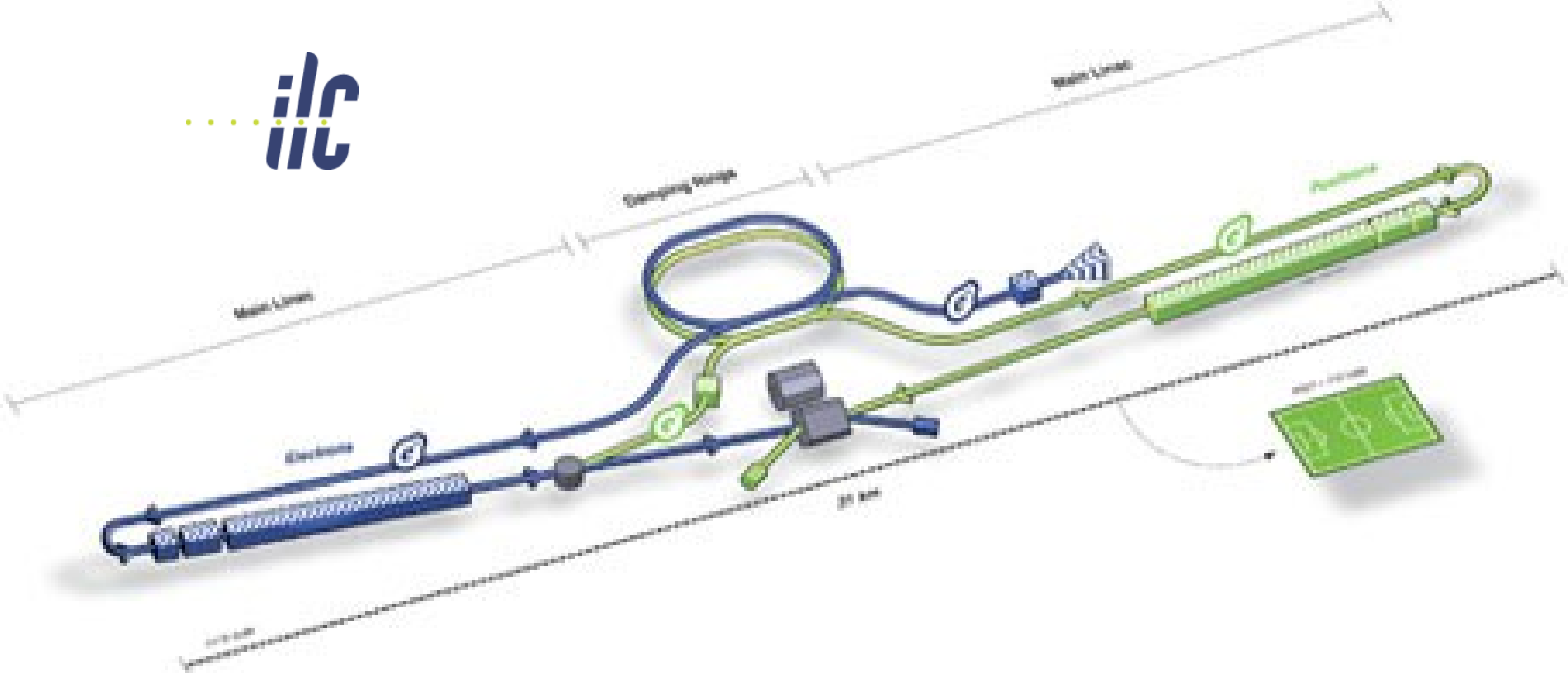
Top and Heavy quark studies at linear colliders

Roman Pöschl
on behalf of the LCC Physics Working Group

Snowmass Process EF03 Meeting 11/6/20

Announcement: The LCC Physics Working Group will soon publish a document called
"ILC Study Questions to Snowmass 2021"
This document will be circulated among the conveners and made public on the arxiv

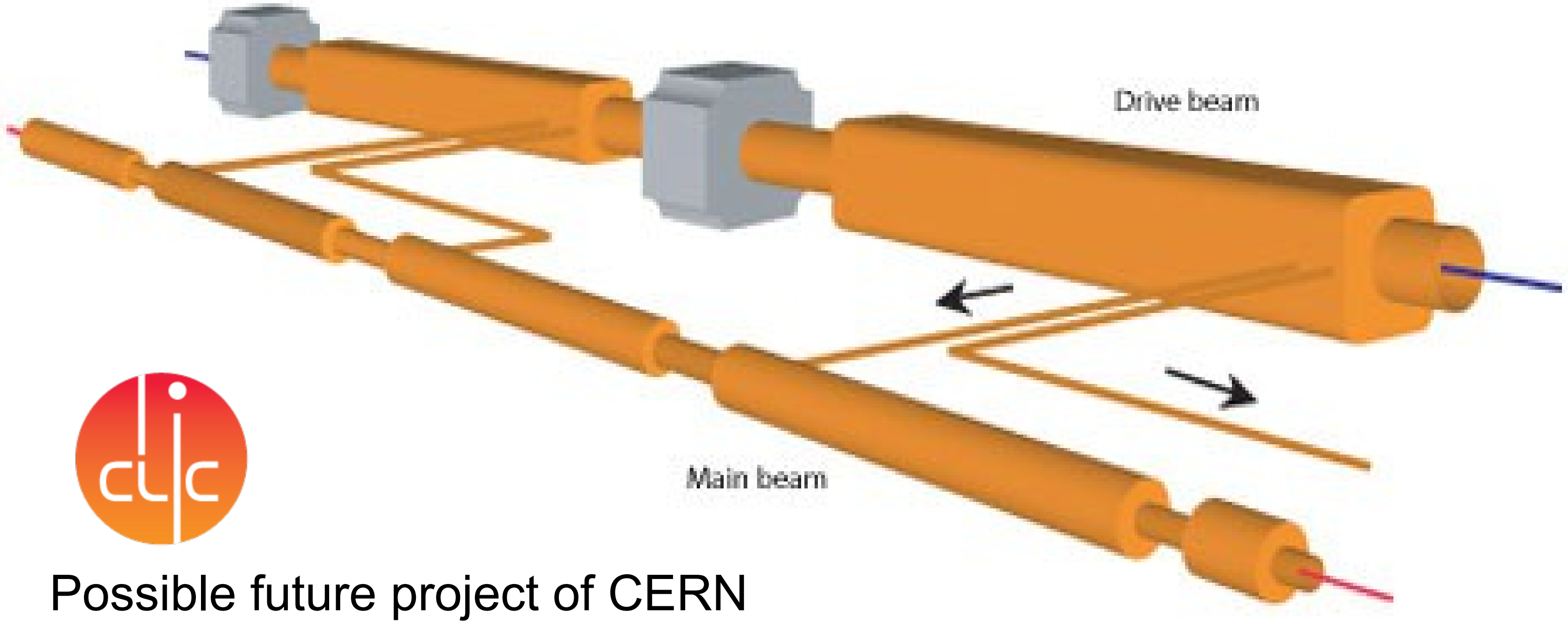
Linear Electron positron colliders



Under discussion in Japanese Gouvernment and inernational community

Energy: 0.1 - 1 TeV
Electron (and positron) polarisation
TDR in 2013
+ DBD for detectors
Footprint 31 km

Initial Energy 250 GeV – Footprint ~20km



Possible future project of CERN

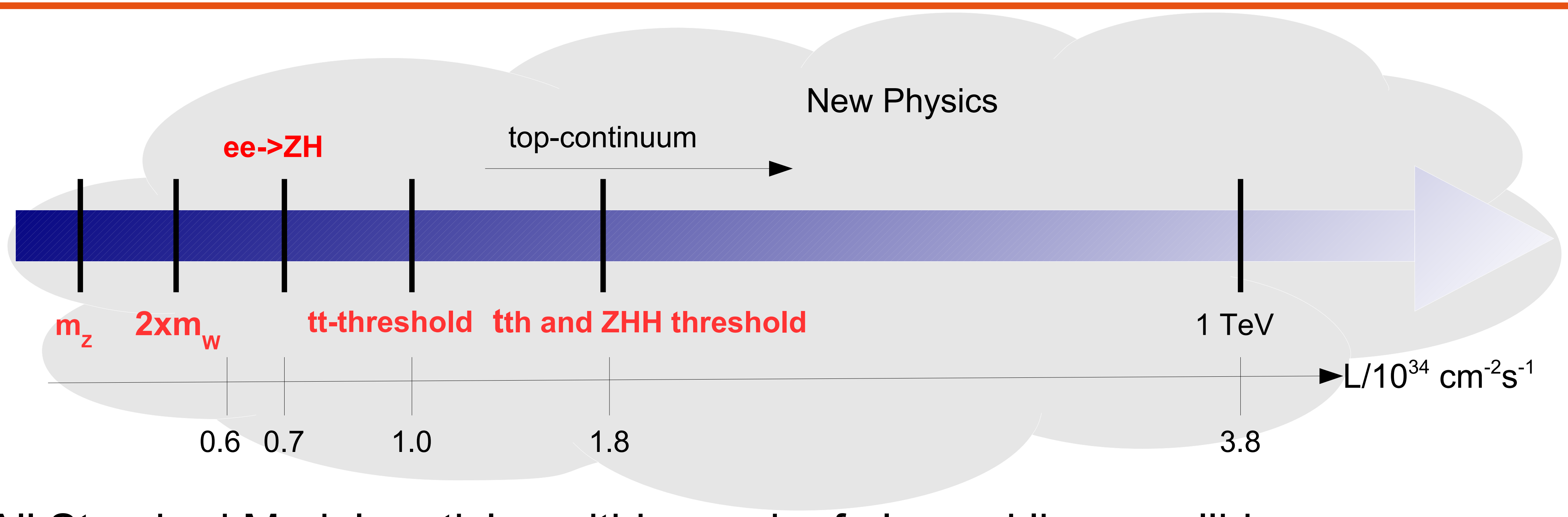
Energy: 0.4 - 3 TeV

CDR in 2012

Footprint 48km

Initial Energy 380 GeV

Linear colliders physics program



All Standard Model particles within reach of planned linear colliders

High precision tests of Standard Model over wide range to detect onset of New Physics

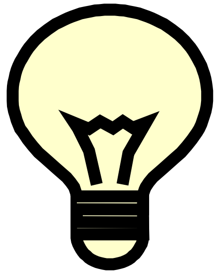
Machine settings can be “tailored” for specific processes

- Centre-of-Mass energy
- Beam polarisation (straightforward at linear colliders)

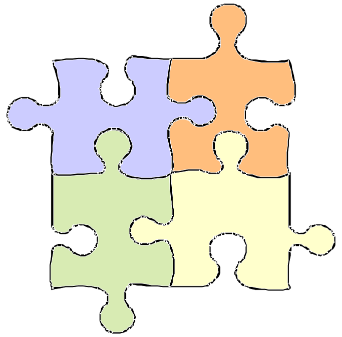
$$\sigma_{P,P'} = \frac{1}{4}\left[(1 - PP')(\sigma_{LR} + \sigma_{RL}) + (P - P')(\sigma_{RL} - \sigma_{LR})\right]$$

Background free searches for BSM through beam polarisation

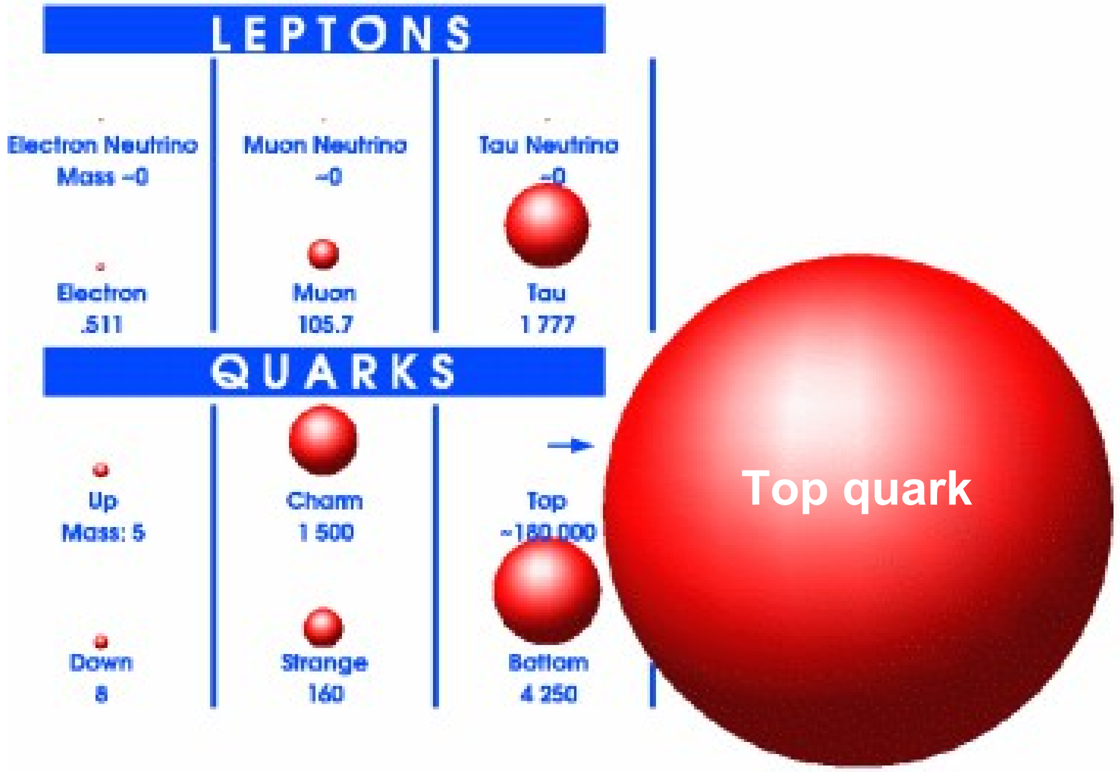
An enigmatic couple



Elementary Scalar?



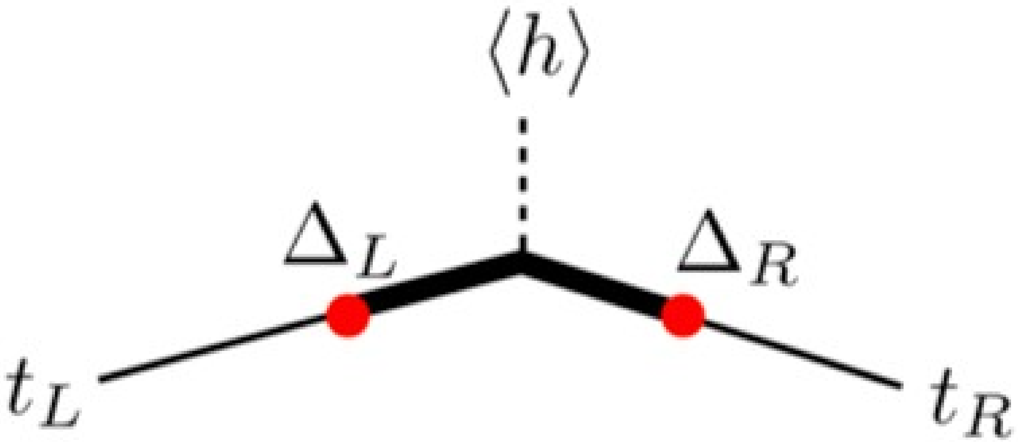
Composite object?



- Higgs and top quark are intimately coupled!
  Top Yukawa coupling O(1) !
  => Top mass important SM Parameter

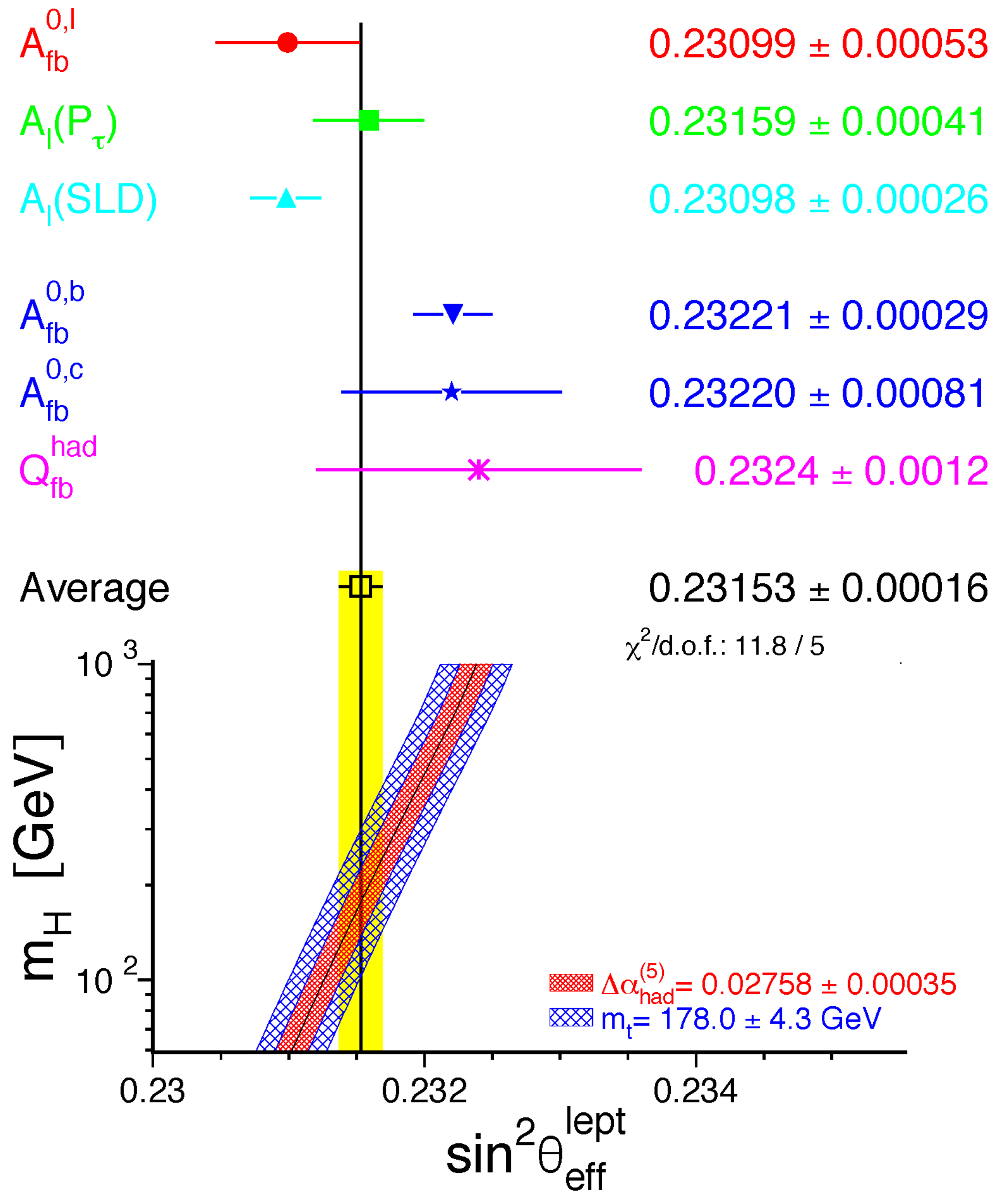
- New physics by compositeness?
  Higgs <u>and</u> top composite objects?

- e+e- collider perfectly suited to decipher both particles



Courtesy of S. Rychkov

Anomalies in LEP/SLD data



Most precise single Individual determination of $\sin^2\theta^\ell_{\mathrm{eff.}}$ from SLC

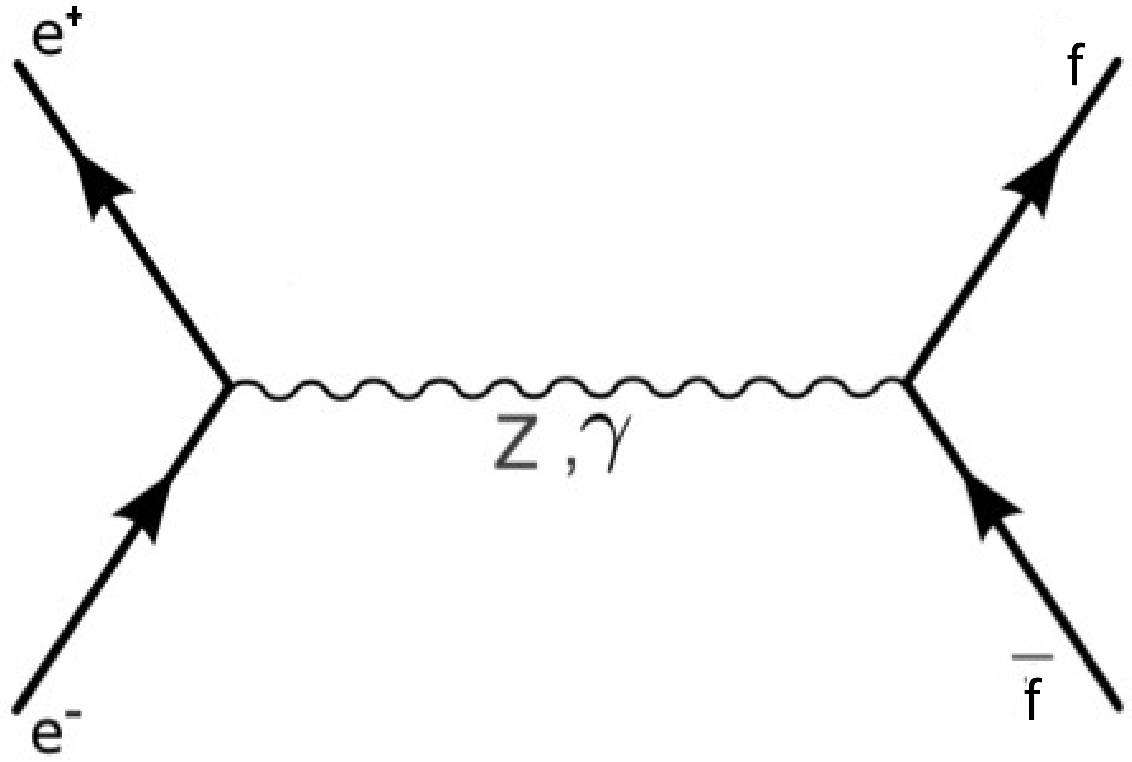
- Left-right asymmetry of leptons

- Most precise measurement of $\sin^2\theta^\ell_{\mathrm{eff.}}$ from forward backward asymmetry A^b_{FB} in ee→bb at LEP

:

- Most precise determinations of $\sin^2\theta^\ell_{\mathrm{eff.}}$ differ significantly
  - Requires verification
  - **Heavy quark effect, effect on all quarks/fermions, no effect at all?**

