Quasi-Elastic (Electron) Scattering from Nuclei

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Why study Quasielastic Scattering from Nuclei ?

New Subatomic Particle Could Help Explain the Mystery of Dark Matter. A flurry of evidence reveals that "sterile neutrinos" are not only real but common, and could be the stuff of dark matter. *Scientific American, Jan 2011*

Accelerator Experiments

Mini-Boone, Minerva, LBNE, T2K,...

Astrophysics and Cosmology

Reactor Experiments

CP, CPT violation
sterile neutrinos &
dark matter, ...

Quasielastic scattering a bit more mundane - but interesting physics, required to describe charge and current propagation in nuclei

Outline:

What is quasi-elastic scattering?

Electron scattering: definitions and simple models

Scaling and super-scaling in the electron data

Longitudinal vs. Transverse EM response

Neutrino Scattering vs. Electron Scattering

Can the inclusive cross-section exceed free nucleons?

Deuteron, ^{3}He, ^{4}He, ...

Microscopic calculations: advantages and limitations

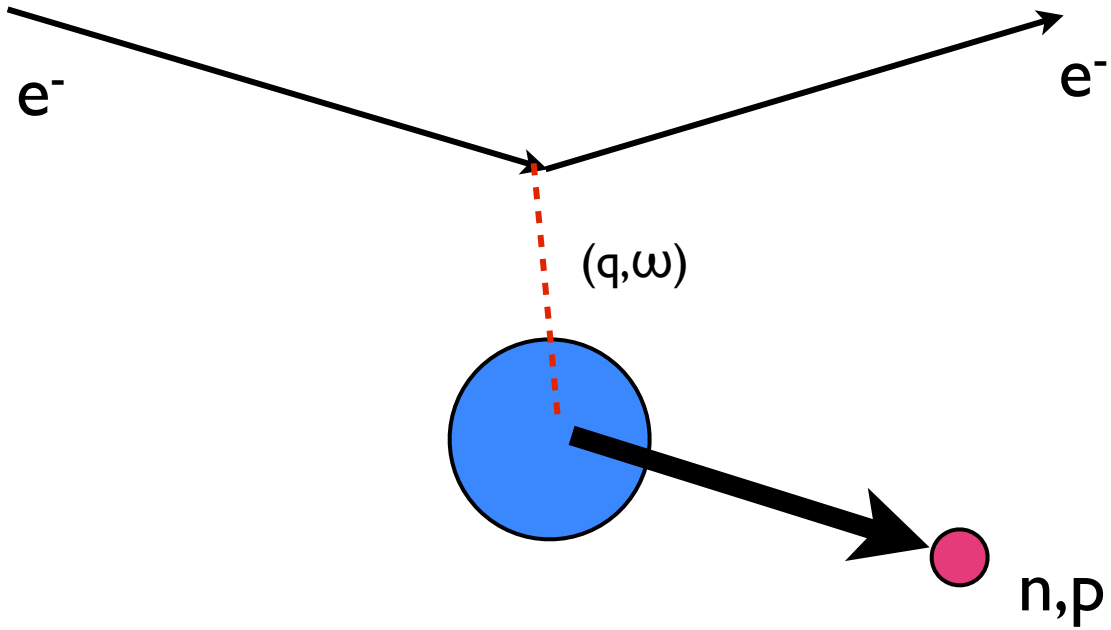
Results of electron scattering calculations

Approaches to neutrino scattering

Future

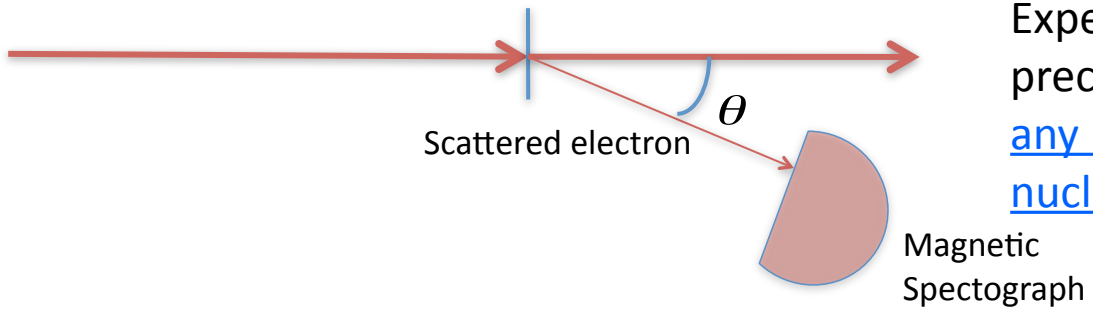
Quasi-elastic Scattering

Inclusive lepton scattering in kinematics
expected to be dominated by knocking
a single (unmeasured) nucleon out of the nucleus



Expect similar response from almost all nuclei,
characterized by initial momentum distribution

Quasi-Elastic Kinematics



Experimentally, q and ω are precisely known without any reference to the nuclear final state

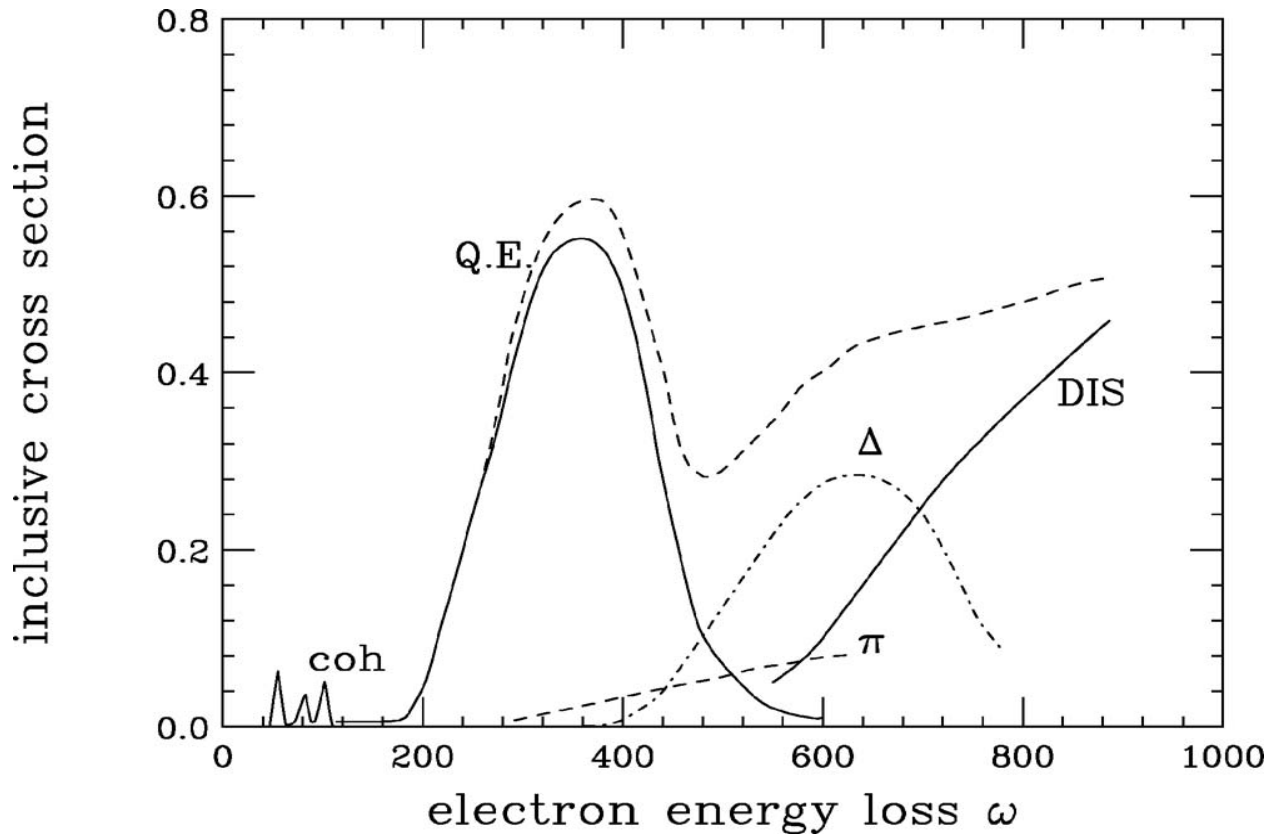


FIG. 1. Schematic representation of inclusive cross section as a function of energy loss.

Why has QES become Important?

Research involving neutrino oscillations has required the extention of QES (CCQE) to neutrino-nucleus.
For $0.3<E_\nu< 3.0$ GeV it is the dominant interaction.

CCQE provides essential information for neutrino oscillations, *flavor and energy*.

CCQE is treated as readily calculable, experimentally identifiable and allows assignment of the neutrino energy.

