Current status of α_s determination on the lattice

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Based on recent review paper

J. Komijani, PP, J. Weber, Prog. Part. Nucl. Phys. 113 (2020) 103788
arXiV:2003:11703 [hep-lat]

Main focus of the talk is on α_s extraction from:

1) Moments of quarkonium correlators
2) Static quark antiquark potential
3) Step scaling with Schrödinger functional

Snowmass Joint EF05 / EF06 Topical Group Meeting, June 30, 2020

General considerations

Calculate a quantity $O(E,a)$ that depends on the energy scale E on the lattice with lattice spacing a. Perform the continuum extrapolation

$$O(E) = \lim_{a\to 0} O(E,a)$$

and compare it with perturbative calculations

$$O(E) = \sum c_n \left(\alpha_s^{\overline{MS}}(E)\right)^n \to \alpha_s^{\overline{MS}}(E).$$

Important ingridient is the determination of a in terms of f_π, f_K or hadron masses can be determined with a precision of 1% or less.

Examples: quarkonium correlators, vacuum polarization (Adler function), static potential, ghos-gluon vertecies
Window problem: $\Lambda_{QCD} \ll E \ll a^{-1}$, difficult to realize in practical calculations
Solution for the window problem: compare with lattice perturbation theory at finite a (small Wilson loops, is lattice pertubration theory reliable ?), step scaling methods (ALPHA collaboration)

Moments of quarkonium correlators

We use moments method pioneered by HPQCD and Karlsruhe group:

$$G_n = \sum_t t^n G(t), \;\; G(t) = a^6 \sum_{\mathbf{x}} (am_{h0})^2 \langle j_5(\mathbf{x},t) j_5(0,0)\rangle \qquad j_5 = \bar{\psi}\gamma_5\psi$$

Calculated continuum perturbation theory to order α_s^3

$$G_n = \frac{g_n(\alpha_s(\mu), \mu/m_h)}{am_h^{n-4}(\mu_m)}$$

To cancel lattice effects consider the reduced moments

$$R_n = \left(\frac{G_n}{G_n^0}\right)^{1/(n-4)}$$

Allison et al, PRD78 (2008) 054513

and similarly on the weak coupling side:

$$R_n = \begin{cases} r_4 & (n=4) \\ r_n \cdot (m_{h0}/m_h(\mu)) & (n \geq 6) \end{cases},$$

$$r_n = 1 + \sum_{j=1}^{3} r_{nj}(\mu/m_h) \left(\frac{\alpha_s(\mu)}{\pi}\right)^j$$

$+$ contribution from condensate

$$\sim \frac{1}{m_h^4} \langle \frac{\alpha_s}{\pi} G^2 \rangle$$

$R_4, \;\; R_6/R_8, R_8/R_{10} \Rightarrow \alpha_s(\mu)$

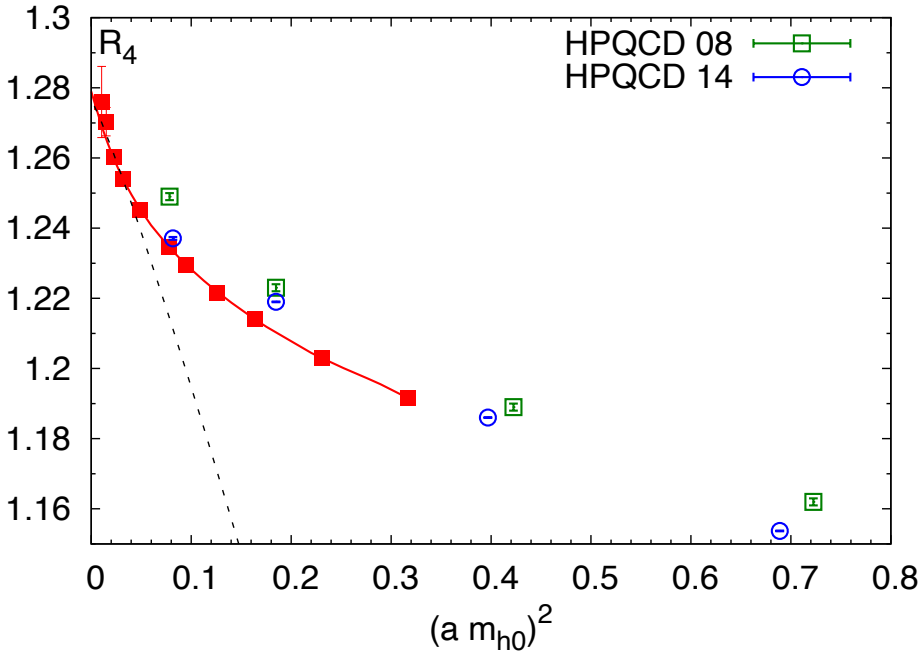
$R_6, \;\; R_8 \;\; R_{10} \Rightarrow m_h(\mu_m)$

