

# Higgs and electroweak theory

2020/08/11  
HCPSS

## Lecture 1

SM gauge group  $SU(3)_c \times SU(2)_L \times U(1)_Y$

$$L_{\text{gauge}} = -\frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu} - \frac{1}{4} W_{\mu\nu}^a W^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$g_{\mu\nu} = \text{diag}(1, -1, -1, -1); p^2 \equiv p_\mu p^\mu = m^2 \text{ (on shell)}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$G_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a + g_s f^{abc} G_\mu^b G_\nu^c$$

structure constants

$$[t^a, t^b] = i f^{abc} t^c$$

$SU(3)$ :  $a = 1 \dots 8$

$$W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g \epsilon^{abc} W_\mu^b W_\nu^c \quad \leftarrow$$

$\uparrow$   
coupling strength

$$D_\mu = \partial_\mu - ig' B_\mu Y - ig W_\mu^a T^a - ig_s G_\mu^a t^a$$

$\uparrow$   
gen's of  $SU(2)$

$\uparrow$   
generators of  $SU(3)$   
(Gell-Mann)

$$T^a = \frac{\sigma^a}{2}$$

Gauge transformations:

$$\psi \rightarrow \psi' = e^{-i\lambda Y} \psi$$

U(1)Y:  $\gamma \rightarrow e \nu L \nu Y \cdots$

$$B_\mu \rightarrow B_\mu + \frac{1}{g'} \partial_\mu Y(x)$$

$SU(2)_L$ :  $\psi \rightarrow \exp[i\lambda_L^a(x) T^a] \psi$        $\lambda_L^a$  infinitesimal

$$W_\mu^a \rightarrow W_\mu^a + \frac{1}{g} \partial_\mu \lambda_L^a(x) + \epsilon^{abc} W_\mu^b \lambda_L^c(x)$$

Mass term for gauge boson:

$$X \rightarrow \mathcal{L} \supset \frac{1}{2} M_B^2 B_\mu B^\mu \quad \text{NOT gauge invariant}$$


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Fermion sector:

$$W_\mu^a T^a u_R = 0$$

$SU(2)_L$ :  $Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$      $u_R$      $d_R$      $L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$      $e_R$

$Y$ :     $1/6$                  $2/3$      $-1/3$                  $-1/2$                  $-1$

Color:    triplet                triplet    triplet                singlet                singlet

Projection operators (for chiral 4-component spinors,

$$P_R \psi \equiv \psi_R \quad P_L \psi \equiv \psi_L$$

$$P_R = \frac{1}{2} (1 + \gamma^5) \quad P_L = \frac{1}{2} (1 - \gamma^5)$$

$$\bar{\psi} P_R = \psi^\dagger \gamma^0 P_R = \psi^\dagger P_L \gamma^0 = (P_L \psi)^\dagger \gamma^0 = \bar{\psi}_L$$

$$\{\gamma^\mu, \gamma^5\} = 0 \quad \gamma^{5+} = \gamma^5 \text{ so } P_L^+ = P_L^-$$

similarly  $\bar{\psi} P_L = \bar{\psi}_R$

$$\mathcal{L}_{\text{Dirac}} = \bar{\Psi}_i \gamma^\mu \gamma^5 \Psi - m \bar{\Psi} \Psi \quad \text{for a generic fermion with mass } m$$

$\uparrow \quad \uparrow$   
 $\Gamma_L^2 + \Gamma_R^2 \quad \Gamma_L^2 + \Gamma_R^2$

$$\Gamma_L + \Gamma_R = 1, \quad \Gamma_L^2 = \Gamma_L, \quad \Gamma_R^2 = \Gamma_R \quad (\gamma^5 \gamma^5 = 1)$$

$$\begin{aligned}\bar{\Psi}_i \gamma_\mu \gamma^\mu \Psi &= \bar{\Psi}_i \gamma_\mu \gamma^\mu \Gamma_L^2 \Psi + \bar{\Psi}_i \gamma_\mu \gamma^\mu \Gamma_R^2 \Psi \\ &= \bar{\Psi}_R \Gamma_R i \gamma_\mu \gamma^\mu \Gamma_L \Psi + \bar{\Psi}_L \Gamma_L i \gamma_\mu \gamma^\mu \Gamma_R \Psi \\ &= \bar{\Psi}_L \Gamma_L i \gamma_\mu \gamma^\mu \Psi_L + \bar{\Psi}_R \Gamma_R i \gamma_\mu \gamma^\mu \Psi_R \quad \leftarrow !\end{aligned}$$

