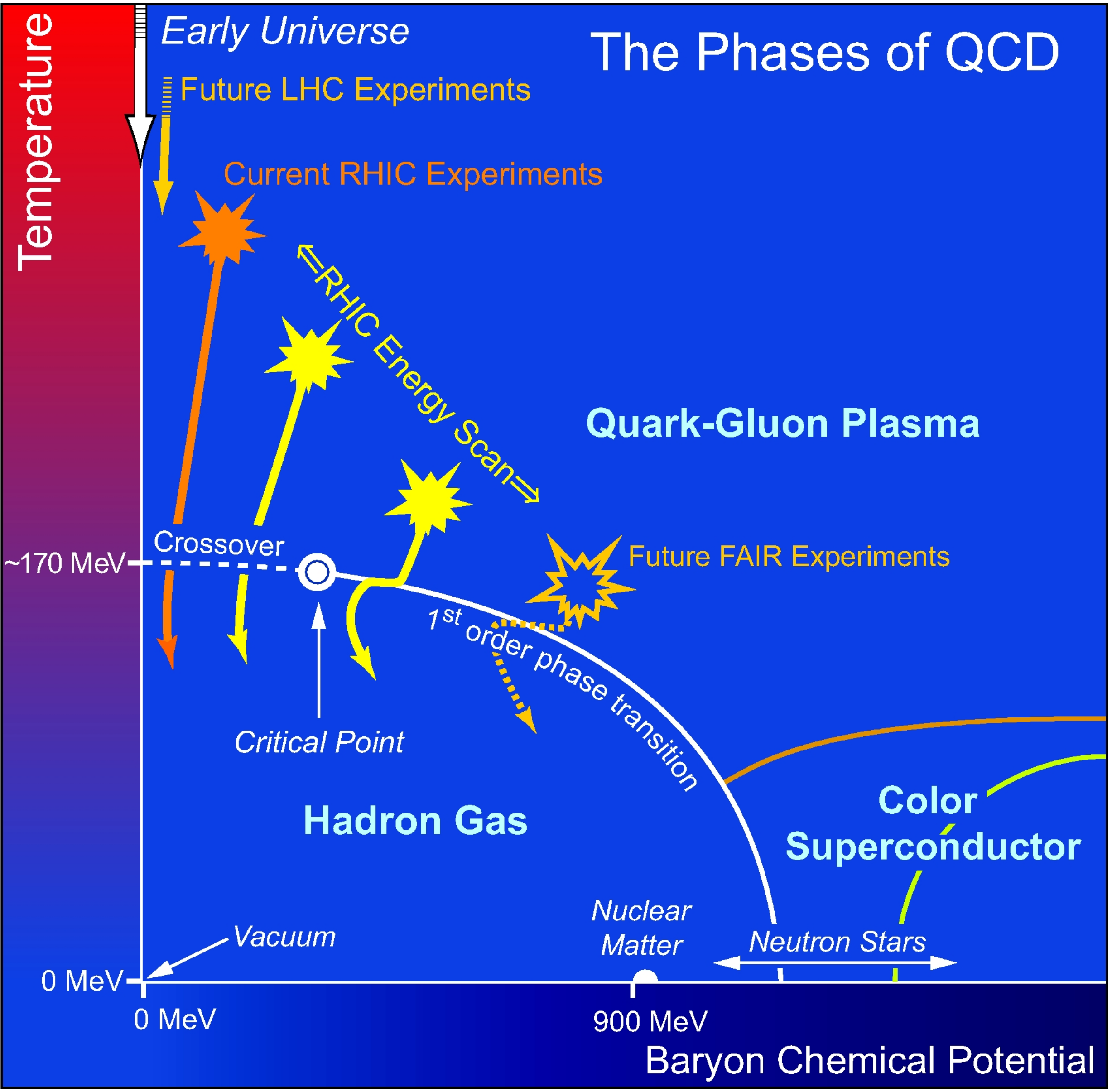
QCD thermodynamics

Frithjof Karsch, BNL



OUTLINE:

- Equation of state and transition temperature
- QCD phase diagram close to the chiral limit
- Charge fluctuations and the RHIC search for the critical point
- Outlook: Thermal QCD on next generation hardware

QCD thermodynamics on appropriate hardware

- the QCD equation of state and calculation of the QCD transition temperature

  requires large zero (and finite) temperature lattices,
  $48^3 \times 64$ lattice generated on leadership class computers (BlueGene/P) at ALCF using USQCD INCITE resources

- studies of the QCD phase diagram at vanishing chemical potential

  moderate size, finite temperature lattices $(\lesssim 48^3 \times 12)$
  use clusters operated for USQCD at JLab and FNAL

- calculating Taylor expansion coefficients for studies of finite density QCD

  large number of matrix inversions on single gauge field configurations are done on GPUs at JLab (and soon at FNAL)

Thermodynamics projects in 2010/11

Equation of state and the transition temperature:

M. Cheng et al., The finite temperature QCD using 2 + 1 flavors of domain wall fermions at Nt = 8, Phys. Rev. D81, 054510 (2010)

M. Cheng et al., Equation of State for physical quark masses, Phys. Rev. D 81, 054504 (2010)

M. Cheng et al., Meson screening masses from lattice QCD with two light and the strange quark, Eur. Phys. J. C71, 1564 (2011)

H.-T. Ding et al., Quark number susceptibilities at high temperature, ongoing

ongoing hotQCD project (INCITE), eg:
A. Bazavov, P. Petreczky, Taste symmetry and QCD thermodynamics with improved staggered fermions, PoS LATTICE2010, 169 (2010)
W. Soldner, Chiral Aspects of Improved Staggered Fermions with 2+1-Flavors from the HotQCD Collaboration, PoS LATTICE2010, 215 (2010)

- type-A cluster and INCITE projects in 2011/12 (R. Soltz)
- type-B cluster project in 2011/12 (H.-T. Ding)

Thermodynamics projects in 2010/11

QCD at non-zero baryon chemical potential or baryon number:

C. DeTar et al., QCD thermodynamics with nonzero chemical potential at Nt=6 and effects from heavy quarks, Phys. Rev. D81, 114504 (2010)

O. Kaczmarek et al., Phase boundary for the chiral transition in (2+1) -flavor QCD at small values of the chemical potential, Phys. Rev. D83, 014504 (2011)

A. Li, A. Alexandru, K.-F. Liu, X. Meng, Finite density phase transition of QCD with Nf=4 and Nf=2 using canonical ensemble method, Phys. Rev. D82, 054502 (2010).

A. Li, A. Alexandru, K.-F. Liu,
Critical point of Nf = 3 QCD from lattice simulations in the canonical ensemble, arXiv:1103.3045 [hep-ph]

- type-A cluster and GPU projects in 2011/12 (S. Mukherjee | P. Hegde)
- type-B cluster project in 2011/12 (A. Li)

...some statistics from SPIRES

- 50 top cited papers in hep-lat during each year in 2006-2010

year	# Thermo papers (WW)	# Thermo papers (US)	rank of the top-cited thermo paper (WW)	rank of the top-cited thermo paper (US)
2006	14	1	2 (MEM)	6 (Quarkonium)
2007	24	8	2 ((2+1)-f, T_c)	4 ((2+1)-f, T_c)
2008	20	6	1 ((2+1)-f, EoS,I)	1 ((2+1)-f, EoS,I)
2009	20	8	1 ((2+1)-f, EoS,I)	1 ((2+1)-f, EoS,I)
2010	19	6	1 ((2+1)-f, EoS,II)	1 ((2+1)-f, EoS,II)

- about 1/3 of the 50 top cited papers in hep-lat deal with topics in QCD thermodynamics
- about 1/3 of these papers involve authors from the US (=USQCD)

most cited US-Thermo papers 2010

1) A. Bazavov et al (hotQCD Collaboration), Equation of state and QCD transition at finite temperature, Phys.Rev.D80, 014504 (2009) [75 citations]

3) M. Cheng et al (RBC-Bielefeld collaboration), The QCD equation of state with almost physical quark masses, Phys.Rev.D77, 014511 (2008) [55 citations]

28) O. Kaczmarek and F. Zantow (Bielefeld-BNL), Static quark anti-quark interactions in zero and finite temperature QCD. I. Heavy quark free energies, running coupling and quarkonium binding, Phys. Rev. D71, 114510 (2005) [31 citations]