Some 40 calculations published since 2005

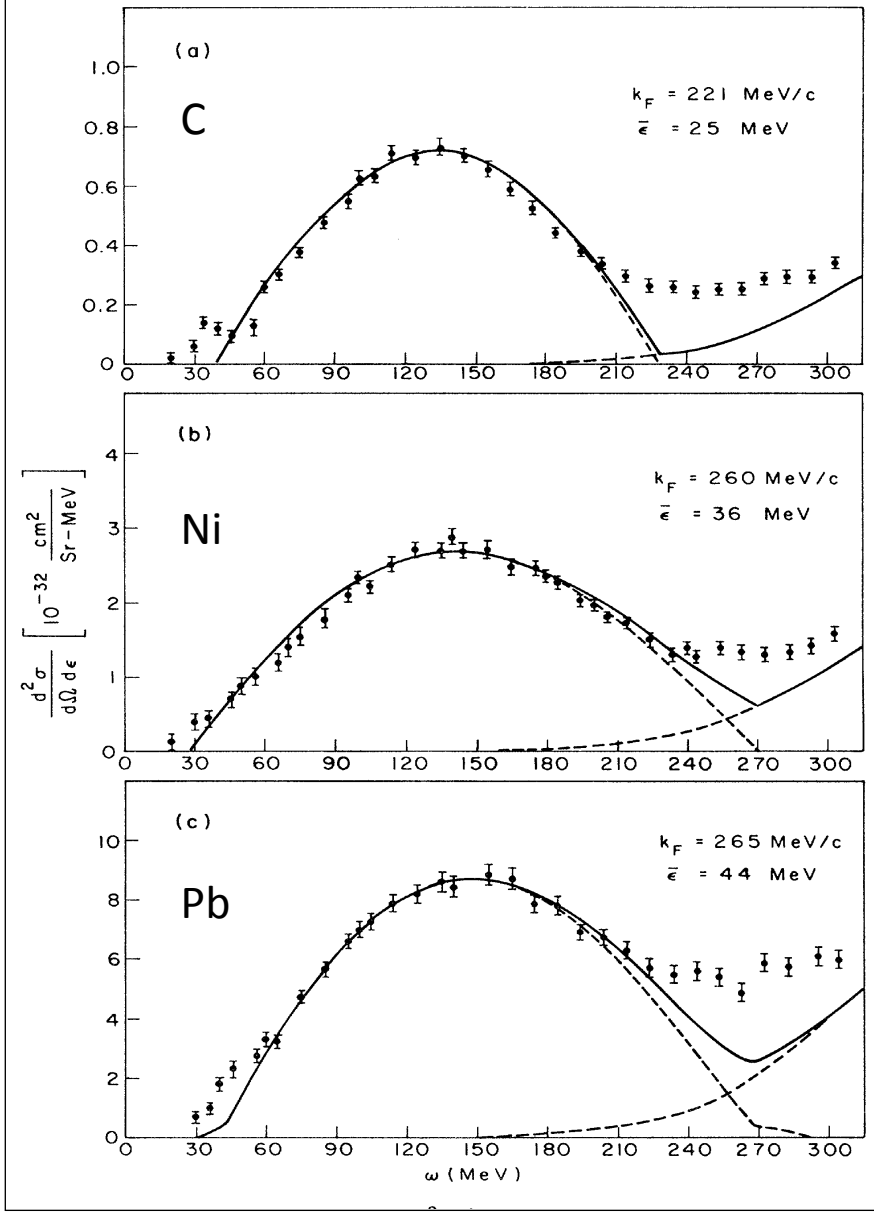
Oscillation period: $1.27\Delta m_{ij}^2(ev^2)\times(L(km)/E_\nu(GeV))$

$\Delta m_{23}^2=10^{-3}$ $L/E\approx10^3$ LBNE

$\Delta m_{2S}^2=1$ $L/E\approx1$ SBNE

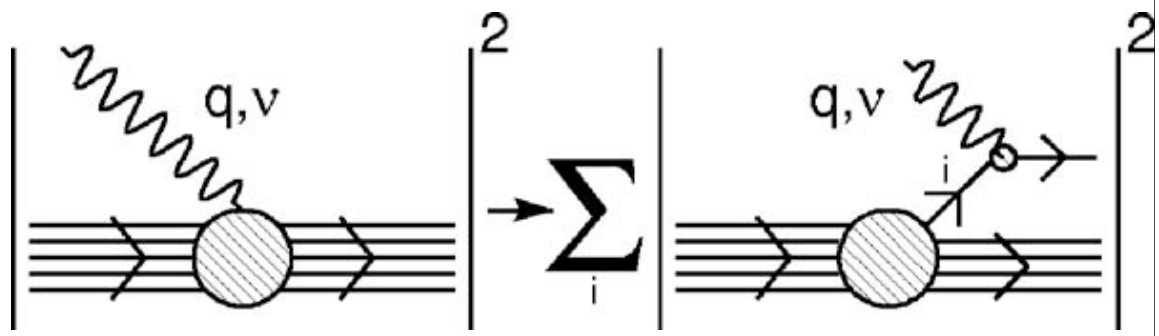
Quasi Elastic Scattering in NP originated with e-Nucleus

Moniz et al PRL 1971



Simple Fermi Gas 2 parameter , S_E, p_F

Impulse Approximation



Parameters change with nucleus
Normalization not entirely clear

Quasi-Elastic Electron Scattering

$$\frac{d\sigma^2}{d\Omega_e d\omega} = \left(\frac{d\sigma}{d\Omega_e}\right)_{Mott} \left\{ \left(\frac{Q}{|\vec{q}|}\right)^4 R_L(|\vec{q}|,\omega) + \left(\frac{1}{2}\left(\frac{Q}{|\vec{q}|}\right)^2 + \tan^2\frac{\theta}{2}\right) R_T(|\vec{q}|,\omega) \right\}$$

$$Q^2 = q^2 - \omega^2 \equiv q^2 - \nu^2$$

$$\tau^2 = \frac{Q^2}{4M^2}$$

$$(d\sigma / d\Omega_e)_{Mott} = \alpha^2 \cos^2(\theta/2)/E\sin^4(\theta/2)$$

$$\langle p', N | J_\mu | p, N \rangle = \frac{i}{\Omega} < \bar{u}_N(p') | [F_1^N(q^2)\gamma_\mu + F_2^N(q^2)\sigma_{\mu\nu}q_\nu] | u_N(p) >$$

$$F_i^{\tau_3}(q^2) = \frac{1}{2}(F_i^S(q^2) + \tau_3 F_i^V(q^2)) \qquad 2mF_2^S(0) = \mu'_p + \mu_n = -0.120$$

$$F_1^S(0) = 1 \qquad F_1^V(0)=1 \qquad 2mF_2^V(0) = \mu'_p - \mu_n = +3.706$$

Nucleon Cross Section

$$\frac{d\sigma}{d\Omega_e} = \sigma_{Mott}\frac{E'}{E_0}\left[\frac{G_E^{N,2}(q^2) + \tau G_M^{N,2}(q^2)}{1+\tau} + 2\tau G_M^2(q^2)\tan^2(\frac{\theta}{2})\right]$$

$$G_E(q^2) = F_1(q^2) + \tau F_2(q^2) \qquad G_M(q^2) = F_1(q^2) + F_2(q^2)$$

Quasi-Elastic Scattering in the Fermi Gas Model

The energy given up by the electron, to nucleon with initial Fermi momentum $\vec{k}$

$$\omega = T_N + E_s + E_R$$

T_N is the kinetic energy of the struck nucleon, E_s the separation energy of the struck nucleon, E_R the recoil kinetic energy of the nucleus.

$$\omega = [(\vec{k} + \vec{q})^2 + m^2]^{\frac{1}{2}} - m + E_s + E_{recoil}$$

$$= [k_{\|}^2 + 2k_{\|}q + q^2 + k_{\perp}^2 + m^2]^{\frac{1}{2}} - m + E_s + E_{recoil}$$

$$neglect\ E_s,\ E_{recoil}, k_{\perp}$$

$$k_{\|} = \sqrt{\omega^2 + 2m\omega} - q \equiv y$$

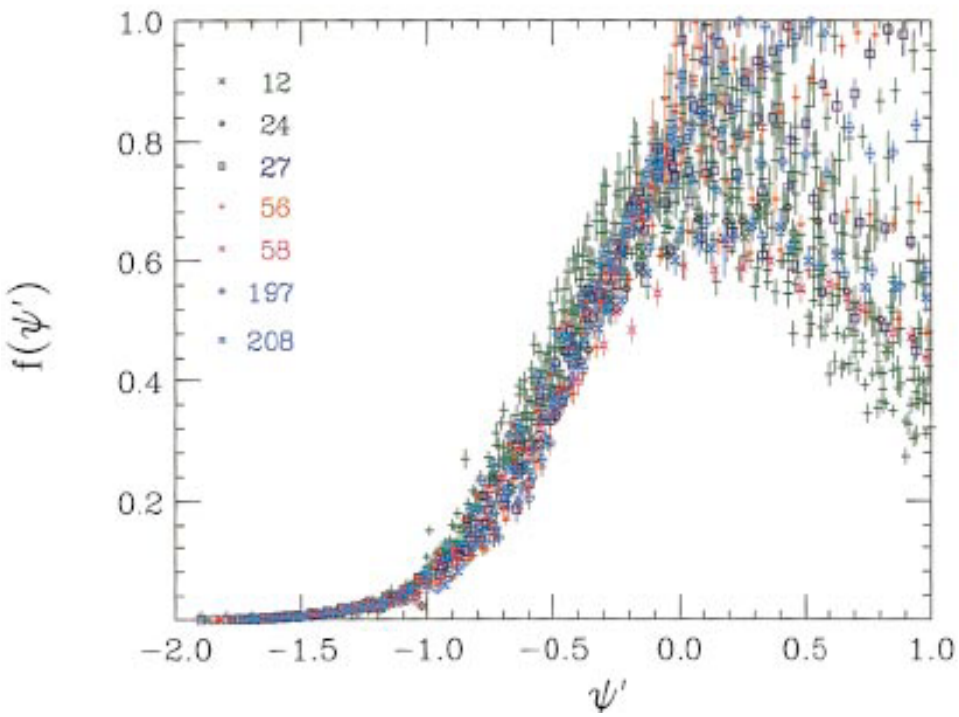
Instead of presenting the data as a function of q and ω, it can be expressed in terms of a single variable y

$$F(y,q) = \left(\frac{d^2\sigma}{d\Omega d\omega}\right)_{EXP} \left(\frac{1}{Z\sigma_{ep}(q) + N\sigma_{en}(q)}\right) \frac{d\omega}{dy}$$

The scaling function $F(y,q)$ is formed from the measured cross section at 3- momentum transfer q, dividing out the incoherent single nucleon contributions at that three momentum transfer.

