Double-beta decay
nuclear matrix elements

Javier Menéndez
University of Barcelona

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Nuclear matrix elements for new-physics searches

Neutrinos, dark matter studied in experiments using nuclei

Nuclear structure physics encoded in nuclear matrix elements key to plan, fully exploit experiments

$0^{\nu}_{\beta\beta}$: 
\[
\left( T^{0\nu\beta\beta}_{1/2} \right)^{-1} \propto g_A^4 \left| M^{0\nu\beta\beta} \right|^2 m_{\beta\beta}^2
\]

Dark matter: 
\[
\frac{d\sigma_{\chi N}}{dq^2} \propto \left| \sum_i c_i \zeta_i \mathcal{F}_i \right|^2
\]

CE$\nu$NS: 
\[
\frac{d\sigma_{\nu N}}{dq^2} \propto \left| \sum_i c_i \zeta_i \mathcal{F}_i \right|^2
\]

$M^{0\nu\beta\beta}$: Nuclear matrix element
$\mathcal{F}_i$ : Nuclear structure factor
Calculating nuclear matrix elements

Nuclear matrix elements needed in low-energy new physics searches

\[ \langle \text{Final} | \mathcal{L}_{\text{leptons-nucleons}} | \text{Initial} \rangle = \langle \text{Final} | \int dx \, j^\mu (x) J_\mu (x) | \text{Initial} \rangle \]

- **Nuclear structure calculation of the initial and final states:**
  - Shell model, QRPA, IBM,
  - Energy-density functional
  - Ab initio many-body theory
  - GFMC, Coupled-cluster, IMSRG...

- **Lepton-nucleus interaction:**
  - Hadronic current in nucleus:
    - phenomenological,
    - effective theory of QCD
$0\nu\beta\beta$ decay nuclear matrix elements

Large difference in nuclear matrix element calculations: factor $\sim 2 - 3$

$$\langle 0^+_f | \sum_{n,m} \tau_n^- \tau_m^- \sum_x H_x^x (r) \Omega^x | 0^+_i \rangle$$

$\Omega^x =$ Fermi ($\mathbb{1}$), GT ($\sigma_n\sigma_m$), Tensor

$H(r) =$ neutrino potential

EDF: large NMEs
QRPA: wider range
NSM: small NMEs
IMSRG ab initio

$^{48}$Ca NME: quite small (no 2b currents)

M. Agostini, G. Benato, J. Detwiler, JM, F. Vissani, in preparation
Correlations: proton-neutron pairing

$0\nu\beta\beta$ NMEs agree without nuclear correlations (very simplistic nuclei)

NMEs too large if proton-neutron pairing correlations are neglected

Related to approximate $SU(4)$ symmetry of the $\sum H(r)\sigma_i\sigma_j\tau_i\tau_j$ operator
Correlations: large configuration space

$^{48}\text{Ca}$ extended configuration space from $pf$ to $sdpf$, 4 to 7 orbitals
dimension $10^5$ to $10^9$

$^{48}\text{Ca}$ $0^+_2$ state lowered by 1.3 MeV
nuclear matrix elements
enhanced only moderately 30%

Iwata et al. PRL116 112502 (2016)

Also small effect of large space
with perturbative calculation
Coraggio et al. PRC 101 044315 (2020)

Likewise, very mild effect
found in $^{76}\text{Ge}$ GCM calculations
Jiao et al. PRC96 054310 (2017)
Correlations: ab initio IMSRG NME for $^{48}\text{Ca}$

Complete nuclear correlations included in ab initio calculations
IMSRG (in-medium similarity renormalization group) for $^{48}\text{Ca}$ NME

Correlations systematically built on collective reference state
Generator coordinate method: deformation, isoscalar pairing

Best calculation reproduces electromagnetic transitions in $^{48}\text{Ti}$

NME $\sim 0.4/30\%$ smaller than nuclear shell model

Yao et al.
PRL 124 232501 (2020)
Ab initio many-body methods

Oxygen dripline using chiral NN+3N forces correctly reproduced ab-initio calculations treating explicitly all nucleons excellent agreement between different approaches

No-core shell model (Importance-truncated)
In-medium SRG
Hergert et al. PRL110 242501(2013)
Self-consistent Green’s function
Cipollone et al. PRL111 062501(2013)
Coupled-clusters
Jansen et al. PRL113 142502(2014)

Energy (MeV)

obtained in large many-body spaces

AME 2012

MR-IM-SRG
IT-NCSM
SCGF
Lattice EFT
CC

Mass Number A
Chiral effective field theory

Chiral EFT: low energy approach to QCD, nuclear structure energies
Approximate chiral symmetry: pion exchanges, contact interactions
Systematic expansion: nuclear forces and electroweak currents

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Weinberg, van Kolck, Kaplan, Savage, Wise, Meißner, Epelbaum...