Two fermion processes



Differential cross sections for (relativistic) di-fermion production*:

$$\frac{d\sigma}{d\cos\theta}(e^-_L e^+_R \rightarrow f\bar{f}) = \Sigma_{LL}(1+\cos\theta)^2 + \Sigma_{LR}(1-\cos\theta)^2$$

$$\frac{d\sigma}{d\cos\theta}(e^-_R e^+_L \rightarrow f\bar{f}) = \Sigma_{RL}(1+\cos\theta)^2 + \Sigma_{RR}(1-\cos\theta)^2$$

*add term ~$\sin^2\theta$ in case of non-relativistic fermions e.g. top close to threshold

Σ_{IJ} are helicity amplitudes that contain couplings g_L, g_R (or F_V, F_A)

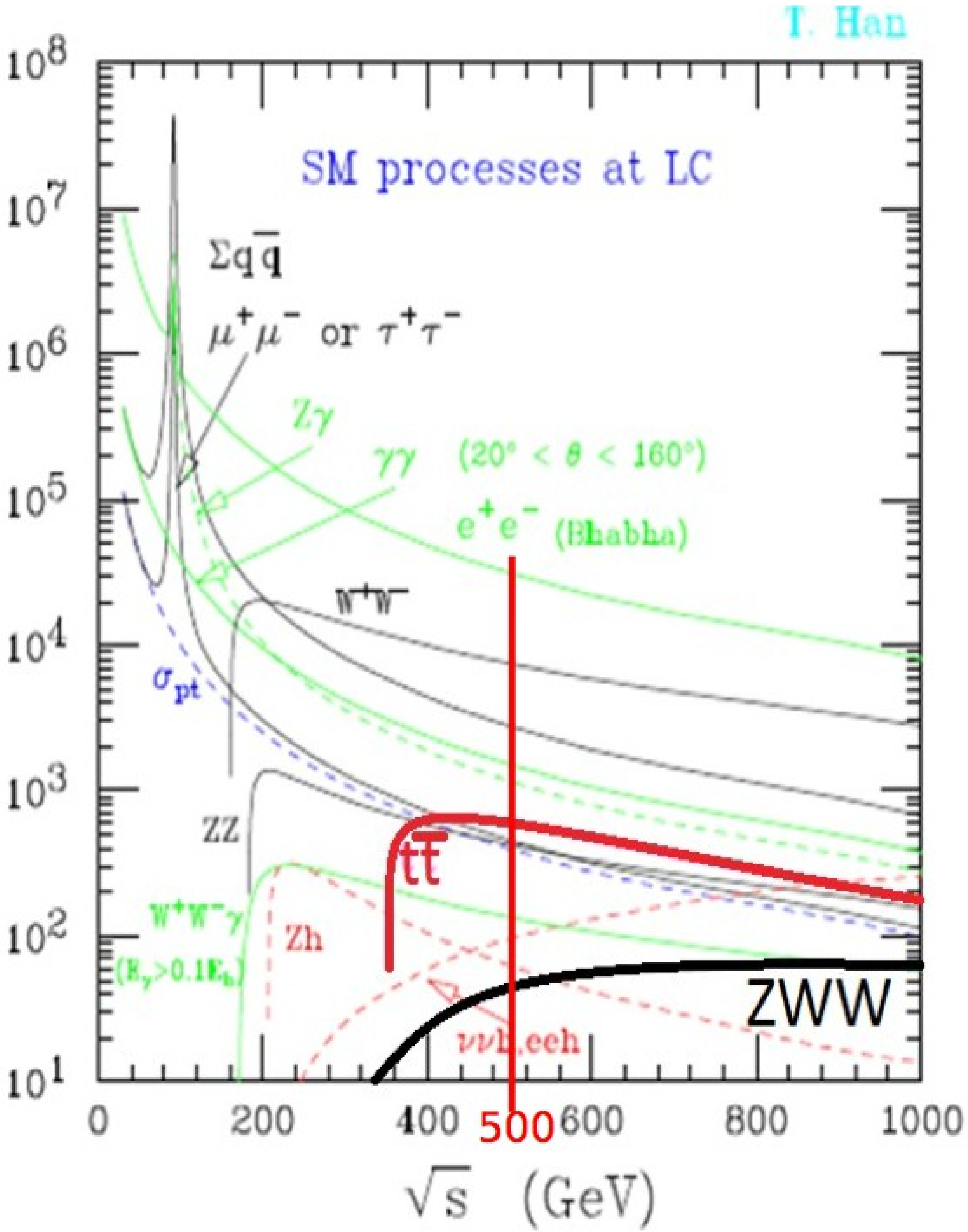
$\Sigma_{IJ} \neq \Sigma_{I'J'}$ => (characteristic) asymmetries for each fermion

Forward-backward in angle, general left-right in cross section

All four helicity amplitudes for all fermions **only available with polarised beams**

Here we focus on tt, bb and cc pair production

Cross sections



$e^+e^- \rightarrow t\bar{t}$: 500 GeV

Channel	$\sigma_{unpol.}$ [fb]	$\sigma_{-,+}$ [fb]	$\sigma_{+,-}$ [fb]
$t\bar{t}$	572	1564	724
$\mu^+\mu^-$	456	969	854
$\sum_{q=u,d,s,c} q\bar{q}$	2208	6032	2793
$b\bar{b}$	372	1212	276
γZ^0	11185	25500	19126
W^+W^-	6603	26000	150
Z^0Z^0	422	1106	582
$Z^0W^+W^-$	40	151	8.7
$Z^0Z^0Z^0$	1.1	3.2	1.22
Single t for $e^+e^- \rightarrow e^-\bar{\nu}_e t\bar{b}$ [11]	3.1	10.0	1.7

352 GeV (unpol)

450 fb

25.2 pb

11.5 pb
865 fb

$e^+e^- \rightarrow b\bar{b}$: 250 GeV

Channel	σunpol fb	σL fb	σR fb
$b\bar{b}$	1756	5629	1394
$\gamma b\bar{b}$ (Z return)	7860	18928	12512
ZZ hadronic with $b\bar{b}$	196	549	236
HZ hadronic with $b\bar{b}$	98	241	152

$e^+e^- \rightarrow c\bar{c}$: 250 GeV

$$\sigma(P_{e^-} = -1, P_{e^+} = +1) \approx 8518\,\text{fb}$$

$$\sigma(P_{e^-} = +1, P_{e^+} = -1) \approx 3565\,\text{fb}$$

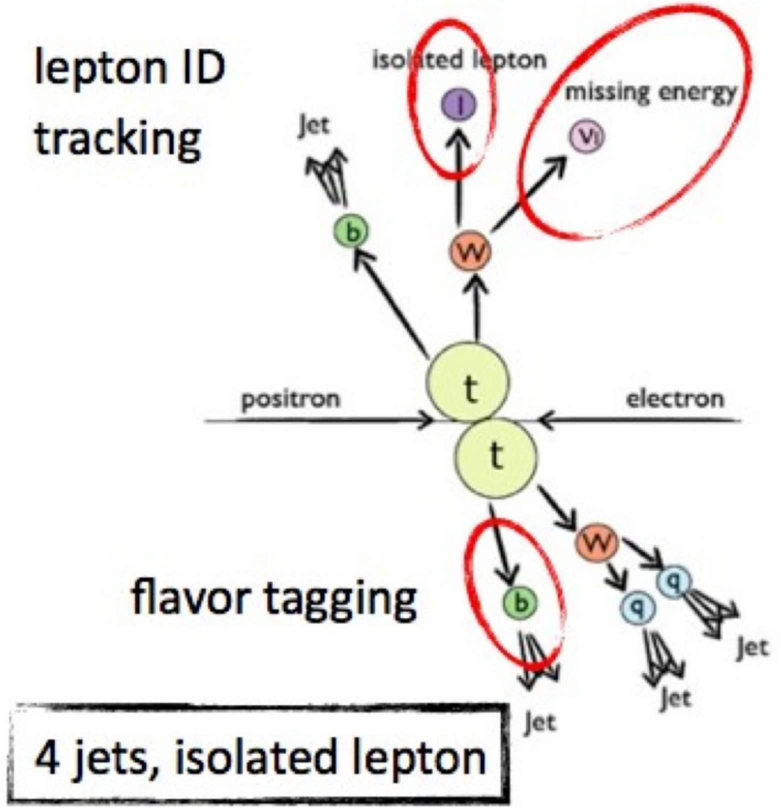
$$\sigma_{unpol.} \approx 3020\,\text{fb}$$

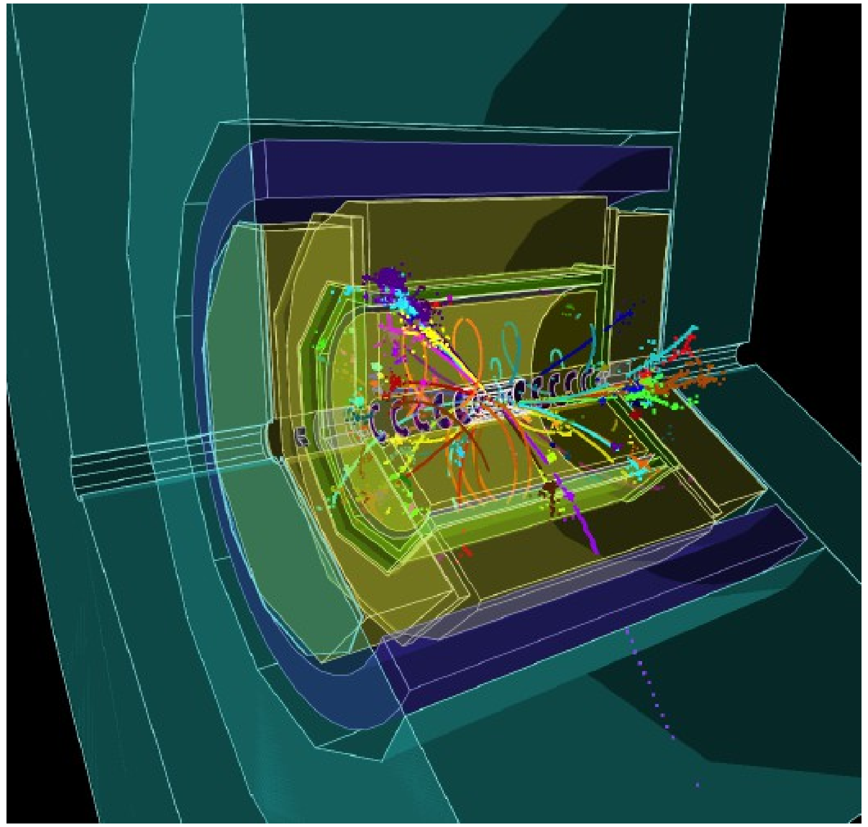
Elements of top quark reconstruction

Three different final states:

1) Fully hadronic (46.2%) → 6 jets
2) Semi leptonic (43.5%) → 4 jets + 1 charged lepton and a neutrino
3) Fully leptonic (10.3%) → 2 jets + 4 leptons

$$t\bar{t} \to (bW)(bW) \to (bqq')(b\ell\nu)$$

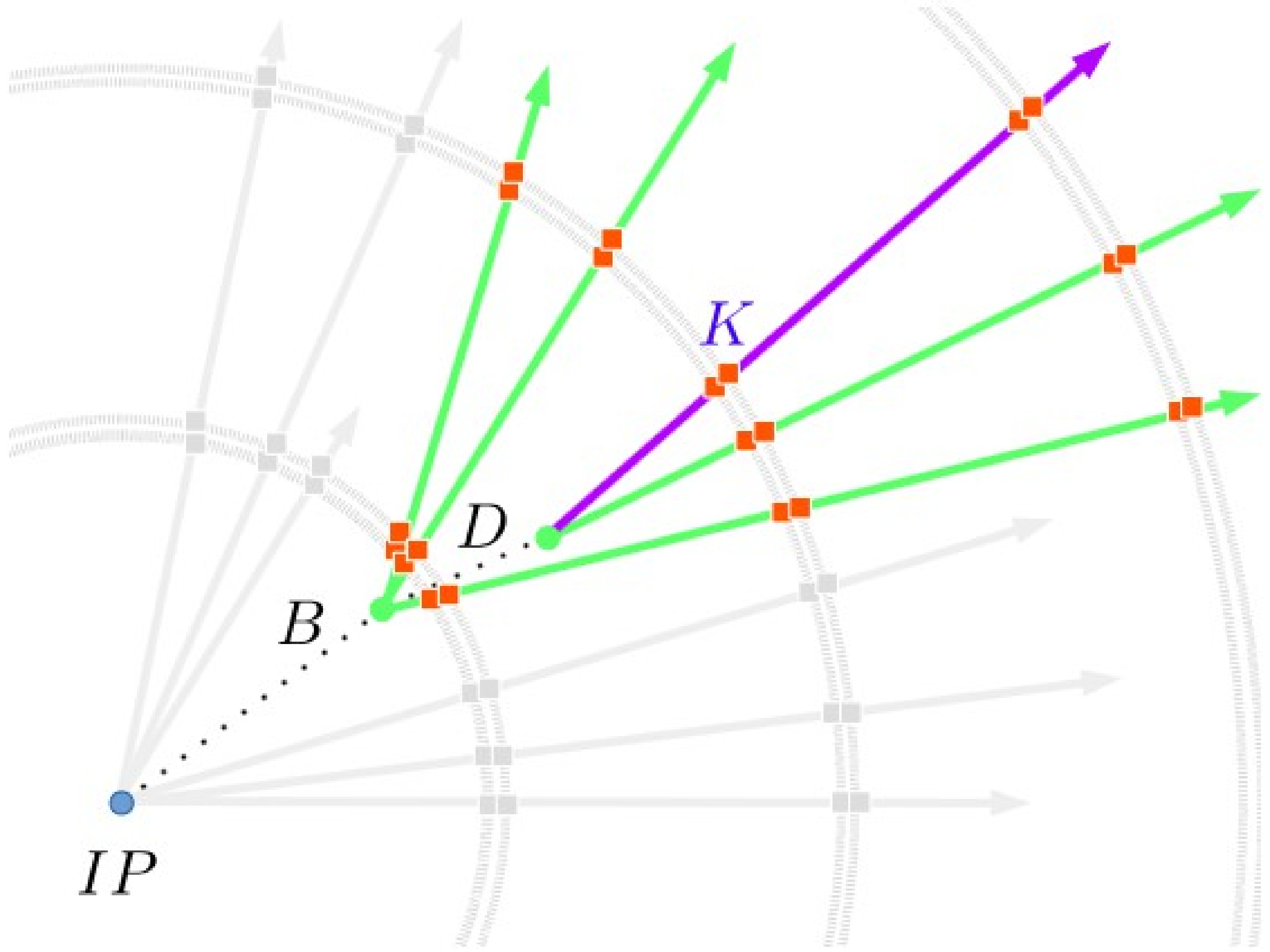




Final state reconstruction uses all detector aspects

Results shown in the following are based on <u>full simulation</u> of LC Detectors

Experimental challenges - Flavor tagging and charge measurment

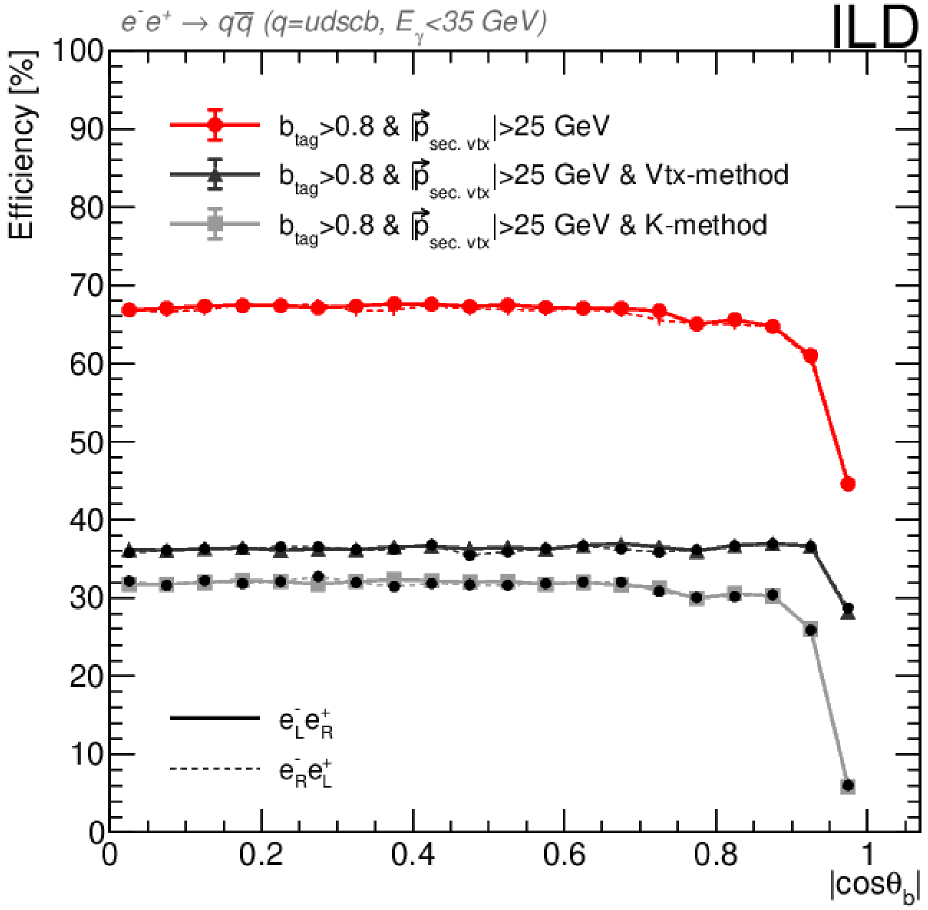


PhD thesis: *S. Bilokin*
A. Irles

- Flavor tagging
  - Indispensable for analyses with final state quarks
- Quark charge measurement
  - Important for top quark studies,
  - indispensable for ee->bb, cc, ss, ...
- Control of migrations:
  - Correct measurement of vertex charge
  - Kaon identification by dE/dx (and more)
- Future detectors can base the entire measurements on double Tagging and vertex charge
  - LEP/SLC had to include single tags and Semi-leptonic events

Typical efficiencies

$e^+e^- \rightarrow b\bar{b}$: 250 GeV



- Individual efficiency for correct b-tag and charge measurements using Vtx and Kaon charge
- Final efficiency ~20% from combination of Vtx and Kaon charge in different/same jets

$e^-_L e^+_R \rightarrow t\bar{t}$ at 500 GeV

General selection cuts	IDR-L	IDR-S
Isolated Lepton	92.1%	92.1%
$btag_1 > 0.8$ or $btag_2 > 0.3$	81.2%	81.1%
Thrust < 0.9	81.2%	81.1%
Hadronic mass	78.2%	78.2%
Reconstructed m_W and m_t	73.4%	73.4%
t quark polar angle spectrum		
$\gamma_t^{had.} + \gamma_t^{\ell} > 2.4$	62.2%	61.8%
$\lvert p_{B,had} \rvert > 15$ GeV	34.5%	33.9%
"$t\bar{t}$ identification"	30.6%	30.2%
b quark polar angle spectrum		
No additional cuts		

$e^-_R e^+_L \rightarrow t\bar{t}$ at 500 GeV

General selection cuts	IDR-L	IDR-S
Isolated Lepton	94.1%	94.0%
$btag_1 > 0.8$ or $btag_2 > 0.3$	84.9%	84.8%
Thrust < 0.9	84.9%	84.8%
Hadronic mass	82.2%	82.3%
Reconstructed m_W and m_t	77.6%	77.5%
t quark polar angle spectrum		
$\gamma_t^{had.} + \gamma_t^{\ell} > 2.4$	64.1%	64.1%
b quark polar angle spectrum		
Vtx+Vtx	10.8%	10.3%

Total cross section
- Typical efficiency 75%
- Independent of beam polarisation

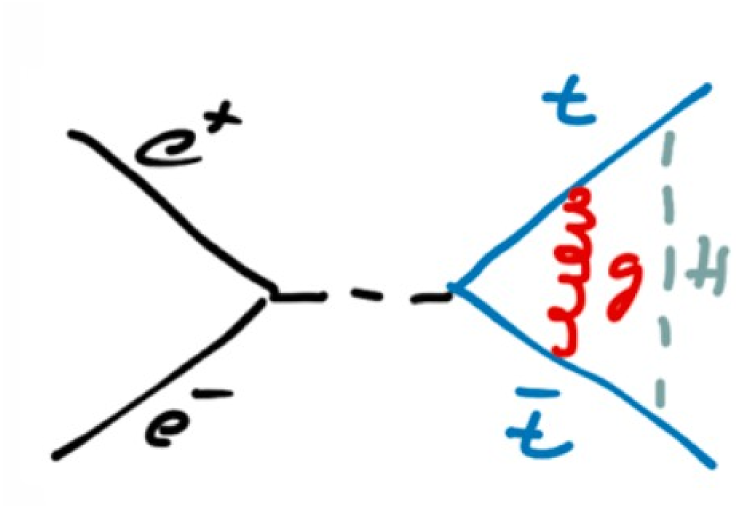
Differential cross section
- Note, difference for different beam polarisations
- Left hand polarisation more vulnerable to migrations
- Requires information from hadronic final state
- Vtx, Kaon as in bb-case