Scaling and Super Scaling in Quasi-Elastic Scattering

The fact that the nuclear density is nearly constant for A ≥ 12 leads one to ask, can scaling results be applied from 1 nucleus to another? W.M. Alberico, et al Phys. Rev. **C38**, 1801 (1988), T.W. Donnelly and I. Sick, Phys. Rev. **C60**, 065502 (1999)



A new dimentionless scaling variable is employed

$$\psi = \frac{y_{RFG}}{k_{Fermi}} = \frac{m_N}{k_{Fermi}}(\lambda\sqrt{1+\tau^{-1}} - \kappa)$$

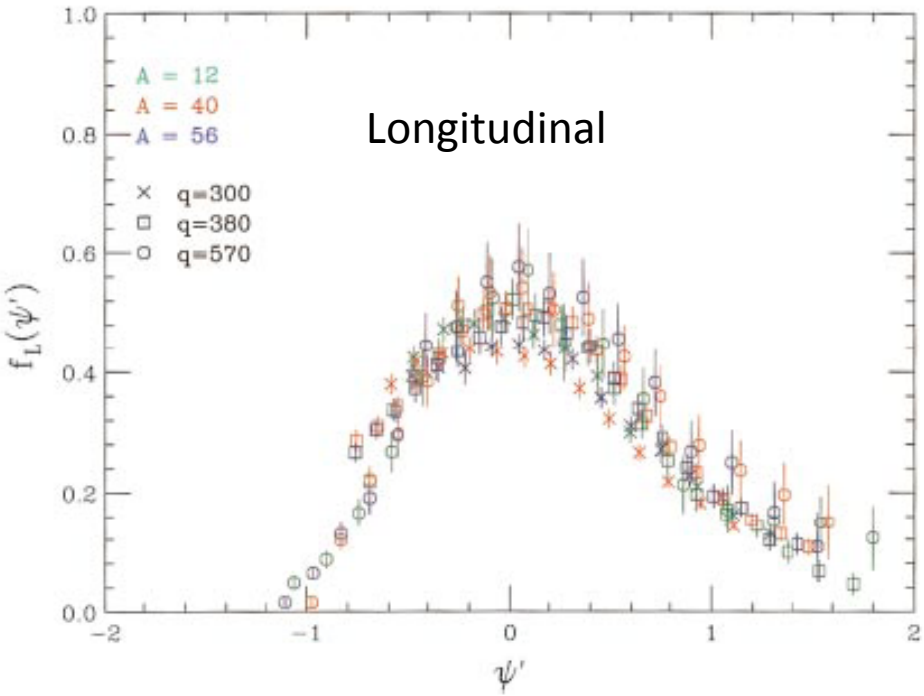
$$\lambda = \omega / 2m_N, \tau = Q^2 / 4m_N^2, \kappa = q / 2m$$

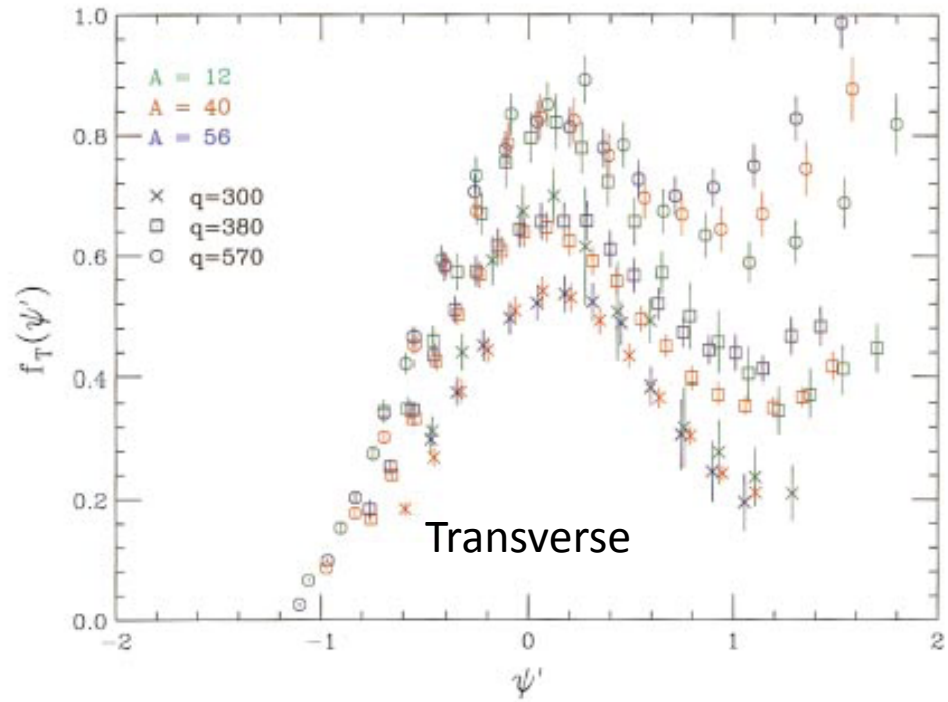
Note linear scale: not bad for

$$\psi < 0$$

Serious divergence above $\psi = 0$

Look more carefully: scaling into its Longitudinal and Transverse Responses Phys. Rev. **C60**, 065502 (1999)





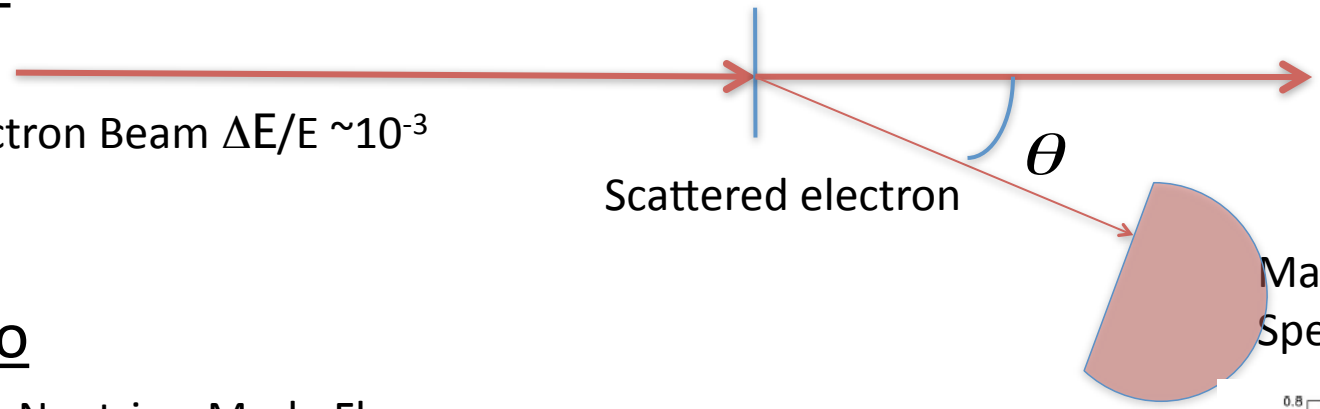
Longitudinal scattering weakly dependent upon nucleus and momentum transfer

Transverse response depends dramatically upon q^2 (up to ~50%): not reproduced in FG model!

Transverse also nearly independent of nucleus.

