Improving positivity bounds on aQGCs

Cen Zhang
Institute of High Energy Physics

Snowmass EF04 Topical Group Community Meeting
July 2 2020, online

Based on ongoing project with Kimiko Yamashita and Shuang-Yong Zhou

- Dim-8 anomalous QGC is a commonly used TH framework to interpret VBS (and tri-boson) results.

$$O_{S,0} = [(D_\mu\Phi)^\dagger D_\nu\Phi] \times [(D^\mu\Phi)^\dagger D^\nu\Phi]$$
$$O_{S,1} = [(D_\mu\Phi)^\dagger D^\mu\Phi] \times [(D_\nu\Phi)^\dagger D^\nu\Phi]$$
$$O_{S,2} = [(D_\mu\Phi)^\dagger D_\nu\Phi] \times [(D^\nu\Phi)^\dagger D^\mu\Phi]$$
$$O_{M,0} = \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right] \times \left[(D_\beta\Phi)^\dagger D^\beta\Phi\right]$$
$$O_{M,1} = \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\nu\beta}\right] \times \left[(D_\beta\Phi)^\dagger D^\mu\Phi\right]$$
$$O_{M,2} = \left[\hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\right] \times \left[(D_\beta\Phi)^\dagger D^\beta\Phi\right]$$
$$O_{M,3} = \left[\hat{B}_{\mu\nu}\hat{B}^{\nu\beta}\right] \times \left[(D_\beta\Phi)^\dagger D^\mu\Phi\right]$$
$$O_{M,4} = \left[(D_\mu\Phi)^\dagger \hat{W}_{\beta\nu} D^\mu\Phi\right] \times \hat{B}^{\beta\nu}$$
$$O_{M,5} = \tfrac{1}{2}\left[(D_\mu\Phi)^\dagger \hat{W}_{\beta\nu} D^\nu\Phi\right] \times \hat{B}^{\beta\mu} + h.c.$$
$$O_{M,7} = \left[(D_\mu\Phi)^\dagger \hat{W}_{\beta\nu}\hat{W}^{\beta\mu} D^\nu\Phi\right]$$

$$O_{T,0} = \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right] \times \mathrm{Tr}\left[\hat{W}_{\alpha\beta}\hat{W}^{\alpha\beta}\right]$$
$$O_{T,1} = \mathrm{Tr}\left[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}\right] \times \mathrm{Tr}\left[\hat{W}_{\mu\beta}\hat{W}^{\alpha\nu}\right]$$
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$$O_{T,5} = \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right] \times \hat{B}_{\alpha\beta}\hat{B}^{\alpha\beta}$$
$$O_{T,6} = \mathrm{Tr}\left[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}\right] \times \hat{B}_{\mu\beta}\hat{B}^{\alpha\nu}$$
$$O_{T,7} = \mathrm{Tr}\left[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}\right] \times \hat{B}_{\beta\nu}\hat{B}^{\nu\alpha}$$
$$O_{T,8} = \hat{B}_{\mu\nu}\hat{B}^{\mu\nu} \times \hat{B}_{\alpha\beta}\hat{B}^{\alpha\beta}$$
$$O_{T,9} = \hat{B}_{\alpha\mu}\hat{B}^{\mu\beta} \times \hat{B}_{\beta\nu}\hat{B}^{\nu\alpha},$$

+ 2 missing operators

- VBS @ HL/HE-LHC: QGC sensitivity ~ TeV scale.

	14 TeV		27 TeV	
	$WZjj$	$W^\pm W^\pm jj$	$WZjj$	$W^\pm W^\pm jj$
f_{S_0}/Λ^4	[-8,8]	[-6,6]	[-1.5,1.5]	[-1.5,1.5]
f_{S_1}/Λ^4	[-18,18]	[-16,16]	[-3,3]	[-2.5,2.5]
f_{T_0}/Λ^4	[-0.76,0.76]	[-0.6,0.6]	[-0.04,0.04]	[-0.027,0.027]
f_{T_1}/Λ^4	[-0.50,0.50]	[-0.4,0.4]	[-0.03,0.03]	[-0.016,0.016]
f_{M_0}/Λ^4	[-3.8,3.8]	[-4.0,4.0]	[-0.5,0.5]	[-0.28,0.28]
f_{M_1}/Λ^4	[-5.0,5.0]	[-12,12]	[-0.8,0.8]	[-0.90,0.90]

(in TeV^{-4}), from HL/HE-LHC report

- SMEFT global fit seems the right way to go, even adding dim-6 operators. > 20 dimensional theory space to explore.

- However, SMEFT is meant to connect EXP data with concrete UV models. Therefore it does not make much sense to study the EFT space which cannot be UV-completed (if we know in advance).
- Particularly relevant at dim-8: positivity bounds tell us which part of the parameter space **cannot** be UV-completed. (e.g. if dim-8 coefficients have wrong signs).

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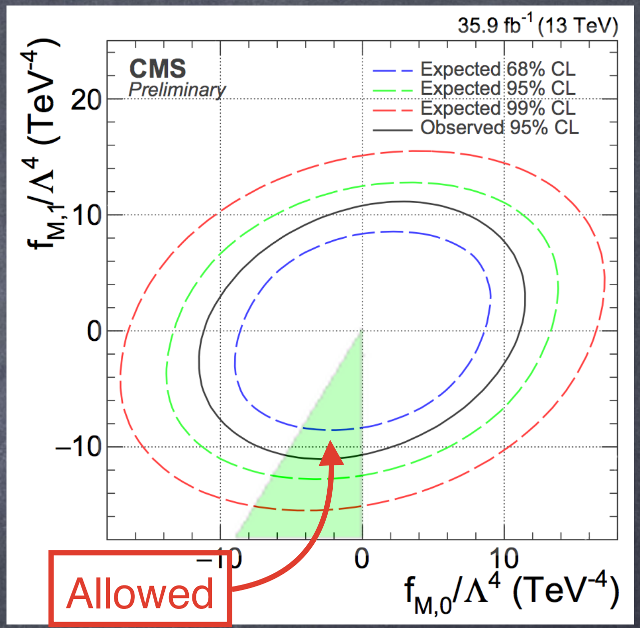
- Currently affects the aQGC parametrization.

[CMS-PAS-SMP-18-001]



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- However, **in the future, more dim-8 effects may become accessible.**

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[Alioli, Boughezal, Mereghetti, Petriello, 2003.11615])

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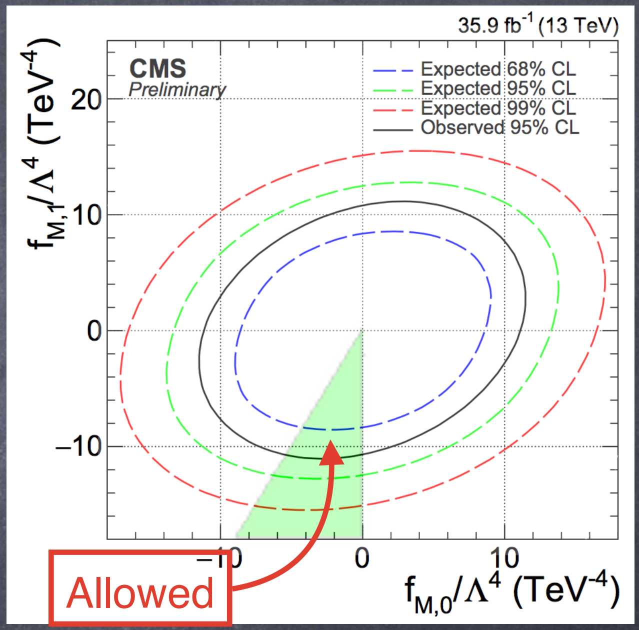

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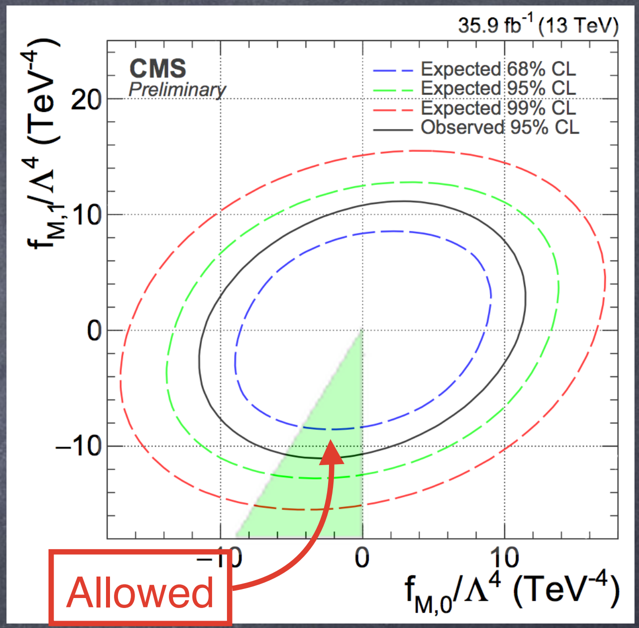

Relevant even for improved EXP precision

- These bounds need to be studied, to identify the meaningful parameter space, to form a consistent interpretation of data within the SMEFT framework, and also to help focus the EXP search.

- The E^4/Λ^4 operators (dim-8 SMEFT operators) need to satisfy "positivity bounds", for a UV completion to exist (with causality, locality, Lorentz invariance…) Certain linear combinations of dim-8 coefficients must be positive, e.g. transversal QGCs: $4C_{T,0} + 4C_{T,1} + 3C_{T,2} + 12C_{T,10} \geq 0$.

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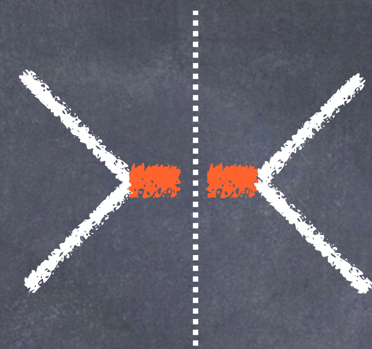
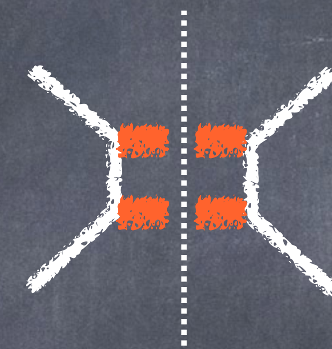
- Still not complete. Room to improve.
- In addition, a new approach has been proposed in [**CZ,** S.-Y. Zhou, 2005.03047].
- We would like to understand the full set of bounds on all QGC operators, to provide TH guidance for future VBS and QGC measurements.

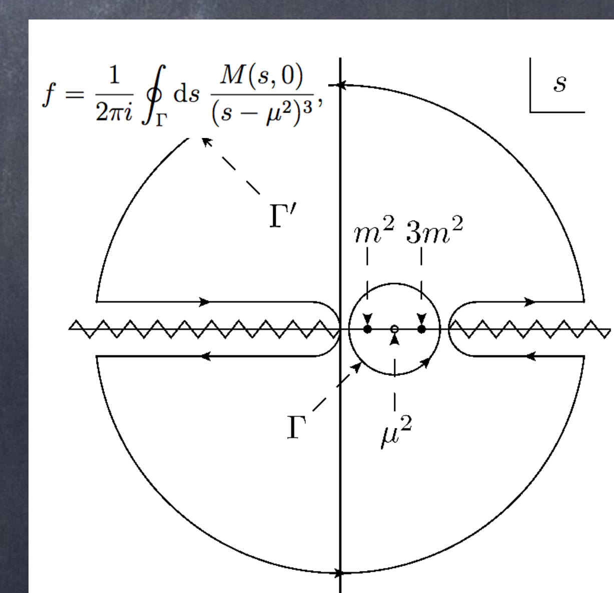
Outline

- Dispersion relation
- The traditional approach (elastic positivity)
- The new approach
- Some preliminary results on transversal QGCs

$\frac{d^2}{ds^2}$  =  +  + … + s<->u crossing

$$\frac{d^2}{ds^2} M_{ij\to kl}\left(s = \frac{1}{2}M^2, t = 0\right)$$

$$= \sum_X{}' \int_{(\epsilon\Lambda)^2}^{\infty} \frac{ds\, M_{ij\to X}(s, \Pi_X) M^*_{kl\to X}(s, \Pi_X)}{\pi\left(s - \frac{1}{2}M^2\right)^3} + (j \leftrightarrow l)$$