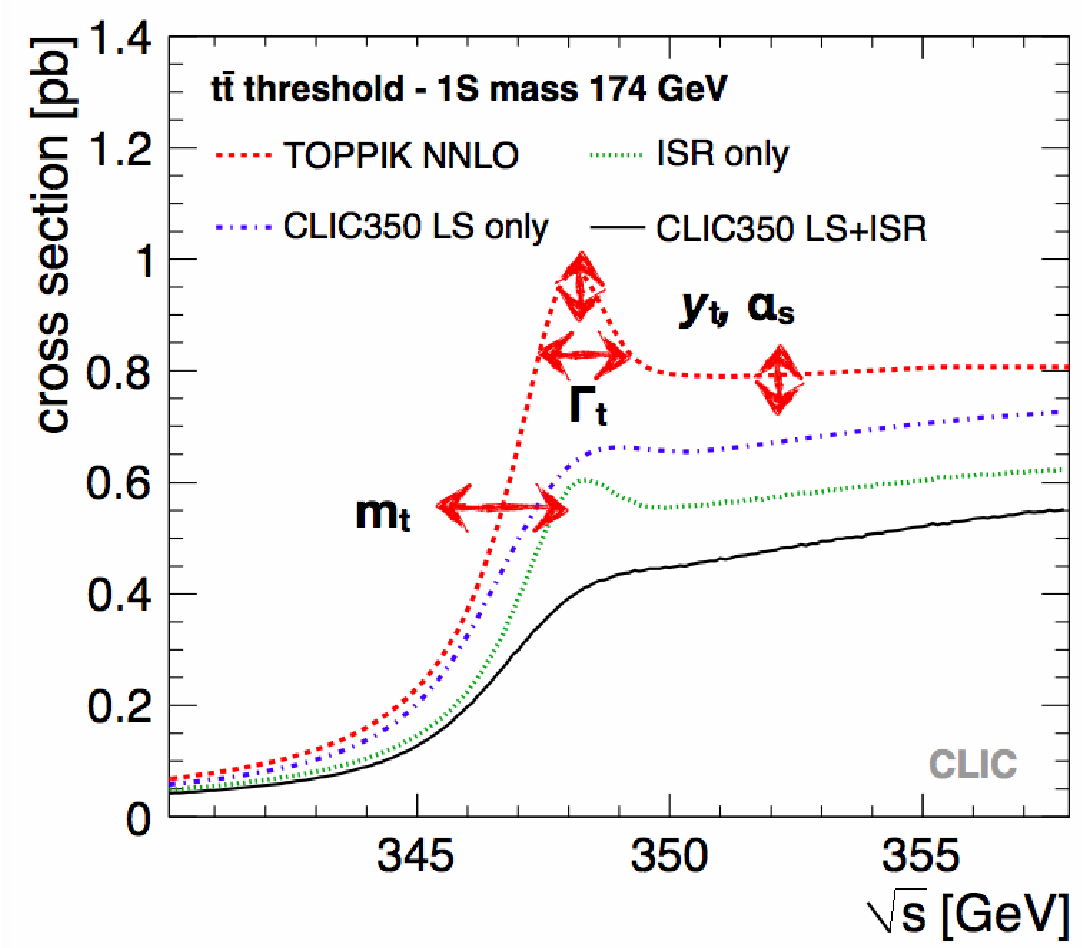
Top pair production at threshold

Small size of ttbar "bound state" at threshold ideal premise for precision physics

Cross section around threshold is affected by several properties of the top quark and by QCD

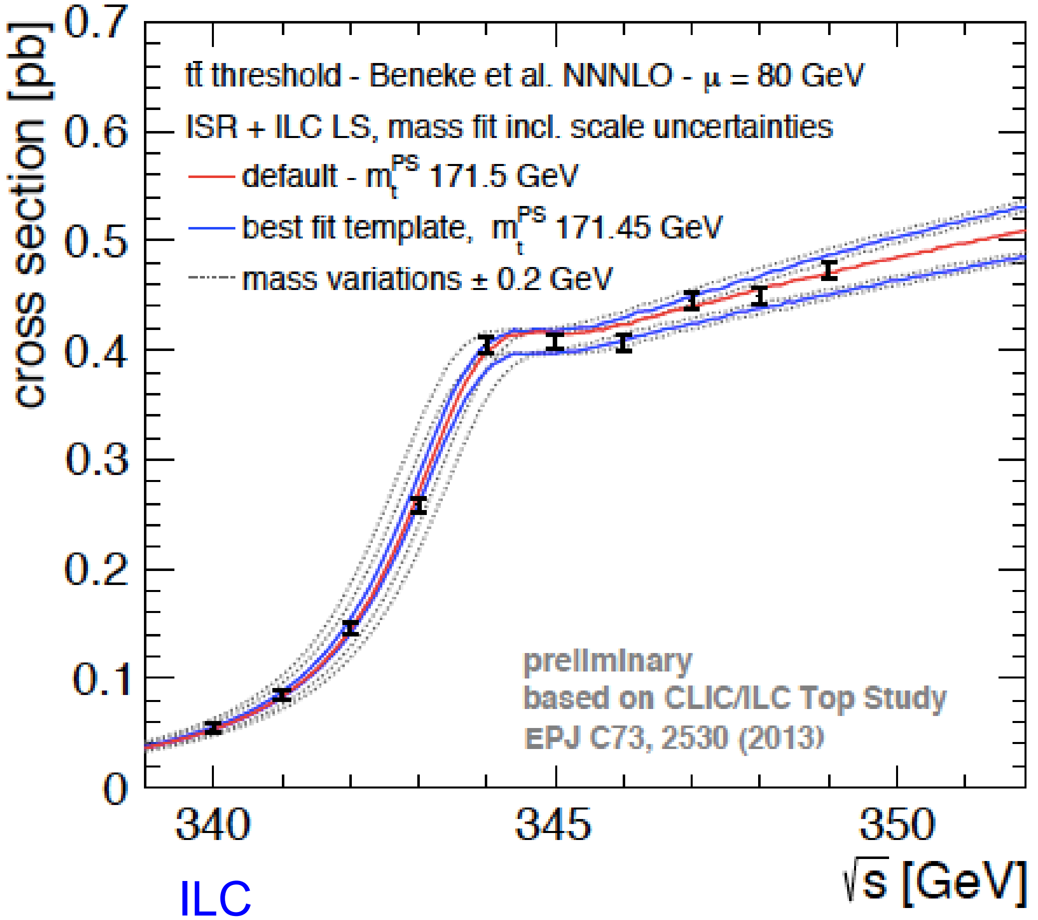
- Top mass, width Yukawa coupling
- Strong coupling constant





- Effects of some parameters are correlated:
- Dependence on Yukawa coupling rather weak,
- Precise external α_s helps

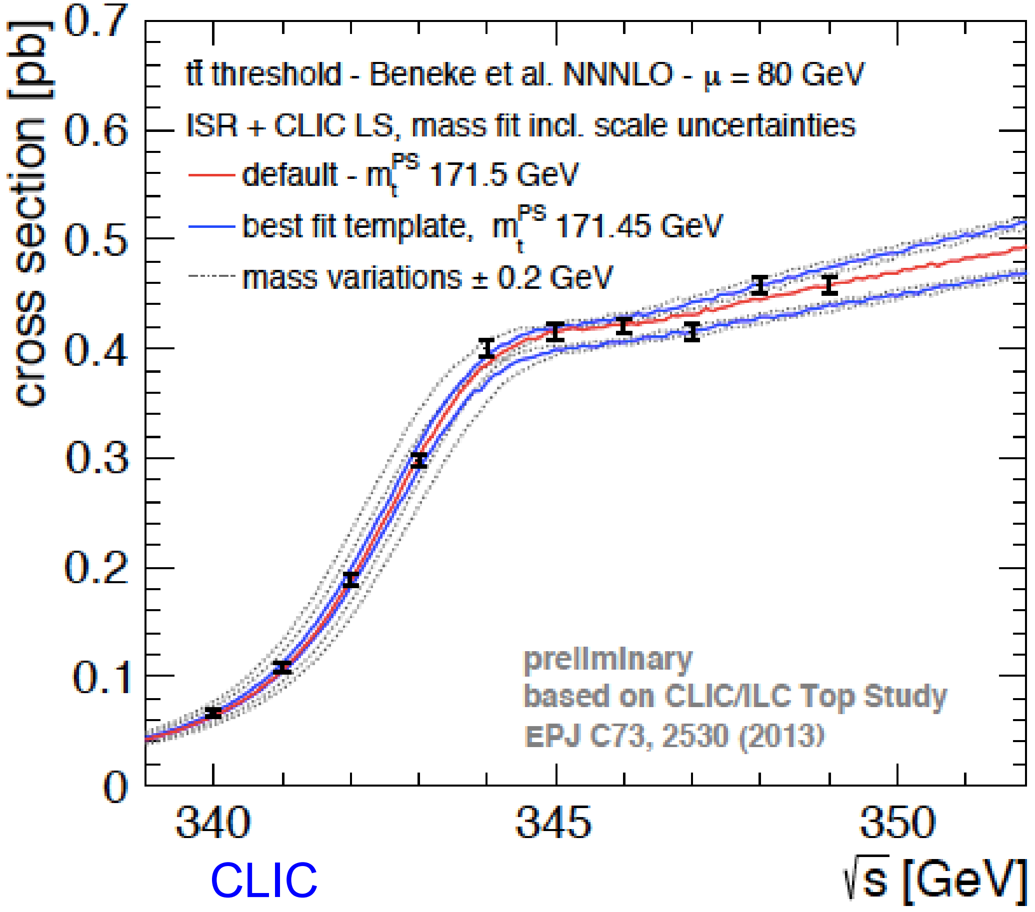
Top pair production at threshold



ILC

Fit uncertainty:
28.5 MeV (18 MeV stat)

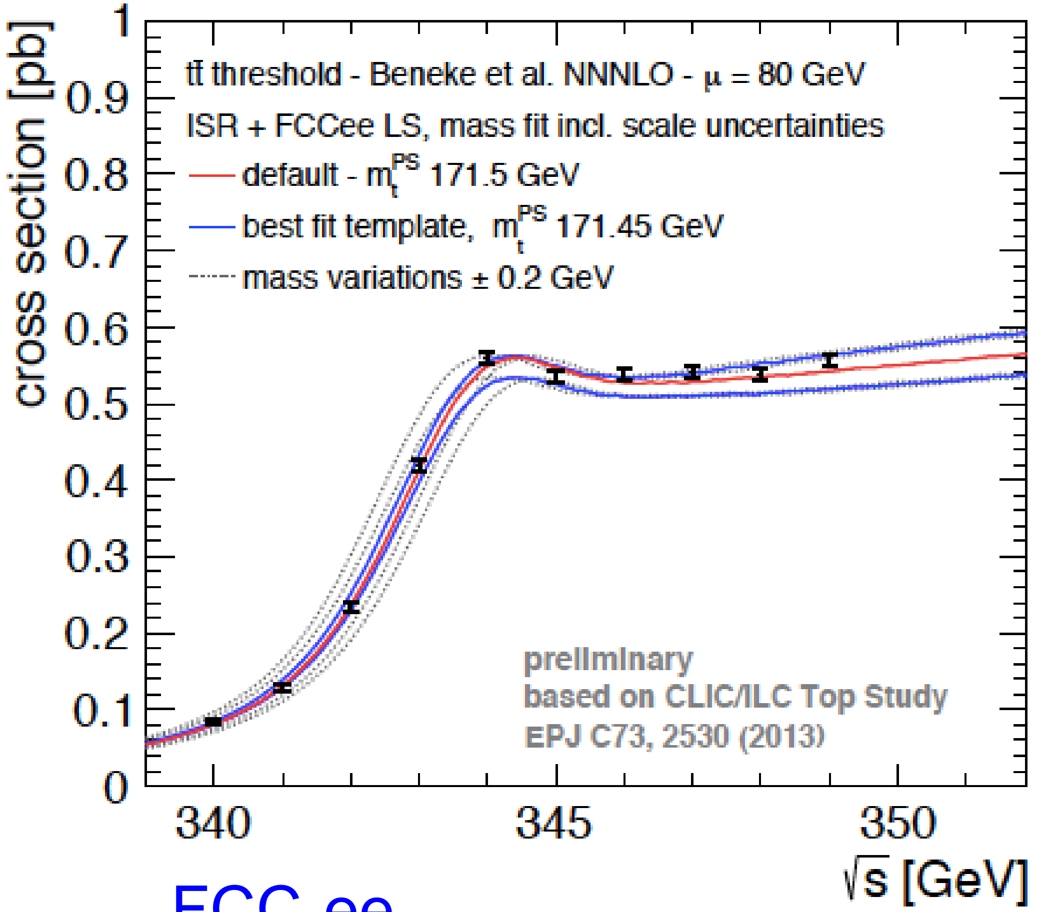
Scale uncertainty:
40 MeV



CLIC

Fit uncertainty:
31 MeV (21 MeV stat)

Scale uncertainty:
42 MeV

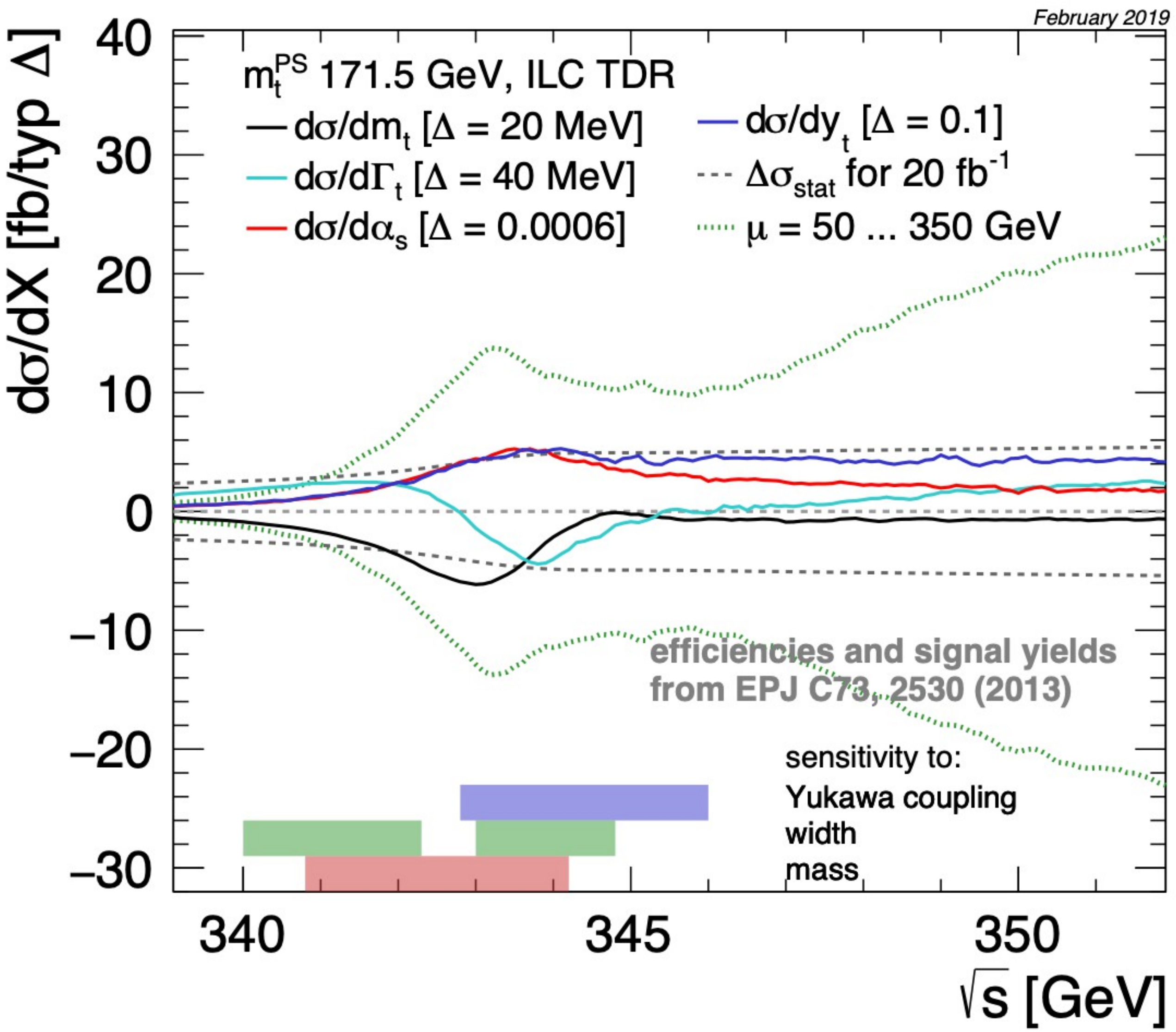


FCC-ee

Fit uncertainty:
27 MeV (15 MeV stat)

Scale uncertainty:
40 MeV

Sensitivity and error breakdown

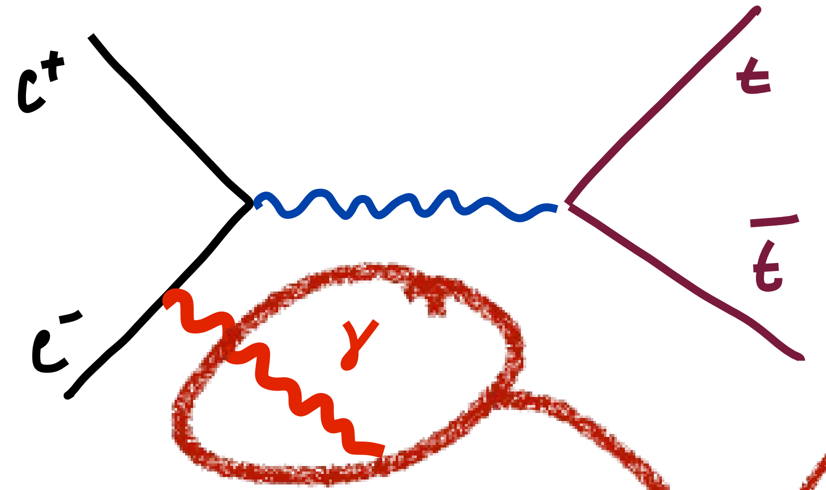


error source	Δm_t^{PS} [MeV]
stat. error (200 fb^{-1})	13
theory (NNNLO scale variations, PS scheme)	40
parametric (α_s, current WA)	35
non-resonant contributions (such as single top)	<40
residual background / selection efficiency	10 – 20
luminosity spectrum uncertainty	<10
beam energy uncertainty	<17
combined theory & parametric	30 – 50
combined experimental & backgrounds	25 - 50
total (stat. + syst.)	40 – 75

- Detailed evaluation of systematic uncertainties
- Multi-parameter fits (mass, width, αs, yt), scan optimization…

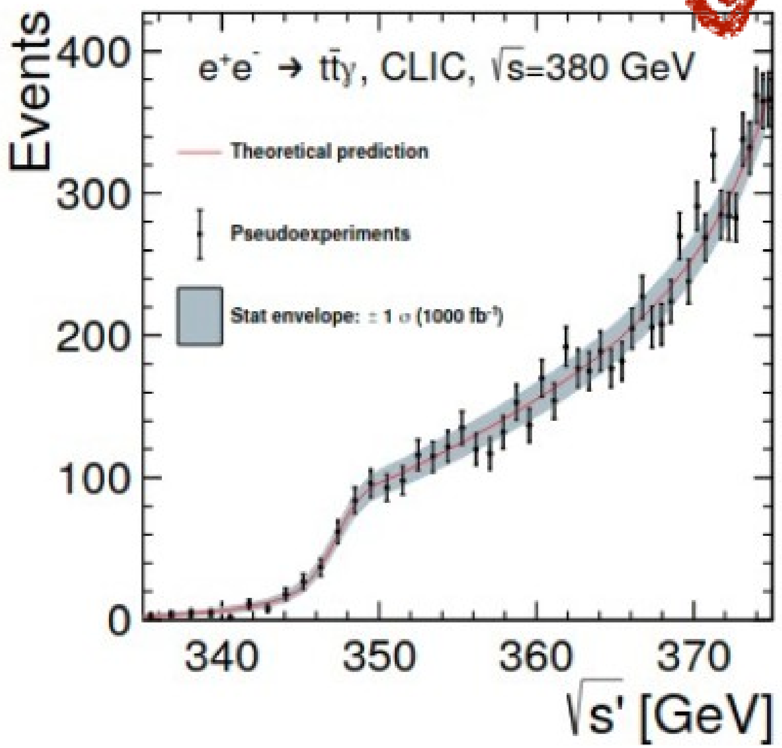
Sensitivity and error breakdown

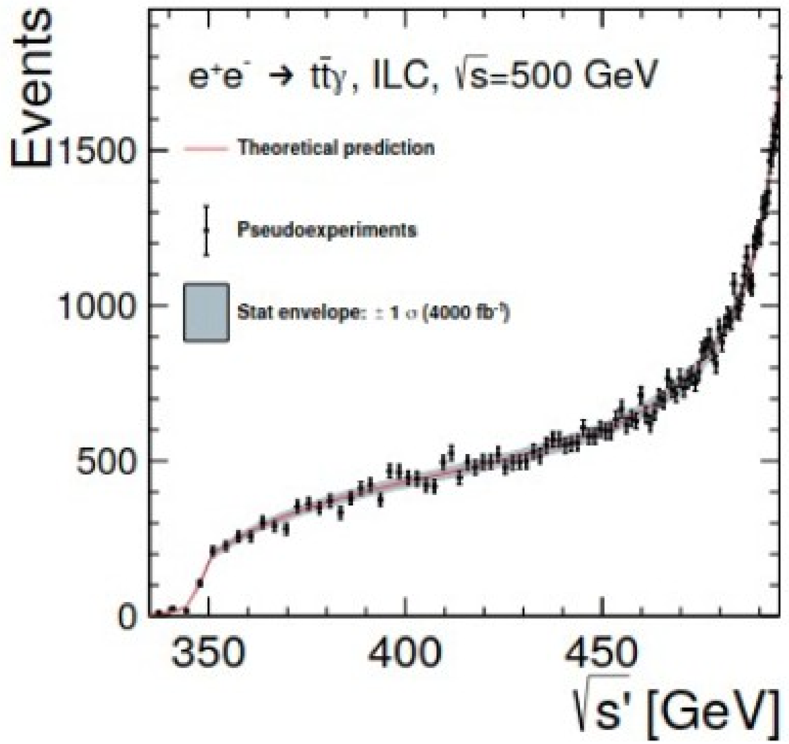
- A new(er) idea to measure the top mass in a theoretically well-defined scheme in high-energy running above the threshold