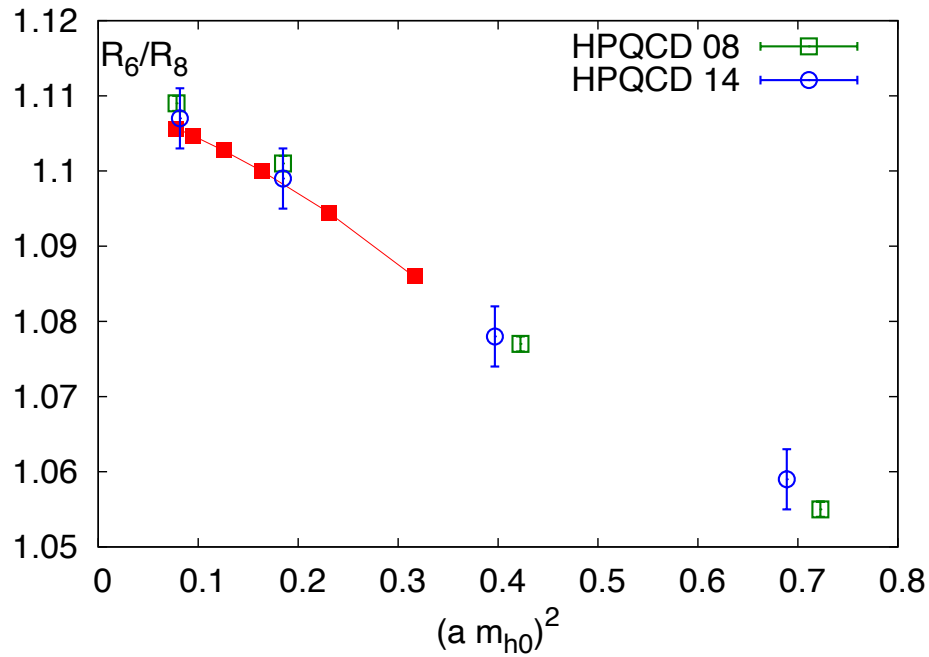
Lattice results on the moments of quarkonium correlators

$$m_h = m_c,\ 1.5m_c,\ 2m_c,\ 3m_c,\ 4m_c,\ m_b$$

Random color wall sources => statistical errors are negligible; Dominant errors are the finite volume errors and errors due to mistuning of the heavy quark mass.