[C. Cheung, G. Remmen, JHEP 16]
[de Rham, Melville, Tolley, Zhou, JHEP 17]

ijkl: particle index

$1 \le i, j, k, l \le n$

$M^2 = m_i^2 + m_j^2 + m_k^2 + m_l^2$

Forward scattering amp, at **low energy** (calculable in EFT)

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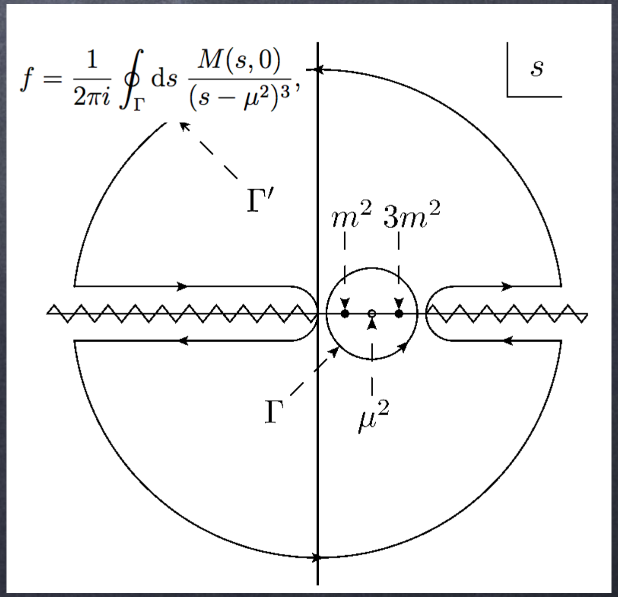

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s<->u crossing

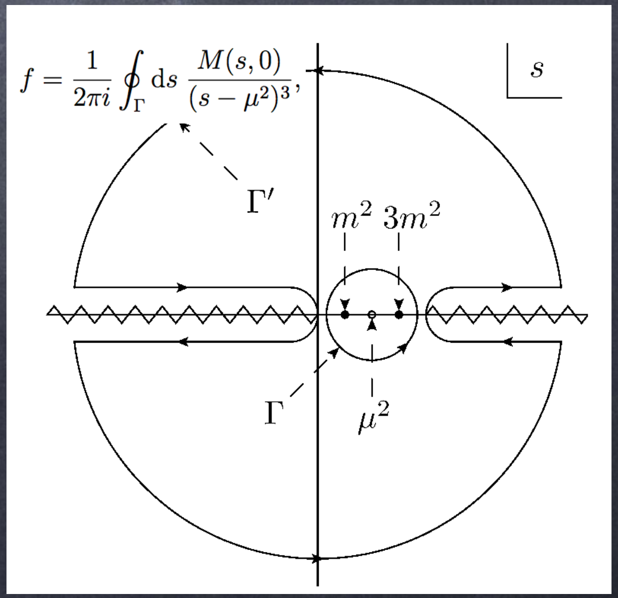
$\epsilon < 1$

X = BSM states

summation & PS integration

Amplitude

of SM -> X



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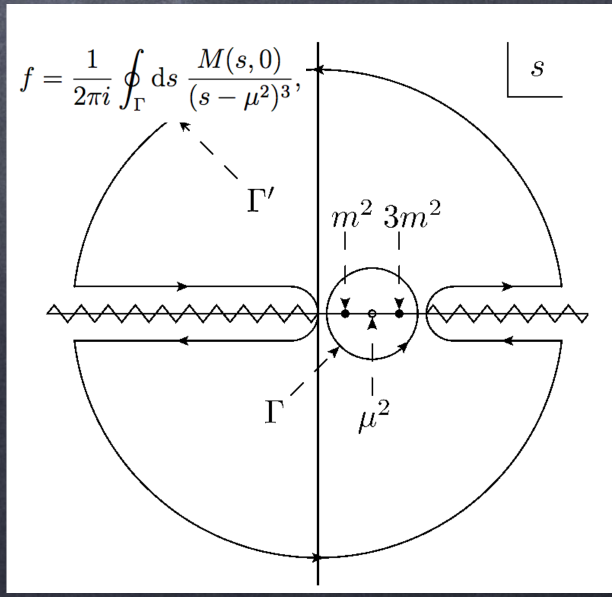
- L.H.S : calculable in EFT
  - At tree level, simply linear combination of $C^{(8)}$
- R.H.S : integration of BSM contribution
- Might think of this as a matching formula.



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$$M^{ijkl} = \sum_X{}' \int_{(\epsilon\Lambda)^2}^{\infty} \frac{d\mu\, {m_X}^{ij} {m_X}^{kl}}{\pi(\mu - \frac{1}{2}M^2)^3} + (j \leftrightarrow l)$$

where $M^{ijkl} \equiv \frac{d^2}{ds^2} M_{ij\to kl}\left(\frac{1}{2}M^2\right), \quad m_X^{ij} \equiv M_{ij\to X}(\mu, \Pi_X)$

M^{ijkl} calculable in SMEFT, e.g. $M^{ijkl} = \sum_\alpha C_\alpha^{(8)}/\Lambda^4 M_\alpha^{ijkl}$

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- Model-independent EFT: m_X^{ij} function on the RHS can take any value in $\mathbb{R}^{n^2}$

- However, M^{ijkl} on LHS cannot take arbitrary values in $\mathbb{R}^{n^4}$ -> bounds on M^{ijkl}, or equivalently, on $C^{(8)}$.

The traditional approach: elastic scattering

$$M^{ijkl} = {\sum_X}' \int_{(\epsilon\Lambda)^2}^{\infty} \frac{d\mu \, {m_X}^{ij} {m_X}^{kl}}{\pi(\mu - \frac{1}{2}M^2)^3} + (j \leftrightarrow l)$$

- When i=k, j=l, RHS -> complete squares >0 i.e. a discrete set of inequalities:

$$M^{ijij} \geq 0$$

M (or C8) must stay inside

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- More generally, consider $u^i v^j u^{*k} v^{*l} \cdot M^{ijkl}, \quad u, v \in \mathbb{C}^n$
  RHS -> $|u \cdot m_X \cdot v|^2 + |u \cdot m_X \cdot v^*|^2 \geq 0$
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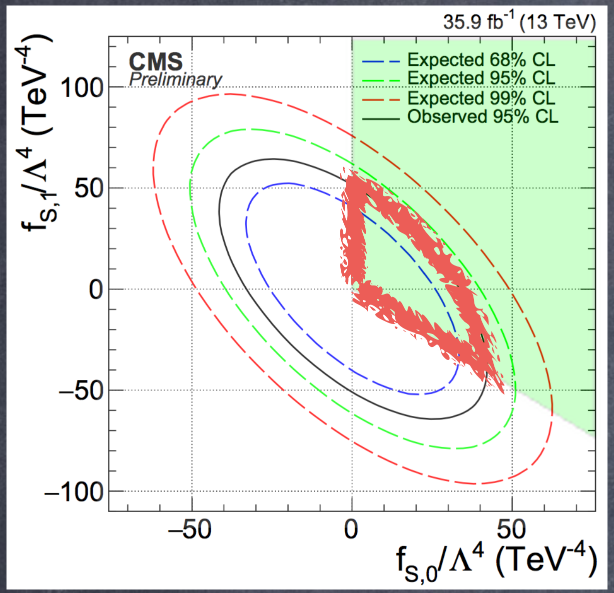
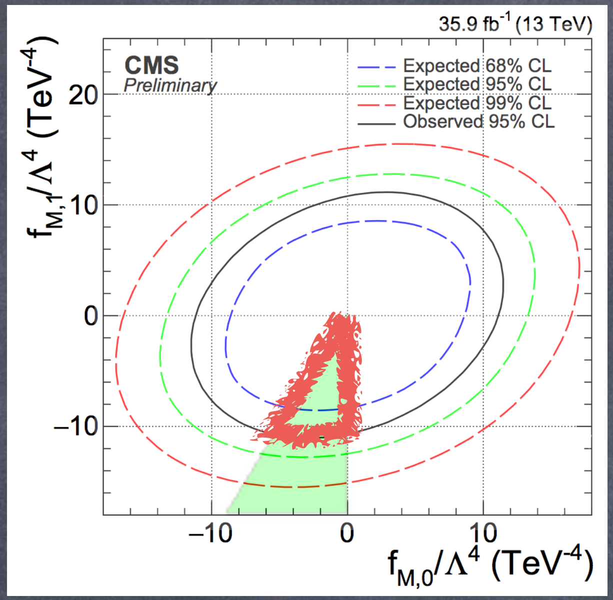
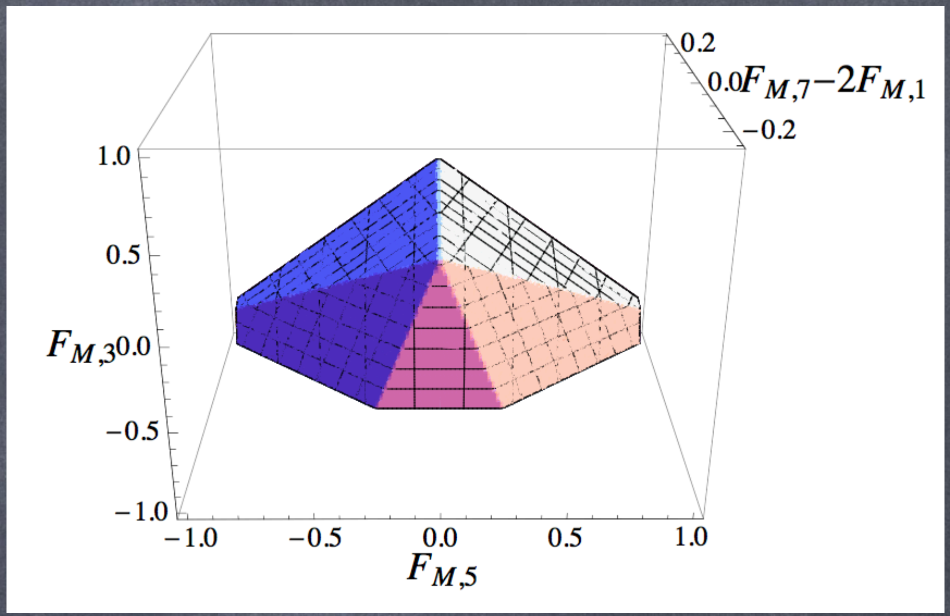
  - This is the elastic scattering between two superposed states (by the u,v vectors). Vary u,v to get the full set of bounds.

The traditional approach: elastic scattering

- [Q. Bi, CZ, S.-Y. Zhou, JHEP 19] Mass eigenstates with superposed helicity states:







- [Remmen, Rodd, JHEP 19] Gauge eigenstates, all superposition of Goldstones:

$$
\begin{aligned}
c_{S,0}+c_{S,1}+c_{S,2} &> 0 \\
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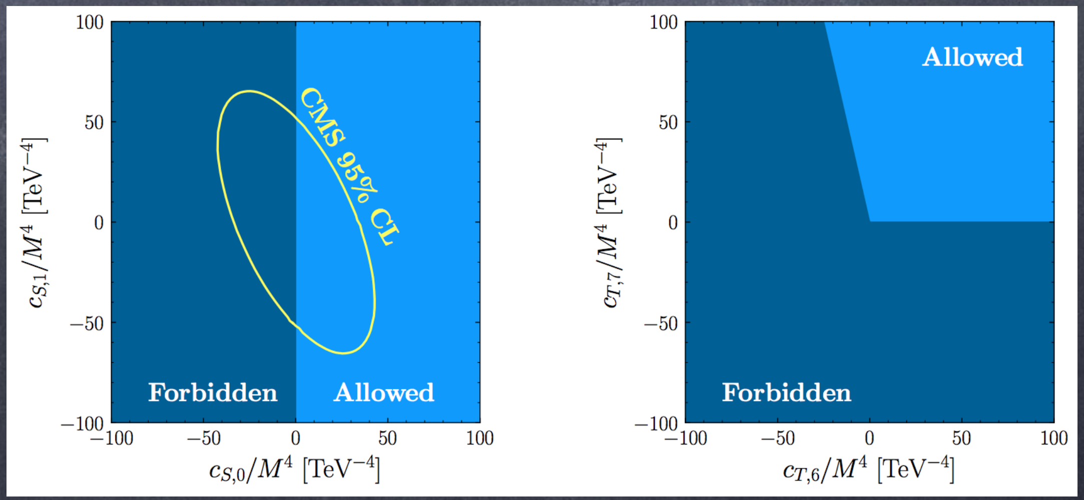
c_{S,0} &> 0.
\end{aligned}
$$

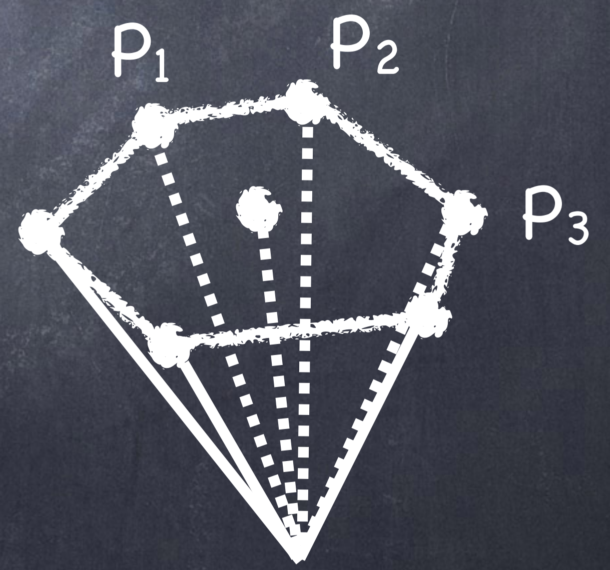


- A lot of room to improve: superposition of gauge components of W/B, together with Goldstones, etc.