2b currents applied to $\nu d$ scattering (SNO), $^3$H $\beta$-decay, $\mu$ moment...

Park, Baroni, Krebs...
Solution of the “$g_A$ quenching” puzzle

$\beta$ decays ($e^-$ capture) challenge for nuclear theory

\[ \langle F | \sum_i [g_A \sigma_i \tau_i^-]^{\text{eff}} | l \rangle , \quad [\sigma_i \tau]^{\text{eff}} \approx 0.7 \sigma_i \tau \]

Phenomenological models need $\sigma_i \tau$ “quenching”


Ab initio calculations including two-body/meson-exchange currents and additional nuclear correlations do not need any “quenching”
Origin of $\beta$ decay “quenching”

Which are main effects missing in conventional $\beta$-decay calculations?

Relatively similar and complementary impact of

- nuclear correlations
- meson-exchange currents

Gysbers et al.
2b currents in $\beta\beta$ decay

In $0\nu\beta\beta$ decay, two weak currents lead to four-body operator when including the product of two 2b currents: computational challenge

Approximate 2b current as effective 1b current

Quenching reduced to $\sim 20\%$ at $p \sim m_\pi$ for $0\nu\beta\beta$ decay

JM et al. PRL107 062501(2011)

Approximate 4b operator as effective 3b operator

Estimated effect $\sim 10\%$

Wang et al. PRC98 031301 (2018)

Estimations suggest that 2b currents smaller in $0\nu\beta\beta$ than $\beta$ decay
Light neutrino exchange: new contact operator

Contact operator suggested to contribute to light-neutrino exchange
Cirigliano et al. PRL120 202001(2018)

\[ T_{1/2}^{-1} = G_{01} \left( g_A^2 M^{0\nu} + g^{NN}_\nu m_\pi^2 M^{0\nu}_{\text{cont}} \right)^2 \frac{m_{\beta\beta}^2}{m_e^2} \]

Unknown value of the hadronic coupling \( g^{NN}_\nu \)!
to be determined experimentally or Lattice QCD calculations (\( \sim g_A \))

Short-range operator similar disagreement than standard NME
larger error bars because uncertainty in short-range dynamics
$0\nu\beta\beta$ mediated by BSM heavy particles

Standard Model extensions
trigger $0\nu\beta\beta$ (heavy $\nu$, $M_R$...)

Effective field theory
master formula
Cirigliano et al JHEP 12 097 (2018)

\[
T^{-1}_{1/2} = G_0 \left( g_A^2 M_0^{\nu} + g_{\nu}^{NN} m^2_\pi M_0^{\nu} \right)^2 \frac{m^2_{\nu\nu}}{m^2_e} \\
+ \frac{m^2_N}{m^2_e} \ddot{G} \ddot{g}^4 \ddot{M}^2 \left( \frac{\nu}{\Lambda} \right)^6 \\
+ \frac{m^4_N}{m^2_e v^2} \dddot{G}' \dddot{g}'^4 \dddot{M}'^2 \left( \frac{\nu}{\Lambda'} \right)^{10} + \cdots ,
\]

Current constraints on
dim-7 ($\sim 1/\Lambda^3$), dim-9 ($\sim 1/\Lambda^5$)
operators: $\Lambda \gtrsim 250 / 5$ TeV

Agostini, Benato, Detwiler, JM, Vissani, in prep.
Tests of nuclear structure

Spectroscopy well described: masses, spectra, transitions, knockout...

Schiffer et al. PRL100 112501 (2009)
Kay et al. PRC79 021301 (2009)
...
Szwec et al., PRC94 054314 (2016)

Rodríguez et al. PRL105 252503 (2010)
...
Vietze et al. PRD91 043520 (2015)
Two-neutrino ECEC of $^{124}$Xe

Predicted $2\nu$ECEC half-life:
shell model error bar largely dominated by “quenching” uncertainty

Shell model, QRPA and Effective theory (ET) predictions
good agreement with XENON1T measurement of $2\nu$ECEC!
Tests of $2\nu\beta\beta$ electron spectrum