PP, J Weber PRD 100 (2019) 034519

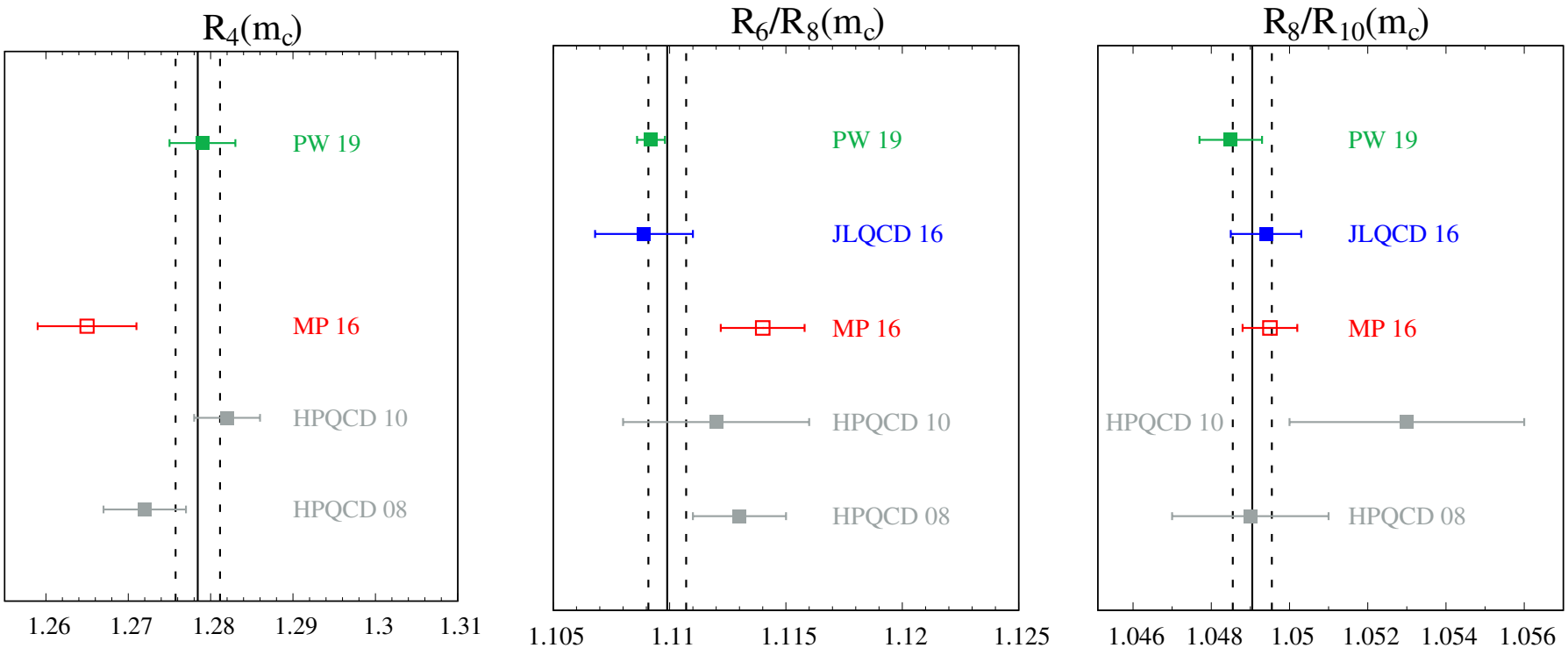




Continuum results are needed but there is a significant dependence on the lattice spacing

Lattice cutoff effects: $\sim \sum_{i=1}^{I} \sum_{j=1}^{J} \alpha_s^i (am_{h0})^{2j}$ $\Rightarrow$ use fits with $I = 2$, $J = 5$ and $\alpha_s = \dfrac{g_0^2}{4\pi u_0^4}$

Continuum results on the moments



HPQCD: 2+1 flavor improved staggered (asqtad) sea + valence HISQ,
Allison et al, PRD 78 (2008) 054513; McNeile, PRD 82 (2010) 034512

JLQCD: 2+1 flavor Domain-Wall Fermions,
Nakayama, Fahy, Hashimoto, PRD 94 (2016) 054507

MP 16: 2+1 flavor HISQ (sea and valence sectors)
Maezawa, PP, PRD PRD 94 (2016) 034504

Discrepancies are understood to be due simple $a^2 + a^4$ extrapolations

Extracting the strong coupling constant in 3f QCD

Natural choice: $\mu = \mu_m = m_h(m_h)$ $\quad R_4,\ R_6/R_8,\ R_8/R_{10} \to \alpha_s(m_h)$

perturbative error: $\pm 5 \times r_{n3}\alpha_s^4$

condensate error: $\langle \frac{\alpha_s}{\pi} G^2 \rangle = (-0.006 \pm 0.012)\ \mathrm{GeV}^4$

$\frac{m_h}{m_c}$	R_4	R_6/R_8	R_8/R_{10}	av.	$\Lambda_{\overline{MS}}^{n_f=3}$ [MeV]
1.0	0.3815(55)(30)(22)	0.3837(25)(180)(40)	0.3550(63)(140)(88)	0.3788(65)	315(9)
1.5	0.3119(28)(4)(4)	0.3073(42)(63)(7)	0.2954(75)(60)(17)	0.3099(48)	311(10)
2.0	0.2651(28)(7)(1)	0.2689(26)(35)(2)	0.2587(37)(34)(6)	0.2649(29)	285(8)
3.0	0.2155(83)(3)(1)	0.2338(35)(19)(1)	0.2215(367)(17)(1)	0.2303(150)	284(48)

perturbative and condensate errors decrease with increasing m_h/m_c

R_8/R_{10} gives systematically lower values of $\alpha_s(m_h)$ except for $m_h = 2m_c$
Take weighted average of R_4, R_6/R_8, R_8/R_{10} results to get the final $\alpha_s(m_h)$