[CZ and S.-Y. Zhou 2005.03047]

The new approach

- Two kinds of symmetries: SM gauge, and SO(2) rotation around the forwards axis. Both act on the particle indices (i,j,k,l)
- Dispersion relation: $M^{ijkl} = {\sum_{X\in r}}' \int_{(\epsilon\Lambda)^2}^{\infty} d\mu \frac{|<X|M|r>|^2}{\pi\left(\mu - \frac{1}{2}M^2\right)^3} P_r^{i(j|k|l)}$ ← Symmetry (Dynamics → $|<X|M|r>|^2$)
- P_r^{ijkl} is the projective operator of an irrep r, obtained by CG coefficients.
- The allowed values of M must be all positive linear combinations of $P_r^{i(j|k|l)}$ i.e. cone($\{P_r^{i(j|k|l)}\}$), a convex cone positively generated by (j,l symmetrized) projectors.
- In practice, we compute all projectors P1, P2, P3,..., which are generators of M, and their convex hull determines the cone.
- Positivity bounds are "facets" of the cone. Knowing the edges, they are obtained by the "vertex enumeration" algorithm.



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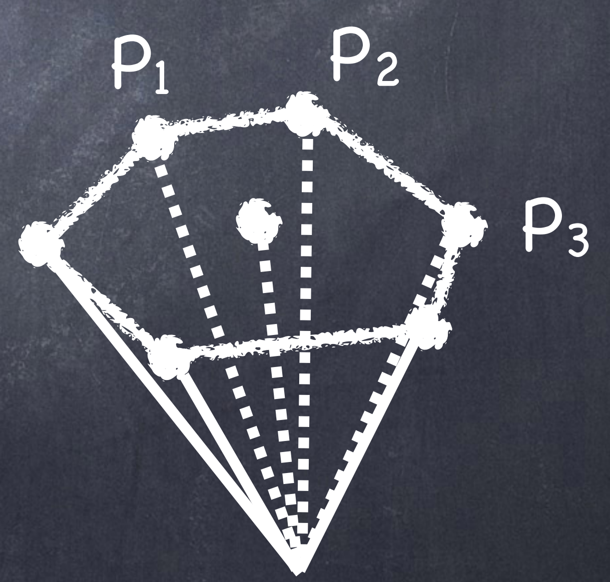

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- For example, consider transversal WW -> WW. Traditional approach (elastic scattering) gives:

bounds	channel $(\|1>+\|2>\to\|1>+\|2>)$
$F_{T,2}\geq 0,$	$1: W_x^1,\ 2: W_y^2$
$4F_{T,1}+F_{T,2}\geq 0,$	$1: W_x^1,\ 2: W_x^2$
$F_{T,2}+8F_{T,10}\geq 0,$	$1: W_x^1+W_y^2,\ 2: W_y^1-W_x^2$
$8F_{T,0}+4F_{T,1}+3F_{T,2}\geq 0,$	$1: W_x^1+W_y^2,\ 2: W_x^1+W_y^2$

- While the new approach gives better bounds [CZ and S.-Y. Zhou 2005.03047]

$$F_{T,2}\geq 0,$$
$$4F_{T,1}+F_{T,2}\geq 0,$$
$$F_{T,2}+8F_{T,10}\geq 0,$$
$$8F_{T,0}+4F_{T,1}+3F_{T,2}\geq 0,$$
$$12F_{T,0}+4F_{T,1}+5F_{T,2}+4F_{T,10}\geq 0,$$
$$4F_{T,0}+4F_{T,1}+3F_{T,2}+12F_{T,10}\geq 0.$$

Cannot be obtained from any elastic channel

For QGCs, we are interested in:

- Bounds: what are the best set of bounds on all QGC, available from the dispersion relation?
- General approaches: establish more concrete and systematic algorithms? which may apply to other operators/channels...
    - Elastic approach: how to determine $u^i v^j u^{*k} v^{*l} M^{ijkl} \geq 0$ w.r.t. all u,v vectors? The determination of a degree-4 polynomial >0 is NP hard. Maybe symmetries could help here.
    - New approach: in case continuous generators show up (like circular cones), how to get bounds (i.e. "vertex enumeration" for cones with curved boundary)?
    - Numerical alternatives?

First step: bounds on transversal QGCs

Transversal operators:

$$O_{T,0} = \mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}]\mathrm{Tr}[\hat{W}_{\alpha\beta}\hat{W}^{\alpha\beta}] \quad O_{T,1} = \mathrm{Tr}[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}]\mathrm{Tr}[\hat{W}_{\mu\beta}\hat{W}^{\alpha\nu}]$$
$$O_{T,2} = \mathrm{Tr}[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}]\mathrm{Tr}[\hat{W}_{\beta\nu}\hat{W}^{\nu\alpha}] \quad O_{T,10} = \mathrm{Tr}[\hat{W}_{\mu\nu}\tilde{W}^{\mu\nu}]\mathrm{Tr}[\hat{W}_{\alpha\beta}\tilde{W}^{\alpha\beta}]$$
$$O_{T,5} = \mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}]\hat{B}_{\alpha\beta}\hat{B}^{\alpha\beta} \quad O_{T,6} = \mathrm{Tr}[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}]\hat{B}_{\mu\beta}\hat{B}^{\alpha\nu}$$
$$O_{T,7} = \mathrm{Tr}[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}]\hat{B}_{\beta\nu}\hat{B}^{\nu\alpha} \quad O_{T,11} = \mathrm{Tr}[\hat{W}_{\mu\nu}\tilde{W}^{\mu\nu}]\hat{B}_{\alpha\beta}\tilde{B}^{\alpha\beta}$$
$$O_{T,8} = \hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\hat{B}_{\alpha\beta}\hat{B}^{\alpha\beta} \quad O_{T,9} = \hat{B}_{\alpha\mu}\hat{B}^{\mu\beta}\hat{B}_{\beta\nu}\hat{B}^{\nu\alpha}$$

Note $O_{T,10}$ and $O_{T,11}$ have been missed in standard QGC parameterization.

Pointed out by [Remmen, Rodd, JHEP 12 (2019) 032]

See also dim-8 basis: [C. Murphy 2005.00059], [H.-L. Li 2005.00008]

Preliminary results: the traditional (elastic approach)

The traditional (elastic) approach: consider the scattering of two superposition of 8 SM modes: $W_x^1, W_y^1, W_x^2, W_y^2, W_x^3, W_y^3, B_x, B_y$

- This is the determination of the positive-definiteness of a 4th-order polynomial with 32 variables. Too hard...

- One solution: assuming the superposition can be factorized in gauge/helicity space,

$$u^i = x^a \alpha^b,\ v^i = y^a \beta^b,\ i = (a, b)$$

with polarization index a and gauge index b.

- The result is conservative, but it converts the problem into "quadratically constrained quadratic programming" problems, can be solved analytically.

E.g.

$$\begin{array}{ll} \text{minimize} & 2b_1 M_{b1} + b_3 M_{b3} + 2b_4 M_{b5} + b_6 M_{b6} + b_7 M_{b7} \\ \text{subject to} & b_7 \geq 0, 0 \leq b_1 \leq b_3, |b_4| \leq 2\sqrt{b_1 b_7}, b_6 \geq 2\sqrt{b_3 b_7} \end{array}$$

Linear bounds:

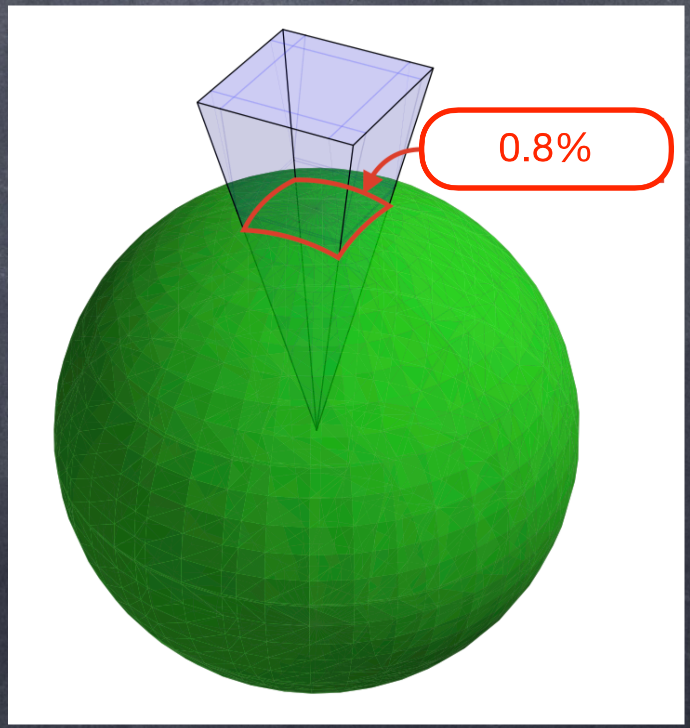
$$2F_{T,0}+2F_{T,1}+F_{T,2}\geq 0$$
$$F_{T,2}+4F_{T,10}\geq 0$$
$$4F_{T,1}+F_{T,2}\geq 0$$
$$F_{T,2}\geq 0$$
$$2F_{T,0}+F_{T,1}+F_{T,2}+2F_{T,10}\geq 0$$
$$2F_{T,8}+F_{T,9}\geq 0$$
$$F_{T,9}\geq 0$$
$$4F_{T,6}+F_{T,7}\geq 0$$
$$F_{T,7}\geq 0$$

Quadratic bounds:

$$4\sqrt{[2(F_{T,0}+F_{T,1})+F_{T,2}](2F_{T,8}+F_{T,9})}\geq \max(0,-2(2F_{T,5}+2F_{T,6}+F_{T,7}),4F_{T,5}+F_{T,7})$$
$$2\sqrt{F_{T,9}(F_{T,2}+4F_{T,10})}\geq \max(0,-(2F_{T,11}+F_{T,7}),2F_{T,11})$$
$$2\sqrt{[4F_{T,10}+4(F_{T,0}+F_{T,1})+3F_{T,2}](4F_{T,8}+3F_{T,9})}\geq |2F_{T,11}+4F_{T,5}+F_{T,7}|$$

+ some cubic bounds...

- The parameter space is constrained to 0.8% of the total (in terms of solid angle)
  - Compared with previous result, 2.1% for S+M+T operators...