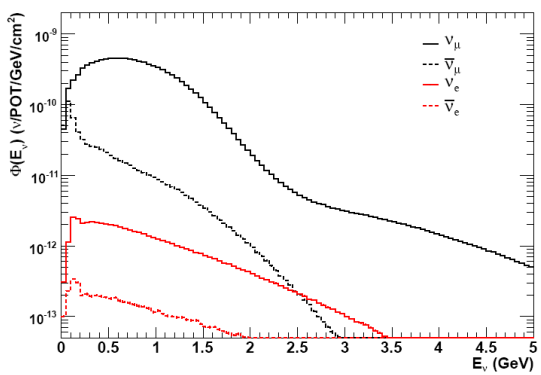
Contrast e-N with ν-N Experiments

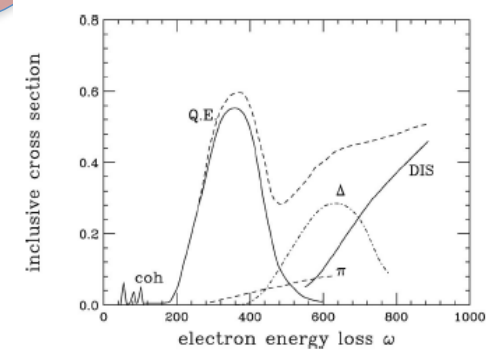
Electron

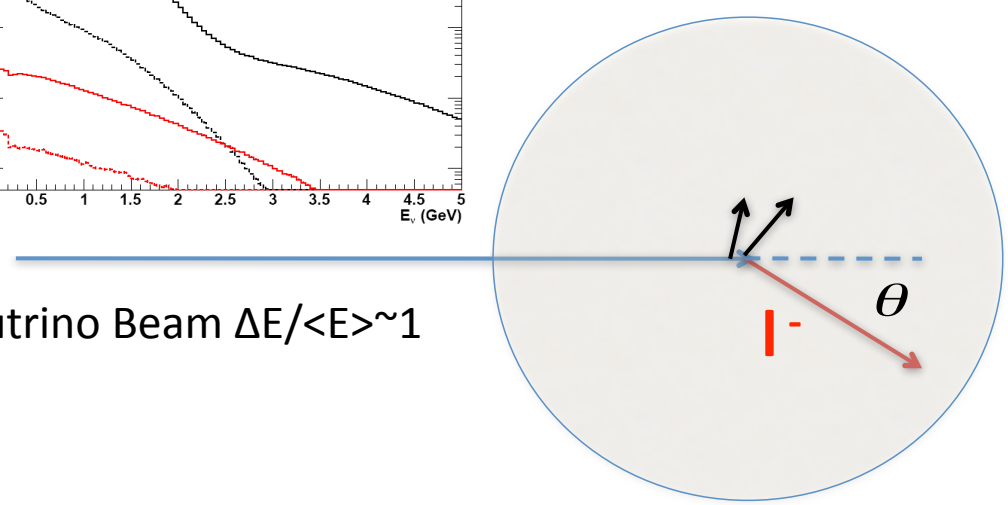


Neutrino

Neutrino-Mode Flux







Neutrino Beam ΔE/<E>~1

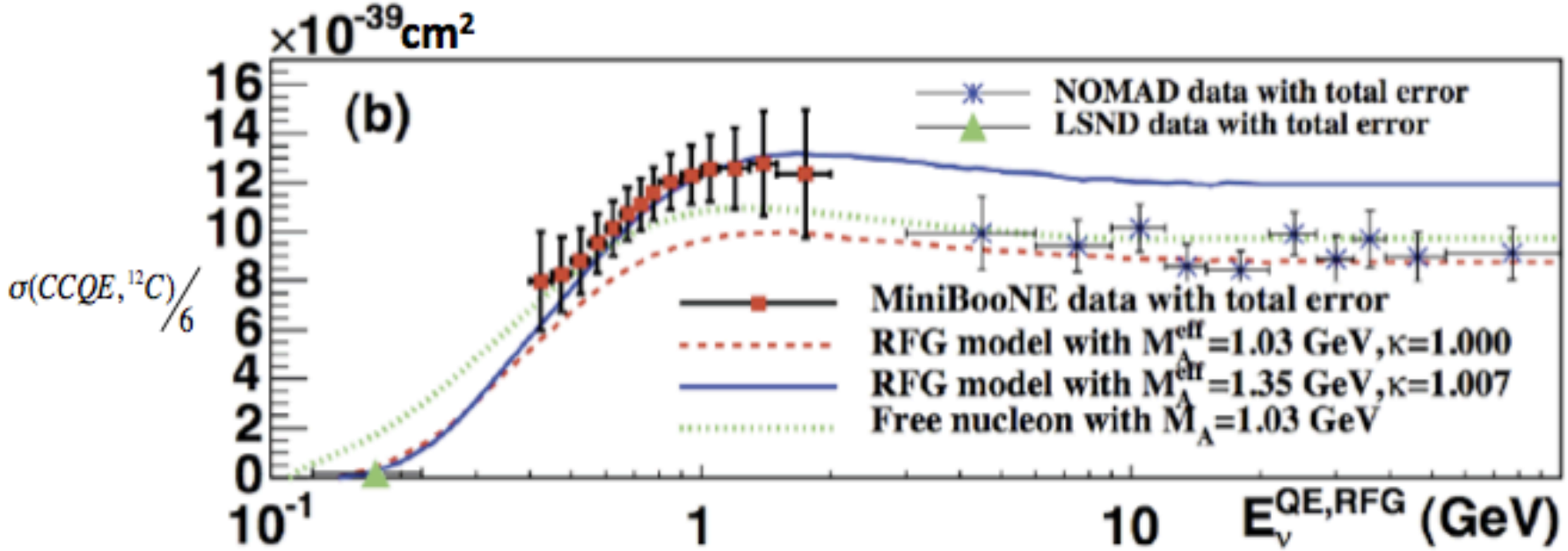
E_ν unknown!

What's q,ω ???

Try to infer from lepton

Physics very similar but experimental conditions vastly different

MiniBooNE's "CCQE" Data for $\nu_\mu + {}^{12}C \rightarrow \mu^- + (7p,5n)$



How can the cross section exceed that of the free neutron???

Omar Benhar et al

arXiv:1006.4783v1 [nucl-th] 24 Jun 2010

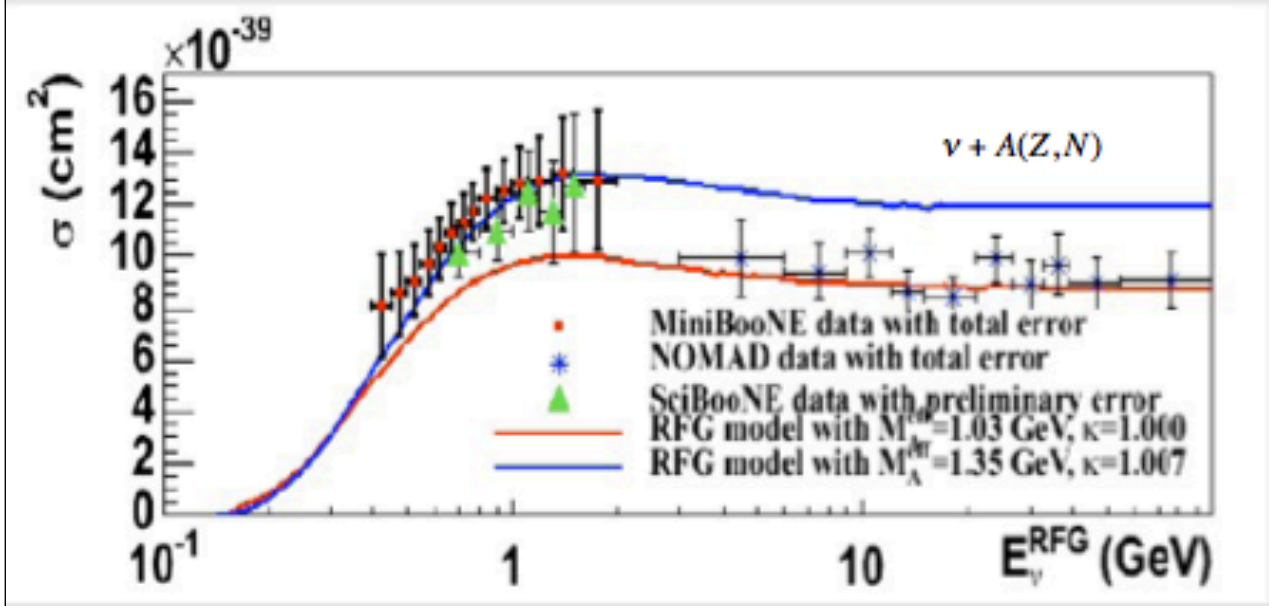
"We analyze the assumptions underlying the treatment of electron scattering data, and argue that the description of neutrino data will require a new paradigm, suitable for application to processes in which the lepton kinematics is not fully determined." **They can only reduce the RFG!**

Other Experimental Results from CCQE

SciBooNE CCQE results

CCQE results from SciBooNE

- SciBooNE: (highly segmented) scibar in Booster nu beam at FNAL (as MiniBooNE)
- (preliminary) results indicate higher cross section as seen by MiniBooNE (arXiv:0909.5647)



Experiment	Target	Cut in Q^2 [GeV^2]	M_A[GeV]
K2K[4]	oxygen	$Q^2 > 0.2$	1.2 ± 0.12
K2K[5]	carbon	$Q^2 > 0.2$	1.14 ± 0.11
MINOS[6]	iron	no cut	1.19 ± 0.17
MINOS[6]	iron	$Q^2 > 0.2$	1.26 ± 0.17
MiniBooNE[7]	carbon	no cut	1.35 ± 0.17
MiniBooNE[7]	carbon	$Q^2 > 0.25$	1.27 ± 0.14
NOMAD[8]	carbon	no cut	1.07 ± 0.07

TABLE I. Recent M_A measurements

arXiv:1007.2195

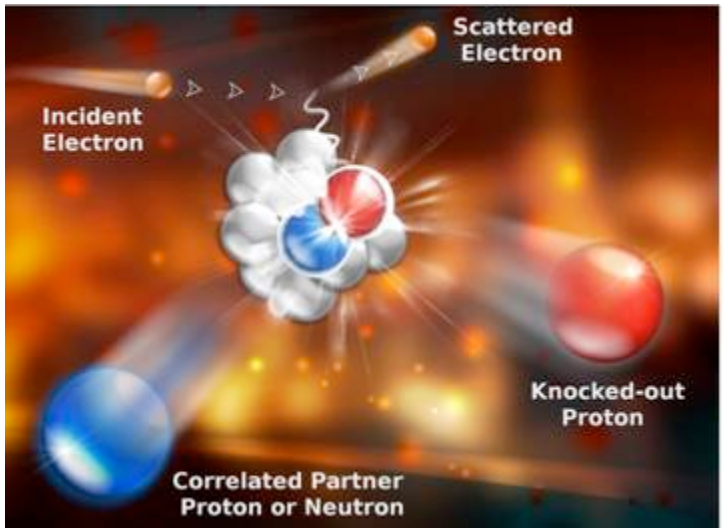
Generally: Large value of M_A required to fit both the Q^2 dependence and the larger than expected yield.

How do we go beyond the Fermi Gas model?

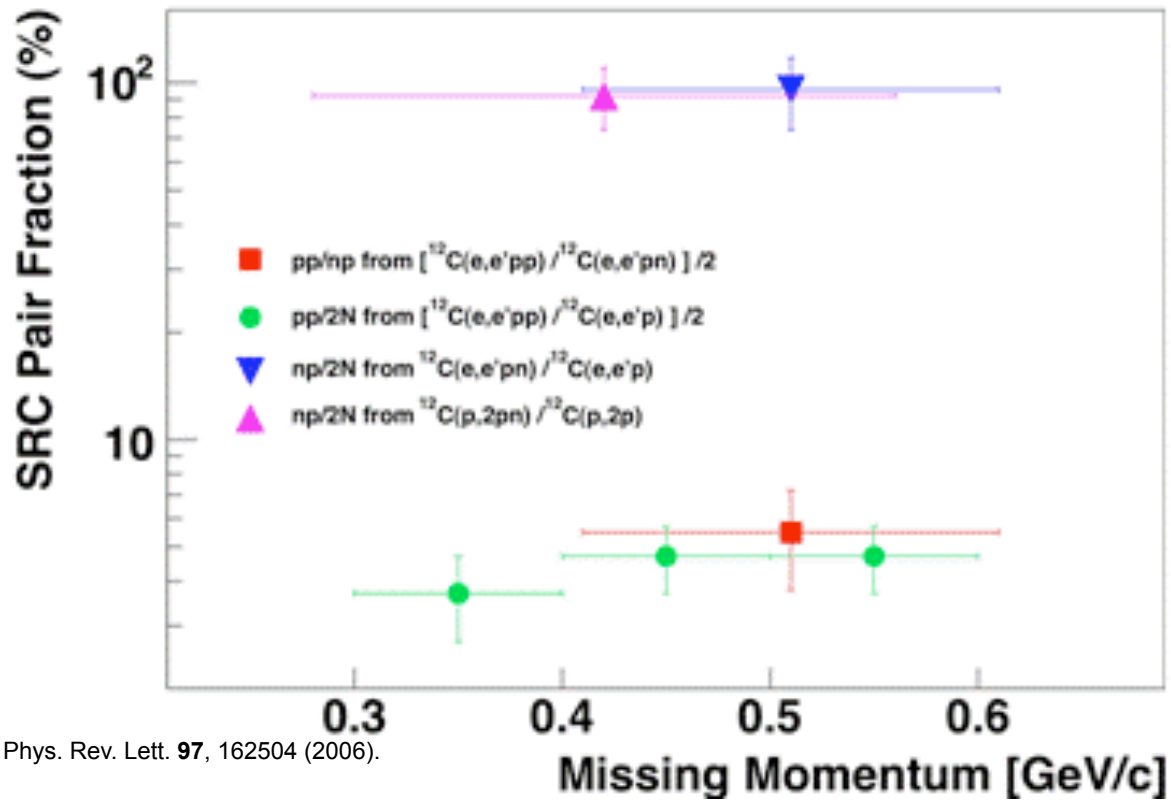
Typical FG models include high-momentum nucleons.