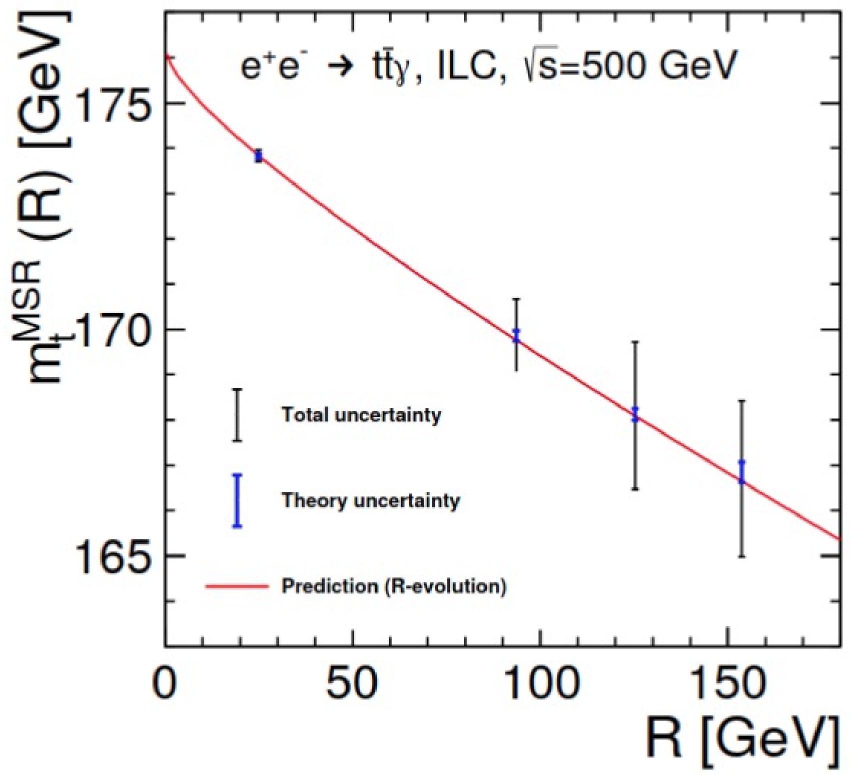
cms energy	CLIC, $\sqrt{s} = 380$ GeV		ILC, $\sqrt{s} = 500$ GeV	
luminosity [fb^{-1}]	500	1000	500	4000
statistical	140 MeV	90 MeV	350 MeV	110 MeV
theory	46 MeV		55 MeV	
lum. spectrum	20 MeV		20 MeV	
photon response	16 MeV		85 MeV	
total	150 MeV	110 MeV	360 MeV	150 MeV

matched NNLO + NNLL calculation, luminosity spectrum folded in explicitly;
Extraction of short distance MSR mass



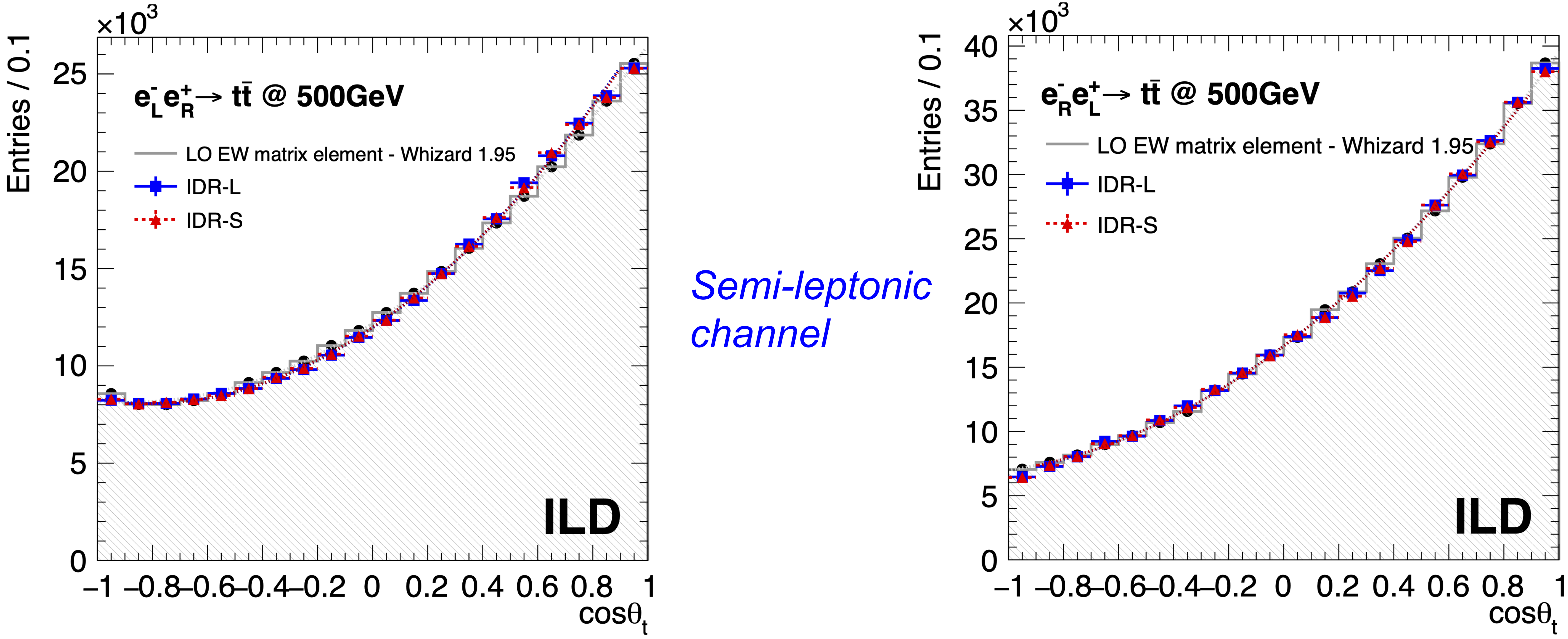


can provide 5σ evidence for scale evolution ("running") of the top quark MSR mass from ILC500 data alone



leetingf

Top quark polar angle spectrum at 500 GeV



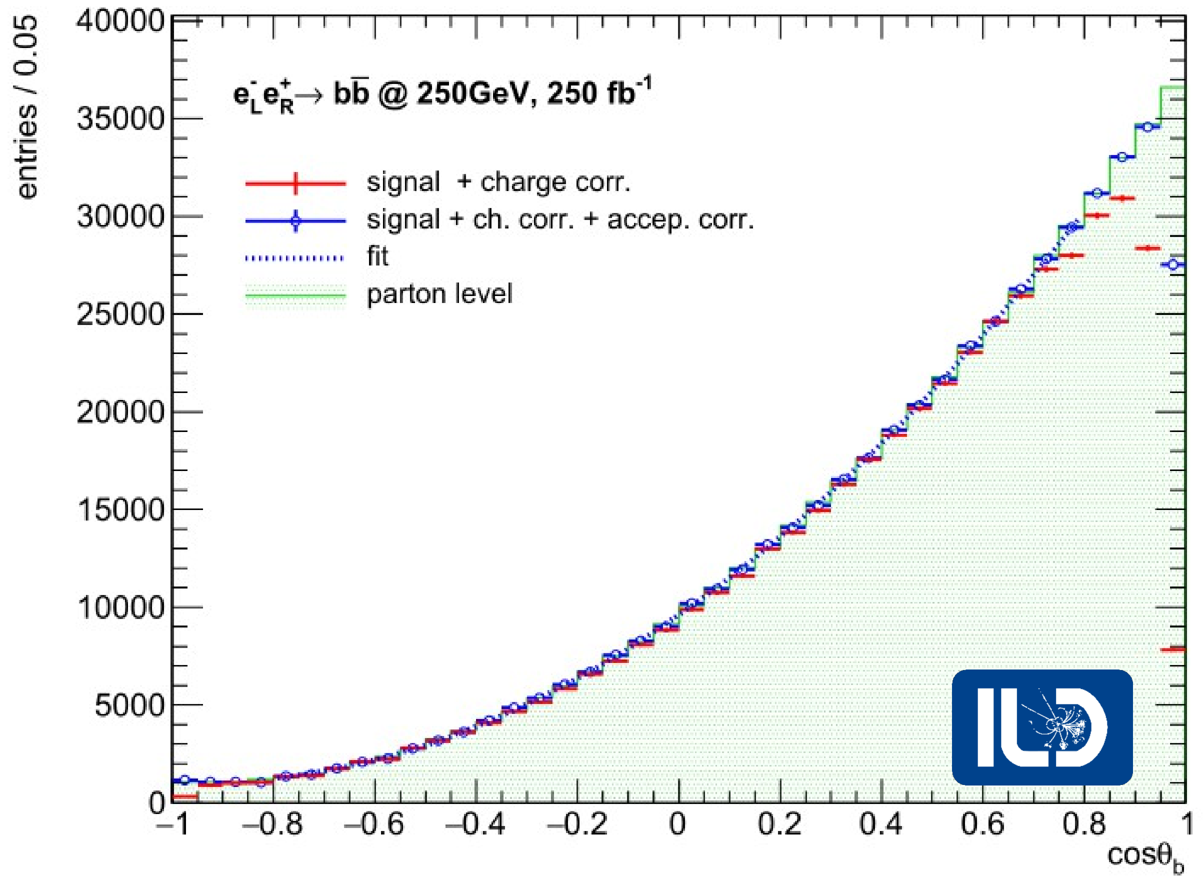
Semi-leptonic channel

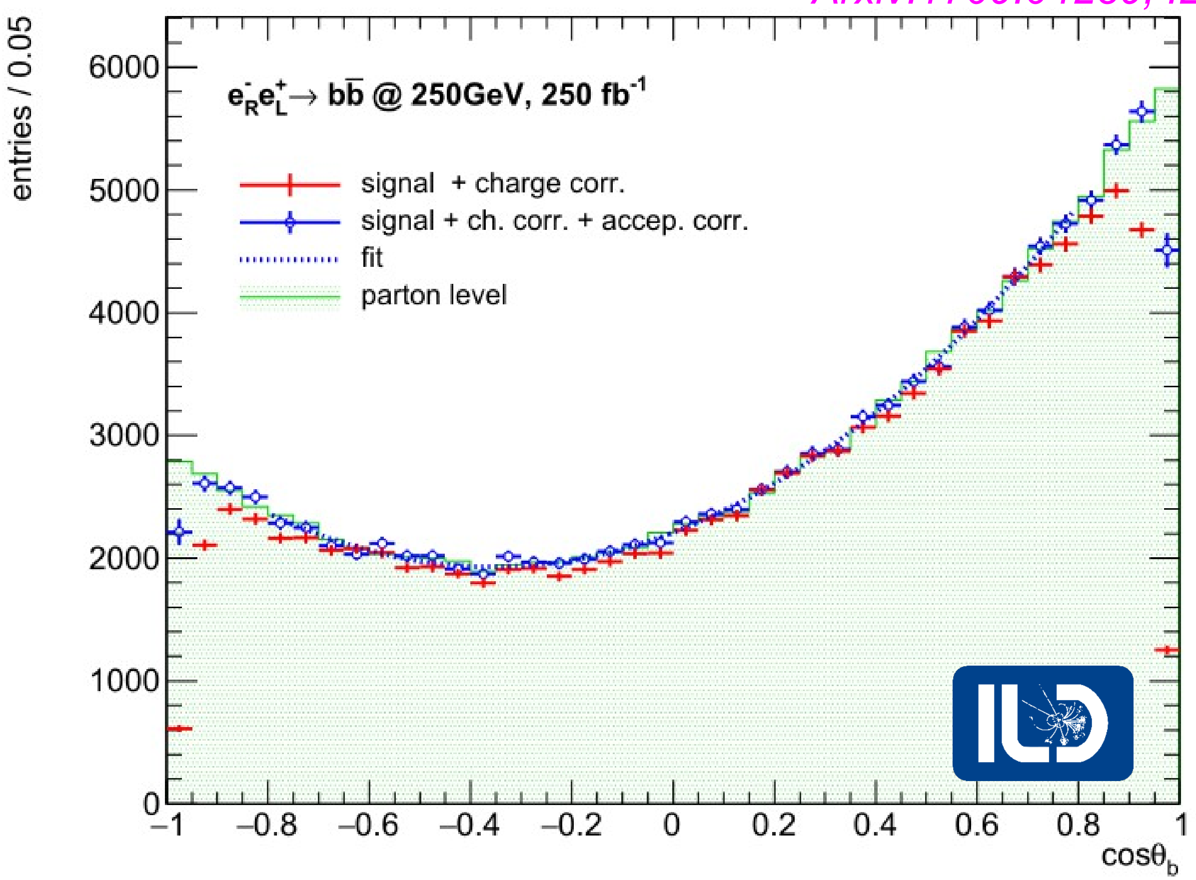
ILD-Note-2019-007

- Integrated Luminosity 4 fb^{-1}
- Exact reproduction of generated spectra
- Statistical precision on cross section: ~0.1%
- Statistical precision on A_{FB}: ~0.5%
  - Can expect that systematic errors will match statistical precision (but needs to be shown)

Decomposing ee→bb – Differential cross section

Arxiv:1709.04289, ILD Paper in progress



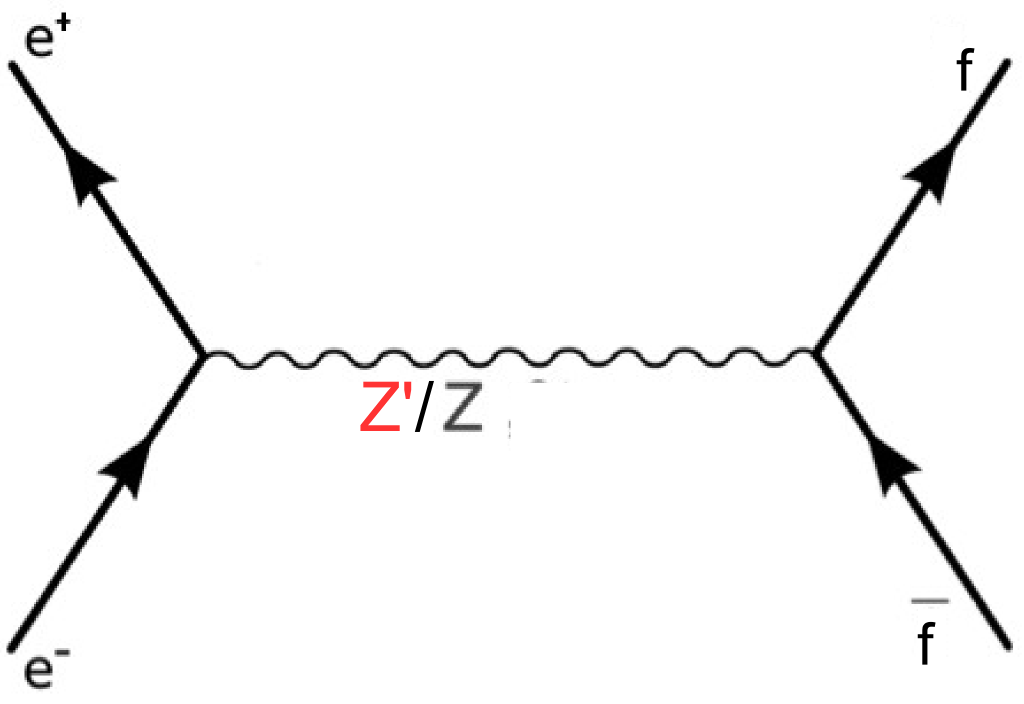


- Full simulation study (with ILD concept), Benchmark reaction
- Long lever arm in cos θ_b to extract from factors or couplings

- Note that the precision will reach the per-mill level -> requires full control over detector performance
- Background can be reduced to a negligible level (see backup) but requires careful treatment of e.g. radiative return events
  - Discussion of all experimental aspects deserves dedicated talk!!!

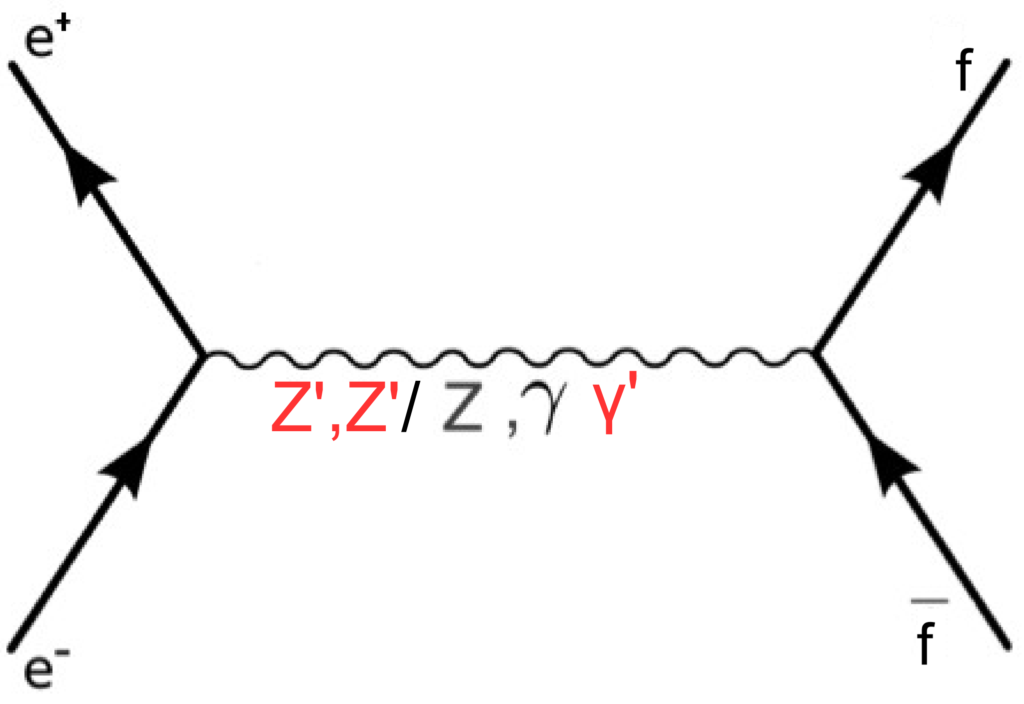
How can the Z-Pole help?

On the Z-pole



- Sensitivity to Z/Z' mixing
- Sensitivity to vector (and tensor) couplings of the Z
  - the photon does not "disturb"

Above the Z-pole

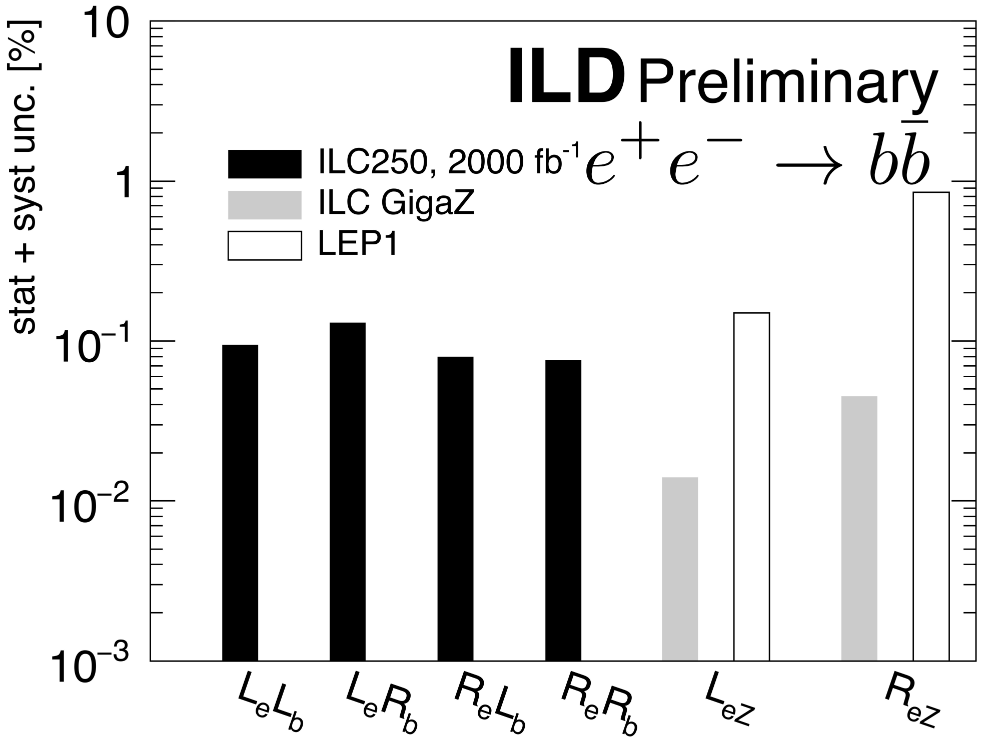


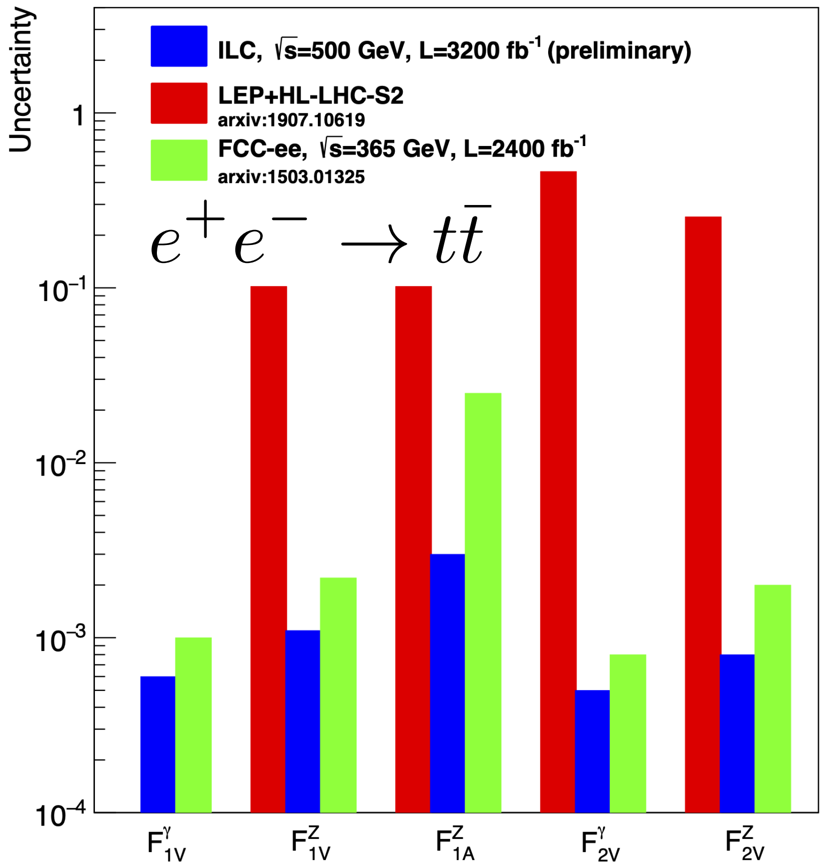
- Sensitivity to interference effects of Z and photon!!
- Measured couplings of photon and Z can be influenced by new physics effects
- Interpretation of result is greatly supported by precise input from Z pole

More on ILC GigaZ Program in EF04 Meeting on Friday 19/6/20

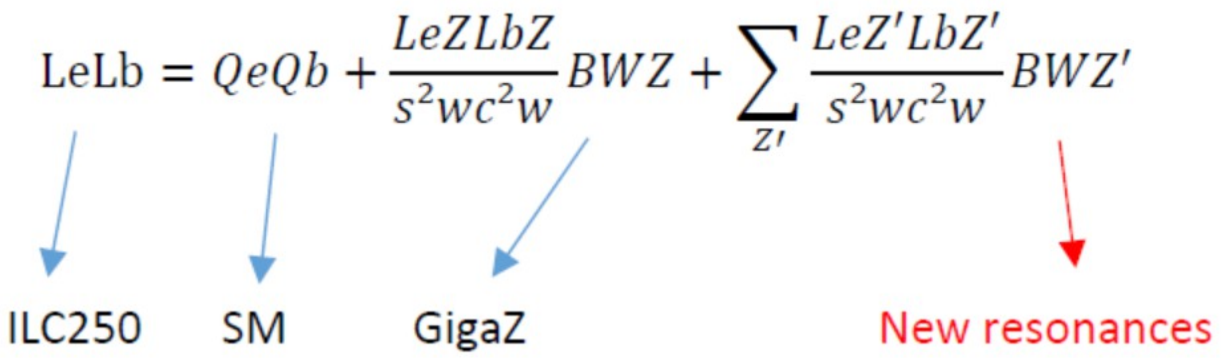
Precision on electroweak form factors and couplings