Preliminary results: the new (extremal) approach

The new approach: need to consider an infinite set of projectors (which are potentially generators), continuously parametrized by r:

$$\vec{E}_1 = (1,0,0,0,0,0,0,0,0,0,0,0,0)$$
$$\vec{E}_2 = (0,1,0,0,0,0,0,0,0,0,0,0,0)$$
$$\vec{E}_3 = (0,0,1,0,0,0,0,0,0,0,0,0,0)$$
$$\vec{E}_4 = (0,0,0,1,0,0,0,0,0,0,0,0,0)$$
$$\vec{E}_5 = \left(-\frac{1}{6},\frac{1}{6},0,0,-\frac{5}{3},0,0,\frac{5}{3},0,0,\frac{5}{6},0,0\right)$$
$$\vec{E}_6 = \left(0,0,-1,1,0,-\frac{3}{4},0,0,\frac{3}{4},0,0,0,1\right)$$
$$\vec{E}_7(r) = (0,0,0,0,1,r,r^2,0,0,0,0,0,0)$$
$$\vec{E}_8(r) = (0,0,0,0,0,0,0,1,r,r^2,0,0,0)$$
$$\vec{E}_9(r) = (0,0,0,0,0,0,0,0,0,0,1,r,r^2)$$
$$\vec{E}_{10}(r) = \left(-\frac{1}{3},\frac{1}{3},-\frac{4r}{3},\frac{4r}{3},-\frac{1}{3},0,-r^2,\frac{1}{3},0,r^2,-\frac{1}{3},0,-\frac{4r}{3}\right)$$
$$\vec{E}_{11}(r) = \left(\frac{1}{2},\frac{1}{2},\frac{r^2}{2},\frac{r^2}{2},-1,-\frac{3r^2}{8},0,-1,-\frac{3r^2}{8},0,-\frac{1}{2},r,-\frac{r^2}{2}\right)$$
$$\vec{E}_{12}(r) = \left(1,0,r^2,0,-2,-\frac{3r^2}{4},0,0,0,0,1,-2r,r^2\right)$$

Question: what is the cone spanned by all these vectors? How to identify its boundary?

Analytically: a tower of linear, quadratic, cubic, ... inequalities.

- So far only able to obtain the first two levels

Linear:

$$F_{T,2} \geq 0$$
$$4F_{T,1} + F_{T,2} \geq 0$$
$$F_{T,2} + 8F_{T,10} \geq 0$$
$$8F_{T,0} + 4F_{T,1} + 3F_{T,2} \geq 0$$
$$12F_{T,0} + 4F_{T,1} + 5F_{T,2} + 4F_{T,10} \geq 0$$
$$4F_{T,0} + 4F_{T,1} + 3F_{T,2} + 12F_{T,10} \geq 0$$
$$4F_{T,6} + F_{T,7} \geq 0$$
$$F_{T,7} \geq 0$$
$$2F_{T,8} + F_{T,9} \geq 0$$
$$F_{T,9} \geq 0$$

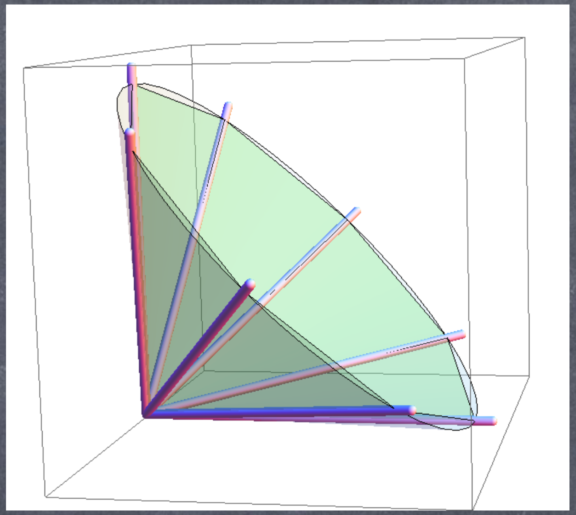
Quadratic:

$$F_{T,9}\left(F_{T,2} + 4F_{T,10}\right) \geq F_{T,11}^2$$
$$16\left(2\left(F_{T,0} + F_{T,1}\right) + F_{T,2}\right)\left(2F_{T,8} + F_{T,9}\right) \geq \left(4F_{T,5} + F_{T,7}\right)^2$$
$$32\left(2F_{T,8} + F_{T,9}\right)\left(3F_{T,0} + F_{T,1} + 2F_{T,2} + 4F_{T,10}\right) \geq 3\left(4F_{T,5} + F_{T,7}\right)^2$$
$$2\sqrt{2}\sqrt{F_{T,9}\left(F_{T,2} + 8F_{T,10}\right)} \geq \max\left(4F_{T,6} + F_{T,7} - 4F_{T,11}, F_{T,7} + 4F_{T,11}\right)$$
$$4\sqrt{\left(8F_{T,0} + 4F_{T,1} + 3F_{T,2}\right)\left(2F_{T,8} + F_{T,9}\right)} \geq \max\left(-8F_{T,5} - F_{T,7}, 8F_{T,5} + 4F_{T,6} + 3F_{T,7}\right)$$
$$4\sqrt{F_{T,9}\left(12F_{T,0} + 4F_{T,1} + 5F_{T,2} + 4F_{T,10}\right)} \geq \max\left(4F_{T,6} + F_{T,7} - 4F_{T,11}, F_{T,7} + 4F_{T,11}\right)$$
$$4\sqrt{6}\sqrt{\left(2F_{T,8} + F_{T,9}\right)\left(12F_{T,0} + 4F_{T,1} + 5F_{T,2} + 4F_{T,10}\right)} \geq \max\left[-3\left(8F_{T,5} + F_{T,7}\right), 3\left(8F_{T,5} + 4F_{T,6} + 3F_{T,7}\right)\right]$$
$$\sqrt{6}\sqrt{\left(4F_{T,8} + 3F_{T,9}\right)\left(6F_{T,0} + 2F_{T,1} + 3F_{T,2} + 6F_{T,10}\right)} \geq \max\left[-3\left(2F_{T,5} + F_{T,11}\right), 3\left(2F_{T,5} + F_{T,7} + F_{T,11}\right)\right]$$
$$2\sqrt{\left(12F_{T,8} + 7F_{T,9}\right)\left(12F_{T,0} + 4F_{T,1} + 5F_{T,2} + 4F_{T,10}\right)} \geq \max\left(-12F_{T,5} - F_{T,7} - 2F_{T,11}, -12F_{T,5} + 4F_{T,6} - F_{T,7} - 2F_{T,11}, -12F_{T,5} - F_{T,7} + 2F_{T,11}, 12F_{T,5} + 4F_{T,6} + 5F_{T,7} + 2F_{T,11}\right)$$

- The parameter space is constrained to 0.687% of the total. (Conservative)

Numerically: might as well directly determine if a given point is included in the convex hull of all projectors (convex inclusion)

- Infinite number of (potential) generators, but numerically, we sample them with a large number (N of order 100~1000) of discrete ones, i.e. polyhedral cone inscribed to a "circular" cone =>

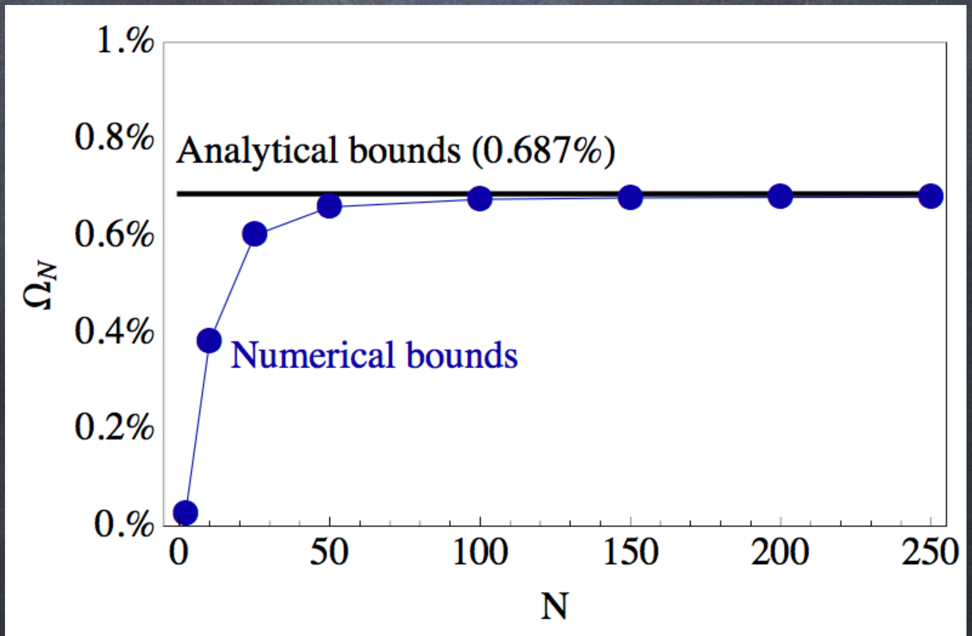


- Determination of inclusion can be turned in to a linear programming problem.

- Volume: ~0.681% (1 - 79.3/N^2)

- **<u>The true volume seems to be 0.681%.</u>**

- Analytical bounds of the first two orders are sufficient.

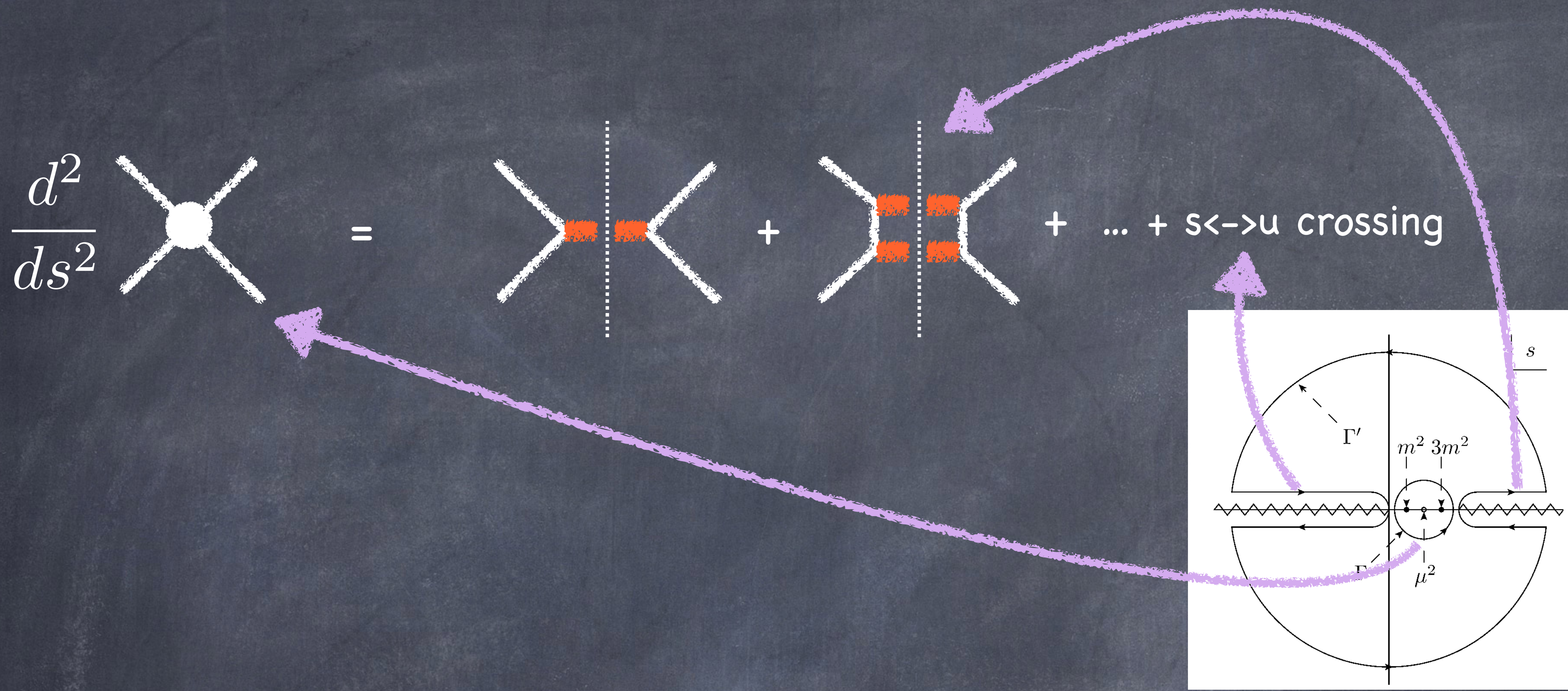


Summary and to-do list:

- 99.32% of transversal QGC parameter space is redundant (not UV-completable)
- New approach to derive analytical bounds on coefficients. Numerical determination also possible. Will apply to the full set of QGCs.
  - Further investigation of analytical approach is needed.
  - Impacts of double insertion of dim-6, SM loops, etc.
- Other interesting questions:
  - Implication on EXP analysis? And global fits? Must be some if >99% parameter space is ruled out.
  - Apart from VBS, other opportunities to directly test positivity nature on dim-8 operators (e.g. at a future ee collider)?

Thank you

Backups



- Derived from: analyticity (from causality), Froissart bound (from locality), optical theorem (unitarity), and Lorentz Invariance.

[Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi, JHEP 06]
[de Rham, Melville, Tolley, Zhou, JHEP 17]
[C. Cheung, G. Remmen, JHEP 16] [Bi, **CZ**, Zhou, JHEP 19]
[G. Remmen, N. Rodd, JHEP 19] and more

$$M^{ijkl} = \sum_X{}' \int_{(\epsilon\Lambda)^2}^{\infty} \frac{d\mu\, m_X{}^{ij} m_X{}^{kl}}{\pi(\mu - \frac{1}{2}M^2)^3} + (j \leftrightarrow l)$$

Positivity bound is one way to learn something (not all):

- When i=k, j=l, RHS -> $\mathrm{Tr}\,(mm^T) \geq 0$
  i.e.

  $$M^{ijij} \geq 0$$

- More generally, consider $u^i v^j u^{*k} v^{*l} \cdot M^{ijkl}, \quad u, v \in \mathbb{C}^n$
  RHS -> $|u \cdot m_X \cdot v|^2 + |u \cdot m_X \cdot v^*|^2 \geq 0$
  i.e.

  $$u^i v^j u^{*k} v^{*l} M^{ijkl} \geq 0$$

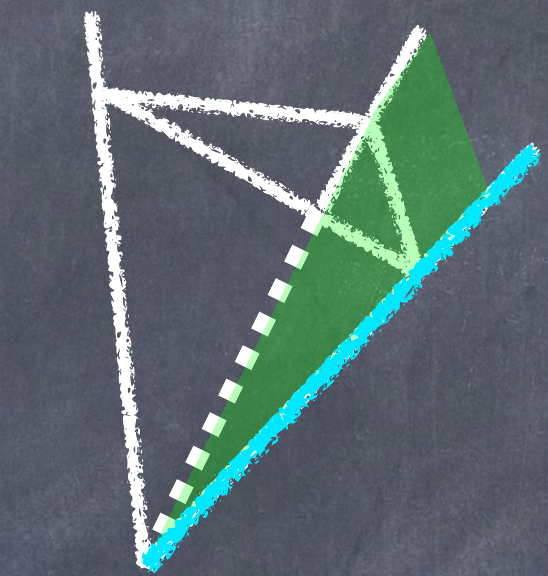
EFT has UV completion -> above degree-4 polynomial of (u,v) is positive semi-definite (PSD)

$$M^{ijkl} = {\sum_X}' \int_{(\epsilon\Lambda)^2}^{\infty} \frac{d\mu\, m_X{}^{ij} m_X{}^{kl}}{\pi(\mu - \frac{1}{2}M^2)^3} + (j \leftrightarrow l)$$

Still, open questions:

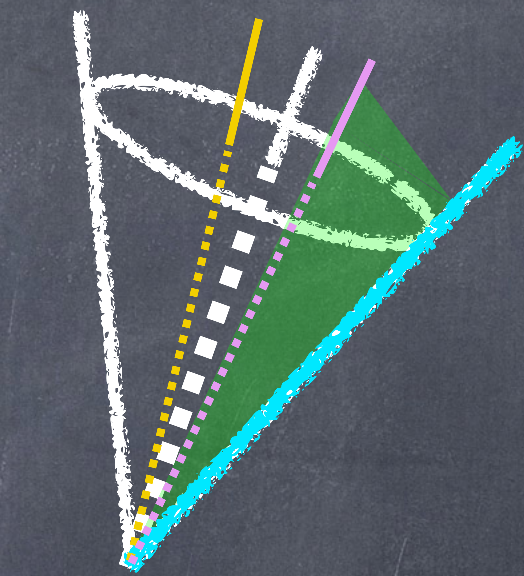
- In practice, e.g. in SMEFT how can one determine the PSDness of $u^i v^j u^{*k} v^{*l} M^{ijkl} \geq 0$, with 100+ variables.
  - A quartic PSD polynomial may NOT be sum of squares. In general an NP-hard problem. Difficult with large n.
- Are there more bounds that can be derived, than using (u,v)?
  - YES. Will show an example...
- Physics interpretation of the bounded EFT space?

- Positivity has the form: $u^iv^ju^iv^jM^{ijkl} \geq 0 \ \Rightarrow \ \sum_\alpha C_\alpha^{(8)} p_\alpha(u,v) \geq 0$
- A set of linear inequality => **Convex Cone**
- Convex Cone has 2 representation:
  - As bounded by **faces**, and
  - As **convex hull** of **extremal rays (ERs)**
  - ERs are the rays that **cannot** be split into two rays in the same cone.
  - Convex hull of {X} is positively generated by elements of {X}.
  - Translate to physics:
    ERs are the generators
    of all UV-completable EFTs!



O

Polyhedral cone



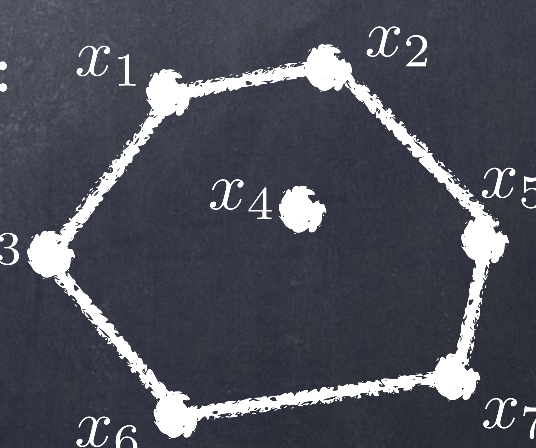
O

Circular cone

e.g. convex hull of x_i:

$$x = \sum_i x_i w_i,$$

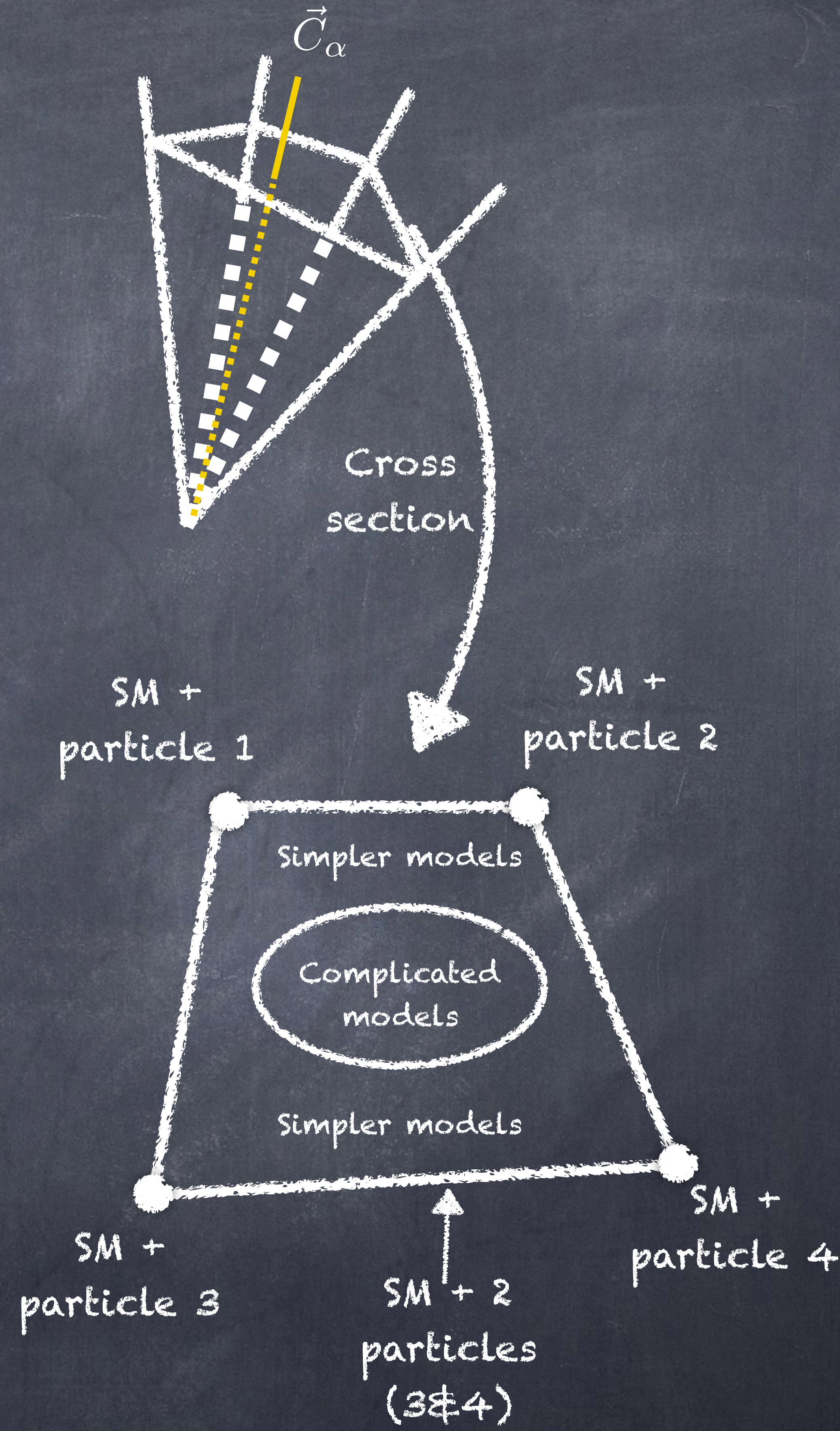
$$w_i \geq 0, \ \sum_i w_i = 1$$



- Consider tree-level UV completion, SM + n particles.
- Integrating out each particle gives a ray within the cone $\vec{C}_\alpha = (C_1, C_2, ...)$
- If n>1, the total cannot be an ER. (ER cannot be split)
- ER corresponds to one-particle SM extension!
    - From which all UV models can be generated.

Heuristically,

- More inner part of the cone tend to correspond to more complicated models, as they are positively weighted sum of outer elements.
- Most outer elements -> ERs, are the most fundamental one-particle extensions.



Points on a k-face correspond to ~k UV particles

- Convex cones are sets closed under addition and positive scalar multiplication.

- The set, C, of all positive linear combinations of elements of X = {x}, is a convex cone, denoted by $C = \mathrm{cone}(X)$

- An element x is an extremal ray of C, if it cannot be split into two other elements in a nontrivial way:

$$\text{if } x = u + v \text{ and } u, v \in C, \text{ then } x = \lambda u \text{ or } x = \lambda v, \lambda > 0$$

- Hahn-Banach separation theorem -> a convex cone is the intersection of half-spaces (supporting planes)

- Krein-Milman theorem: a salient cone C is a convex hull of its ERs.

- Salient: if the cone C does not contain a straight line.
  e.g. $c \in \mathcal{C} \Rightarrow -c \notin \mathcal{C}$ (unless c=0)

- The set of PSD matrices is a convex cone. It's ERs are rank-1 symmetric matrices (1D projectors), $M^{ij} = m^i m^j$

- The set of PSD matrices can be written as $\mathrm{cone}(m^i m^j)$

$$M^{ijkl} = \sum_X{}' \int_{(\epsilon\Lambda)^2}^{\infty} \frac{d\mu \, m_X{}^{ij} m_X{}^{kl}}{\pi(\mu - \frac{1}{2}M^2)^3} + (j \leftrightarrow l)$$

Let C be the set of all possible M^{ijkl}. C is a **salient convex cone**:

$$\mathcal{C} = \text{cone}\left(\left\{m^{ij}m^{kl} + m^{il}m^{kj}, \ m \in \mathbb{R}^{n^2}\right\}\right)$$

Salient because:

$\delta^{ik}\delta^{jl}M^{ijkl} \geq 0$

$\forall M \in \mathcal{C}$

Instead of finding positivity bounds, might just directly look for the ERs, and take the convex hull.

If there is no (j<->l) term, $\mathcal{C}' = \text{cone}\left(\left\{m^{ij}m^{kl}, \ m \in \mathbb{R}^{n^2}\right\}\right)$

- c^{ijkl} being ER in C', is a necessary condition for $c^{ijkl}+c^{ilkj}$ to be ER in C.
- First find ERs of C', then add (j<->l) to get potential ER (PER) of C, then discard the non ER ones.

$$\mathcal{C}' = \text{cone}\left(\left\{m^{ij}m^{kl},\ m \in \mathbb{R}^{n^2}\right\}\right)$$

is the cone of n^2xn^2 PSD matrices. ERs are simply $m^{ij}m^{kl}$, or **1-D projectors.**

(Integration of ERs) = ER, implies all ERs are parallel!