Experiments from Jlab/BNL probe high-momentum pairs

http://www.aps.org/units/dnp/research/correlations.cfm



np vs. pp high-momentum pairs in Carbon



[3] E. Piasetzky, M. Sargsian, L. Frankfurt, M. Strikman and J. W. Watson, Phys. Rev. Lett. **97**, 162504 (2006).
[4] A. Tang et al., Phys. Rev. Lett. **90**, 042301 (2003).
[5] R. Subedi et al., Science **320**, 1476 (2008).
[6] R. Shneor et al, Phys. Rev. Lett. **99**, 072501 (2007).

Beyond the Fermi Gas Model

$$R_{L,T}\ (q,\omega) = \int d\omega\ \langle 0|\ j^{\mu}(x',t')\ \exp[i\omega(t'-t) + i\mathbf{q}\cdot(x'-x)]\ j_{\mu}(x,t)\ |0\rangle$$

Non-relativistic (Quantum Mechanical) reduction

$$R_L\ (q,\omega) = \sum_f \langle 0|\ \rho^{\dagger}(\mathbf{q}\ |f\rangle\ \langle f|\ \rho(\mathbf{q})\ |0\rangle\ \delta(E_f - E_0 - \omega)$$

$$R_T\ (q,\omega) = \sum_f \langle 0|\ \mathbf{j}^{\dagger}(\mathbf{q})\ |f\rangle\ \langle f|\ \mathbf{j}(\mathbf{q})\ |0\rangle\ \delta(E_f - E_0 - \omega)$$

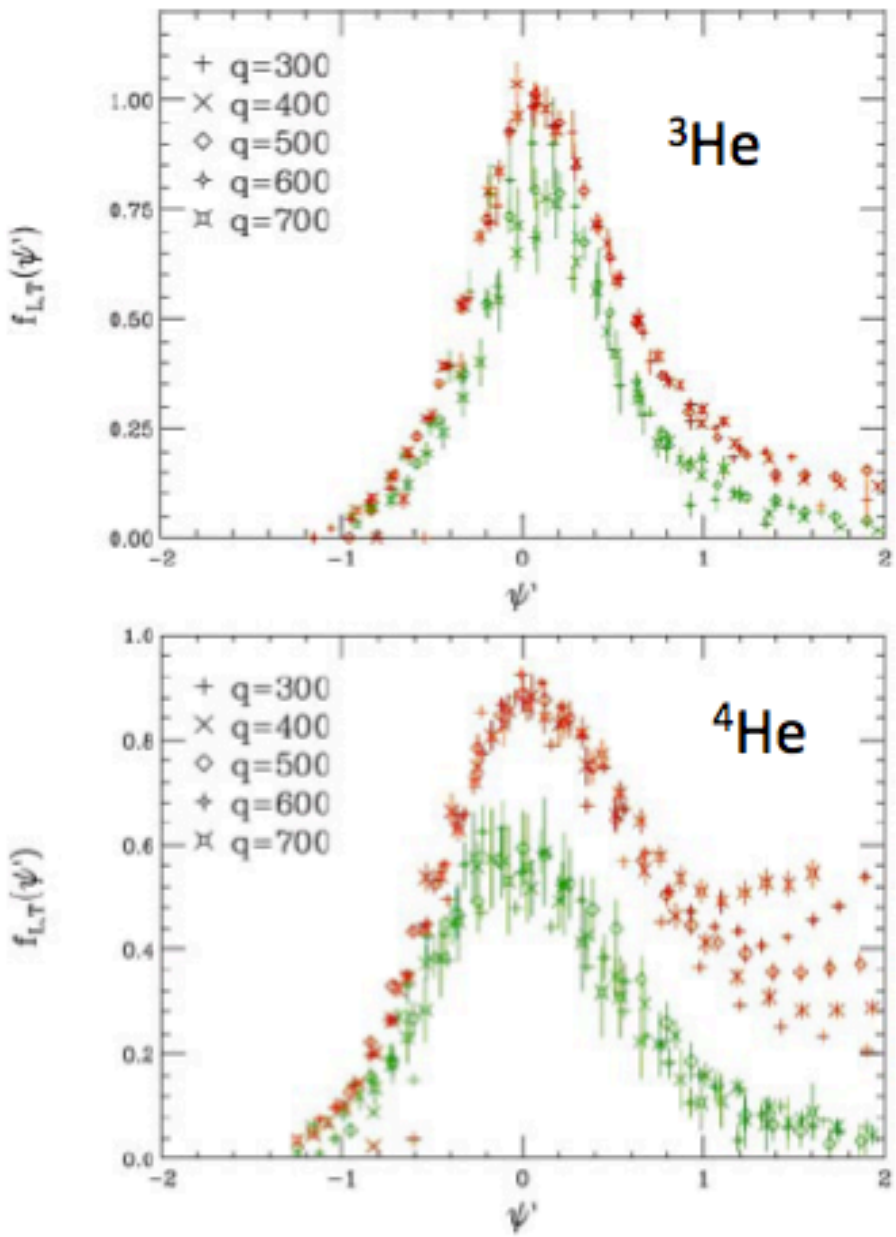
Limitations: Includes only virtual pions, non-relativistic

Advantages: Includes virtual pions (via current operators), complete initial and final state interactions

Can the response exceed that of free nucleons?

Electron Scattering in the lightest nuclei: A = 2,3,4,...

PR C65, 024002 (2002) Investigated the increased transverse response between ^{3}He and ^{4}He



The plots to the left show the longitudinal and transverse response for e + ^{3,4}He QE scattering as a function of the scaling variable ψ'.

$$\psi' = \frac{1}{k_F}\left(\sqrt{\omega^2 + 2m\omega} - q\right)$$

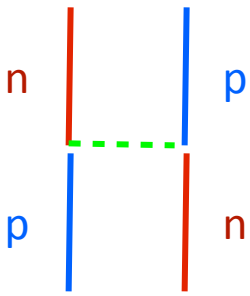
$$at\ \psi'=0\quad \omega=\sqrt{m^2+q^2} - m = T_N$$

Which is the energy transfer to a free nucleon at rest.

$$\psi' < 0, \omega < T_N \quad \psi' > 0, \omega > 0$$

Nuclear Interaction

$$H = \sum_i T_i + \sum_{i<j} V_{ij} + \ldots$$



Pions carry charge and current and typical momenta ~ 1-2 fm^{-1}

Nuclear Current Operators

$$\rho(\mathbf{q}) = \sum_i \rho_i(\mathbf{q}) + \ldots$$

$$\mathbf{j}(\mathbf{q}) = \sum_i \mathbf{j}_i(\mathbf{q}) + \sum_{i<j} \mathbf{j}_{ij}(\mathbf{q})$$

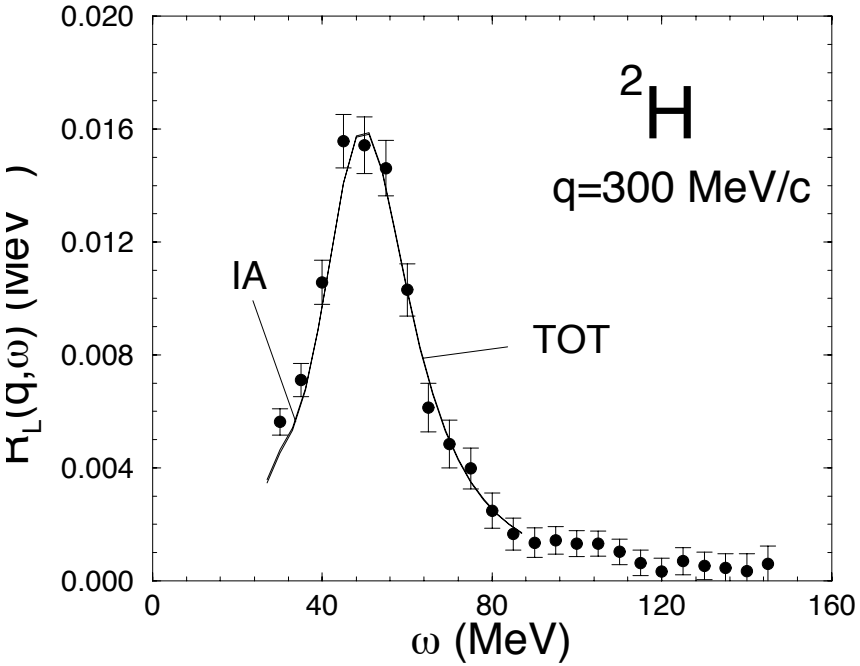
Two-body vector currents required even at non-relativistic order

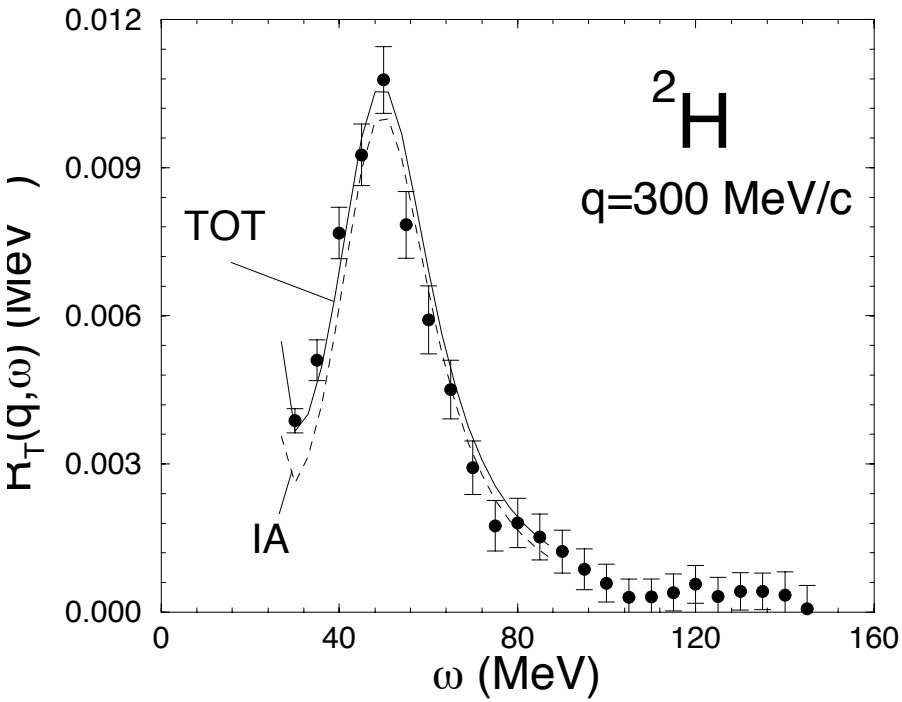
Electron Scattering from the Deuteron

The deuteron is special, average pair separation is 4 fm!