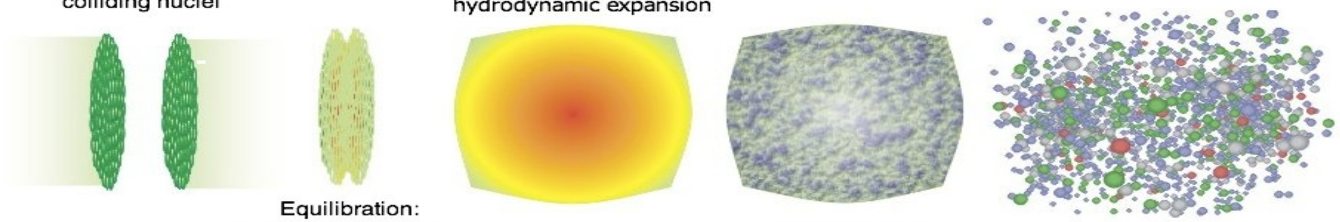
29) M. Cheng et al (RBC-Bielefeld collaboration), The QCD equation of state for physical quark masses, Phys. Rev. D81, 054504 (2010) [29 citations]

33) M. Cheng et al (RBC-Bielefeld Collaboration), The Transition temperature in QCD, Phys. Rev. D74, 054507 (2006) [29 citations]

46) S. Datta et al, Behavior of charmonium after deconfinement, Phys. Rev. D69, 094507 (2004) [25 citations]

QCD equation of state and the QCD (phase) transition temperature

EoS controls hydrodynamic expansion of matter created in HIC



hotQCD collaboration: systematic study of cut-off and quark mass dependence of thermodynamic observables; use asqtad and HISQ actions with different light/strange quark mass ratios (m_l/m_s) and different values of the cut-off $aT \equiv 1/N_\tau$

$$m_l/m_s = 0.1,\ 0.05,\ 0.025\ \Rightarrow\ 100\text{MeV} \lesssim m_\pi \lesssim 200\text{MeV}$$

most T>0 lattices on cluster

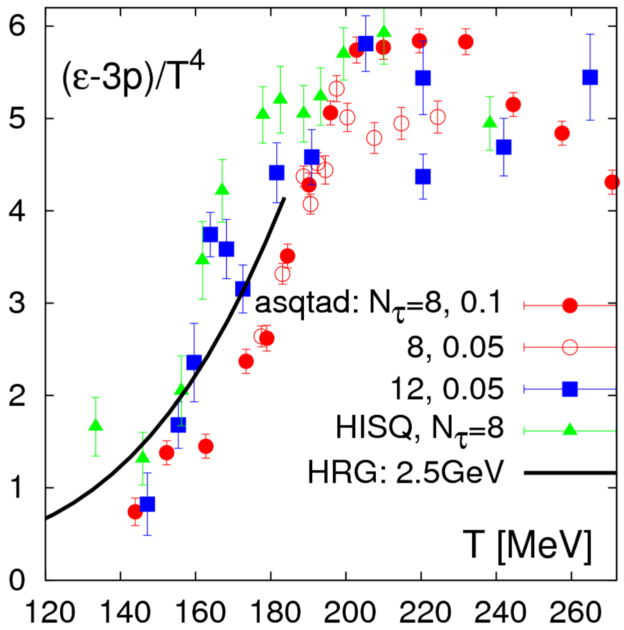
$$N_\tau = 6,\ 8,\ 12$$
$$N_\sigma = 32,\ 48$$

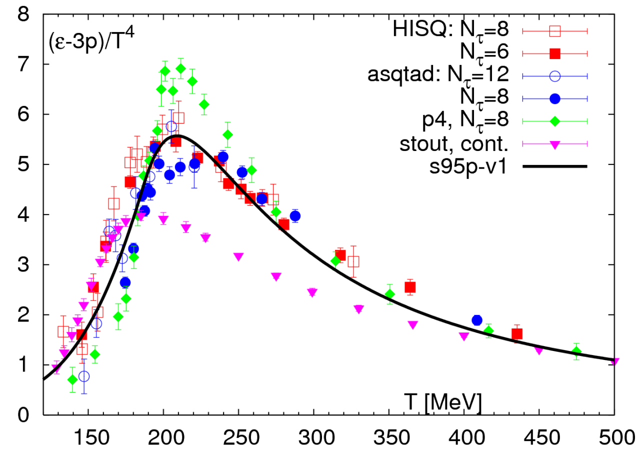
large T=0 lattices on BlueGene P (INCITE)

$$48^4,\ 48^3 \times 64$$

QCD equation of state and the QCD (phase) transition temperature

trace anomaly closer to the continuum limit



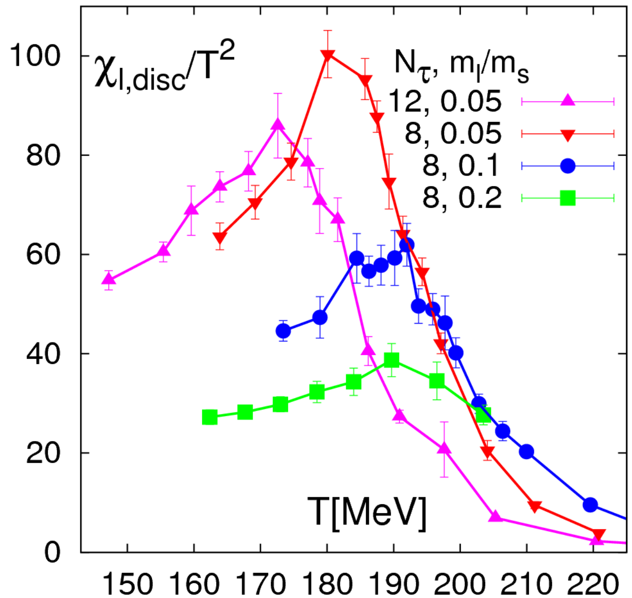


A. Bazavov et al. (HotQCD), PRD80, 014504 (2009)

hotQCD vs. Budapest-Wuppertal

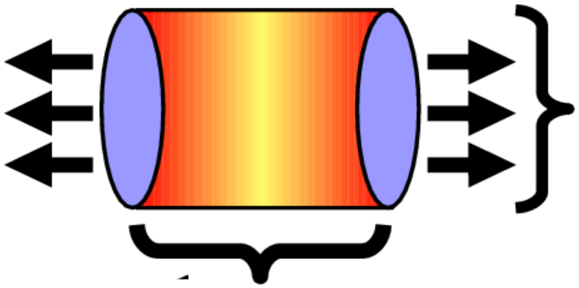
hotQCD preliminary

chiral susceptibility



quark mass and cut-off dependence of the transition temperature

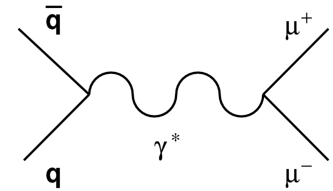
Thermal Dileptons
probing the structure of the QGP



$$V = \pi R^2 \Delta y$$

thermal dilepton rate:

$$\frac{\mathrm{d}W}{\mathrm{d}\omega \mathrm{d}^3p \mathrm{d}^4x} = \frac{5}{9}\frac{\alpha_{em}^2}{6\pi^3}\frac{\rho_V(\omega,\vec{p},T)}{\omega^2(\mathrm{e}^{\omega/T}-1)}$$



thermal emission during expansion:

$$\frac{\mathrm{d}W}{\mathrm{d}\omega \mathrm{d}^3p} = \pi R^2 \Delta y \frac{5}{9}\frac{\alpha_{em}^2}{6\pi^3}\int_{\tau_0}^{\tau_c}\mathrm{d}\tau \frac{\rho_V(\omega,\vec{p},T(\tau))}{\omega^2(\mathrm{e}^{\omega/T(\tau)}-1)}$$

$T(\tau)$

from EoS using Bjorken model or BETTER 3d-hydro

future:

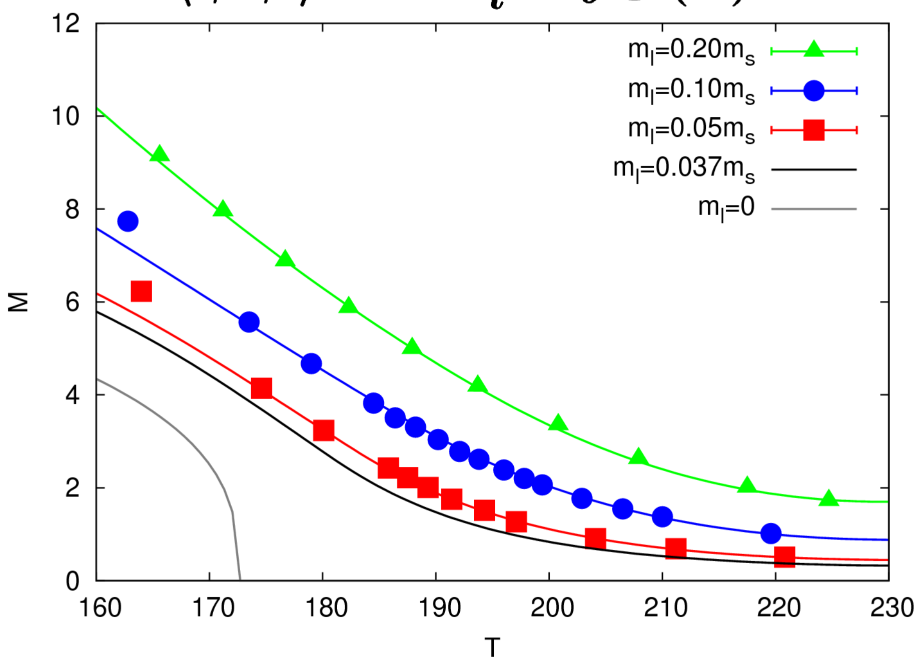
need information on temperature and momentum dependence of the vector spectral function

Transition temperature and O(N) scaling

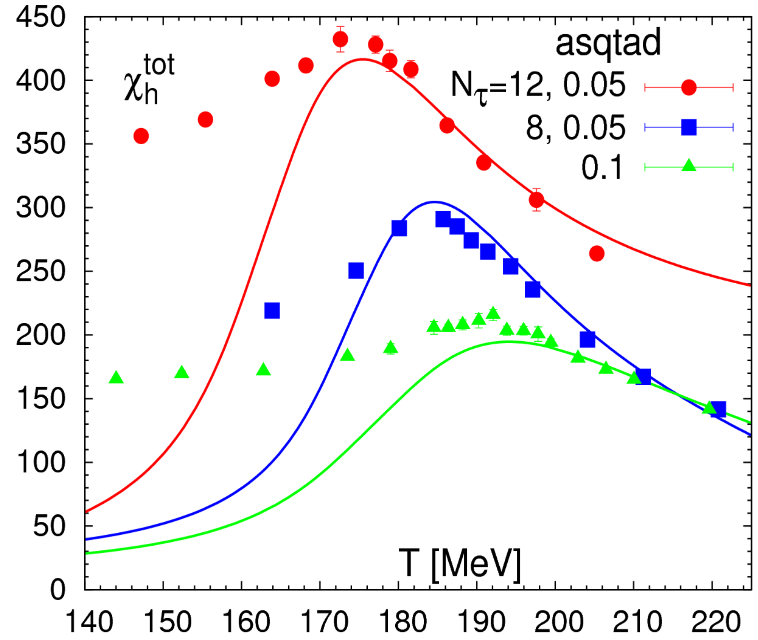
quark mass dependence of the chiral condensate: fit to O(N) scaling functions → **parameter free comparison with chiral susceptibility asqtad, Nt=8, 12**

$$z = z_0 \left| \frac{T - T_c}{T_c} \right| \left(\frac{m_l}{m_s} \right)^{-1/\beta\delta}$$

$$\langle \bar{\psi}\psi \rangle \sim m_l^{1/\delta} f_G(z)$$



$$\chi_h \sim m_l^{1/\delta - 1} f_\chi(z)$$



The curvature of the critical line

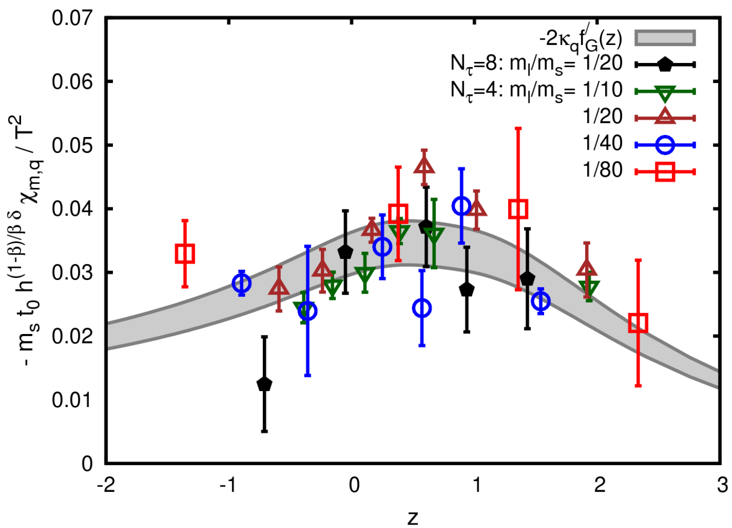
BNL-Bielefeld-GSI, Phys. Rev. D83 (2011) 014504

S. Mukherjee, 2010 GPU project

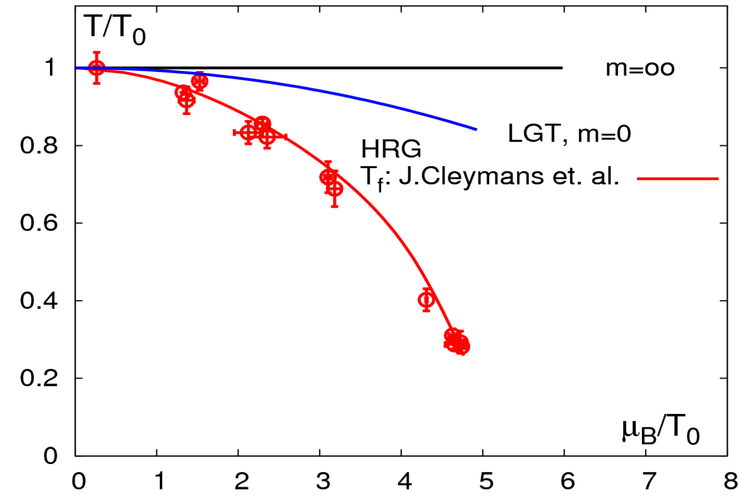
- "thermal" fluctuations of the order parameter

$$t \equiv \frac{1}{t_0}\left(\left(\frac{T}{T_c}-1\right) - \kappa_q \left(\frac{\mu_q}{T}\right)^2\right) \ , \ z = t/h^{1/\beta\delta}$$

$$M_b = h^{1/\delta} f_G(z) \ , \ \chi_{m,q} = \partial^2 M_b / \partial(\mu_q/T)^2 = \frac{2\kappa_q}{t_0 T_c} h^{(\beta-1)/\delta\beta} f'_G(z)$$



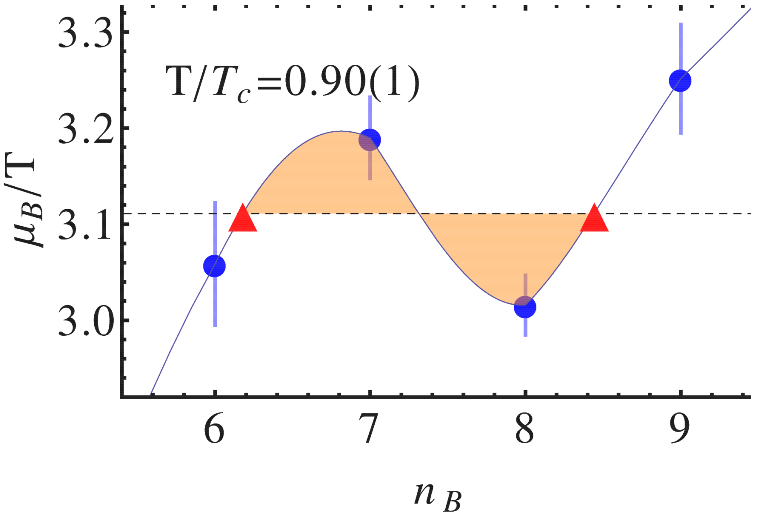
compare to freeze-out curve:

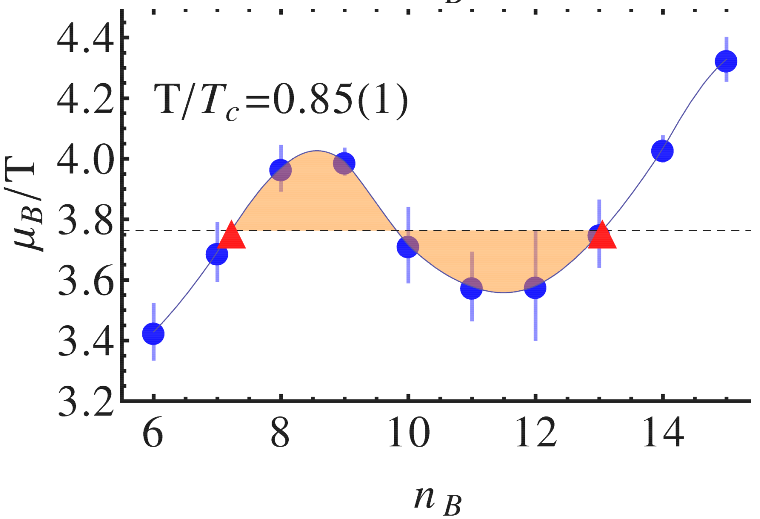


$$\kappa_q = 0.059 \pm 0.006$$

Finite density QCD calculations in the canonical ensemble

Maxwell construction





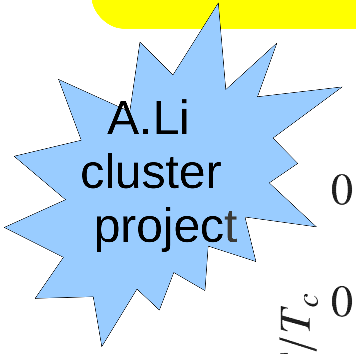
$16^3 \times 4$: 72 parameter sets

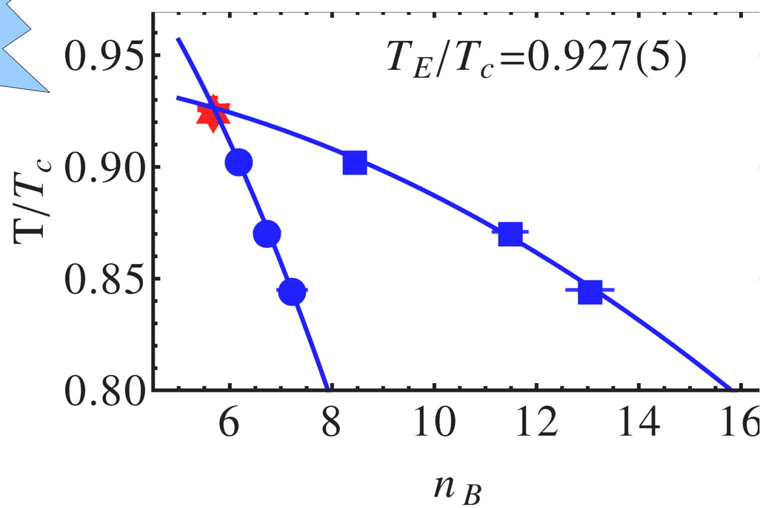
3-f QCD:
simulations at fixed baryon number

determinant ratio

$$\frac{1}{T}\langle\mu_B\rangle_B = -\ln\langle\gamma(U)\rangle_0/\langle\alpha(U)\rangle_0$$

phase !

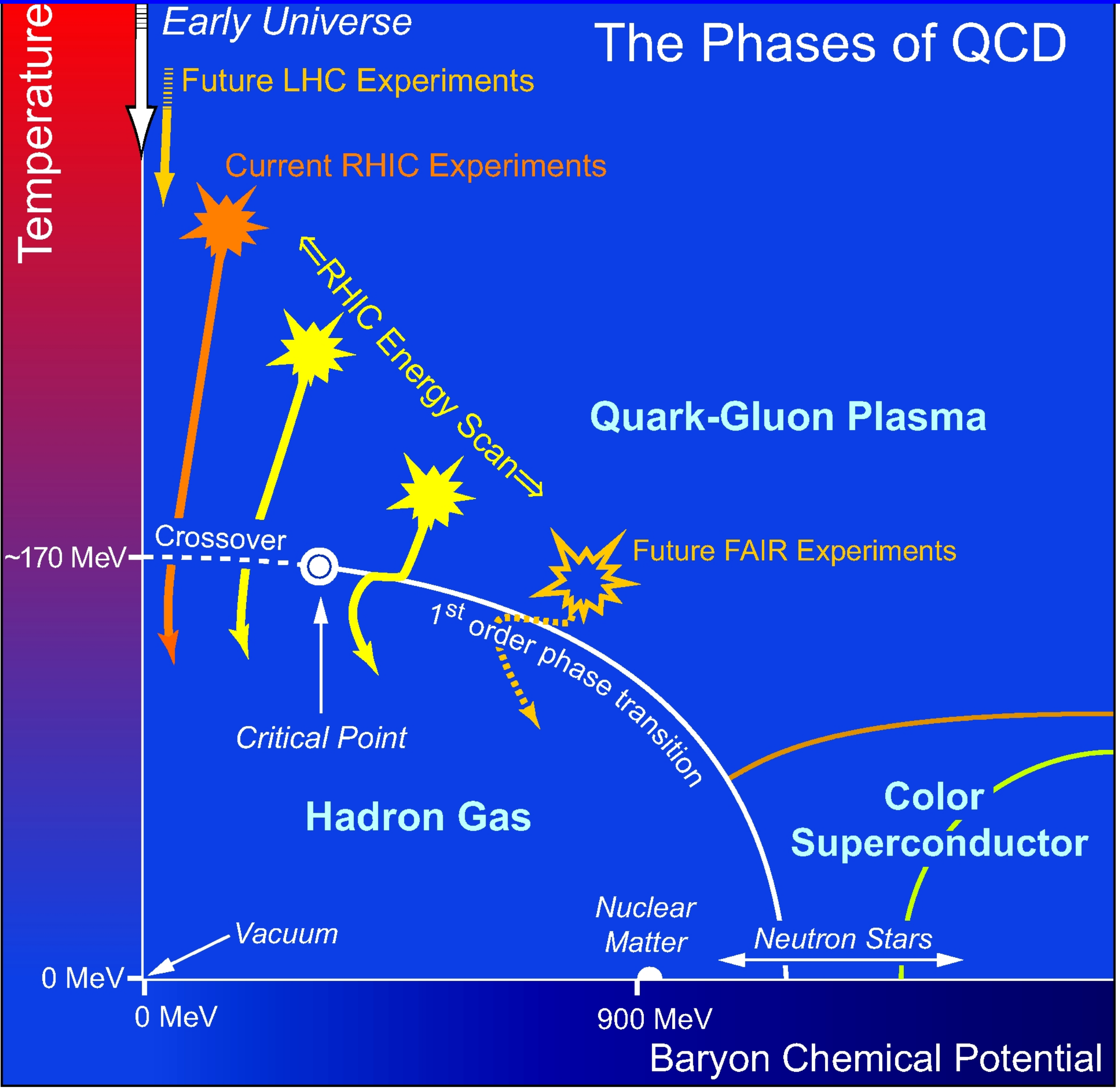




A. Li, A. Alexandru, K-F Liu, arXiv:1103.3045

A. Li, A. Alexandru, K-F Liu, X. Meng, PRD82, 054502 (2010)

QCD phase diagram
(close to the chiral limit)



RHIC low energy runs:

$$\sqrt{s} = (9 - 200)\mathrm{GeV}/A$$

- charge fluctuations along the freeze-out line
- higher moments of charge fluctuations, e.g.