Physics interpretation? For M^{ijkl} to be ER:

$$M^{ijkl} = \sum_X{}' \int_{(\epsilon\Lambda)^2}^{\infty} \frac{d\mu\, {m_X}^{ij} {m_X}^{kl}}{\pi(\mu - \frac{1}{2}M^2)^3} + (j \leftrightarrow l)$$

m_X is a function of s and phase space of X, Π_X.

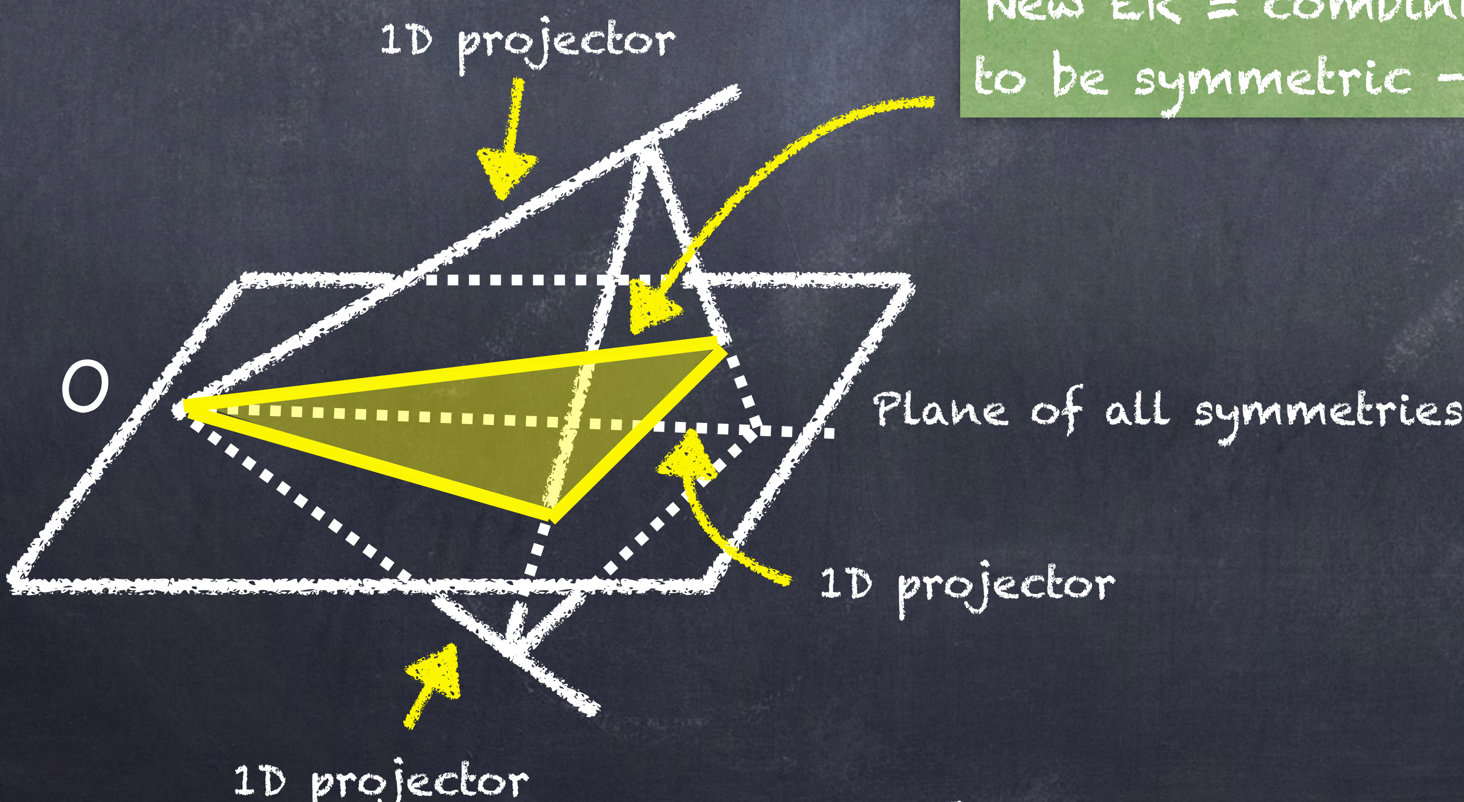
- (Integration of ERs) = ER implies that <u>all ERs are parallel.</u>
- i.e., m_X can only have a factorized dependence on s, Π_X.

$$m_X^{ij}(s, \Pi_X) = f(s, \Pi_X) m^{ij}$$

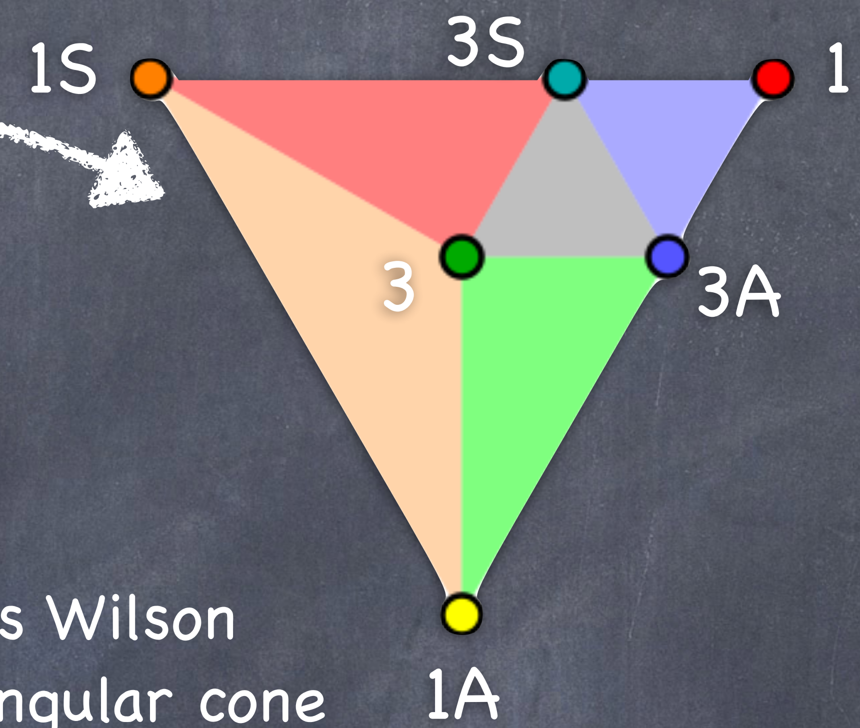
- **Simplest case: X is a one particle state -> Summation and integration vanish. I.e. <u>PERs are one particle extensions of SM.</u>**

- SMEFT has a number of symmetries
  - Internal (e.g. gauge) symmetries of i,j,k,l
  - Rotation around forward direction, SO(2) of transverse polarization.
- With symmetries, instead of 1-D projectors, the PERs are projectors of the irrep of $\mathbf{r}_i \times \mathbf{r}_j$. (Obtain from CG coefs)
- **PERs are one multiplet (w.r.t. SM symmetries) particle extensions of SM.**

New ER = combining 1D projectors to be symmetric -> irrep projectors



The Higgs triangular cone



The space of 4-Higgs Wilson coefficients form a triangular cone

- 3 HHHH operators

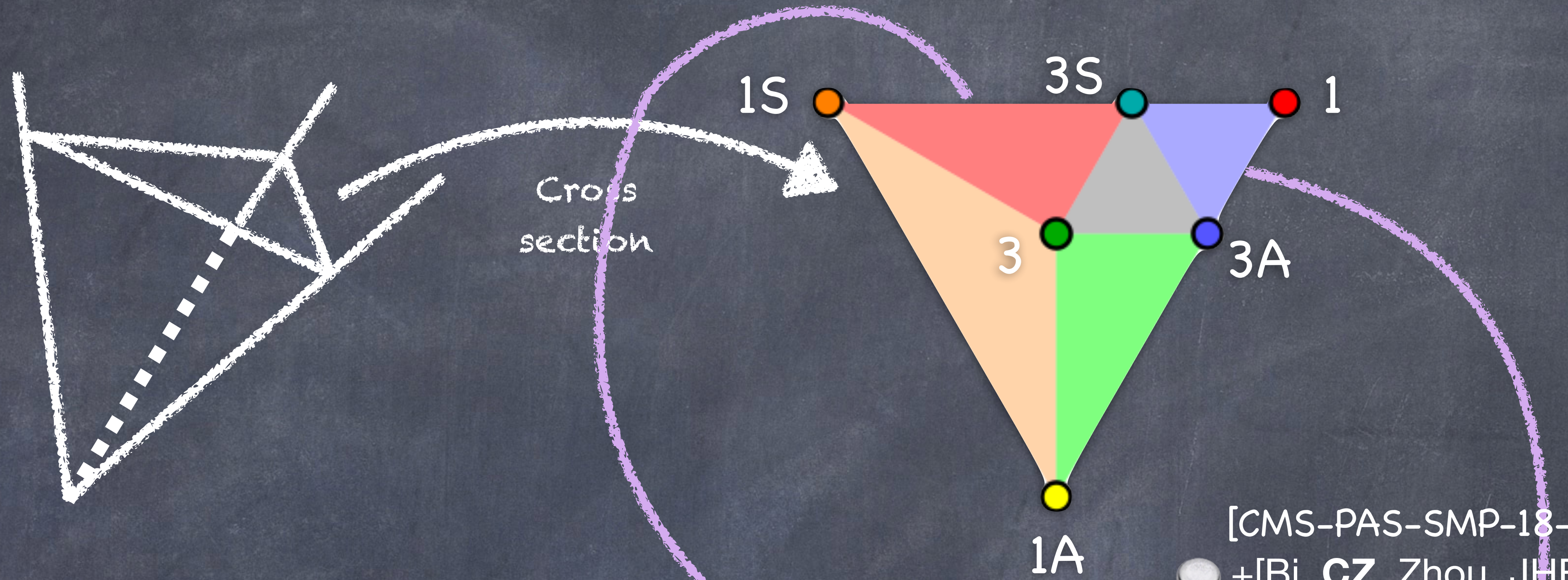
$$
\begin{aligned}
O_{S,0} &= [(D_\mu\Phi)^\dagger D_\nu\Phi] \times [(D^\mu\Phi)^\dagger D^\nu\Phi] \\
O_{S,1} &= [(D_\mu\Phi)^\dagger D^\mu\Phi] \times [(D_\nu\Phi)^\dagger D^\nu\Phi] \\
O_{S,2} &= [(D_\mu\Phi)^\dagger D_\nu\Phi] \times [(D^\nu\Phi)^\dagger D^\mu\Phi]
\end{aligned}
$$

- HH can form 6 irreps.
- Each can be generated by integrating out "1 particle"

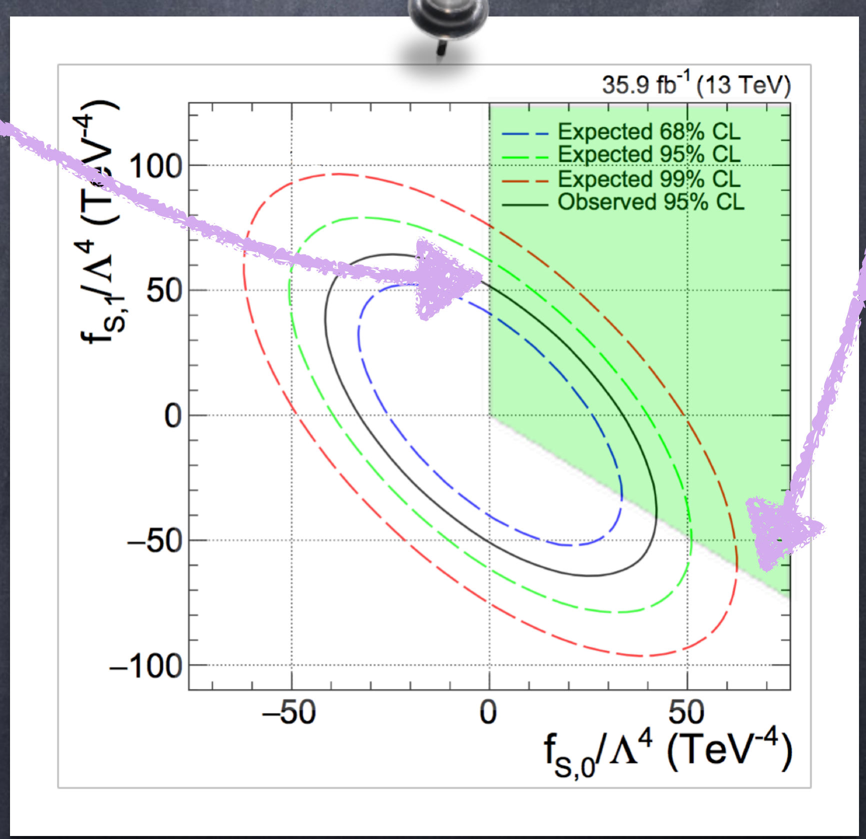
$$
\begin{aligned}
\mathcal{L} = {} & g_1(H^T\epsilon \overleftrightarrow{D}_\mu H)V_1^{\mu\dagger} + g_{1S}(H^\dagger H)S_1 \\
& + ig_{1A}(H^\dagger \overleftrightarrow{D}_\mu H)V_2^\mu + g_3(H^T\epsilon\tau^I H)S_2^{I\dagger} \\
& + g_{3S}(H^\dagger\tau^I H)S_3^I + ig_{3A}(H^\dagger\tau^I \overleftrightarrow{D}_\mu H)V_3^{\mu I} + h.c.
\end{aligned}
$$