`typical' nucleon momentum small

small 2-body current, FSI effects





QE scattering well known, can use deuterium to calibrate neutrino flux

Examine Overall Strength

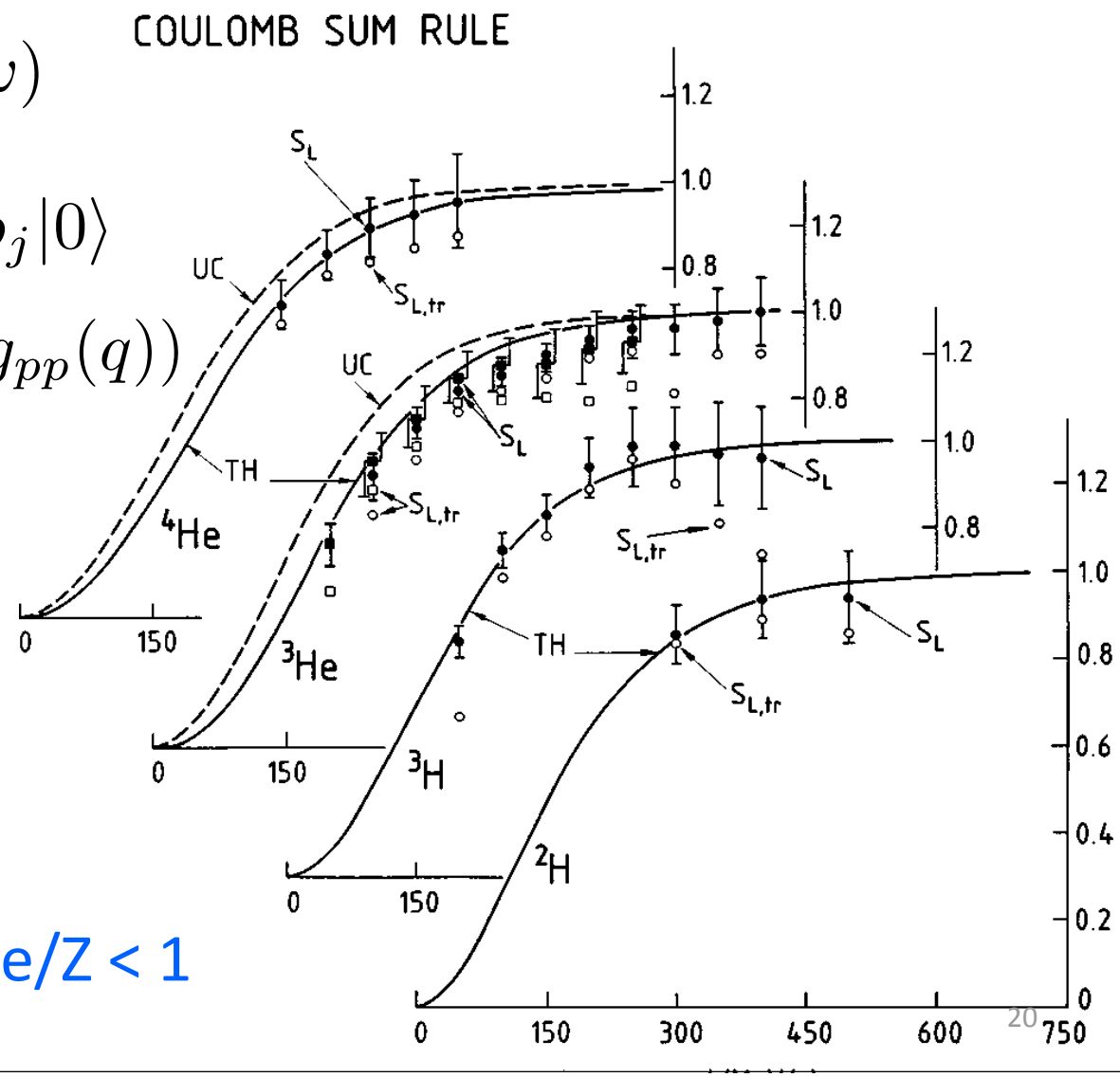
Sum rules calculable as ground-state expectation value

$$R_L\ (q,\omega) = \sum_f \langle 0|\ \rho^\dagger(\mathbf{q}\ |f\rangle\ \langle f|\ \rho(\mathbf{q})\ |0\rangle\ \delta(E_f - E_0 - \omega)$$

$$S_L(q,\omega)\ = \int d\omega\ R_L(q,\omega)$$

$$= \sum i,j\ \langle 0|\rho_i^\dagger\ \rho_j|0\rangle$$

$$= Z(1 + (Z-1)g_{pp}(q))$$

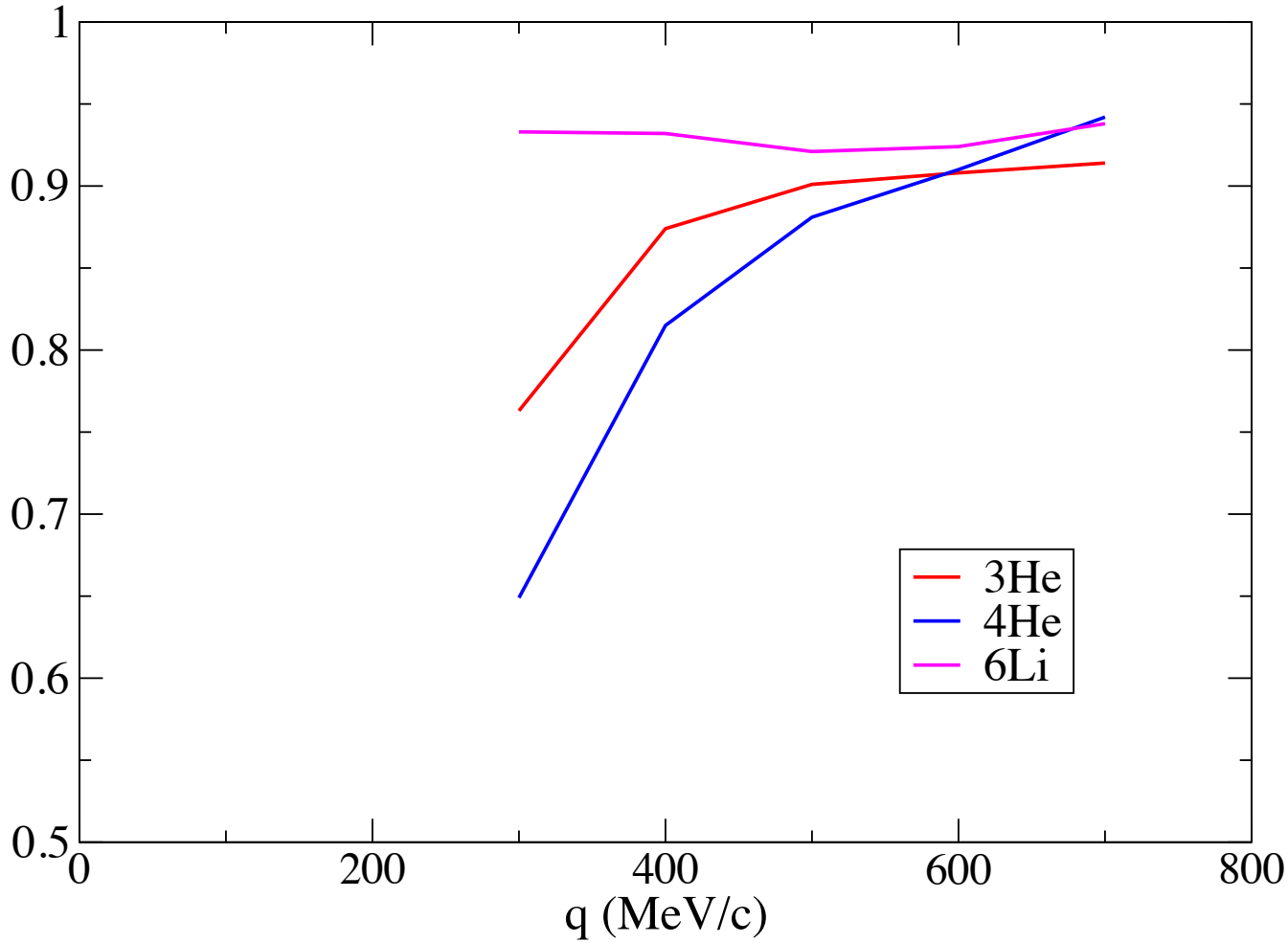


Coulomb Sum:
counts protons plus
p-p correlations + ...