Skewness

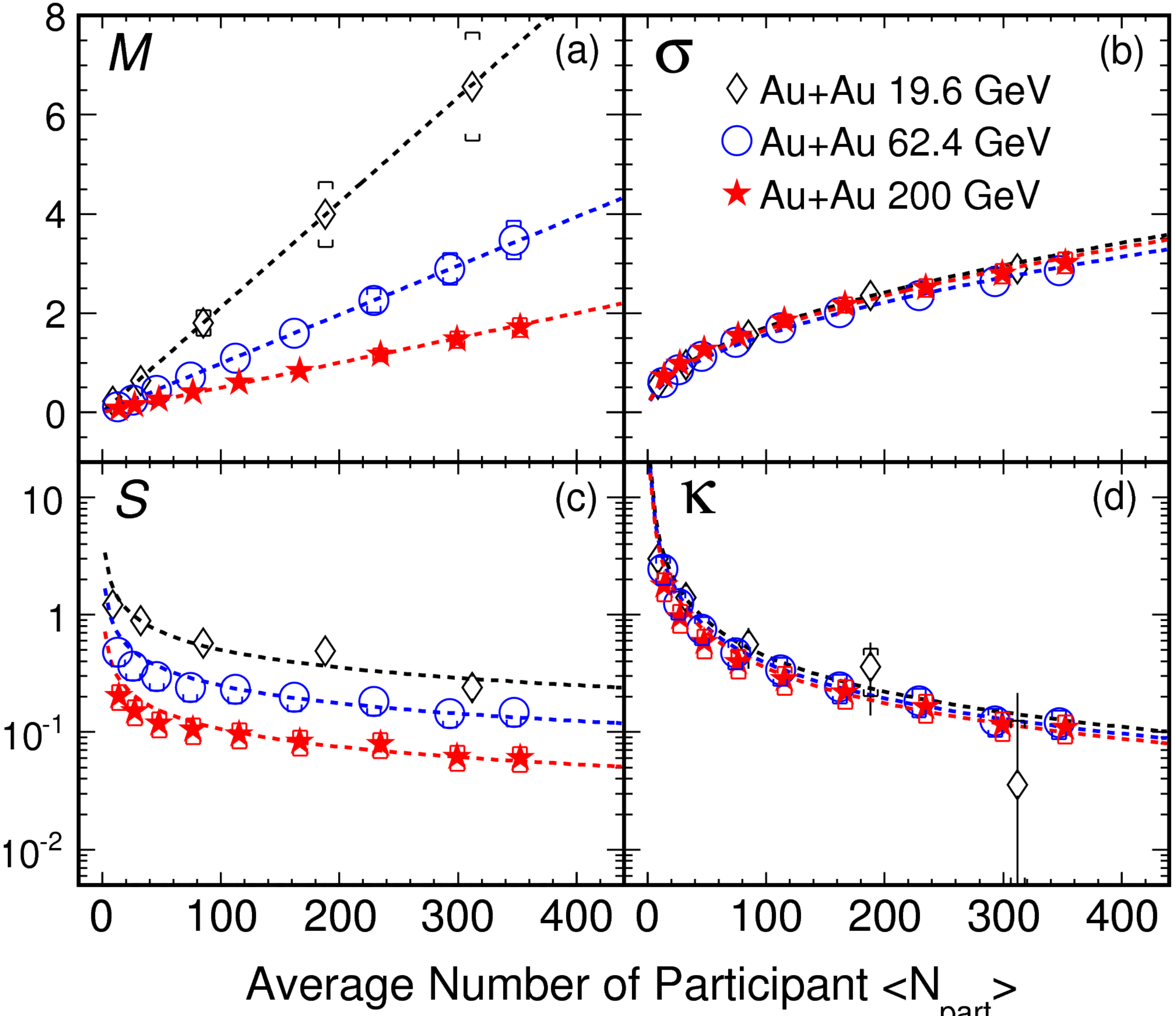
$$S_q \equiv \langle(\delta N_q)^3\rangle/\sigma_q^3$$

Kurtosis

$$\kappa_q \equiv \langle(\delta N_q)^4\rangle/\sigma_q^4 - 3$$

Mean, variance, skewness & kurtosis

STAR results from RHIC low energy run



M. M. Aggarwal et al. (STAR Collaboration), Phys. Rev. Lett. 105 (2010) 22302

Generalized Quark number susceptibilities

- Taylor expansion of the pressure

$$\frac{p}{T^4} = \sum_{n=0}^{\infty} \frac{1}{n!} \chi_{B,0}^{(n)}(T) \left(\frac{\mu_B}{T}\right)^n$$

- generalized quark number susceptibilities

$$\chi_{B,0}^{(n)} = \frac{1}{VT^3} \frac{\partial^n \ln Z}{\partial(\mu_B/T)^n}\bigg|_{\mu_B=0}$$

- Taylor expansion of quark number susceptibilities

$$\chi_{B,\mu}^{(n)} = \sum_{k=0}^{\infty} \frac{1}{k!} \chi_{B,0}^{(k+n)}(T) \left(\frac{\mu_B}{T}\right)^k$$