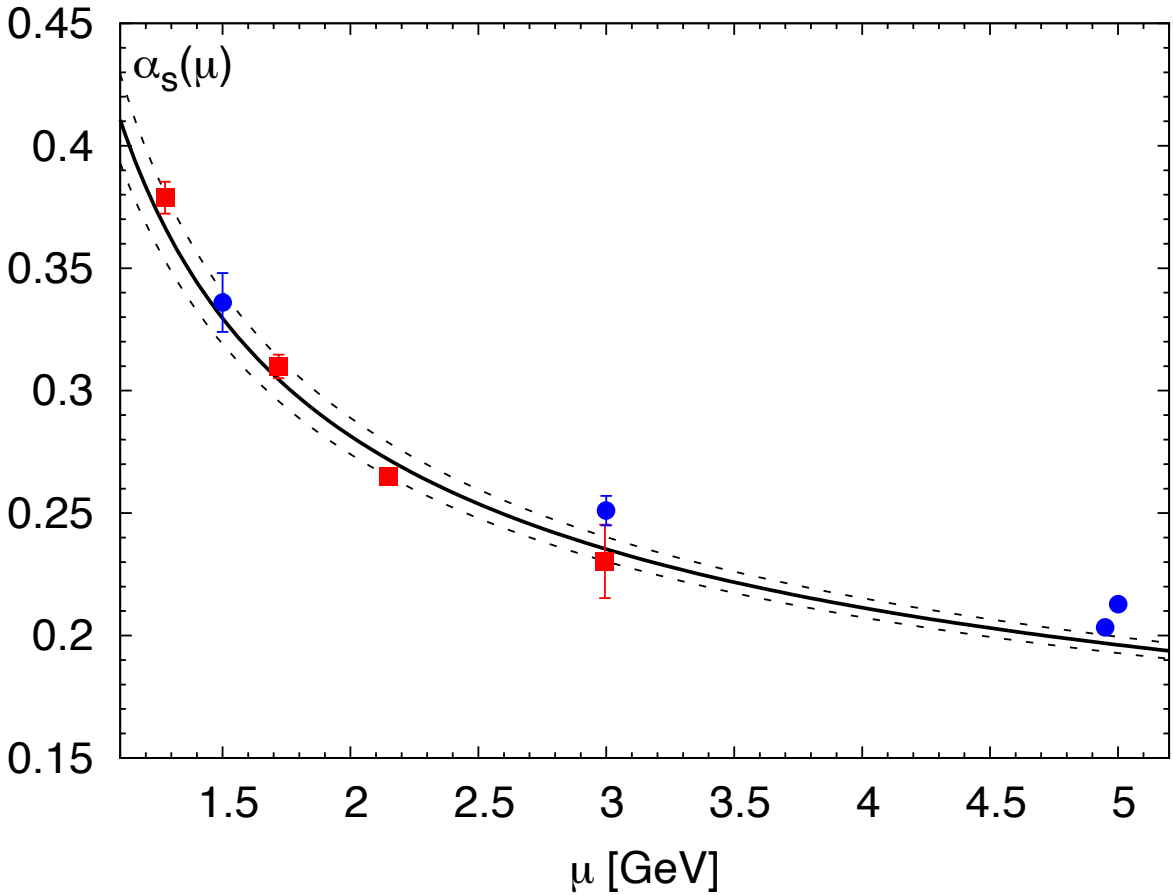
$$\alpha_s(m_h),\ R_6,\ R_8,\ R_{10} \to m_h(m_h) \to \Lambda_{\overline{MS}}^{n_f=3}$$

Problems with continuum extrapolations ?

$m_h = 2m_c$ result is an outlier; weighted average + spread :

$$\Lambda_{\overline{MS}}^{n_f=3} = 298 \pm 16\ \mathrm{MeV} \qquad \alpha_s(M_Z) = 0.1159(12)$$