Note: Coulomb Sum rule/Z < 1

Sum Rules in Light Nuclei

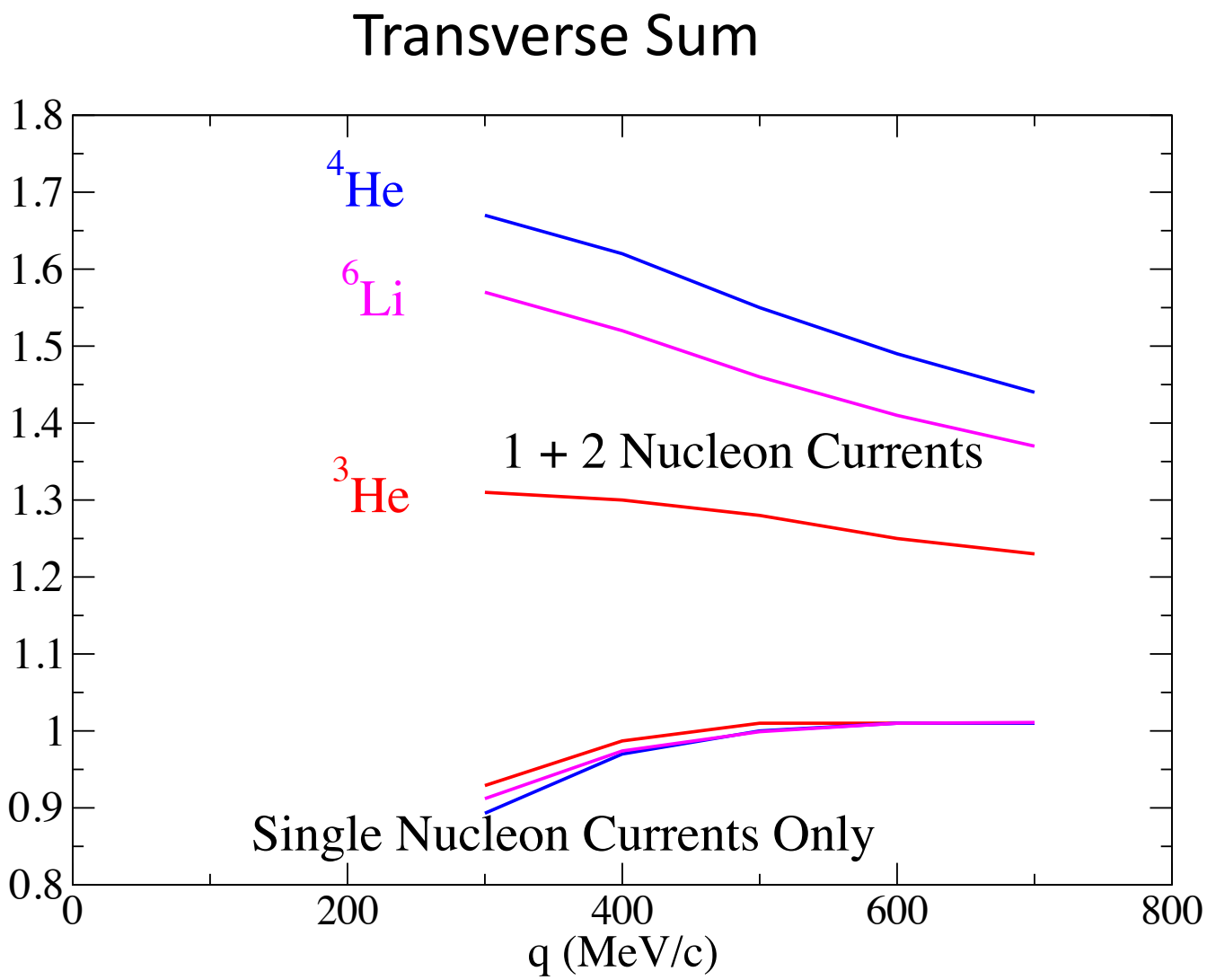
Longitudinal (Coulomb) Sum



Longitudinal Sum ~ 1

JC, Jourdan, Schiavilla, Sick
PRC 2002

Sum Rules in Light Nuclei



Transverse Sum > 1 !!!

Euclidian Response Functions:

PR C, 65, 024002 (cont.)

$$\tilde{E}(|\vec{q}|,\tau) = \int_{\omega_{th}}^{\infty} e^{-(\omega - E_0)\tau} R_{T,L}(|\vec{q}|,\omega) d\omega$$

$\tilde{E}(|\vec{q}|,\tau)$ can be calculated as follows

$$\tilde{E}_L(|\vec{q}|,\tau) = \langle 0 | \rho(\vec{q}) e^{-(H - E_0)\tau} \rho(\vec{q}) | 0 \rangle - e^{-\frac{q^2 \tau}{2Am}} \langle 0(\vec{q}) | \rho(\vec{q}) | 0 \rangle$$

$$\tilde{E}_T(|\vec{q}|,\tau) = \langle 0 | \vec{j}_T(\vec{q}) e^{-(H - E_0)\tau} \vec{j}_T(\vec{q}) | 0 \rangle - e^{-\frac{q^2 \tau}{2Am}} \langle 0(\vec{q}) | \vec{j}_T(\vec{q}) | 0 \rangle$$

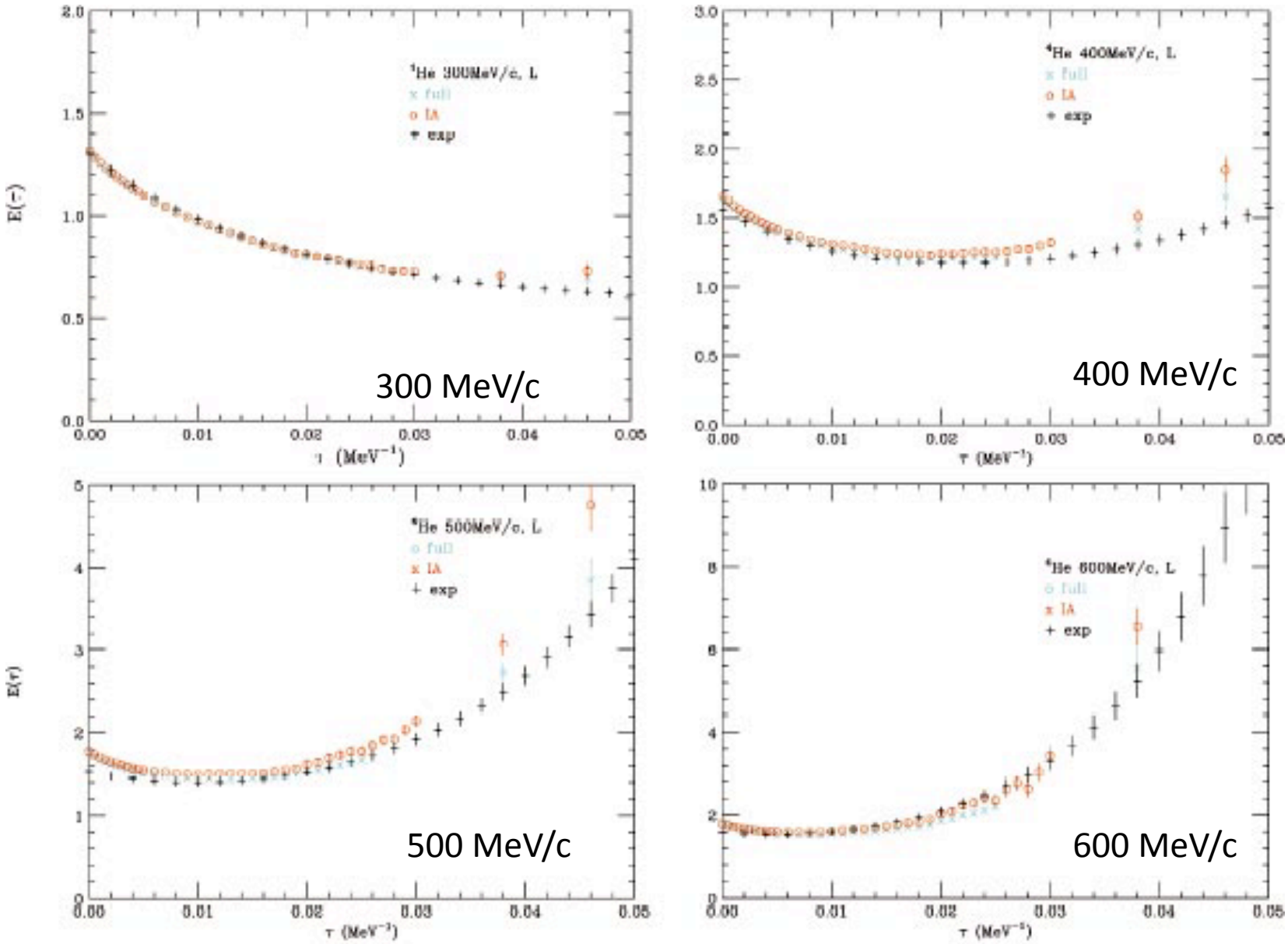
$$E_{T,L}(q,\tau) = \frac{e^{\frac{q^2 \tau}{2m}}}{(1 + Q^2 / \Lambda^2)^{-4}} \tilde{E}_{T,L}(q,\tau)$$

Is presented, removing the trivial kinetic energy dependence of the struck nucleon, and the Q^2 dependence of the nucleon FF