Arxiv:1709.04289, ILD Paper in progress





Couplings are order of magnitude better than at LEP

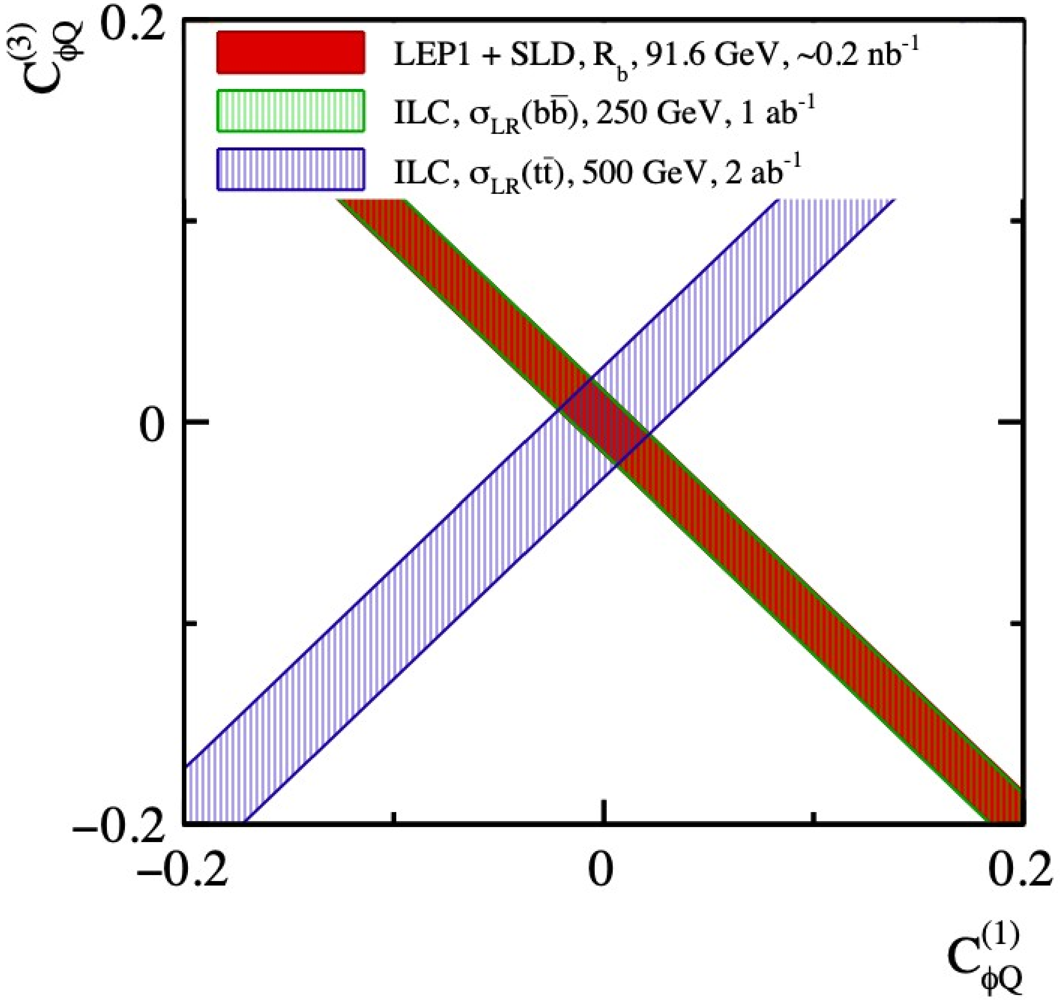


- e+e- collider way superior to LHC (√s = 14 TeV)
- Final state analysis at FCCee
  - Also possible at LC => Redundancy
- Two remarks:
  - 500 GeV is nicely away from QCD Matching regime
    - Less systematic uncertainties
  - Axial form factors are ~ and benefit therefore from higher energies

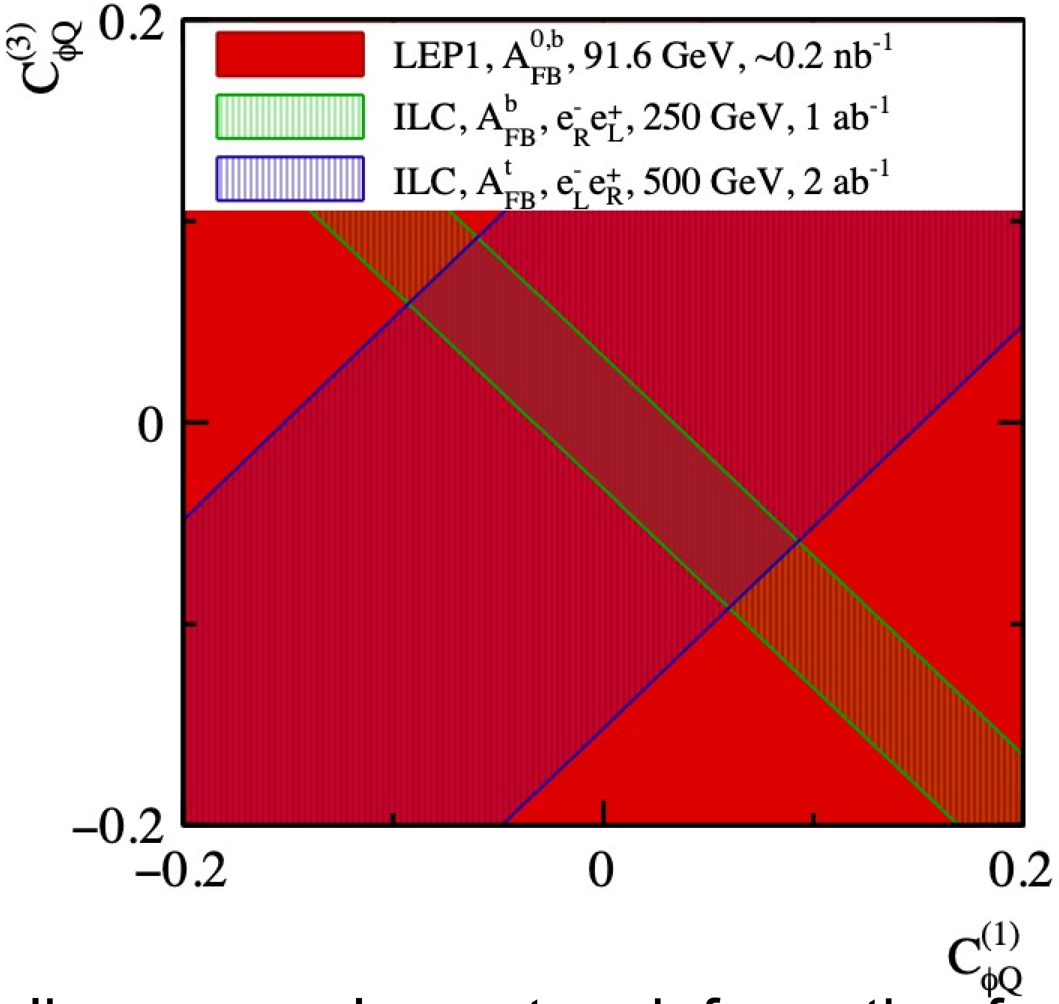
Full disentangling of helicity structure for all fermions only possible with polarised beams!!

Interplay b/t

From cross section



From forward-backward asymmetry

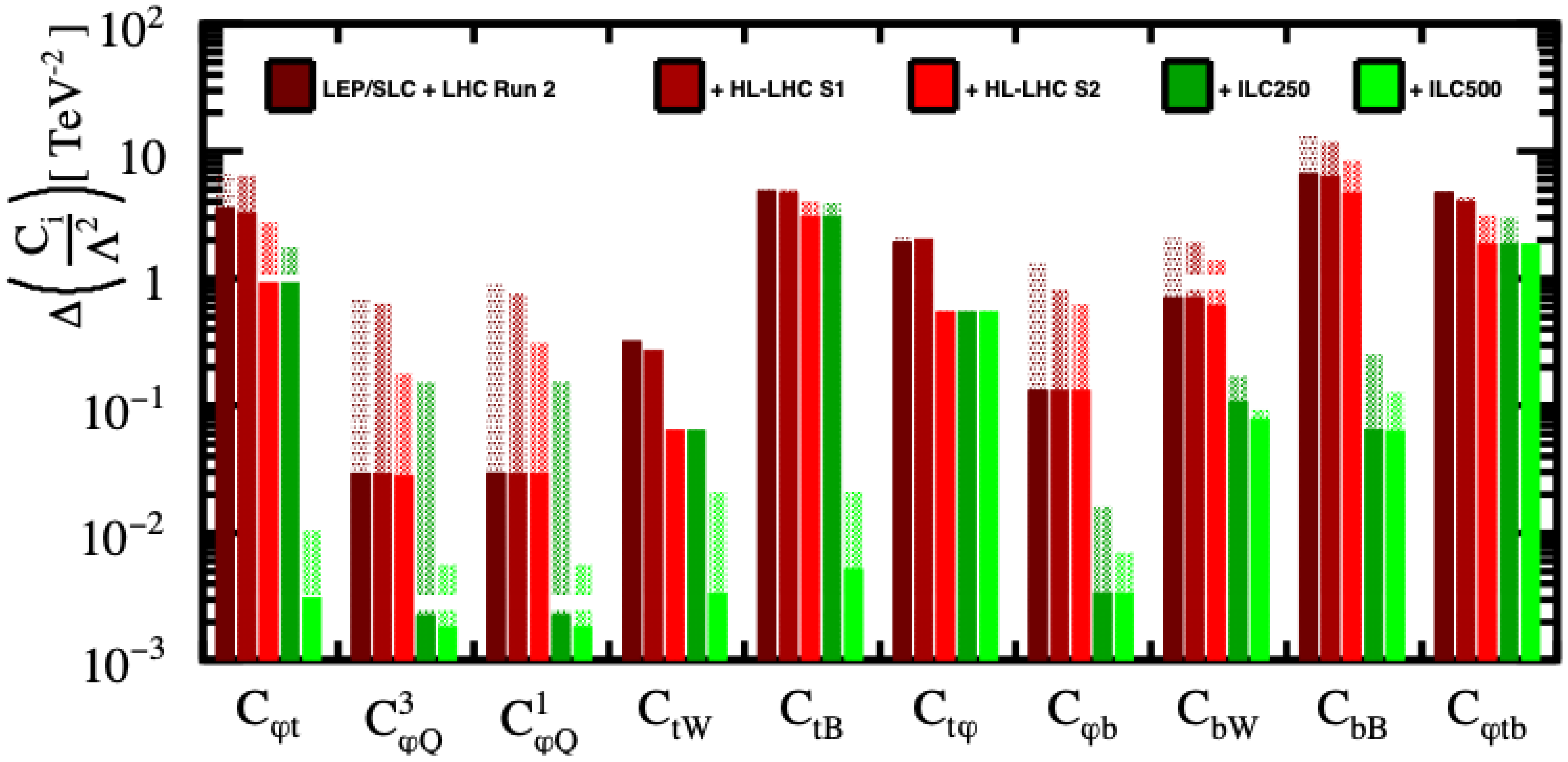


- Measurement of bottom and top observables delivers complementary information for EFT operators
- ILC@250 GeV comparable to LEP in terms of cross section => Constrain on g_{Lb}
- ILC@250 GeV drastically better than LEP in terms of AFB => Constrain on g_{Rb}
  - How would the picture look with GigaZ precisions?

arxiv:1907.10619

Electroweak top couplings EFT-operators

arxiv:1907.10619



Mapping between FF and EFT Coefficients

$$F^Z_{1V} = \frac{\frac{1}{4} - \frac{2}{3}s^2_W}{s_W c_W} - \frac{m_t^2}{\Lambda^2}\frac{1}{2 s_W c_W}\left[C^V_{\varphi q} = C^{(33)}_{\varphi u} + (C^{1(33)}_{\varphi q} - C^{3(33)}_{\varphi q})\right],$$

$$F^Z_{1A} = \frac{-\frac{1}{4}}{s_W c_W} - \frac{m_t^2}{\Lambda^2}\frac{1}{2 s_W c_W}\left[C^A_{\varphi q} = C^{(33)}_{\varphi u} - (C^{1(33)}_{\varphi q} - C^{3(33)}_{\varphi q})\right],$$

$$F^Z_{2V} = 4\frac{m_t^2}{\Lambda^2}\left[C^R_{uZ} = \mathrm{Re}\{c^2_W C^{(33)}_{uW} - s^2_W C^{(33)}_{uB}\}/s_W c_W\right],$$

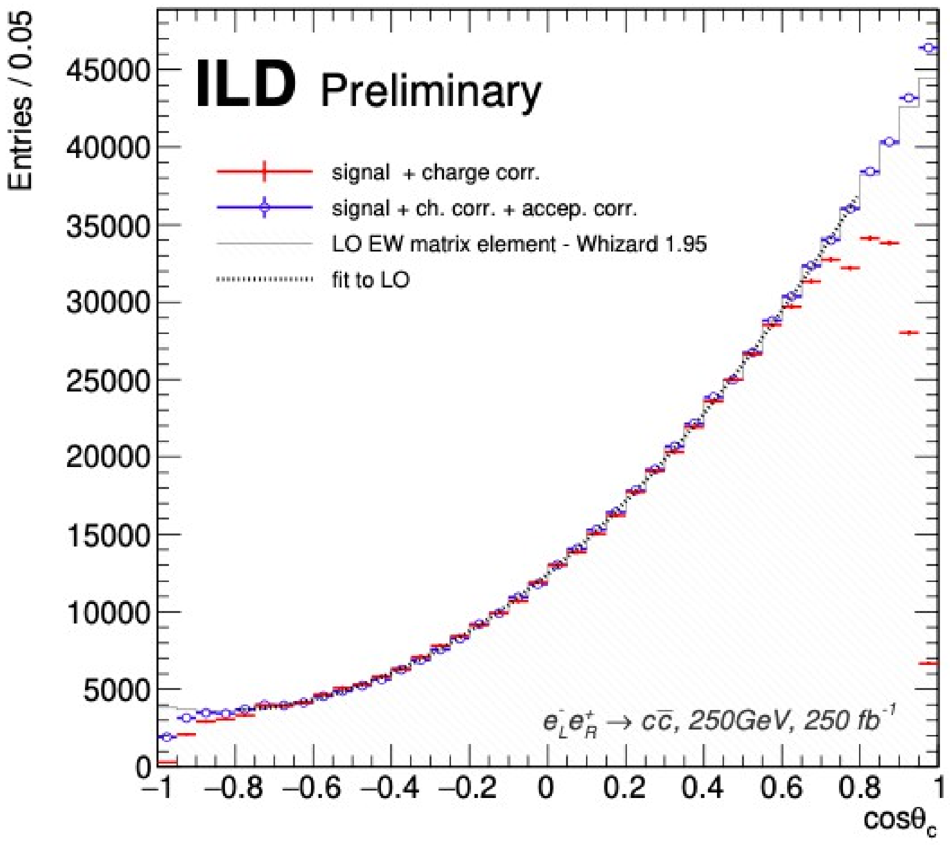
$$F^Z_{2A} = 4\frac{m_t^2}{\Lambda^2} i \left[C^I_{uZ} = \mathrm{Im}\{c^2_W C^{(33)}_{uW} - s^2_W C^{(33)}_{uB}\}/s_W c_W\right],$$

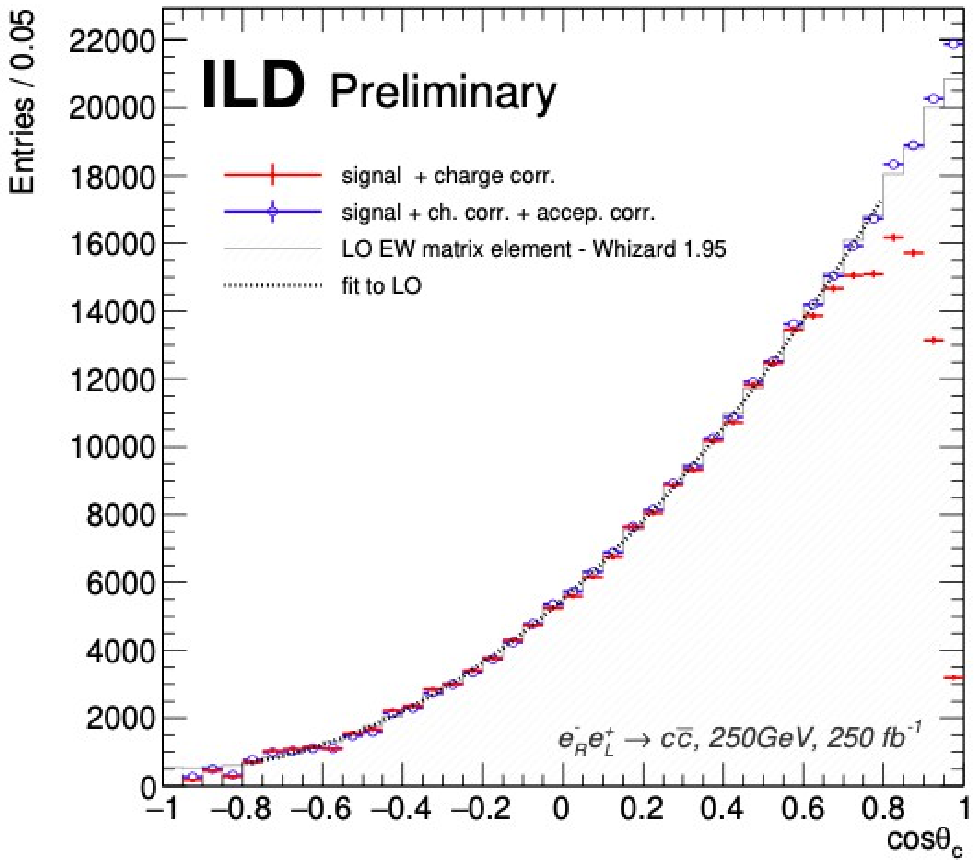
arxiv:1807.02121

- Translation of results into EFT language confirm superiority of e+e- w.r.t. LHC
- Several operators benefit already from 250 GeV running
- Top specific operators constrained by running at 500 GeV

What about lighter quarks – Differential cross section ee->cc

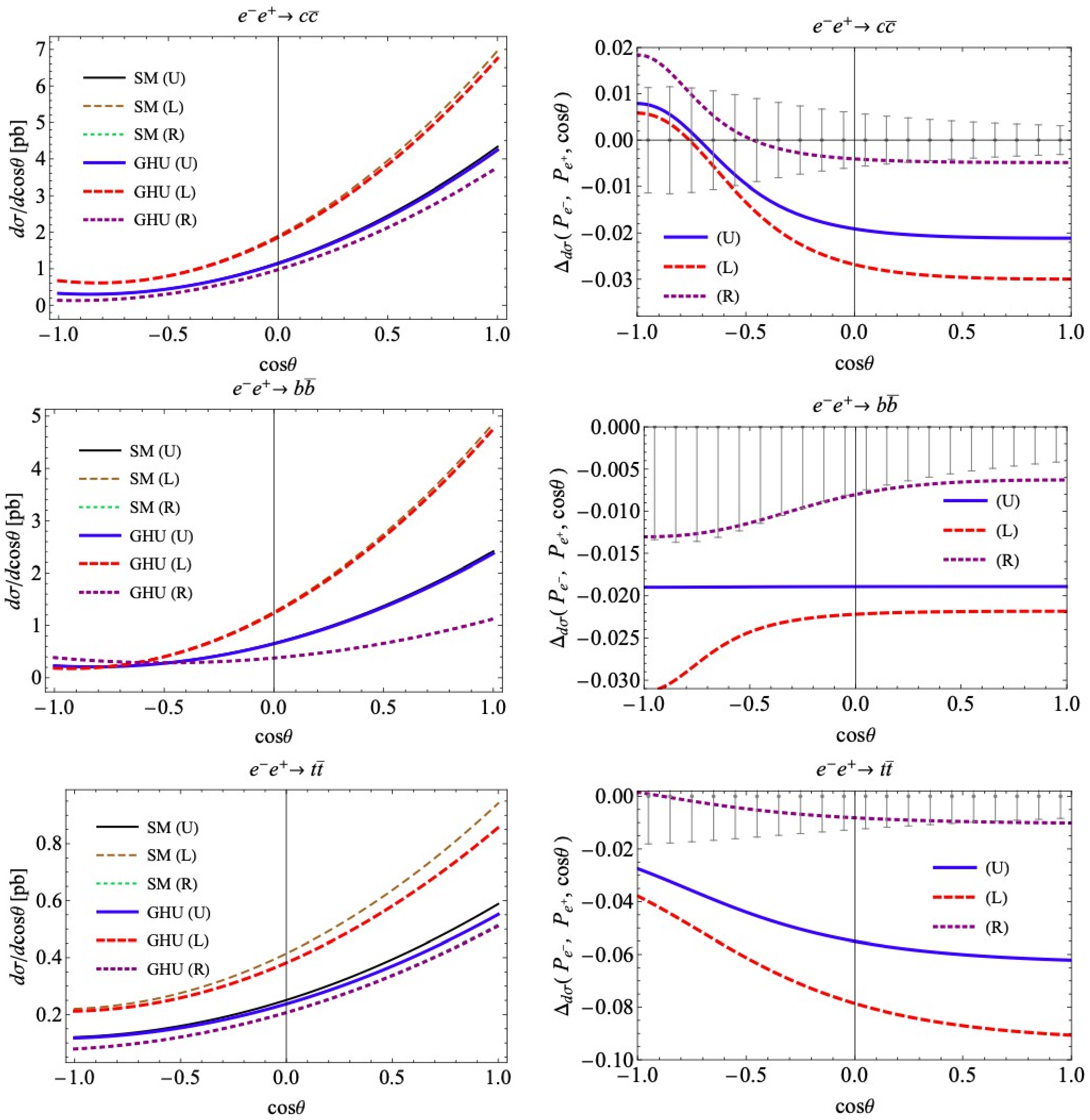
arxiv:2002.05805





Full simulation study (with ILD concept)
Long lever arm in cos θ_c to extract from factors or couplings