Skewness

$$S_B \sigma_B \equiv \frac{\chi_{B,\mu}^{(3)}}{\chi_{B,\mu}^{(2)}}$$

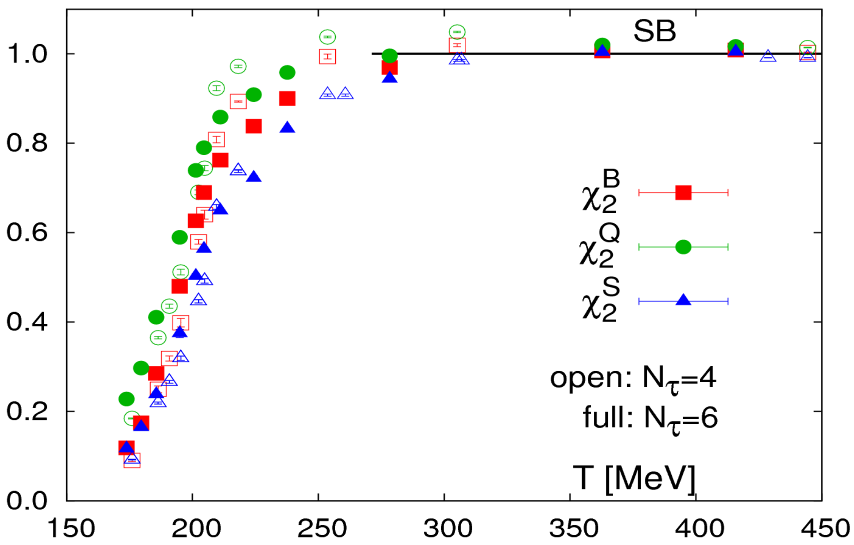
Kurtosis

$$\kappa_B \sigma_B^2 \equiv \frac{\chi_{B,\mu}^{(4)}}{\chi_{B,\mu}^{(2)}}$$

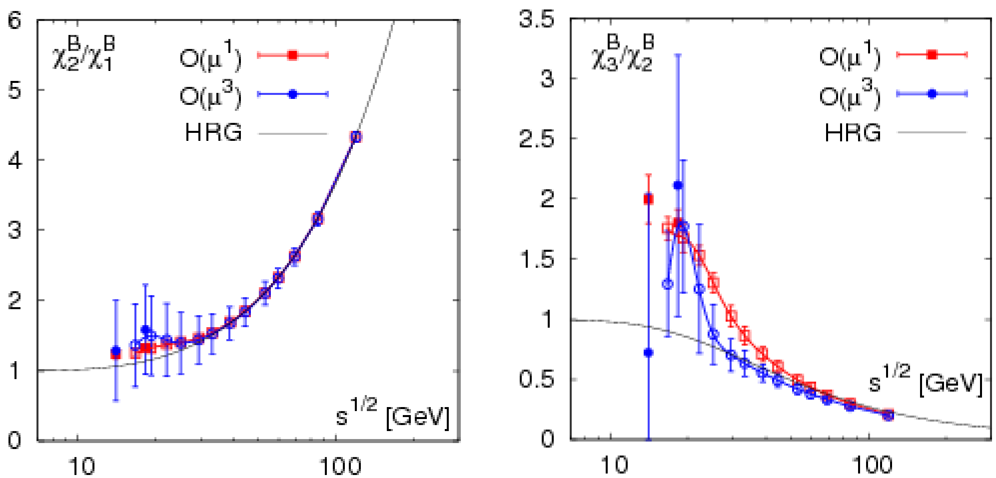
P. Hegde, GPU project

The RHIC low energy runs

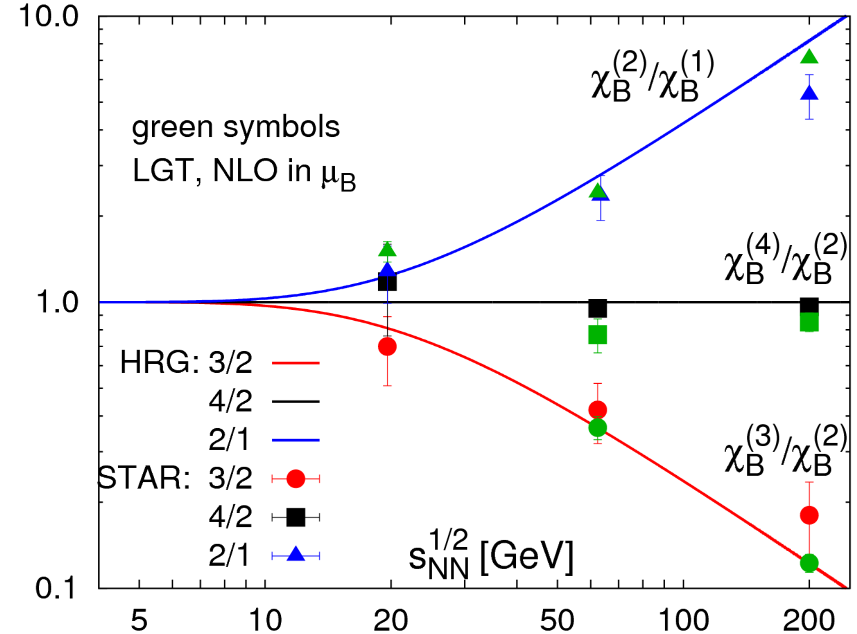
quark number susceptibilities



BNL-Bielefeld-GSI, Phys. Rev. D79 (2009) 074505



Moments of charge fluctuations



HRG(lines): FK, K Redlich, PL B695 (2011) 136
lattice (green): C. Schmidt, arXiv:1007.5164
Experiment: STAR: arXiv:1004.4959

charge fluctuations at freeze-out agree with HRG model predictions and lattice calculations

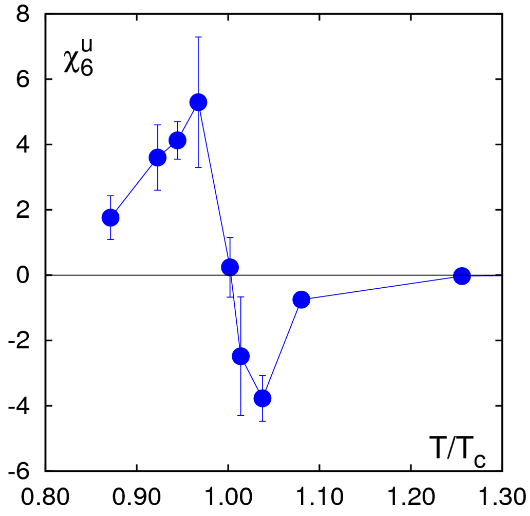
Higher moments of charge fluctuations at RHIC and LHC

- higher moments (e.g. 6th order) are drastically different in QCD close to criticality and in a hadron resonance gas, e.g.

$$\frac{\chi_B^{(6)}}{\chi_B^{(2)}} = \begin{cases} = 1 & \text{, hadron resonance gas} \\ < 0 & \text{, QCD at the crossover transition} \end{cases}$$

$$\chi_{B,0}^{(6)} \sim A_{\pm} \left| \frac{T - T_c}{T_c} \right|^{-(1-\alpha)}$$

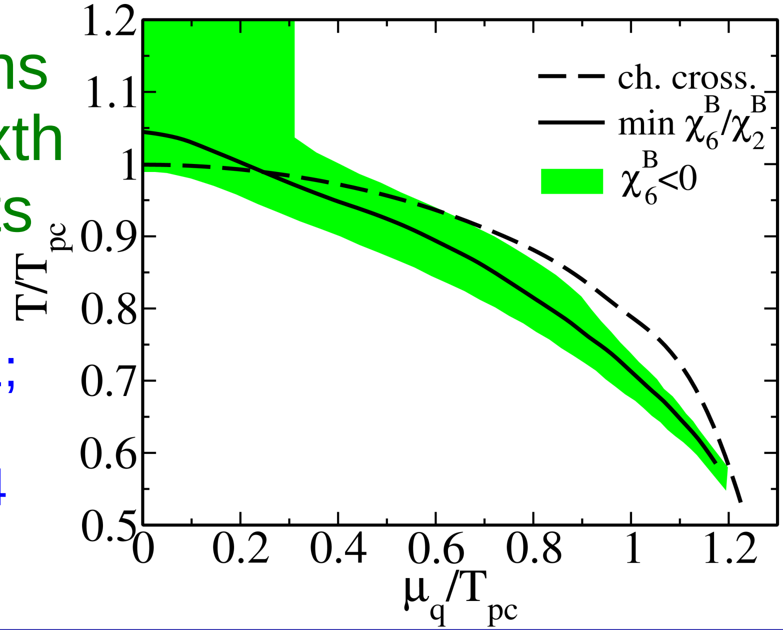
electric charge fluctuations



M. Cheng et al., PRD 79 (2009) 074505

green = regions of negative sixth order moments

B. Friman et al., arXiv:1103.3511; J. Engels, FK, arXiv:1105.0584



Outlook: Thermodynamics on next generation hardware

- getting ready for petascale-thermodynamics
  - – thermodynamics with chiral fermions
  - – spectral properties of light and heavy quark correlation functions

↑

state-of-the-art: quenched QCD on large thermal lattices

$$\mathbf{128^3 \times 48}$$

H.-T. Ding et al., Thermal dilepton rate and electrical conductivity: An analysis of vector current correlation functions in quenched lattice QCD, Phys. Rev. D83, 034504 (2011)

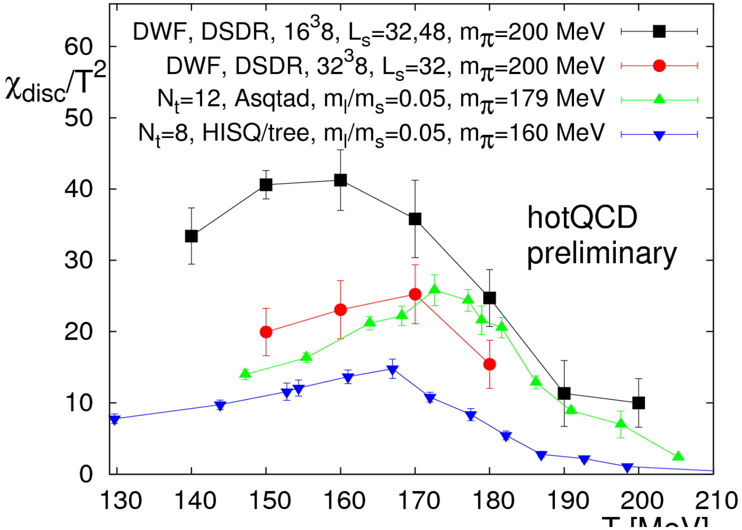
Thermodynamics with Domain Wall Fermions

- using a chiral fermion formulation for thermodynamic calculations allows to perform a more rigorous analysis of the role of the axial $U_A(1)$ symmetry close to T_c ;
- chiral fermions have an exact $SU_L(2) \times SU_R(2)$ flavor symmetry