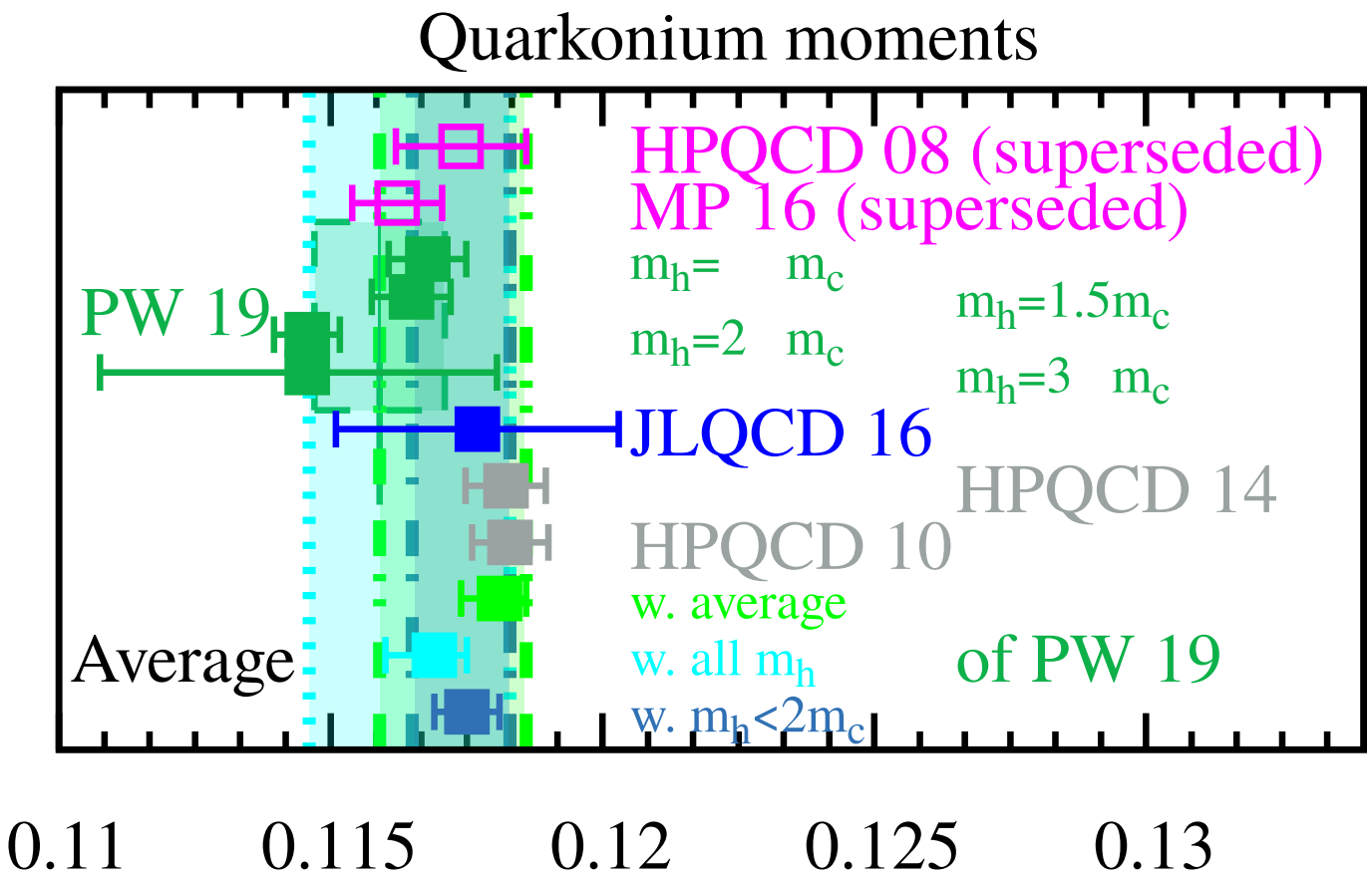
The lattice results on the running coupling constant



Our result on α_s from the moments agrees with the α_s from the static potential
Bazavov et al, PRD 90 (2014) 074038

but is lower than HPQCD result from the moments
Allison et al, PRD 78 (2008) 054513; McNeile et al, PRD 82 (2010) 034512;
Chakraborty et al, PRD 91 (2015) 054508

Summary of α_s from moment of quarkonium correlators



Tension between the new and HPQCD results

exclude $m_h = 2m_c$ results when taking the weighted average

$$\alpha_s(M_Z) = 0.1175^{+0.0008}_{-0.0010}, \quad \chi^2/df = 5.24/4$$

Static quark anti-quark potential in perturbation theory

Potential from pNRQCD, ultrasoft logs, renormalon

$$E_0(r) = V_s(r,\nu,\mu) + \delta_{US}(r,\nu,\mu) + RS(\rho)$$

Problem : Either we have a large log(*vr*) *or* *r*-dependent renormalon term (large uncertainty*)*

Solution: First take the derivative in *r* (RS term is gone) the re-sum the large logarithm *v=1/r*
⇒calculate the force:

Necco, Sommer, PLB 523 (2001) 135, Sumino, PRD 65 (2002) 054003

$$F(r,\frac{1}{r}) = \frac{C_F}{r^2}\alpha_s(1/r)[1 + \frac{\alpha_s(1/r)}{4\pi}\left(\tilde{a}_1 - 2\beta_0\right) + \frac{\alpha_s^2(1/r)}{(4\pi)^2}\left(\tilde{a}_2 - 4\tilde{a}_1\beta_0 - 2\beta_1\right)$$
$$+\frac{\alpha_s^3(1/r)}{(4\pi)^3}\left(\tilde{a}_3 - 6\tilde{a}_2\beta_0 - 4\tilde{a}_1\beta_1 - 2\beta_2\right) + \frac{\alpha_s^3(1/r)}{(4\pi)^3}a_3^L \ln\frac{C_A\alpha_s(1/r)}{2} + \mathcal{O}(\alpha_s^4, \alpha_s^5\ln\alpha_s)\Big]$$

see e.g. Garcia i Tormo, MPLA 28 133028 for a review

$$E_0(r) = \int_{r^*}^{r} dr' F(r') + const$$

Bazavov, Brambilla, Gacia i Tormo, PP, Soto, Vairo, PRD 90 (2014) 074038

Bazavov, Brambilla, Gacia i Tormo, PP, Soto, Vairo, Weber, PRD 100 (2019) 114511

Static quark anti-quark potential on the lattice and α_s

Highly Improved Staggered Quark (HISQ) action, 0.025 fm $< a <$ 0.06 fm

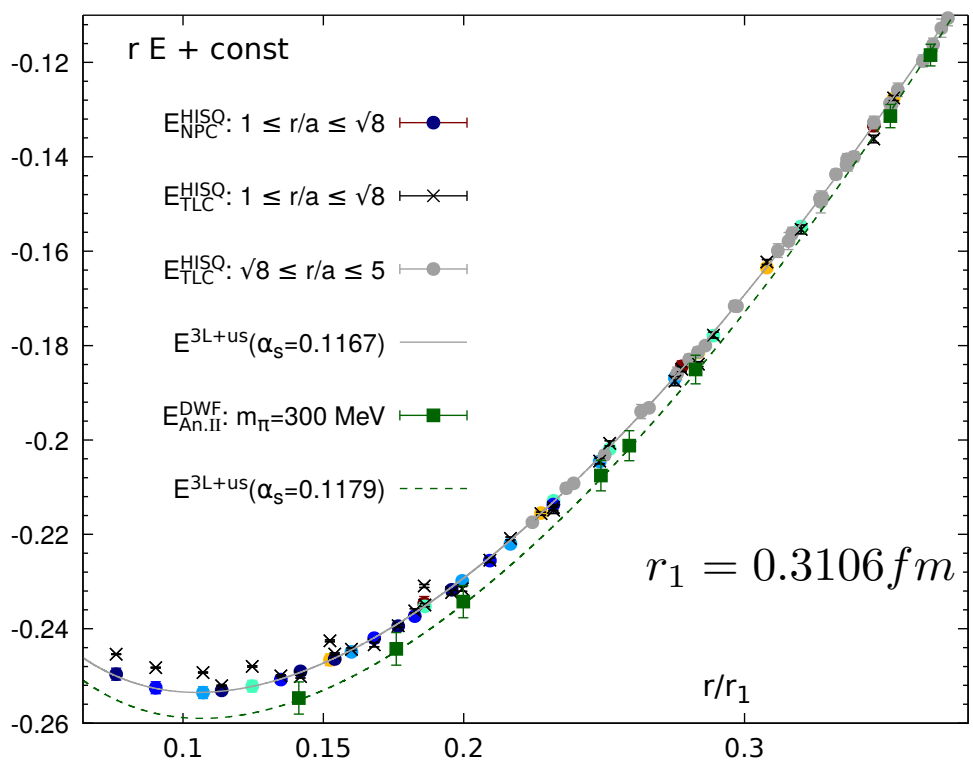
$\alpha_s(M_Z) = 0.11660^{+0.00110}_{-0.00056}$ Bazavov et al, PRD 100 (2019) 114511