Why lighter quarks? – e.g. GUT Inspired Grand Higgs Unification Model



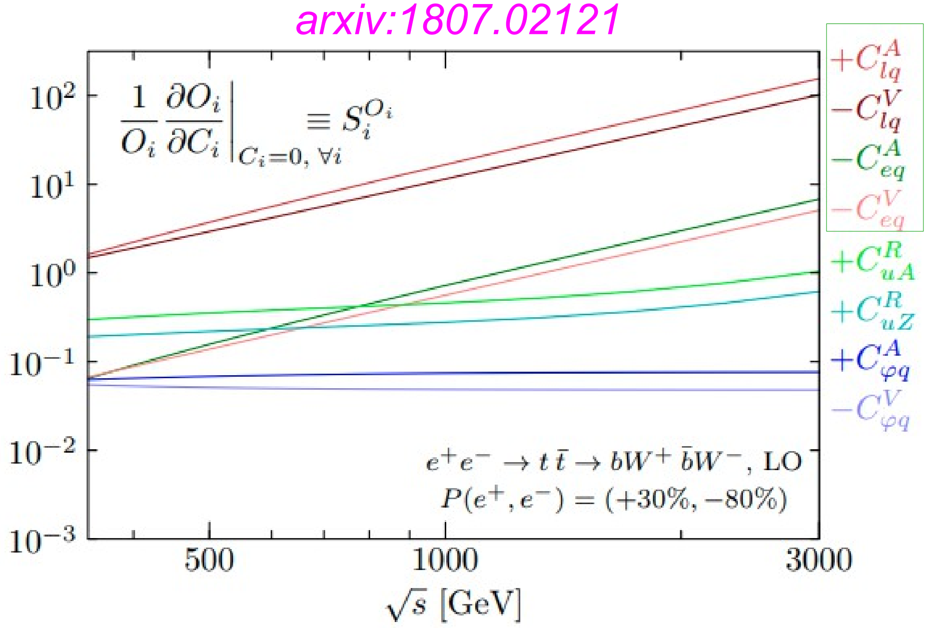
arxiv:2006.02157

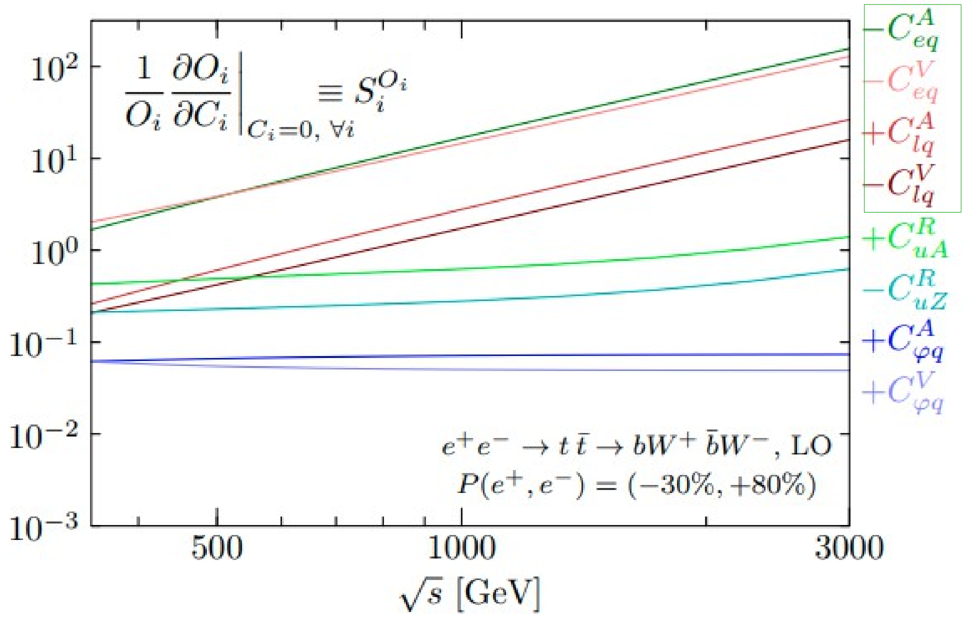
- Model parameter is Hosotani angle θ_H yielding the Higgs-Potential as consequence of Aharanov-Bohm Phase in 5th dimension
- Model defined in Randall-Sundrum warped extra dimensions
  - KK excitations of gauge bosons and new bosons modify fermion couplings
- Predictions for ILC
  - m_{KK} = 13 TeV and θ_H = 0.1
- Deviations from SM of the order of a few %
  - Effects measurable already at 250 GeV
  - Effects amplified by beam polarisations
  - Effects for tt, bb and cc (and other light fermions)
- One concrete example for importance to measure full pattern of fermion couplings
  - Full pattern only available with beam polarisation

Effects at higher energies

Development of EFT Operators

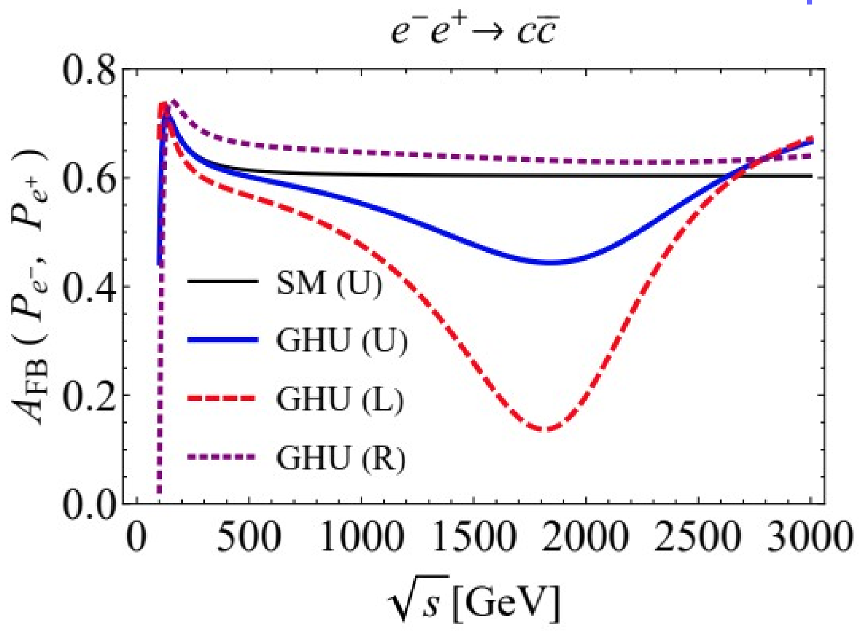
arxiv:1807.02121

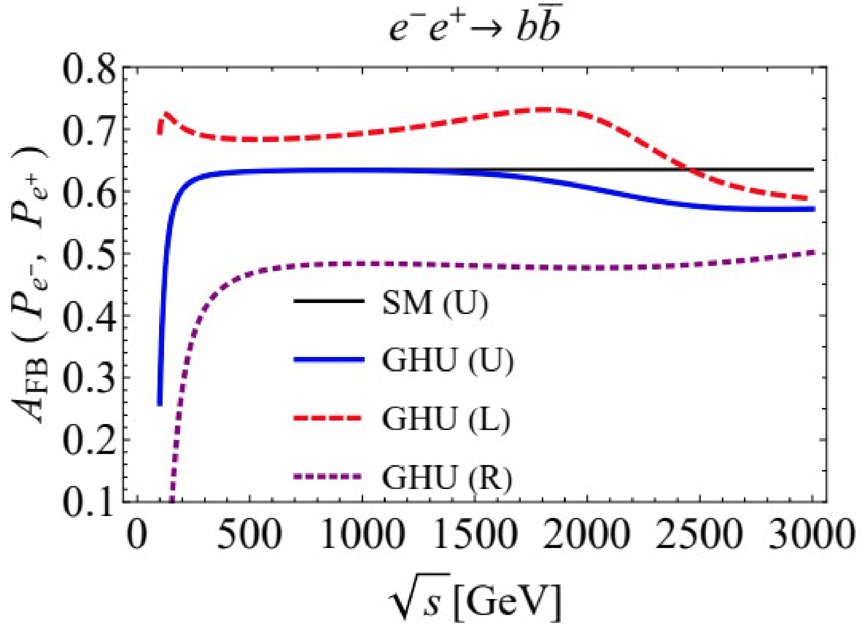


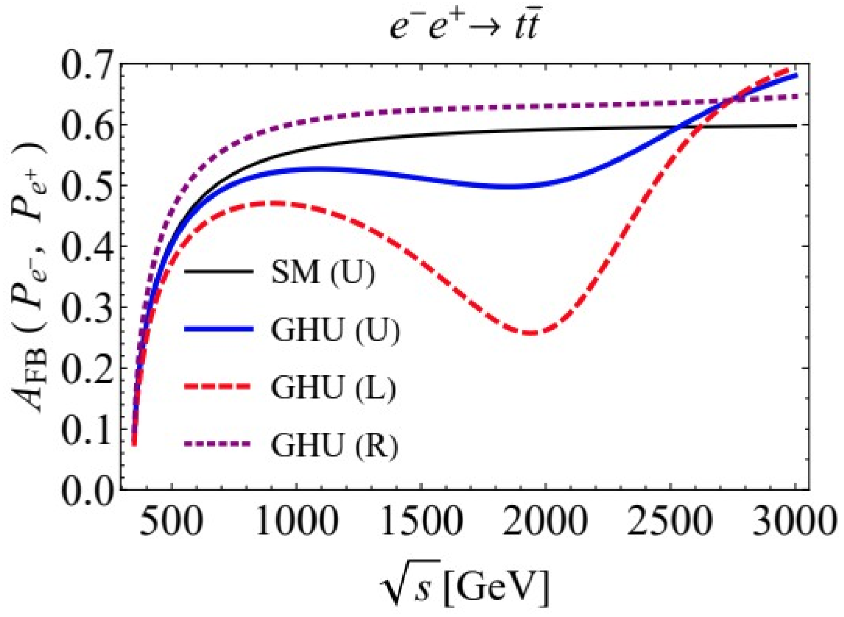


Increased sensitivity to operators representing four-fermion interactions

GUT Inspired GHU Model







- Effects amplified at higher energies
- Different patterns for different beam polarisations (L, U, R)
- Different patterns for different fermions

Summary and outlook

- Linear colliders are ideally suited for precision measurements of two-fermion final states

- Measurement of top mass to a precision of ~50 MeV in clean environment
  - Flexibility in energy allow for complementary methods
  - Threshold scan and radiative events
  - Watch our for new ideas later today

- Linear colliders will have the answer whether new physics acts on heavy doublet (t,b) only or on all fermions

- Will/would proble helicity structure of electroweak fermion couplings over at least one order of magnitude in energy (Z-Pole -> ~1 TeV)
  - Achievable experimental precisions ~0.1 - 1%
  - Effects may become already visible at 250 GeV stage for b quark and c quarks (and other light fermions)
  - Amplification of effects at higher energies
  - Clear and unique pattern thanks to polarised beams

- Active phenomenological studies in terms of global analyses (EFT) and concrete models

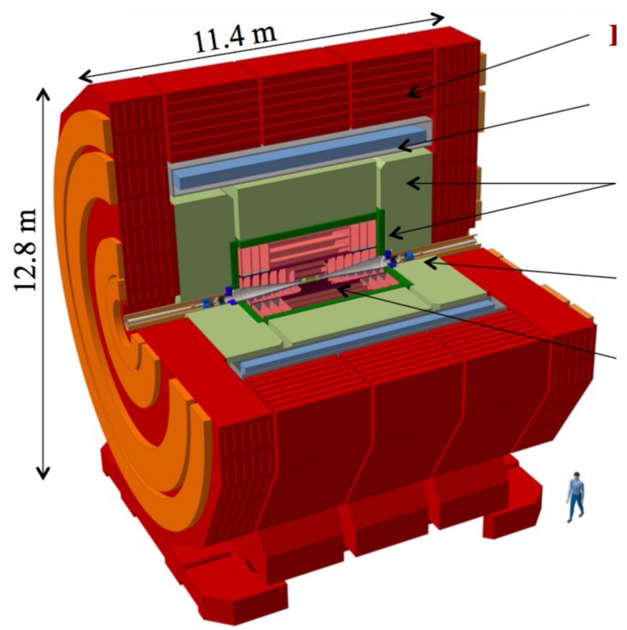
- Main challenge at future machines will be the control of systematic errors
  - Experimentally
    - Vertex charge and particle ID
    - PFO for final state jets
  - Theoretically (not discussed)
    - Need at least NLO electroweak predictions (and MC programs) for correct interpretation of results

Backup

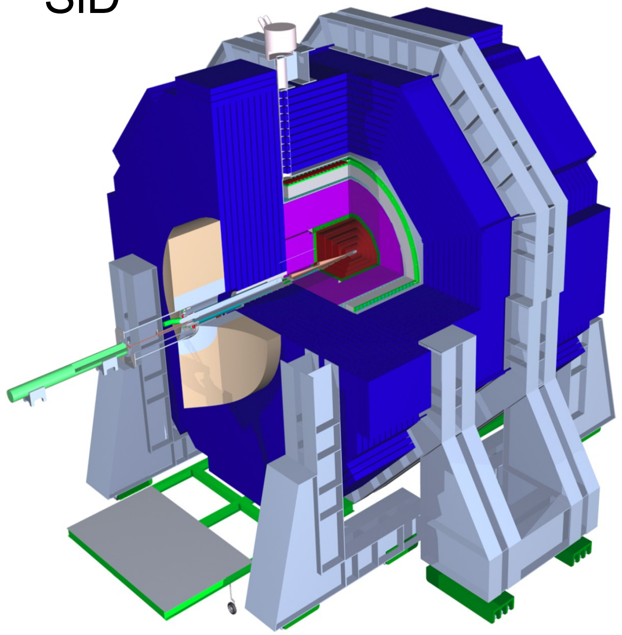
Detector requirements

e+e- detector concepts for linear colliders
Preferred solution Particle Flow Detectors

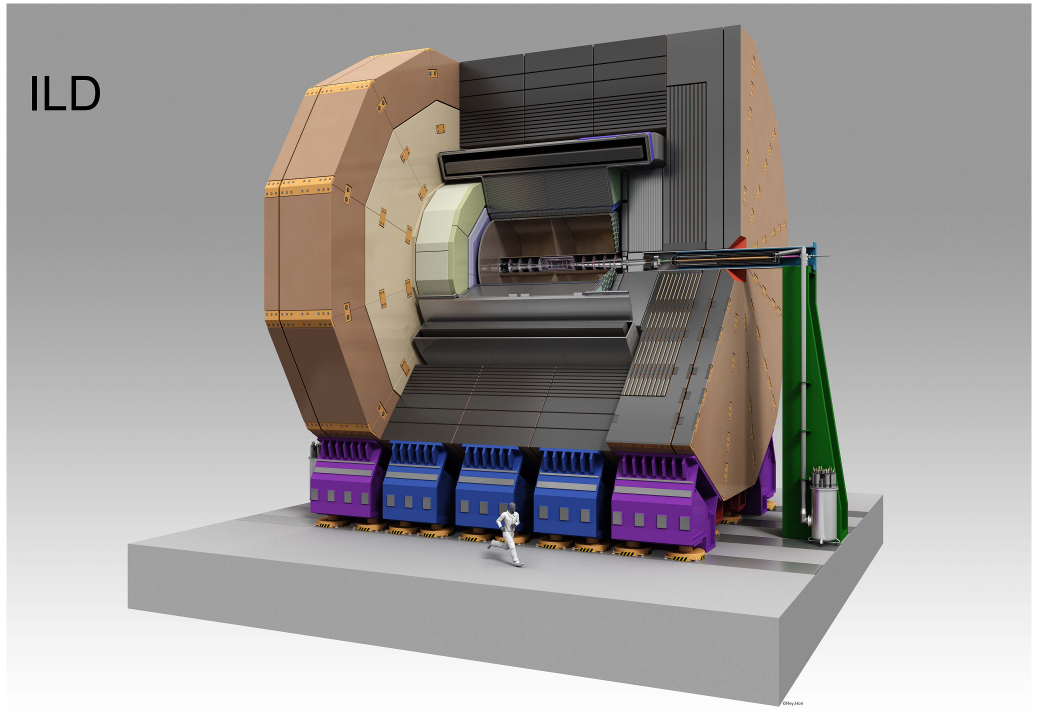
CLIC Detector



SiD



ILD



B= 4T

B= 5T

B= 3.5T

Highly granular calorimeters

Central tracking with silicon

Central tracking with TPC

Inner tracking with silicon

Detector requirements

Track momentum: $\sigma_{1/p}$ < 5 x 10^{-5}/GeV (1/10 x LEP)

(e.g. Measurement of Z boson mass in Higgs Recoil)

Impact parameter: $\sigma_{d0} < [5 \oplus 10/(p[\mathrm{GeV}]\sin^{3/2}\theta)]$ μm (1/3 x SLD)

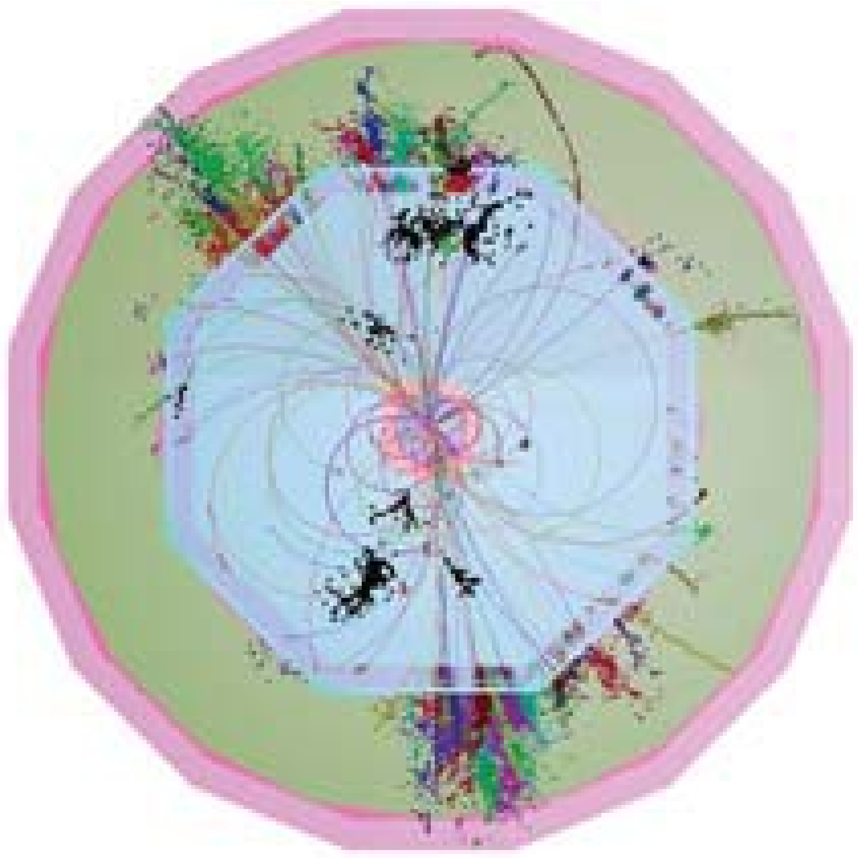
(Quark tagging c/b)

Jet energy resolution : $dE/E = 0.3/(E(\mathrm{GeV}))^{1/2}$ (1/2 x LEP)

(W/Z masses with jets)

Hermeticity : θ_{min} = 5 mrad

(for events with missing energy e.g. SUSY)



Final state will comprise events with a large number of charged tracks and jets(6+)

- High granularity
- Excellent momentum measurement
- High separation power for particles