Shape of $\beta\beta$ spectrum constrains matrix element ratio $\xi_{31}^{2\nu} = M_{GT-3}^{2\nu}/M_{GT}^{2\nu}$

Theory deficiencies in $M_{GT}^{2\nu}$ fixed adjusting $g_A$ ("queching")

$\xi_{31}^{2\nu}$ measurement test theoretical models
KamLAND-Zen coll. in $^{136}$Xe

Shell model, most QRPA $\xi_{31}^{2\nu}$ predictions consistent with 90% confidence limit
KamLAND-Zen et al. PRL122 192501 (2019)

Similar study in $^{82}$Se by CUPID-0: $2\nu\beta\beta$ single-state dominance
only lowest-lying intermediate $^{82}$Br $1^+$ state, not predicted by theory!

CUPID-0, PRL123 262501 (2019)
Double Gamow-Teller strength distribution

Measurement of Double Gamow-Teller (DGT) resonance in double charge-exchange reactions $^{48}$Ca(pp,nn)$^{48}$Ti proposed in 80’s
Auerbach, Muto, Vogel... 1980’s, 90’s

Recent experimental plans in RCNP, RIKEN ($^{48}$Ca), INFN Catania

Promising connection to $\beta\beta$ decay, two-particle-exchange process, especially the (tiny) transition to ground state of final state

Shell model calculation
Shimizu, JM, Yako, PRL120 142502 (2018)

$$B(DGT^-; \lambda; i \rightarrow f) = \frac{1}{2J_i + 1} \left\langle ^{48}\text{Ti} \left| \left[ \sum_i \sigma_i \tau_i^- \times \sum_j \sigma_j \tau_j^- \right]^\lambda \right| ^{48}\text{Ca}_{gs} \right\rangle^2$$
Correlation of $0\nu\beta\beta$ decay to DGT transitions

Double GT transition to ground state $M_{\text{DGT}} = \langle F_{gs} \mid \sum_i \sigma_i \tau_i^- \times \sum_j \sigma_j \tau_j^- \rangle^0 \mid I_{gs} \rangle^2$

very good linear correlation with $0\nu\beta\beta$ decay nuclear matrix elements

Double Gamow-Teller correlation with $0\nu\beta\beta$ decay holds across nuclear chart
Shimizu, JM, Yako
PRL120 142502 (2018)

Common to shell model energy-density functionals interacting boson model, disagreement to QRPA

Experiments at RIKEN, INFN Catania may access DGT transitions
Similar $0\nu\beta\beta$ and $\gamma\gamma$ decays?

Explore correlation between $0\nu\beta\beta$ and $\gamma\gamma$ decays, focused on double-M1 transitions

\[
M_{M1M1}^{\gamma\gamma} = \sum_k \frac{\langle 0_f^+ | \sum_n (g_n^{l} I_n + g_n^{s} \sigma_n)^{IV} \rangle \langle 1_k^+ (IAS) | \sum_m (g_m^{l} I_m + g_m^{s} \sigma_m)^{IV} \rangle | 0_i^+ (DIAS) \rangle}{E_k - (M_i + M_f)/2}
\]

Similar initial and final states but both in same nucleus for electromagnetic transition

M1 and GT operators similar, physics of spin operator M1 also angular momentum

Different energy denominator

Romeo, JM, Peña-Garay, in progress
Correlation of $0\nu\beta\beta$ decay to $\gamma\gamma$ transitions

Good linear correlation between $\gamma\gamma$ M1M1 and $0\nu\beta\beta$ decays obtained with nuclear shell model, holds for nuclei with $A \sim 60 - 140$

Includes $\beta\beta$ nuclei: $^{76}\text{Ge}$, $^{82}\text{Se}$, $^{130}\text{Te}$, $^{136}\text{Xe}$

Romeo, JM, Peña-Garay, in progress

Measurements of $\gamma\gamma$ M1M1 decays could inform $0\nu\beta\beta$ NMEs!
Summary

Nuclear matrix elements key for the design of next-generation tonne-scale $0\nu\beta\beta$ decay experiments

- Present nuclear matrix element calculations still disagree by factor 2 – 3
- Ab initio calculations solve much of $\beta$ decay “quenching” problem, ab initio $^{48}\text{Ca}$ NME quite small extension to heavier $\beta\beta$ nuclei soon
- Tests to $2\nu\beta\beta$ lifetimes, electron spectra promising correlations of $0\nu\beta\beta$ with double-GT and $\gamma\gamma$ transitions
- Similar uncertainties in NMEs for BSM exchange of heavy particles