Domain Wall Fermion (DWF) discretization : 0.044 fm $< a <$ 0.08 fm

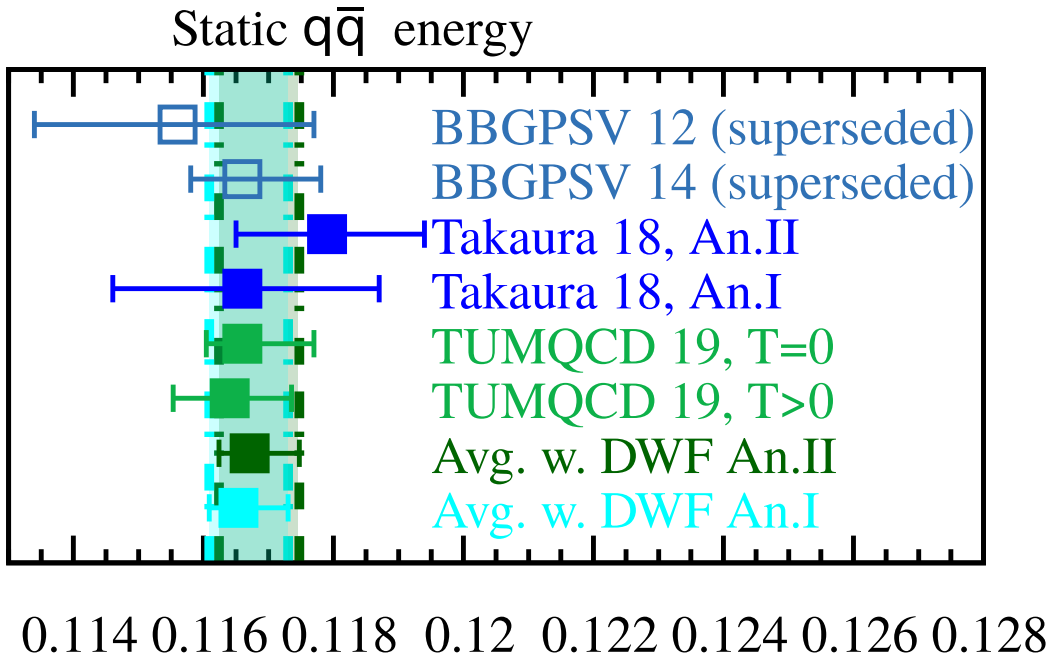
$\alpha_s(M_Z) = 0.1166^{+0.0021}_{-0.0020}$ (An.I)

$\alpha_s(M_Z) = 0.1179^{+0.0015}_{-0.0014}$ (An.II)

Takaura et al, JHEP 04 (2019) 15; PLB 789 (2018) 598



Summary of α_s from quark anti-quark potential



$$\alpha_s(M_Z) = 0.11654^{+0.00076}_{-0.00045}, \ \chi^2/df = 0.040/2, \quad \text{with DWF An. I}$$

$$\alpha_s(M_Z) = 0.11671^{+0.00076}_{-0.00047}, \ \chi^2/df = 0.914/2, \quad \text{with DWF An. II}$$

Step-scaling approach

Define $\alpha_s^S(1/L)$ on the lattice with size L with specified boundary condition and follow the running of $\alpha_s^S(1/L)$ when L is decreased such that $L \gg a$.
$L \ll 1/\Lambda_{QCD}$:

No window problem

$$\alpha_s^{\overline{MS}}(1/L) = \alpha_s^S(1/L)(1 + c_1\alpha_s^S(1/L) + c_2\left(\alpha_s^S(1/L)\right)^2 + \ldots)$$

Start with $\alpha_s^S(1/L_{max})$ with $L_{max} \sim 1$ fm and study the change in α_s^S when