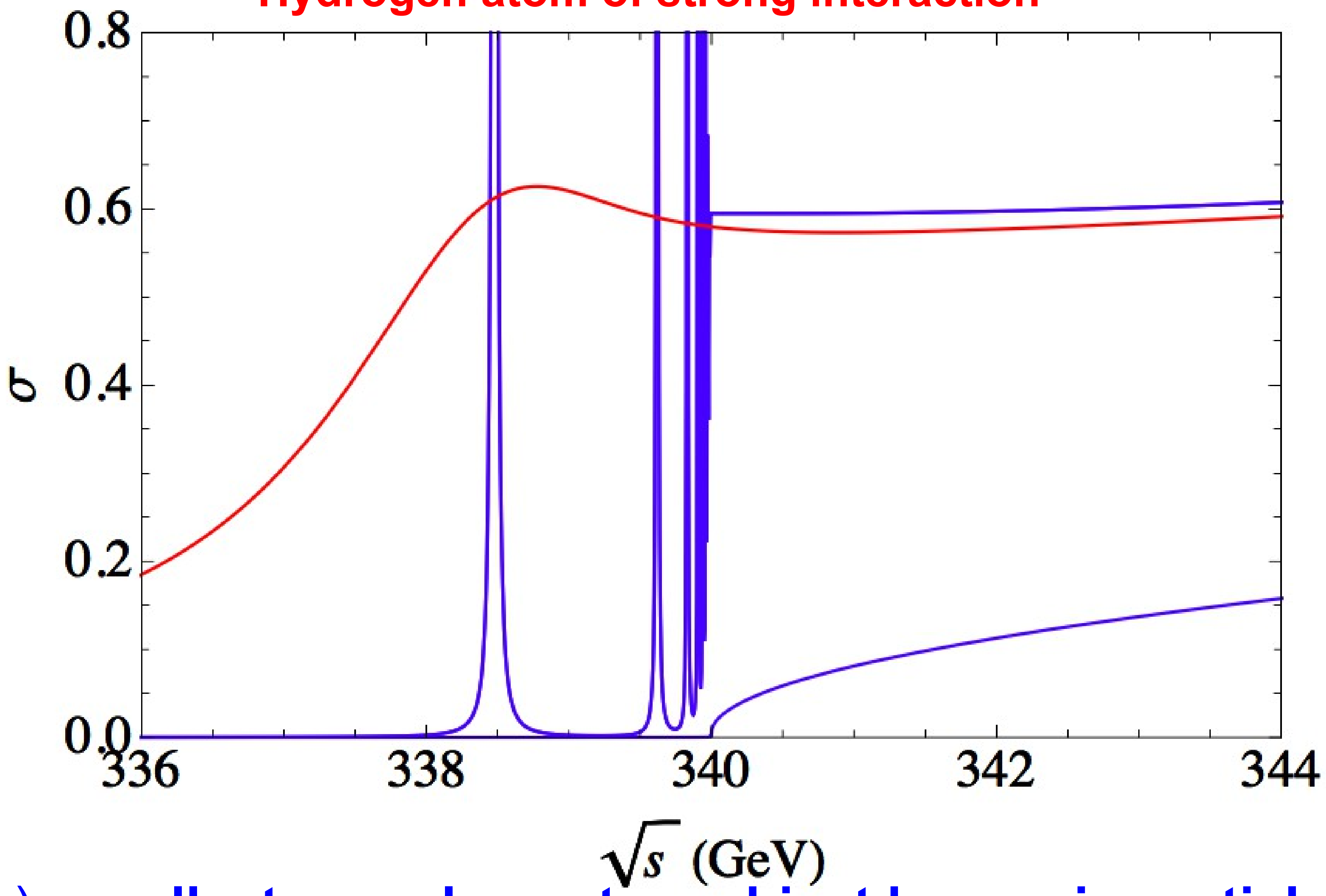
Particle Flow Detectors

Detector Concepts: ILD, SiD and CLICdp

Top pair production at threshold

"Bound states" at tt threshold

Hydrogen atom of strong interaction

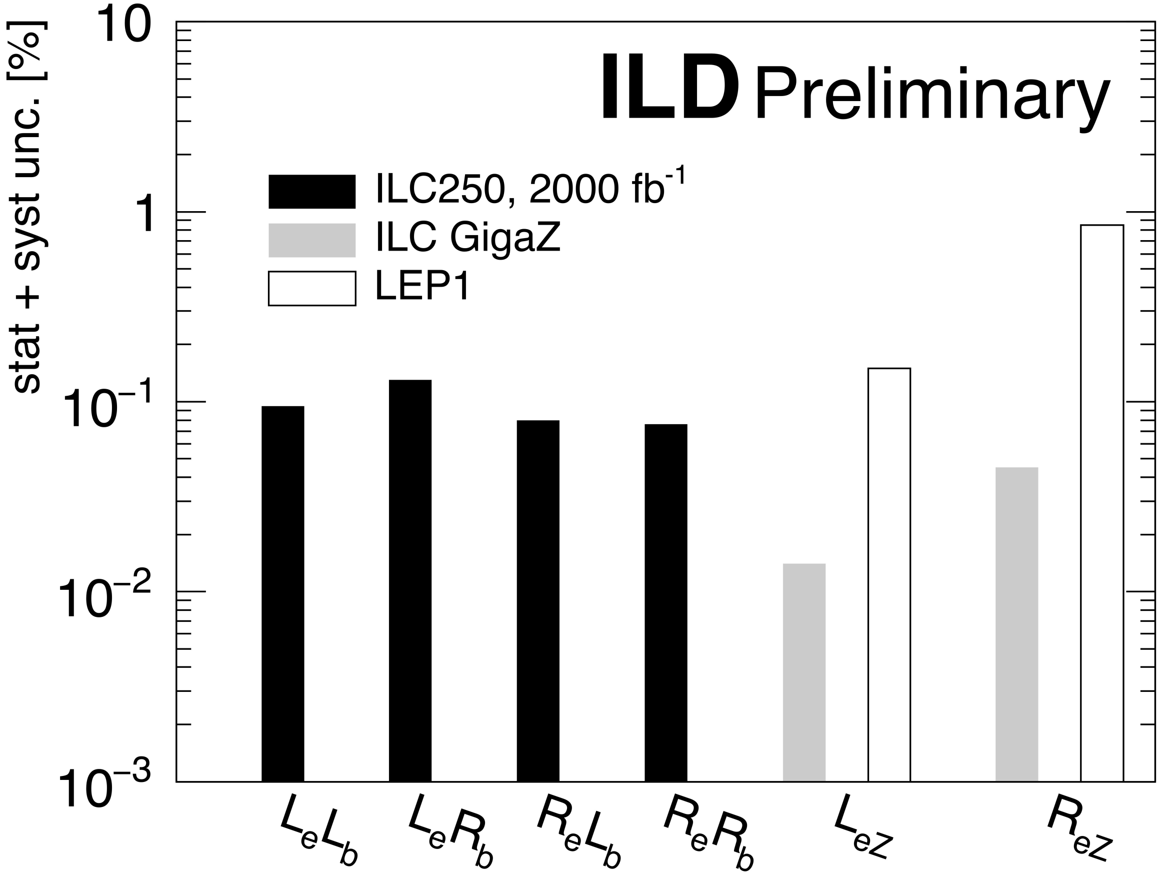


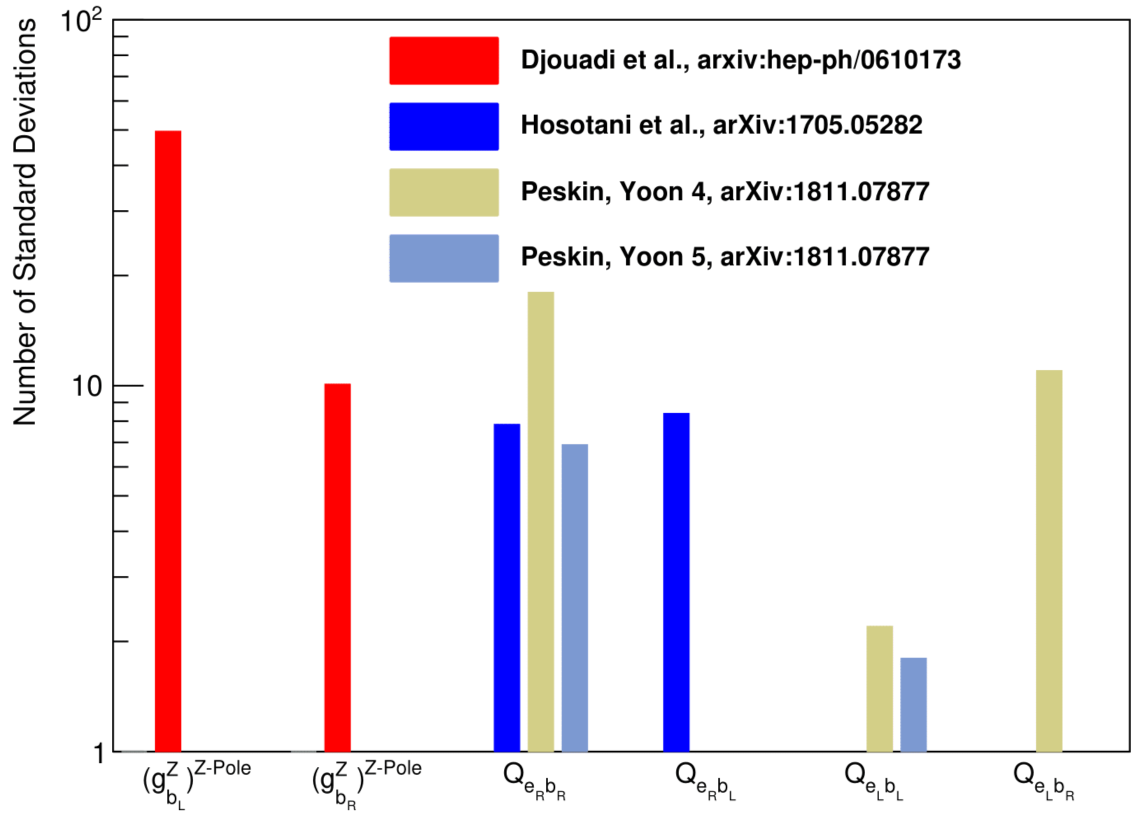
- Size O(10^{-17}m), **smallest non-elementary object known in particle physics**
  Small scale => Free of confinement effects => Ideal premise for precision calculations
  Measurement of (a hypothetical) 1^3S_1 State

- Decay of top quark smears out resonances in a well defined way

Precision on couplings and helicity amplitudes and physics reach

Example b-couplings (same observation for c-couplings, arxiv:2002.05805)





Couplings are order of magnitude better than at LEP

- In particular right handed couplings are much better constrained

New physics can also influence the Zee vertex

- in 'non top-philic' models

Full disentangling of helicity structure for all fermions only possible with polarised beams!!

Impressive sensitivity to new physics in Randall Sundrum Models with warped extra dimensions

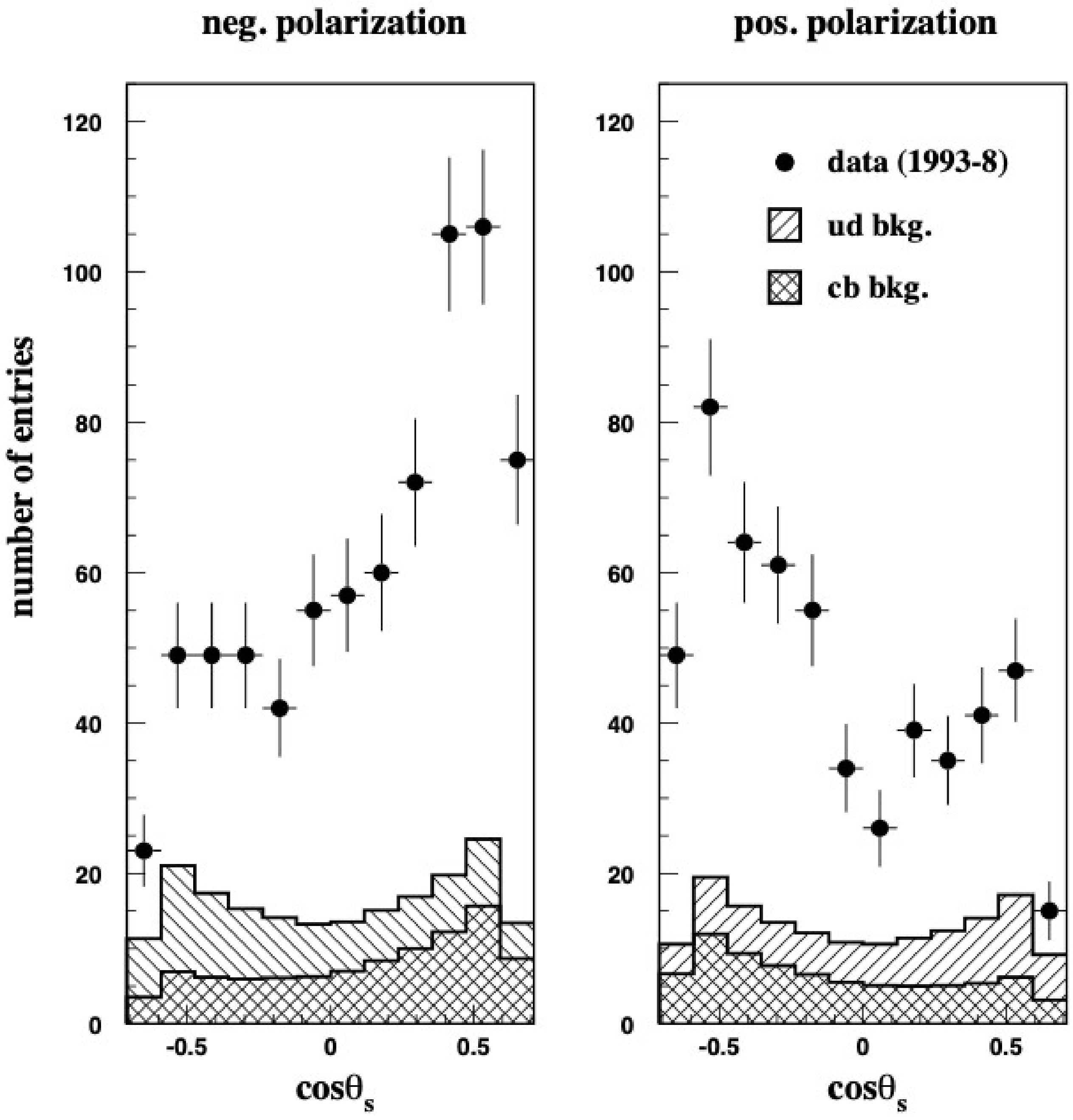
- **Complete tests only possible at LC**
- **Discovery reach O(10 TeV)@250 GeV and O(20 TeV)@500 GeV**

Pole measurements critical input

- Only poorly constrained by LEP

And tomorrow ?

ee -->ss: SLD Analysis at Z Pole



- Extend the heavy quark analyses to light quarks to get full picture
- Optimise vertexing and particle ID (i.e .Kaon ID with full simulation studies

Beam polarisation and disentangling

With two beam polarisation configurations

$P(e^-) = \pm 80\%$ $\qquad$ $P(e^+) = \mp 30\%$

There exist a number of observables sensitive to chiral structure, e.g.

$$\sigma_I$$

x-section

$$A^t_{FB,I} = \frac{N(\cos\theta > 0) - N(\cos\theta < 0)}{N(\cos\theta > 0) + N(\cos\theta < 0)}$$

Forward backward asymmetry

$$(F_R)_I = \frac{(\sigma_{t_R})_I}{\sigma_I}$$

Fraction of right handed top quarks

⇩

Extraction of relevant unknowns

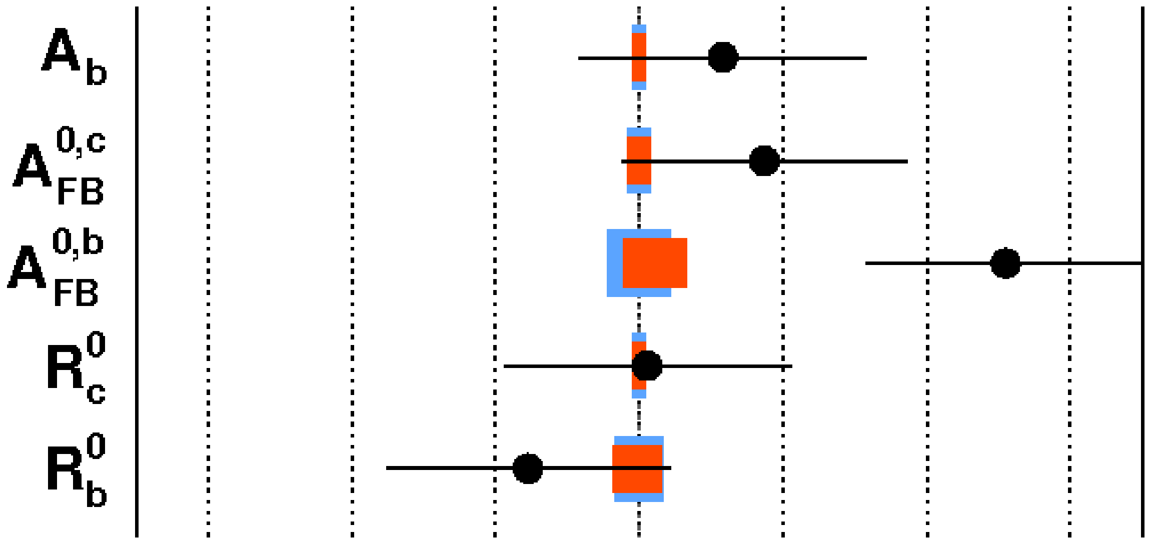
$$F^\gamma_{1V},\ F^Z_{1V},\ F^\gamma_{1A} = 0,\ F^Z_{1A}$$
$$F^\gamma_{2V},\ F^Z_{2V}$$

or equivalently

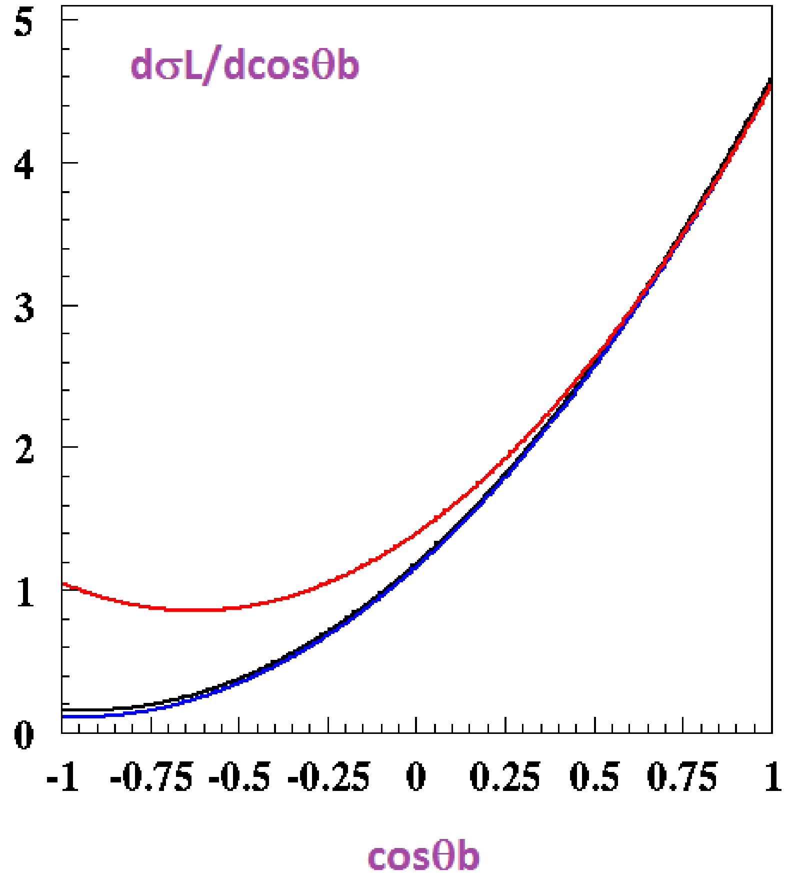
$$g^\gamma_L,\ \ g^\gamma_R,\ \ g^Z_L,\ \ g^Z_R$$

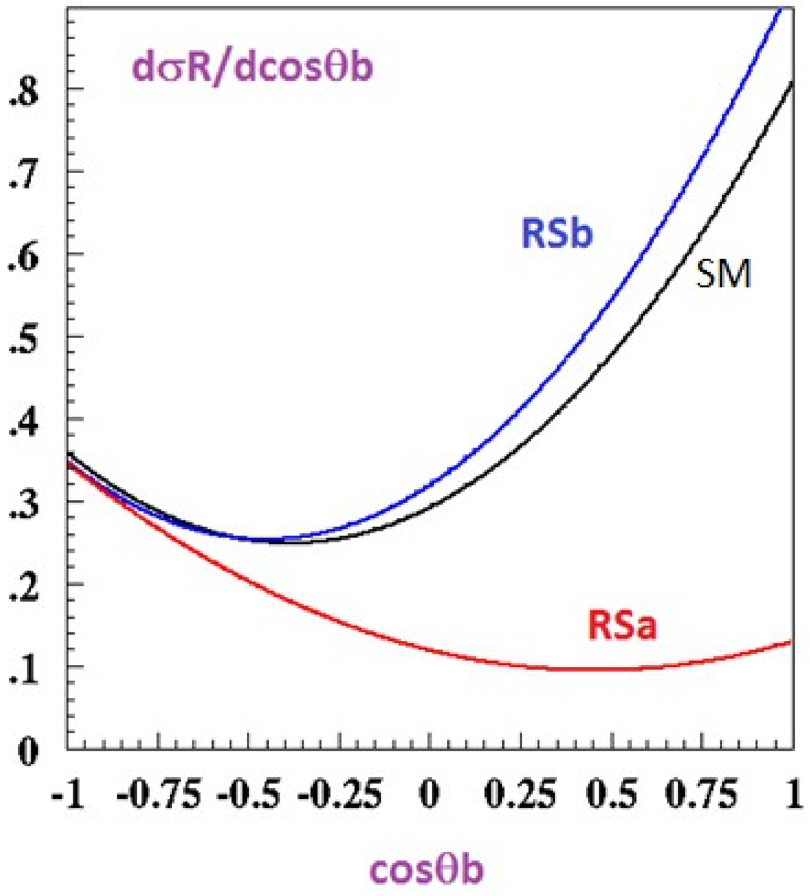
LEP Anomaly on A_{FB}^b

~3σ in heavy quark observable A_{FB}^b



ee->bb@250 GeV





Randall Sundrum Models Djouadi/Richard '06

- Is tension due to underestimation of errors or due to new physics?
- High precision e+e- collider will give final word on anomaly
- In case it will persist polarised beams will allow for discrimination between effects on left and right handed couplings (Remember $Zb_l b_l$ is protected by cross section)
- Note that also B-Factories report on anomalies

Why this luxury?