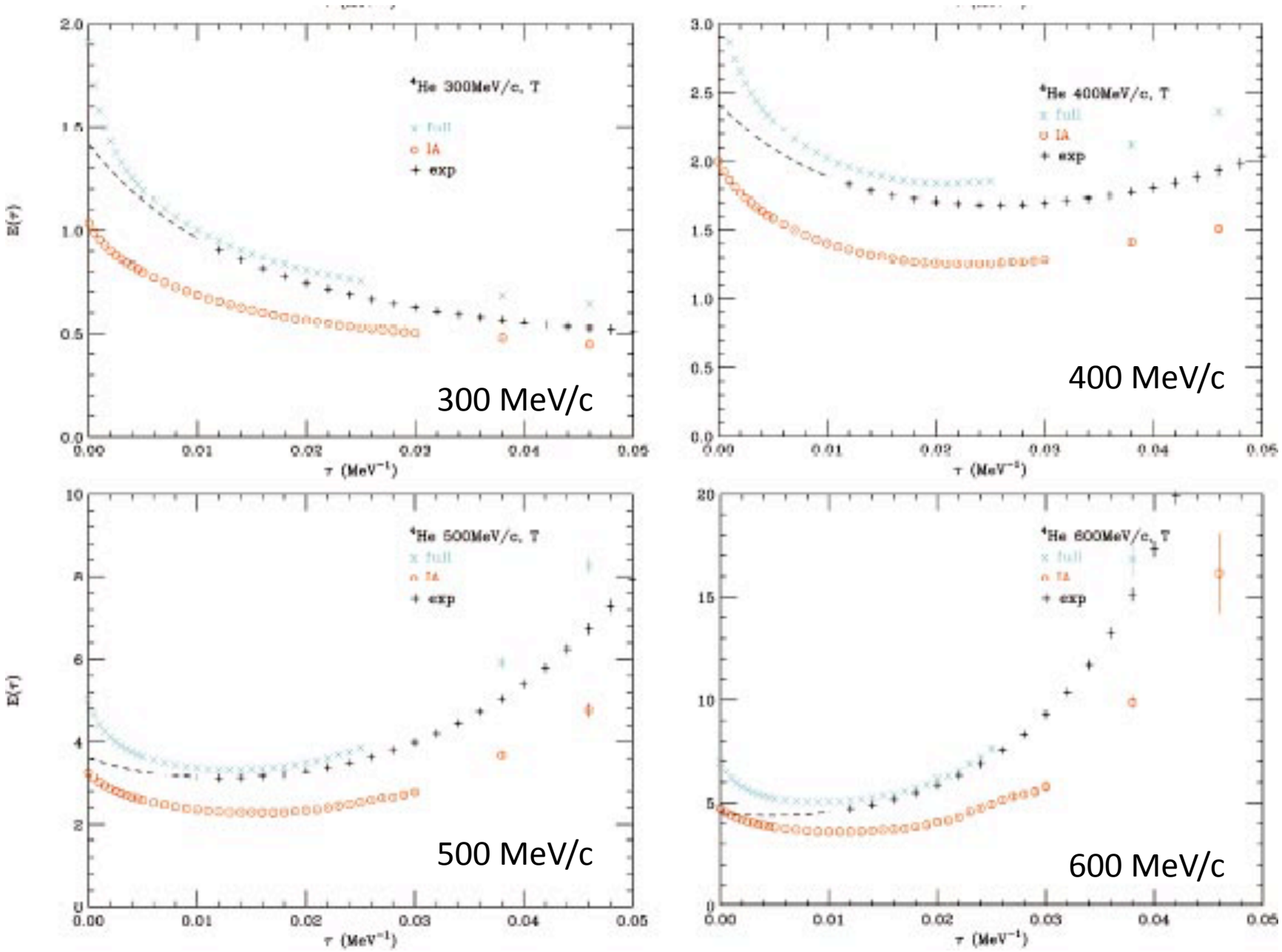
At $\tau = 0$ get sum rule

Large $\tau \Rightarrow$ low energy (eventually elastic) response

^{4}He Longitudinal Response



^{4}He Transverse Response



Further Info from PR C65 024002

Effect is due to n-p pairs

TABLE V. ^{4}He transverse sum rule: contribution of *pp* and *nn* pairs.

q (MeV/c)	1	1+2	1+2 *pp* or *nn* only
300	0.915	1.65	0.919
400	0.980	1.59	0.987
500	1.01	1.53	1.02
600	1.01	1.47	1.03
700	1.01	1.41	1.03

Small effect of 2-body currents evaluated in the Fermi Gas:

TABLE VII. Excess-strength contributions ΔS_L and ΔS_T to the Fermi-gas sum rules from terms involving two-nucleon currents.

q (MeV/c)	ΔS_L	ΔS_T
300	0.004	0.114
400	0.007	0.081
500	0.011	0.066
600	0.017	0.060
700	0.024	0.056

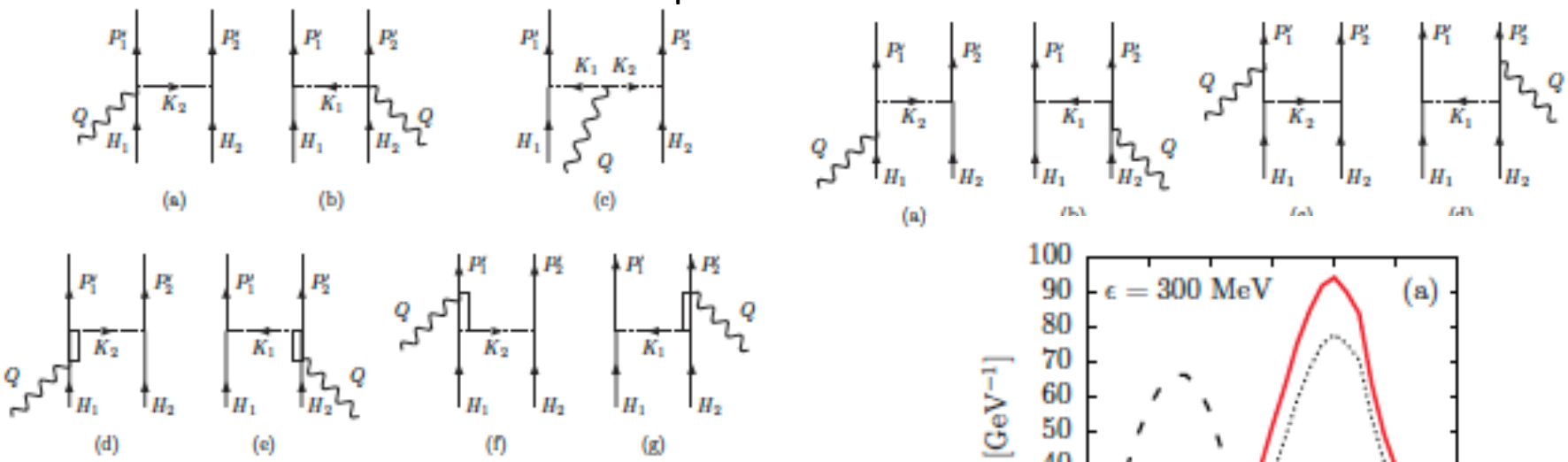
Some More Evidence

Amaro, et al, PHYSICAL REVIEW C **82, 044601 (2010)**

Meson Exchange Diags.

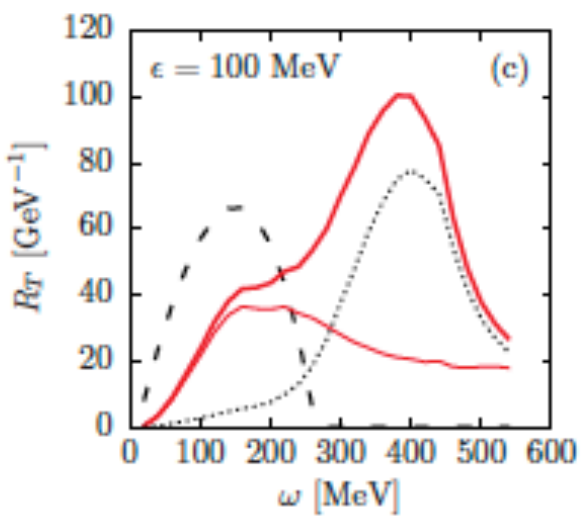
2p-2h fin. sts.

Correlation Diags.

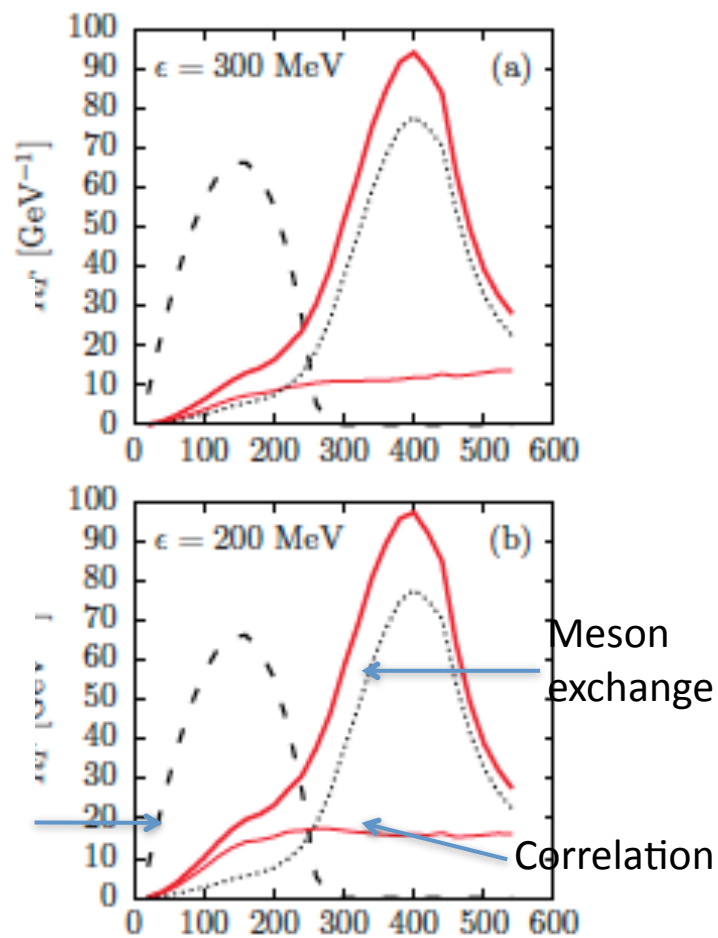


<u>Electron Scattering</u>

^{56}Fe, q=0.55GeV/c



One body RFG



Summary

- Theorists and Experimentalists must more carefully specify what they mean by QE.

- Fermi Gas Model inadequate for ν-nucleus inclusive scattering

- Realistic models required: have to include all the following: initial and final state correlations and 2-body currents

- Both flux and cross-sections are important

- Theory and experiment on e-N transverse Response Functions might be a very fruitful avenue: severe constraints on matrix elements.

Conclusion

Differences between neutrino and anti-neutrino cross sections(yields) should be better understood before (at least while) large and expensive ($1B) CP violation searches are undertaken.

Relatively modest effort could yield a big return.

Extraction of neutrino energy in QE kinematical regime is a very important issue.