$$L_{max} \to \nu^{-1} = L_{max}/2^n, \; n = 2, 3, 4 \ldots$$

In practice Schrödinger functional (SF) boundary conditions are used

$$A_k(x)|_{x_0=0} = C_k(\eta), \; A_k(x)|_{x_0=L} = C'_k(\eta), \; k = 1, 2, 3$$

and the corresponding coupling is defined as

$$\partial_\eta \langle S_{eff} \rangle|_{\eta=0} = \frac{12\pi}{g_{SF}^2}$$

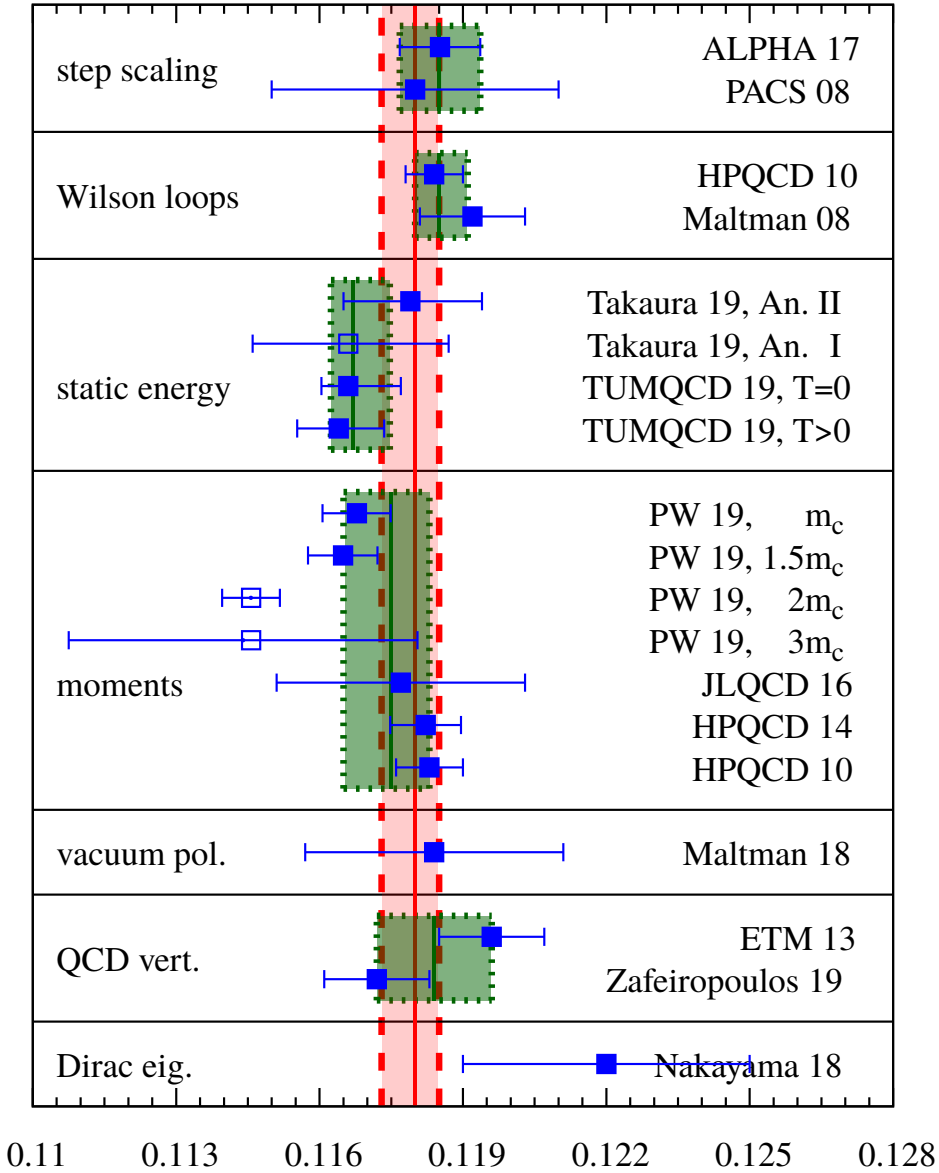
Lüscher et al, NPB 384 (1992) 168

Scales up to $\nu \sim 70$ GeV have been reached (pert. theory is reliable) !

$\alpha_s(M_Z) = 0.11852(84)$

Bruno et al (ALPHA Collaboration), PRL 119 (2017) 102001

Summary of α_s from the lattice



Form pre-averages for each of the quantities and estimate the errors to account the spread in the determinations from different lattice groups. Averaging over pre-averages gives:

$$\alpha_s(M_Z) = 0.11803^{+0.00047}_{-0.00068}, \quad \frac{\chi^2}{df} = \frac{6.5}{6}$$

w/o static pot. :

$$\alpha_s(M_Z) = 0.11838^{+0.00044}_{-0.00048}, \quad \frac{\chi^2}{df} = \frac{2.8}{5},$$

w/o Dirac eig. :

$$\alpha_s(M_Z) = 0.11802^{+0.00046}_{-0.00069}, \quad \frac{\chi^2}{df} = \frac{4.8}{5},$$

w/o step sclaing :

$$\alpha_s(M_Z) = 0.11791^{+0.00054}_{-0.00074}, \quad \frac{\chi^2}{df} = \frac{6.4}{5},$$

Conclusions

- Lattice determination of the strong coupling constant is more precise than the PDG average
- Different lattice approaches are consistent with each other
- To obtain more reliable determination tensions between different results within as single lattice approach need to better understood
- Higher order perturbative calculations are needed to reduce the uncertainty (e.g. for static potential and moments of quarkonium correlators)
- Step scaling is likely the most reliable method to determine the strong coupling constant and it would be desirable to have calculations by different groups using this method