- 6 PERs, 3 are linearly independent, 3 are extremal

The Higgs triangular cone



[CMS-PAS-SMP-18-001]
+[Bi, **CZ**, Zhou, JHEP 19]

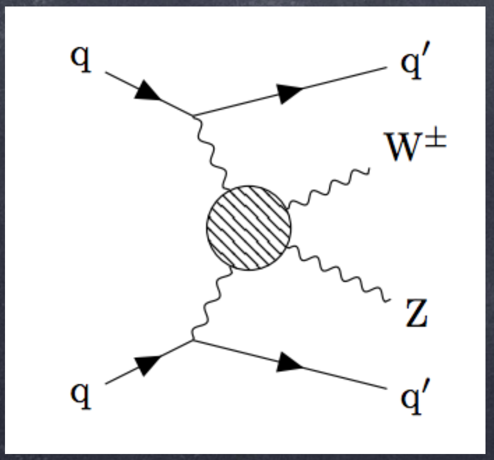


We learned

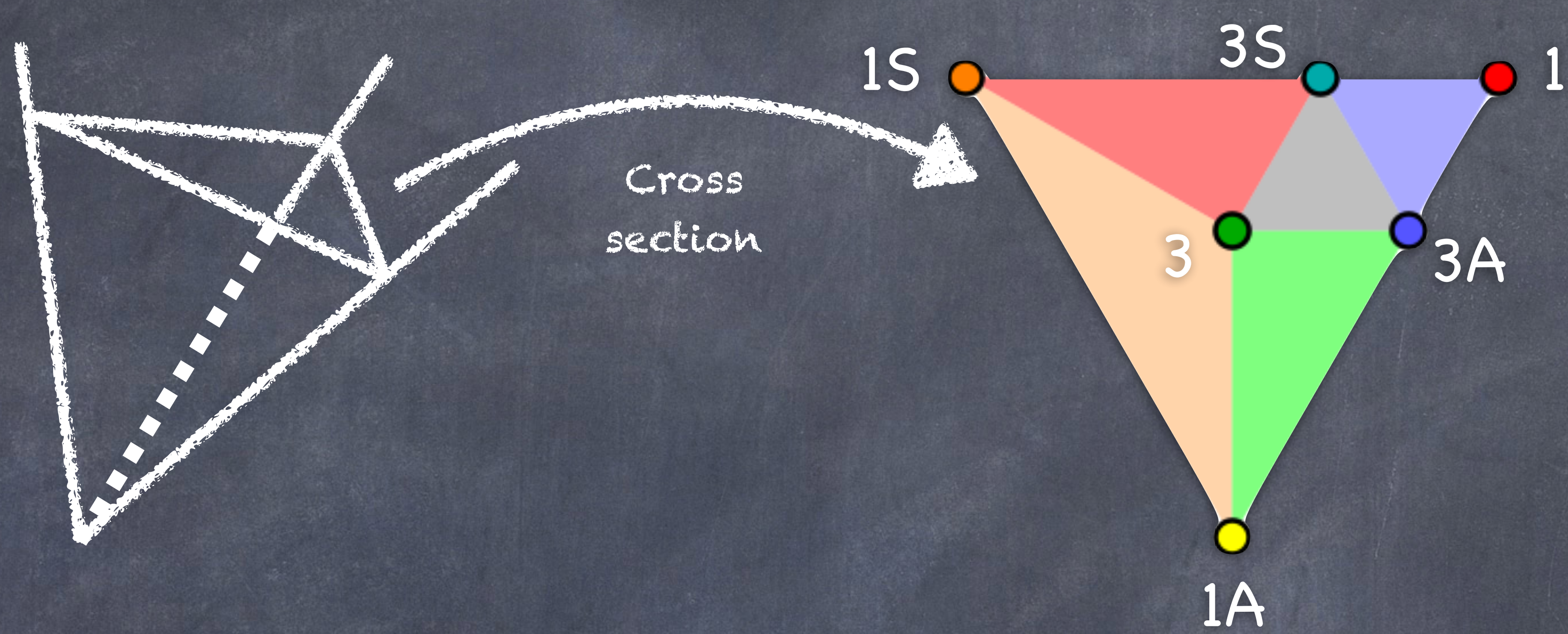
- Positivity bounds: faces of this cone

$$f_{S,0} \geq 0$$
$$f_{S,0} + f_{S,2} \geq 0$$
$$f_{S,0} + f_{S,1} + f_{S,2} \geq 0$$

(i.e. longitudinal 4-gauge boson couplings)



The Higgs triangular cone

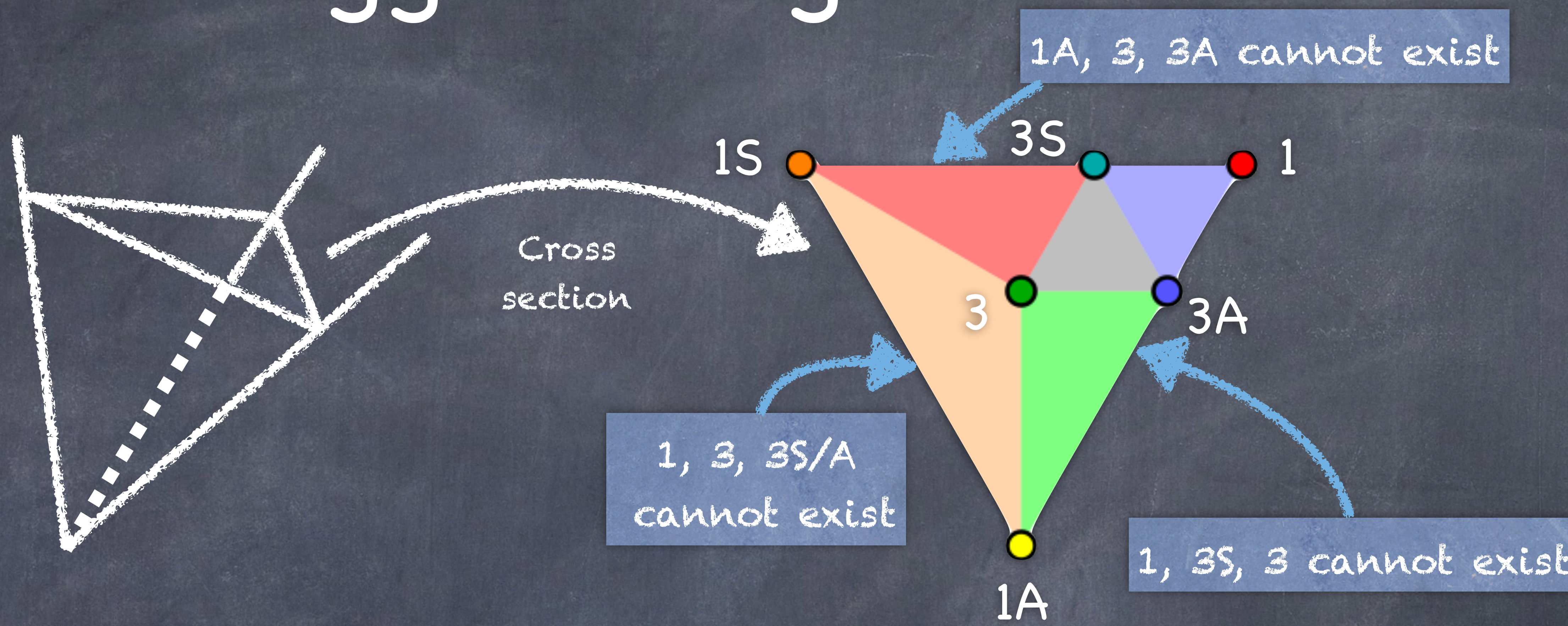


- Infer UV state from measurements, from measured coefficients $C=(C_1,C_2,\ldots)$
- E.g. if C = ER => UV is uniquely determined.
- E.g. C in blue region => particle 1 must exist.
- Can be quantified, e.g. setting lower bound etc.

1, 1S, 1A, 3, 3S, 3A are defined by

$$\mathcal{L} = g_1(H^T\epsilon \overleftrightarrow{D}_\mu H)V_1^{\mu\dagger} + g_{1S}(H^\dagger H)S_1 + ig_{1A}(H^\dagger \overleftrightarrow{D}_\mu H)V_2^\mu + g_3(H^T\epsilon\tau^I H)S_2^{I\dagger} + g_{3S}(H^\dagger\tau^I H)S_3^I + ig_{3A}(H^\dagger\tau^I \overleftrightarrow{D}_\mu H)V_3^{\mu I} + h.c.$$

The Higgs triangular cone

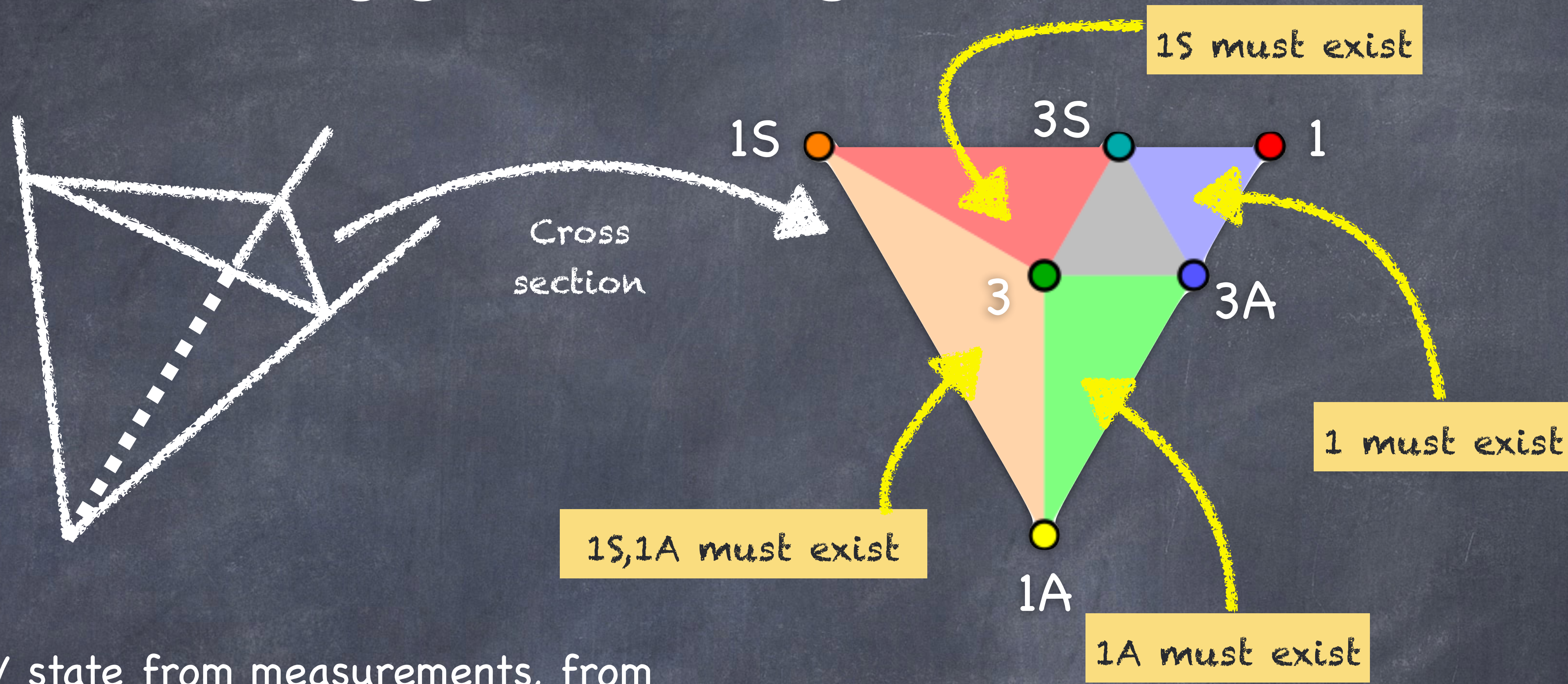


- Infer UV state from measurements, from measured coefficients $C=(C_1,C_2,\ldots)$
- E.g. if C = ER => UV is uniquely determined.
- E.g. C in blue region => particle 1 must exist.
- Can be quantified, e.g. setting lower bound etc.