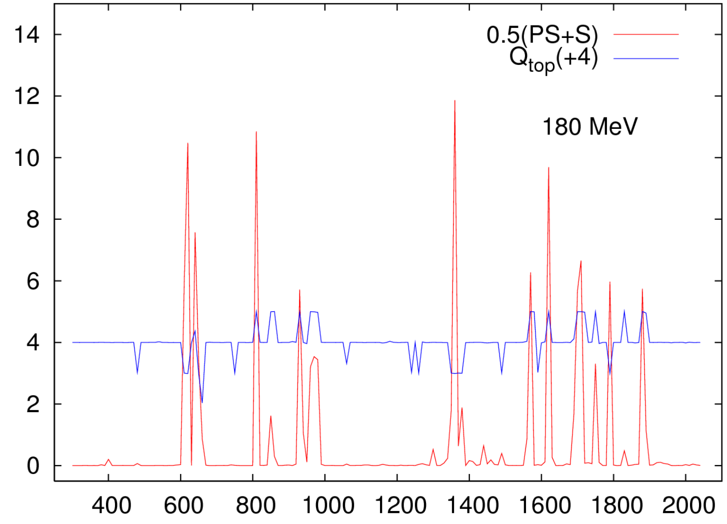
ongoing project of hotQCD on BG/P;

preparatory work of RIKEN/BNL for thermodynamics on BlueGene/Q

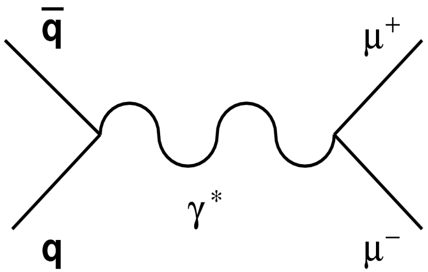
chiral susceptibility



topology and axial anomaly



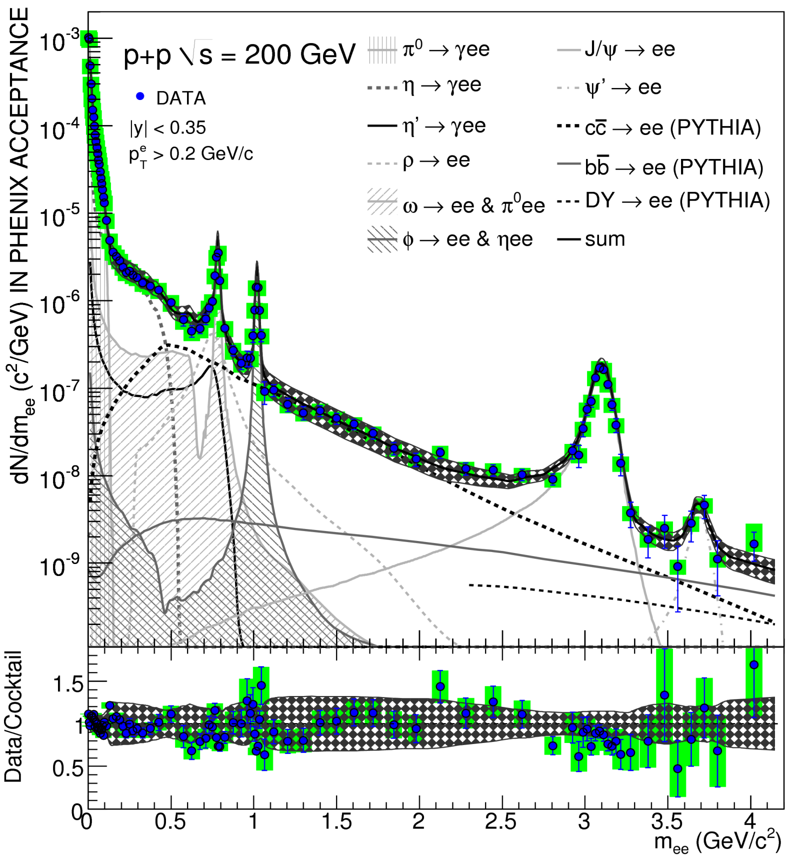
Thermal Dilepton and Photon rates and transport coefficients in the QGP

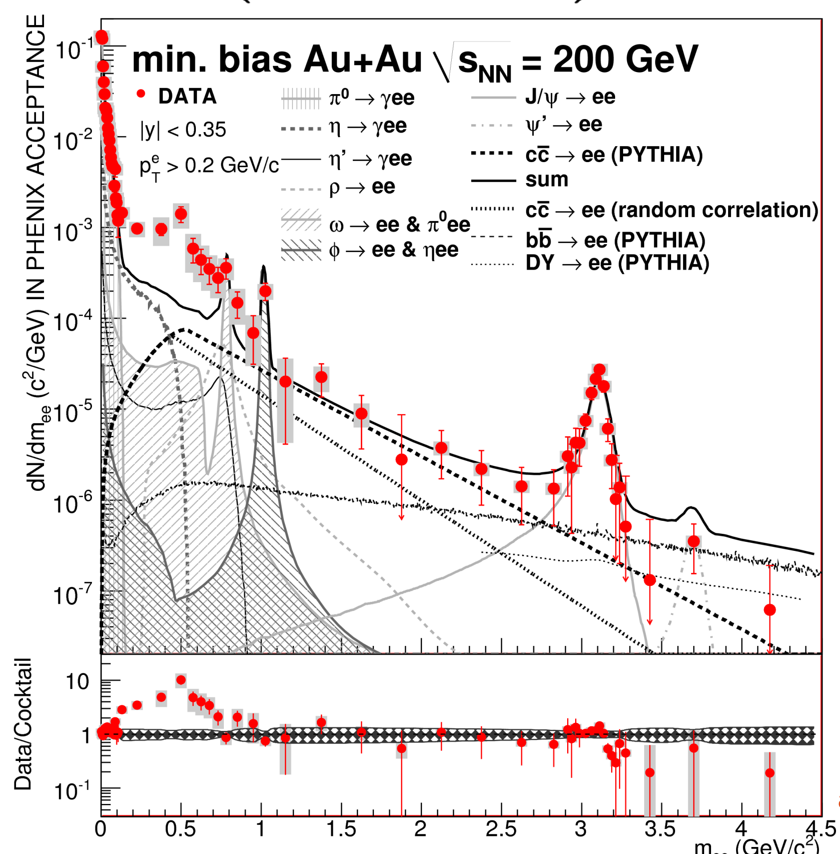


Thermal dilepton rate:

vector spectral function

$$\frac{dW}{d\omega d^3p} = \frac{5}{9}\frac{\alpha_{em}^2}{6\pi^3}\frac{\rho_V(\omega,\vec{p},T)}{\omega^2(e^{\omega/T}-1)}$$





arXiv:0912.0244v1 [

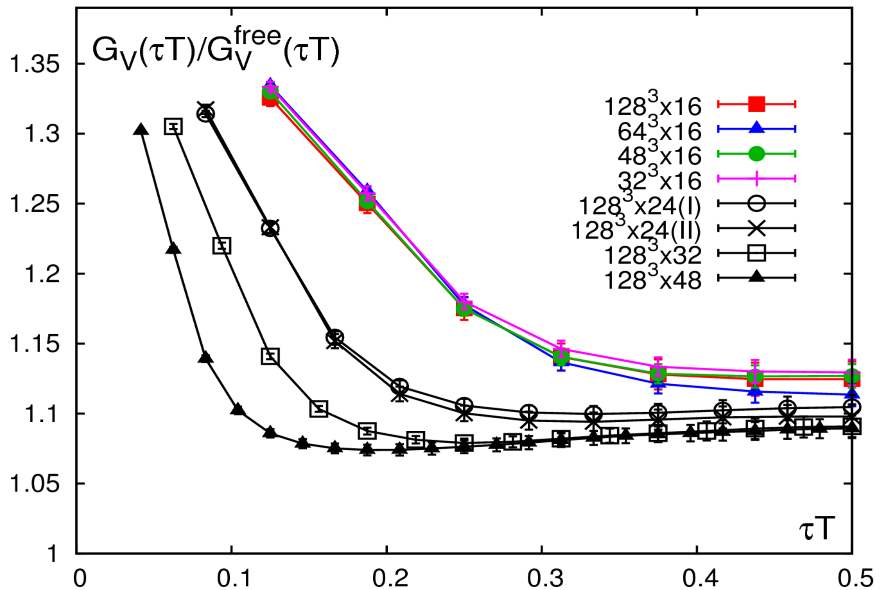
A. Adare et al. (PHENIX Collaboration), Phys. Rev. C81, 034911 (2010)