Beam spot size

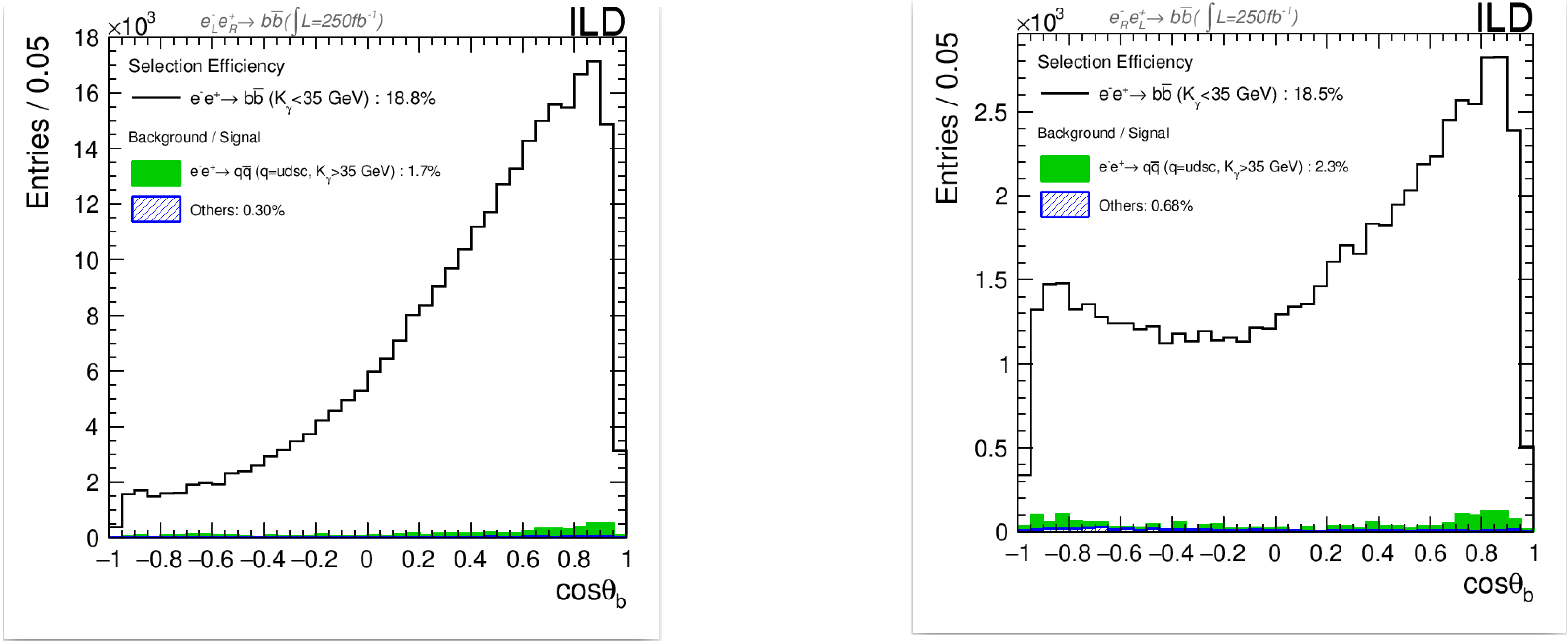
	FCCee	ILC	SLC	LEP
σ_x[nm]	13700	516	1500	200000
σ_y[nm]	36	7.7	500	2500

Source SLC, LEP, PDG

LEP >> SLC >> ILC

ee-->bb – Signal and background

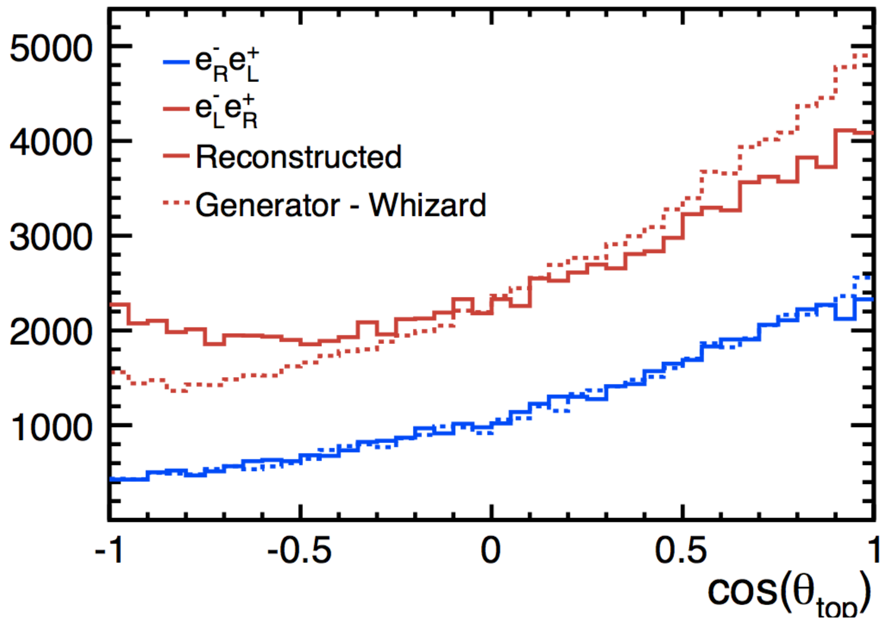
Arxiv:1709.04289, ILD Paper in progress



- Background levels can be kept at very small level
- However, these type of analyses seek per-mille level precision

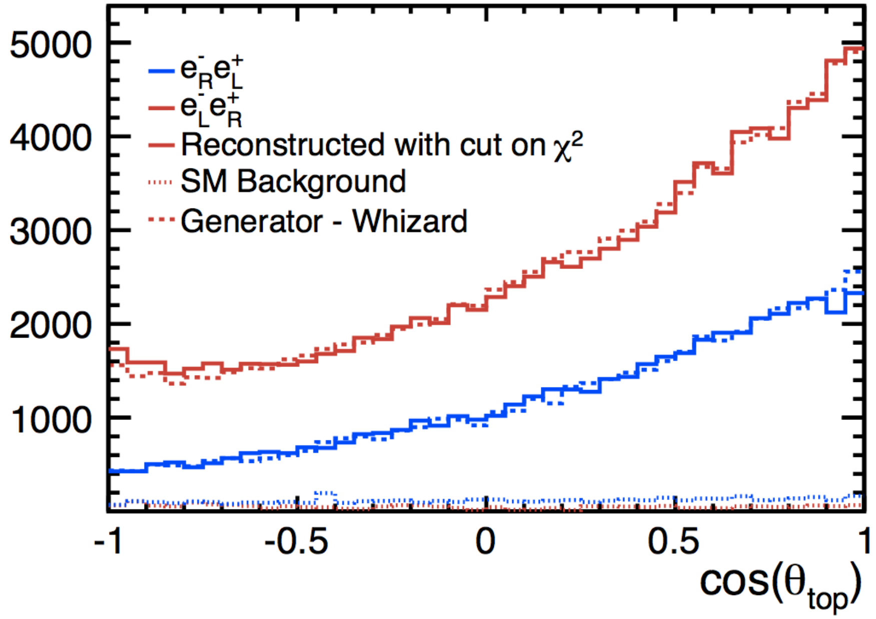
ee-->bb – Signal and background

arxiv:1505.06020



Ambiguities in case of
left handed electron beams
Due to V-A structure at ttX vertex

Precise reconstruction of θ_{top}
in case of right handed electron beams



Remedy to address ambiguities:
Select cleanly reconstructed events by χ^2 analysis
or
Reconstruction of b quark charge

Precise reconstruction for both beam polarisations

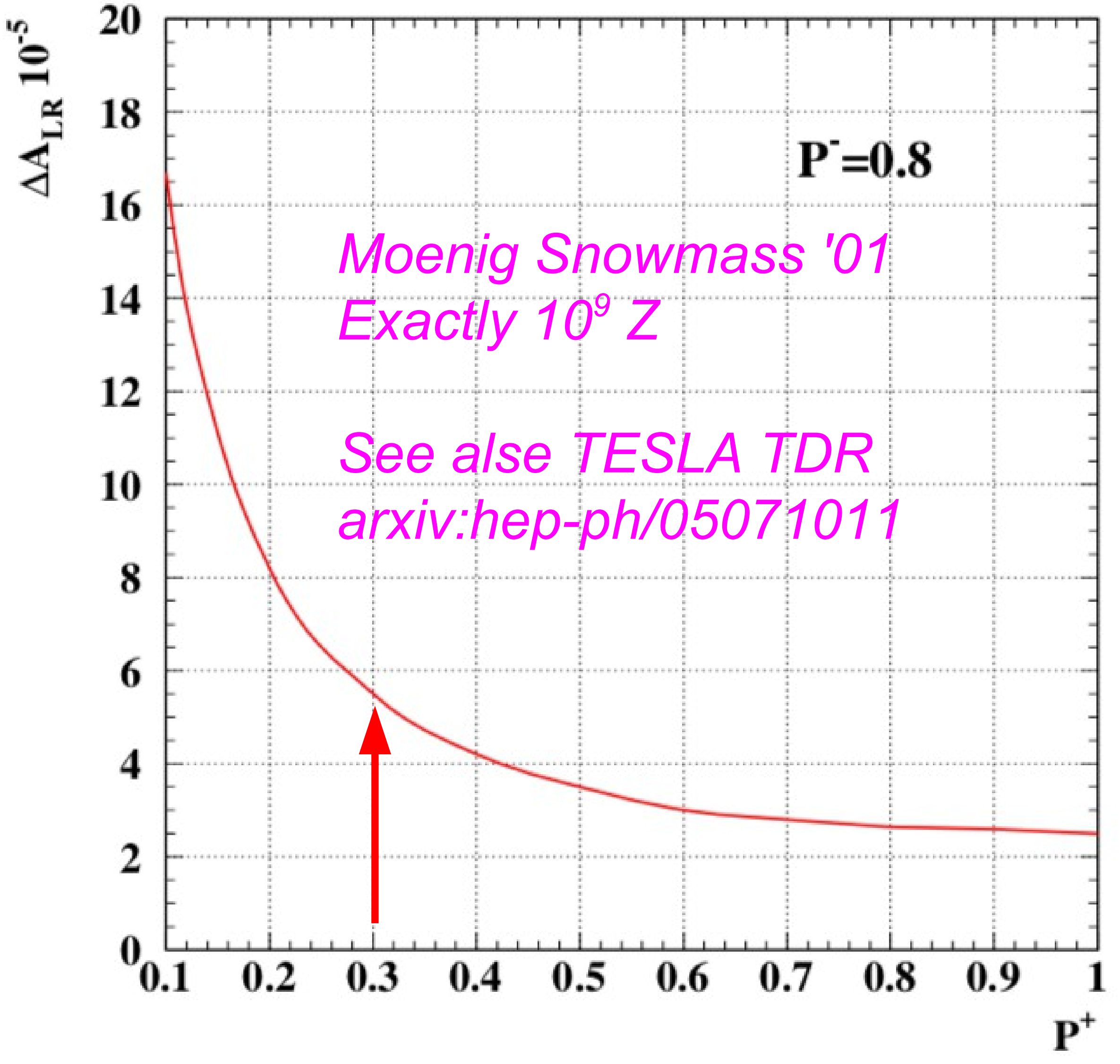
- Efficiency Penalty for e_L
- ε_{tot}: e_R ~ 50%, e_L ~ 30%

Results:

$\mathcal{P}_{e^-}, \mathcal{P}_{e^+}$	$(\delta\sigma/\sigma)_{stat.}$ [%]	$(\delta A^t_{FB}/A^t_{FB})_{stat.}$ [%]
$-0.8, +0.3$	0.47	1.8
$+0.8, -0.3$	0.63	1.3

A_{LR} at GigaZ?

Blondel scheme: $$A_{\rm LR} = \sqrt{\frac{(\sigma_{++}+\sigma_{-+}-\sigma_{+-}-\sigma_{--})(-\sigma_{++}+\sigma_{-+}-\sigma_{+-}+\sigma_{--})}{(\sigma_{++}+\sigma_{-+}+\sigma_{+-}+\sigma_{--})(-\sigma_{++}+\sigma_{-+}+\sigma_{+-}-\sigma_{--})}}$$



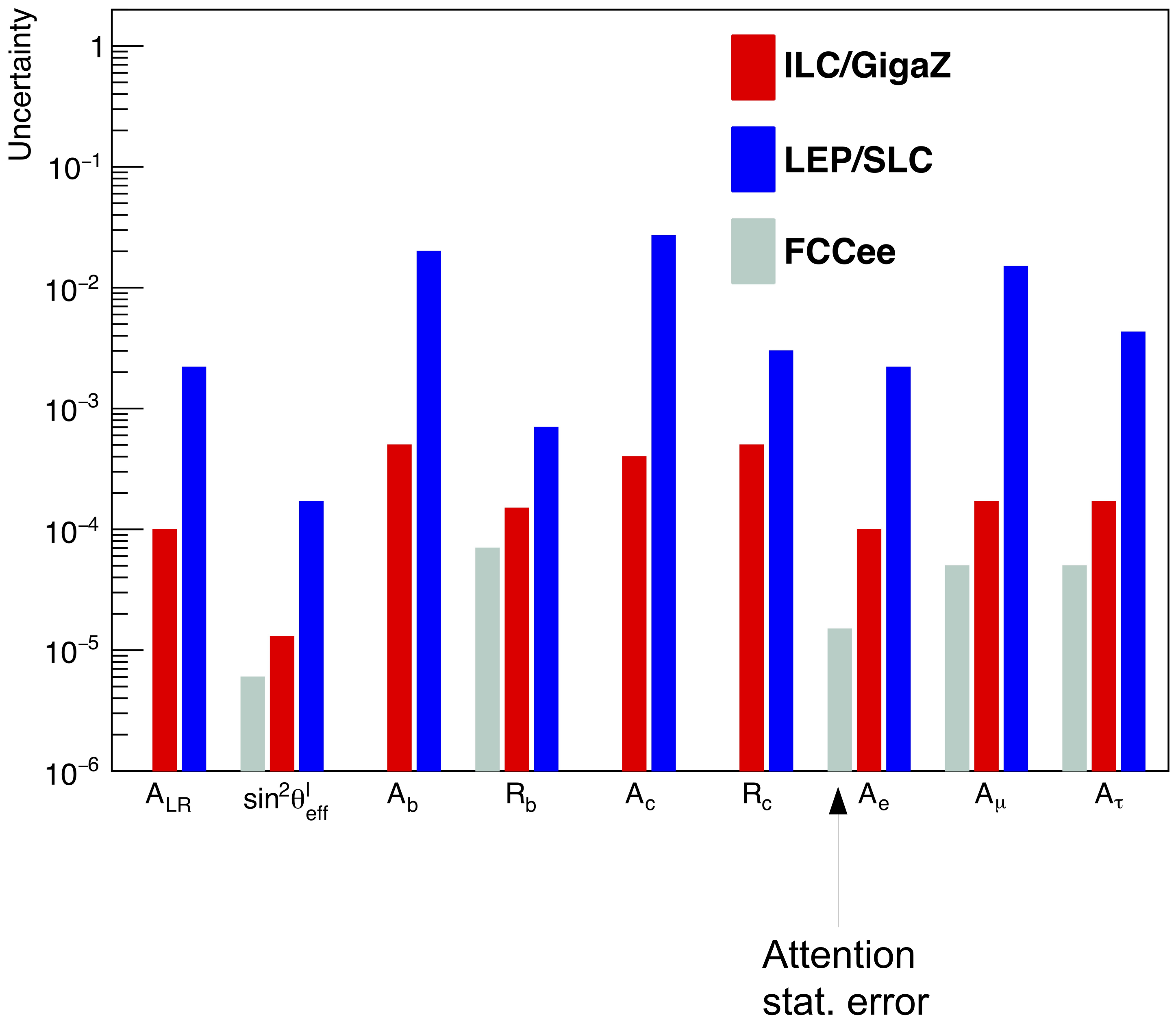
Blondel scheme independent of polarimeter precision

- Assumes perfect spin flip for polarised beams
- Residuals must be monitored by polarimeter
- Residual uncertainty of $\Delta A_{LR} = 0.5\text{x}10^{-4}$ seems possible
- The more positron polarisation the better (see backup)
- Don't forget energy dependency ($dALR/d\sqrt{s} \sim 2\text{x}10^{-5}/MeV$)

Precision $\boldsymbol{\Delta A_{LR} = 1\text{x}10^{-4}}$ is a realistic assumption for GigaZ

=> $$\delta \sin^2\theta^{\ell}_{\rm eff.} \sim 1.3 \cdot 10^{-5}$$

Precision on relevant observables



Precise measurement of $\sin^2\theta^\ell_{\rm eff}$.

- Ten times better than LEP/SLD and competitive with FCC
- Polarisation compensates for ~30 times luminosity
- ... and A_{LR} at LC can benefit from hadronic Z decays
- **No assumption on lepton universality at LC**

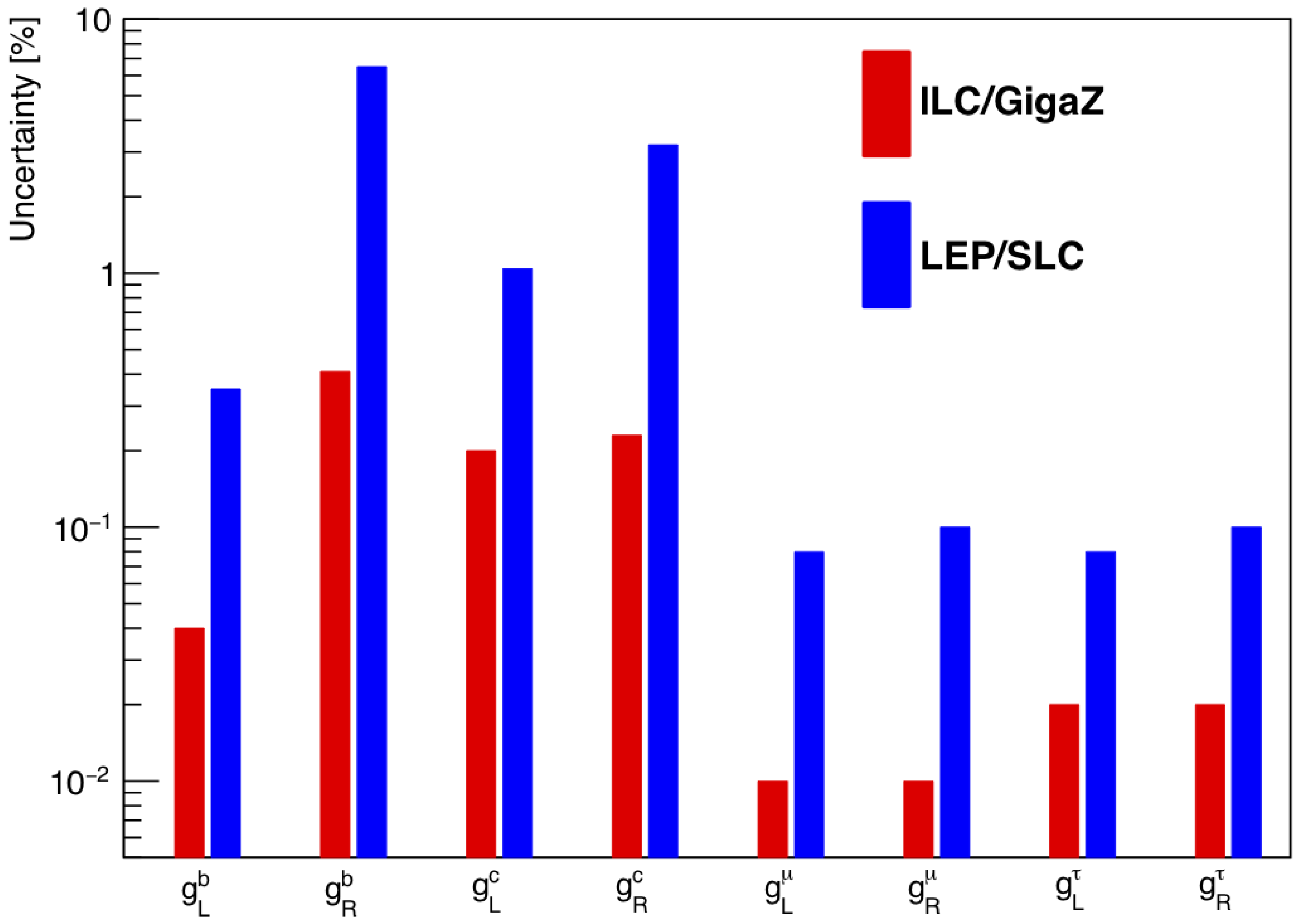
Complete test of lepton universality

- Precisions of order 0.05%

Note excellent measurement of quark asymmetries

- See above for ee->bb at 250 GeV

Precision on electroweak couplings



Rates and asymmetries

Partial fermion width:

$$R_f = \frac{N_f}{N_{had}} = \frac{(g_f^L)^2 + (g_f^R)^2}{\sum_{i=1}^{n_q}[(g_i^L)^2 + (g_i^R)^2]}$$

Sensitive to sum of coupling constants
Available at linear and circular colliders

Left-right asymmetry:

$$A_{LR} = \frac{1}{|\mathcal{P}_{eff.}|}\frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = \mathcal{A}_e = \frac{(g_f^L)^2 - (g_f^R)^2}{(g_i^L)^2 + (g_i^R)^2} \sim 1 - 4\sin^2\theta^\ell_{eff.}$$

Direct sensitivity to Zee vertex
Only available at linear colliders due to beam polarisation
Circular colliders need auxiliary measurement
- e.g. $P_\tau \sim A_e$

Forward-backward asymmetry:

$$A^f_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{3}{4}\mathcal{A}_e\mathcal{A}_f \quad \text{for} \quad \mathcal{P}_e = 0.$$

“Classical” observable to study P-violating effects in ee->ff
Available at circular and linear colliders
Without beam polarisation interpretation is **always model dependent**

Left-right-forward-backward asymmetry:

$$A^f_{FB,LR} = \frac{(\sigma_F - \sigma_B)_L - (\sigma_F - \sigma_B)_R}{(\sigma_F + \sigma_B)_L + (\sigma_L + \sigma_l)_R} = -\frac{3}{4}\mathcal{A}_f$$

Combination of asymmetries above
Only available linear colliders due to beam polarisation
Direct and model independent measurement of A_f

Rates and asymmetries

Helicity amplitudes can be analysed in several ways (not mutually exclusive):

Oblique Parameters W, Z:

$$Q_{e_i f_j} = Q_e^\gamma Q_f^\gamma + \frac{g_{e_i}^Z g_{f_j}^Z}{\sin^2\theta_W \cos^2\theta_W} \frac{s}{s - M_Z^2 + \mathrm{i}\Gamma_Z M_Z} + \frac{s}{m_W^2} f_{i,j}(W,Y)$$

Contact interactions with e.g. compositeness scale Λ:

$$Q_{e_i f_j} = Q_e^\gamma Q_f^\gamma + \frac{g_{e_i}^Z g_{f_j}^Z}{\sin^2\theta_W \cos^2\theta_W} \frac{s}{s - M_Z^2 + \mathrm{i}\Gamma_Z M_Z} + \frac{g_{contact}^2}{2\Lambda^2} \eta_{e_i f_j}$$

New propagators in concrete models of new physics:

$$Q_{e_i f_j} = Q_e^\gamma Q_f^\gamma + \frac{g_{e_i}^Z g_{f_j}^Z}{\sin^2\theta_W \cos^2\theta_W} \frac{s}{s - M_Z^2 + \mathrm{i}\Gamma_Z M_Z} + \sum \frac{g_{e_i}^{Z'} g_{f_j}^{Z'}}{\sin^2\theta_W \cos^2\theta_W} \frac{s}{s - M_{Z'}^2 + \mathrm{i}\Gamma_{Z'} M_{Z'}}$$

Always with I,j being the helicities of the initial state electron e and the final state fermion f

Heavy quark measurements @ GigaZ

- There is a strong motivation to measure electroweak heavy quark couplings at the ILC
- New physics models predict deviations and b and c quarks are at the cross roads between 'top-philic' and 'non-top-philic' models
- Remember also LEP anomaly on A^b_{FB}
- ILC with GigaZ is a unique opportunity for a complete set of measurements and an unambiguous interpretation of the results
- Relevant observables at GigaZ are A_b (see above) and

$$R_q = \frac{N_q}{N_{had}} = \frac{\Gamma_q}{\Gamma_{had}} = \frac{(g_q^L)^2 + (g_q^R)^2}{\sum_{i=1}^{n_q}[(g_i^L)^2 + (g_i^R)^2]}$$

  - Here Γ_{had} is constrained by the fact that all hadrons are produced from the known quark species i.e. $R_b+R_c+R_{uds} = 1$ and has therefore no error, but the g_i are correlated to fulfill this constraint
  - The <u>measured</u> Γ_{had}, which is sensitive to the experimental Z mass resolution has to be considered as a consistency check