1, 1S, 1A, 3, 3S, 3A are defined by

$$\mathcal{L} = g_1 (H^T \epsilon \overleftrightarrow{D}_\mu H) V_1^{\mu\dagger} + g_{1S} (H^\dagger H) S_1 + i g_{1A} (H^\dagger \overleftrightarrow{D}_\mu H) V_2^\mu + g_3 (H^T \epsilon \tau^I H) S_2^{I\dagger} + g_{3S} (H^\dagger \tau^I H) S_3^I + i g_{3A} (H^\dagger \tau^I \overleftrightarrow{D}_\mu H) V_3^{\mu I} + h.c.$$

The Higgs triangular cone

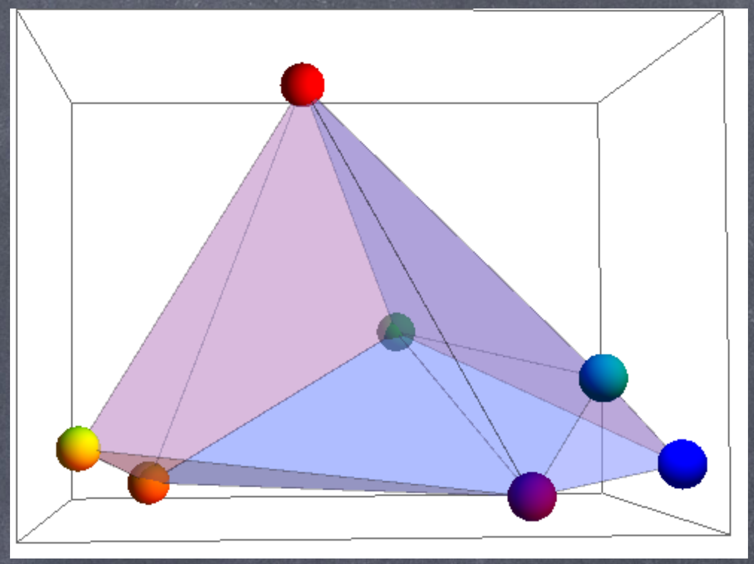


- Infer UV state from measurements, from measured coefficients $C=(C_1,C_2,\ldots)$
- E.g. if C = ER => UV is uniquely determined.
- E.g. C in blue region => particle 1 must exist.
- Can be quantified, e.g. setting lower bound etc.

1, 1S, 1A, 3, 3S, 3A are defined by

$$\mathcal{L} = g_1(H^T\epsilon \overleftrightarrow{D}_\mu H)V_1^{\mu\dagger} + g_{1S}(H^\dagger H)S_1 + ig_{1A}(H^\dagger \overleftrightarrow{D}_\mu H)V_2^\mu + g_3(H^T\epsilon\tau^I H)S_2^{I\dagger} + g_{3S}(H^\dagger\tau^I H)S_3^I + ig_{3A}(H^\dagger\tau^I \overleftrightarrow{D}_\mu H)V_3^{\mu I} + h.c.$$

W-boson polyhedral cone



- The W boson has 6 components. [3 of SU(2), 2 of SO(2)].
- 9 PERs, 8 are extremal, 5 linearly independent. -> 5D polyhedral cone with 8 edges.
- Operators:

$$O_{T,0} = \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right] \times \mathrm{Tr}\left[\hat{W}_{\alpha\beta}\hat{W}^{\alpha\beta}\right]$$
$$O_{T,1} = \mathrm{Tr}\left[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}\right] \times \mathrm{Tr}\left[\hat{W}_{\mu\beta}\hat{W}^{\alpha\nu}\right]$$
$$O_{T,2} = \mathrm{Tr}\left[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}\right] \times \mathrm{Tr}\left[\hat{W}_{\beta\nu}\hat{W}^{\nu\alpha}\right]$$

and more...

- Bounds (on transverse QGCs) are **tighter than** positivity from $u^i v^j u^{*k} v^{*l} M^{ijkl} \geq 0$

$$C_{T,2} \geq 0,\ 4C_{T,1} + C_{T,2} \geq 0,$$
$$C_{T,2} + 8C_{T,10} \geq 0,\ 8C_{T,0} + 4C_{T,1} + 3C_{T,2} \geq 0,$$
$$12C_{T,0} + 4C_{T,1} + 5C_{T,2} + 4C_{T,10} \geq 0,$$
$$4C_{T,0} + 4C_{T,1} + 3C_{T,2} + 12C_{T,10} \geq 0.$$

Cannot be derived from uvuv.M

W-boson polyhedral cone

- Consider $T^{ijkl} = T^{ilkj} \in \mathbb{R}^{(6^4)},\ T^{(ij),(kl)} \succeq 0$ is 36x36 matrix

$$T^{ijkl} m^{ij} m^{kl} = T^{ijkl} m^{il} m^{kj} \geq 0 \quad \Rightarrow \quad T^{ijkl} M^{ijkl} \geq 0$$

- If $T^{ijkl} \neq \sum_{\alpha} \rho_\alpha u^i_\alpha v^j_\alpha u^k_\alpha v^l_\alpha,\ \rho_\alpha \geq 0$

  then we have a new bound not covered by $u^i v^j u^{*k} v^{*l} M^{ijkl} \geq 0$

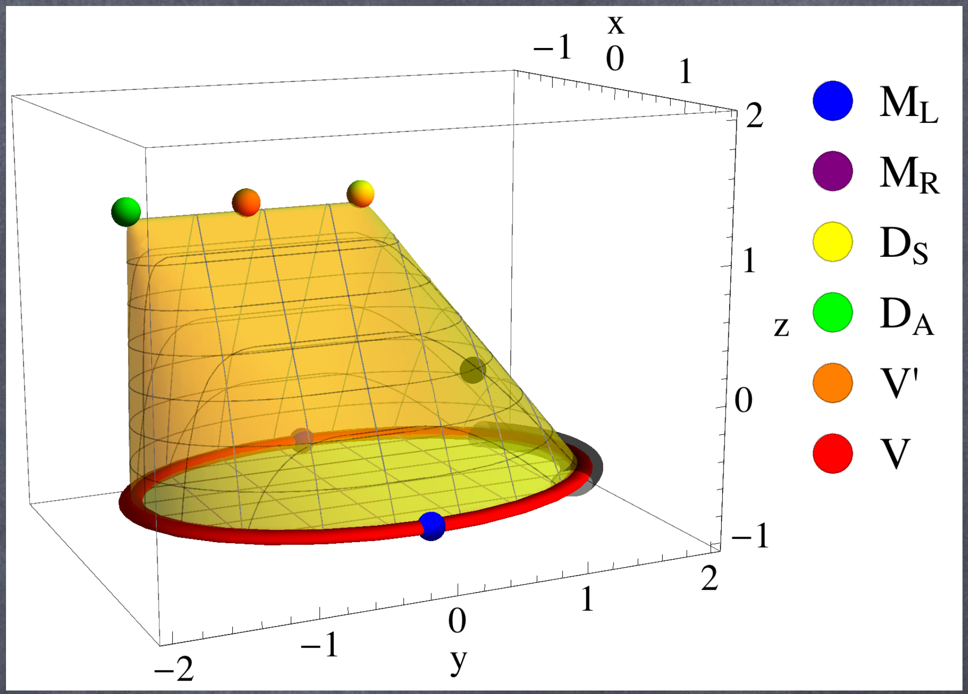
- The ER approach gives two such matrices =>

- Eigenvalues are 15,10,10,10,6,6,6,6,6,6,6,6,6,6,5,2,2,2,2,2 plus 16 0's.

- Same T's apply to other theories.

$$\begin{pmatrix}
\frac{9}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{7}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{3}{2} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{3}{2} & 0 & 0 & \frac{3}{2} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{3}{2} \\
0 & 0 & 0 & \frac{9}{2} & 0 & 0 & 0 & 0 & -\frac{3}{2} & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & \frac{7}{2} & 0 \\
0 & 0 & 0 & 0 & \frac{3}{2} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{3}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{9}{2} & 0 & 0 & 0 & 0 & -\frac{3}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{7}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{3}{2} & 0 & 0 & 0 & 0 \\
\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{9}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{7}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{3}{2} & 0 & 0 & 0 & 0 & \frac{9}{2} & 0 & 0 & 0 & 0 & \frac{7}{2} & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 \\
0 & 0 & -\frac{3}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{2} & 0 & 0 & -\frac{3}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{2} \\
0 & 0 & 0 & 0 & 0 & -\frac{3}{2} & 0 & 0 & 0 & 0 & \frac{9}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{3}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{3}{2} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{3}{2} & 0 & 0 & \frac{3}{2} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{3}{2} \\
0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & \frac{7}{2} & 0 & 0 & 0 & 0 & \frac{9}{2} & 0 & 0 & 0 & 0 & -\frac{3}{2} & 0 \\
\frac{3}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{7}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{9}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & -\frac{3}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{7}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{9}{2} & 0 & 0 \\
0 & 0 & 0 & \frac{7}{2} & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & -\frac{3}{2} & 0 & 0 & 0 & 0 & \frac{9}{2} & 0 \\
0 & 0 & -\frac{3}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{2} & 0 & 0 & -\frac{3}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{2} \\
0 & -\frac{3}{2} & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\
\frac{7}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{3}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{3}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{3}{2} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{3}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & \frac{7}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 \\
\frac{3}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{7}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & -\frac{3}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{7}{2} & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{3}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{7}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{3}{2} & 0 & 0 & 0 \\
0 & -\frac{3}{2} & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{3}{2} & 0 & 0 & 0 & 0 \\
\frac{7}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{2} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{7}{2} & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

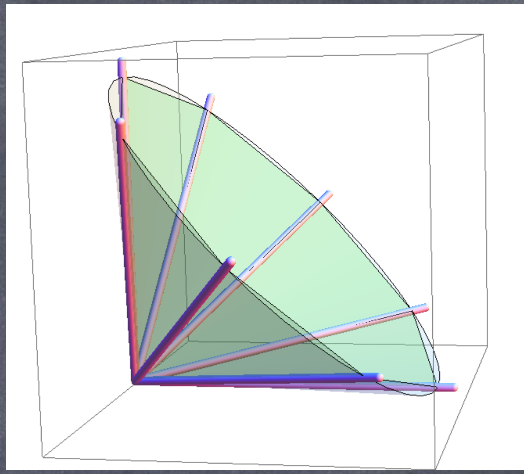
The Fermion cone



- Chiral fermions, FL and FR. 4 F^4DD operators. May couple to new state via
  - Majorana-like scalar coupling (ML, MR)
  - Dirac-like scalar coupling (DS, DA)
  - Vector coupling from same chirality (V) and opposite (V')
- May infer UV state and couplings: Assume the black dot is measured. If the small arc (black) on V is removed, the convex hull of the rest PERs does not contain the point.
  - => V/A type coupling must exist, |gA/gV| < 0.35

Numerically: might as well directly determine if a given point is included in the convex hull of all ERs (convex inclusion)

- Infinite number of (potential) ERs, but numerically, we sample them with a large number (N of order 100~1000) of discrete ERs, i.e. polyhedral cone inscribed to a "circular" cone.



- The inclusion determination is equivalent to a linear programming:

$$\begin{aligned} &\text{minimize} && 0 \\ &\text{subject to} && \textstyle\sum_i w_i \vec{e}_{N,i} = \vec{f},\ w_i \geq 0 \end{aligned}$$

where e's are the ERs, f is the given point, w's are real numbers

- Can be done efficiently with classic programming algorithms.

- Volume: ~0.681% (1 - 79.3/N²)

- **<u>The true volume seems to be 0.681%.</u>**