Vector Meson Spectral Function
Thermal Dilepton

Vector meson correlation function:

$$G_V(\tau,\vec{p},T) = \int_0^\infty \frac{d\omega}{2\pi}\, \rho_V(\omega,\vec{p},T)\, \frac{\cosh(\omega(\tau - 1/2T))}{\sinh(\omega/2T)}$$

$$\vec{p} = 0 \ , \ T = 1.5 T_c$$



BNL-Bielefeld-GSI, Vector current correlator
in quenched QCD, Phys.Rev. D83 (2011) 034504

quenched QCD, clover action:

volume dependence

$$N_\sigma^3 \times 16 \ , \ N_\sigma = 48 - 128$$

cut-off dependence

$$128^3 \times N_\tau \ , \ N_\tau = 16 - 48$$

quark mass dependence

$$m_q^{\overline{MS}}/T = 0.02,\ 0.11$$

Thermal Dileptons & electrical conductivity

Thermal dilepton rate:

vector spectral function

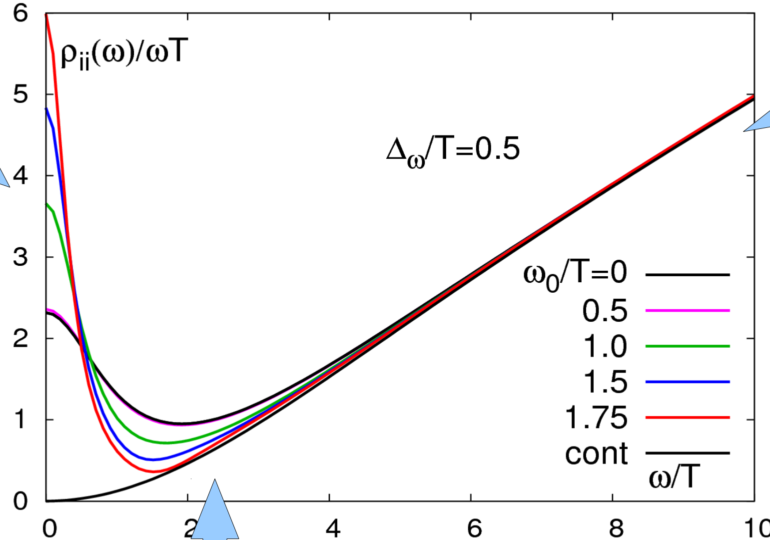
$T = 1.5T_c$

$$\frac{\mathrm{d}W}{\mathrm{d}\omega \mathrm{d}^3p} = \frac{5}{9}\frac{\alpha_{em}^2}{6\pi^3}\frac{\rho_V(\omega,\vec{p},T)}{\omega^2(\mathrm{e}^{\omega/T}-1)}$$

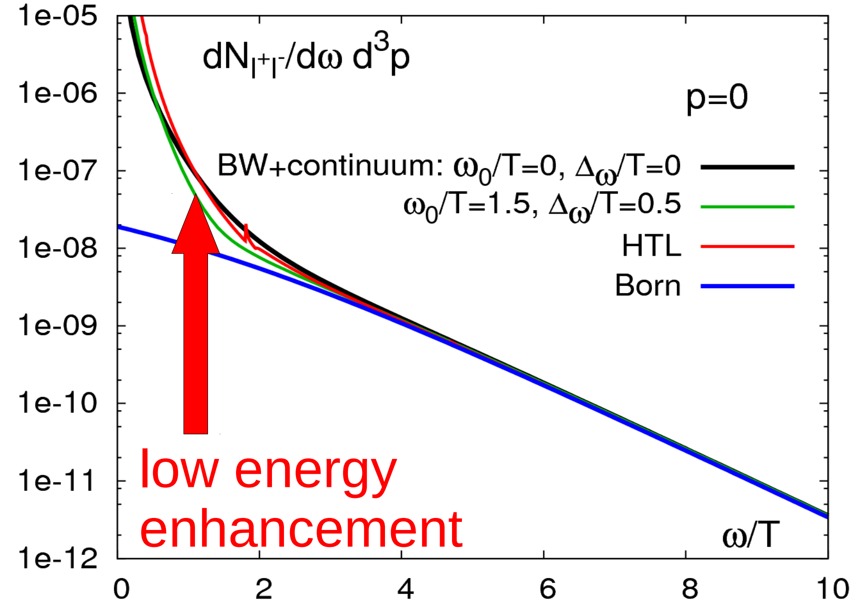
electrical conductivity

$$\sigma/T = \lim_{\omega\to 0} \rho_{ii}(\omega)/(6\omega T)$$

HTL resummation, perturbation theory



Breit-Wigner ansatz



low energy enhancement

Conclusions

- LGT calculations start to produce quantitative predictions on QCD thermodynamics that provide input to the interpretation of heavy ion experiments

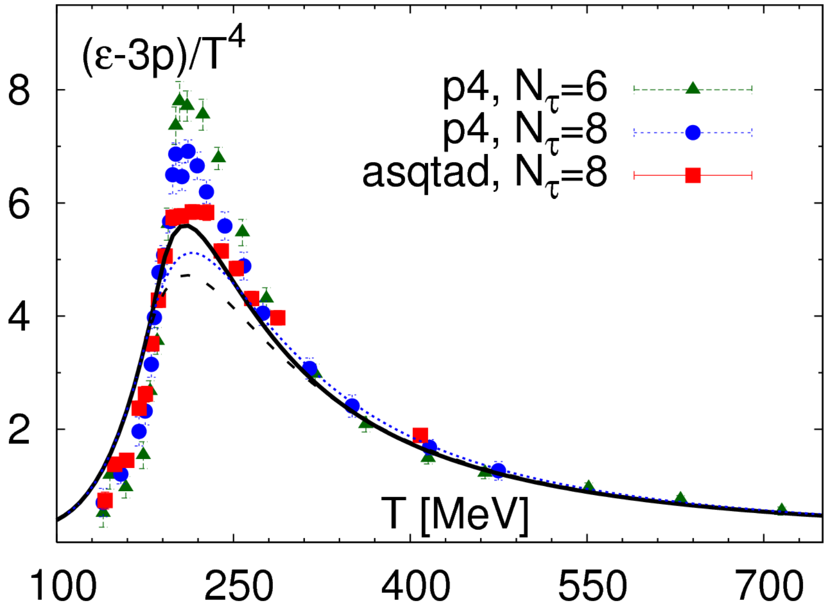
  EoS, Tc, transport coefficients, spectral functions,
  phase boundary, charge fluctuations,.....

- How sensitive is the QCD transition with physical quark masses to universal properties at the chiral phase transition?
- How does the phase transition vary with chemical potentials?

- Use staggered fermion action with reduced taste violation (HISQ) with physical quark masses close to the cont. limit
- Use chiral fermion formulations (DWF) in thermodynamic calculations

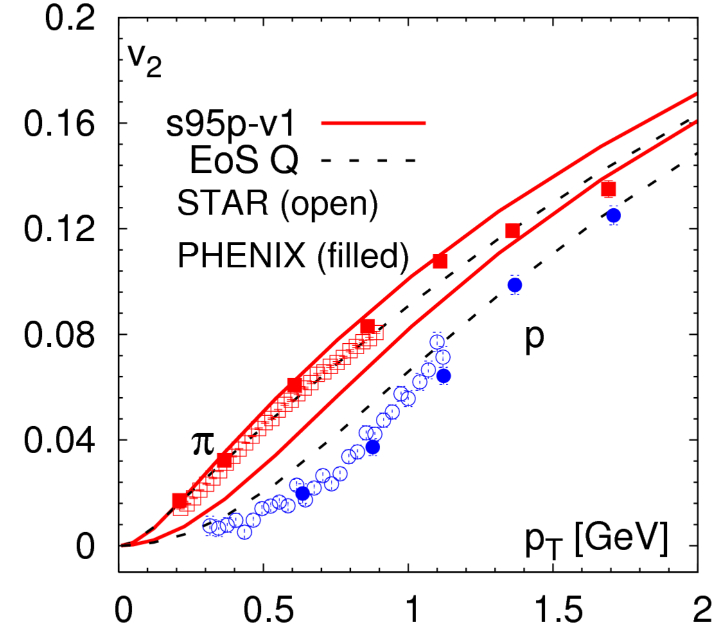
QCD equation of state and the QCD (phase) transition temperature

trace anomaly and its parametrization from L-QCD



A. Bazavov et al. (HotQCD), PRD80, 014504 (2009)

hydrodynamic modelling of elliptic flow spectra



P. Huovinen, P. Petreczky, NP A837, 26 (2010)