$$\begin{aligned}-m \bar{\Psi} \Psi &= -m \bar{\Psi} \Gamma_L^2 \Psi - m \bar{\Psi} \Gamma_R^2 \Psi \\ &= -m \bar{\Psi}_R \Psi_L - m \bar{\Psi}_L \Psi_R \quad \leftarrow !! \\ &\qquad\qquad\qquad \underbrace{\quad}_{= \text{h.c. of 1st term}}\end{aligned}$$

Mass term for any SM fermion is not gauge invariant because gauge charges of  $\Psi_L$  and  $\Psi_R$  are different

### SM Higgs Mechanism

New ingredient: scalar field, complex

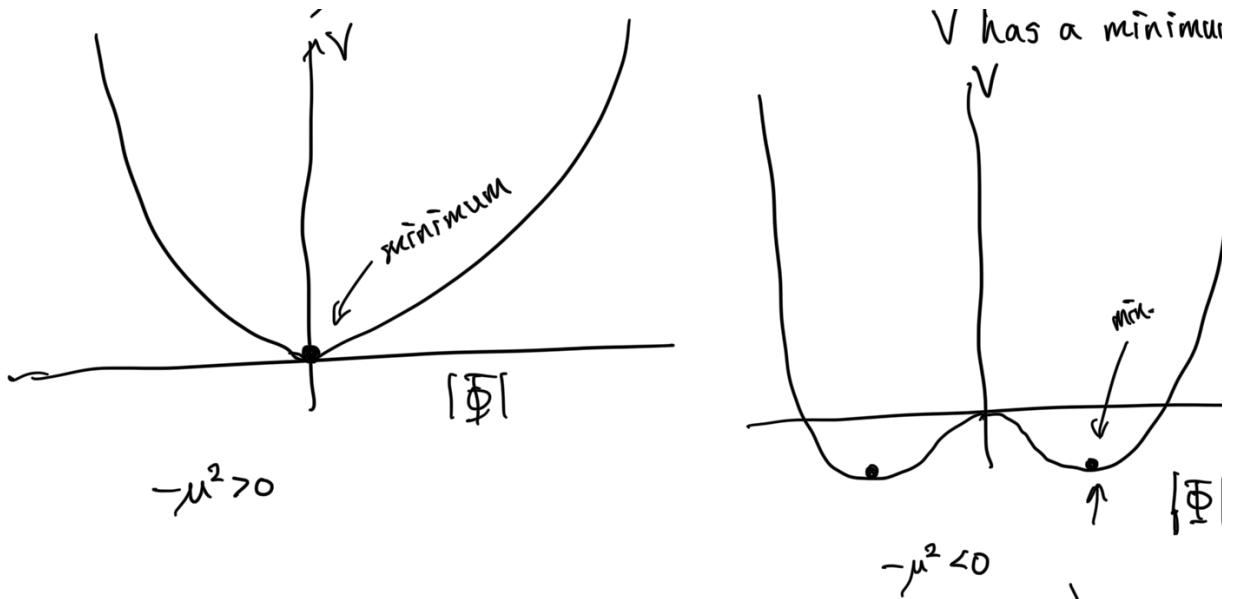
$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}, \quad Y = \frac{1}{2}$$

color singlet

$$\mathcal{L}_\Phi = (\bar{\mathcal{D}}_\mu \Phi)^+ (\bar{\mathcal{D}}^\mu \Phi) - V(\Phi) + \mathcal{L}_{\text{Yukawa}}$$

Scalar potential:

$$V = -\mu^2 \bar{\Phi}^+ \Phi + \lambda (\bar{\Phi}^+ \Phi)^2 \quad \lambda \text{ positive so th.}$$



$$\bar{\Phi} \rightarrow \exp\left[i\lambda_y(x) Y\right] \bar{\Phi}$$

$\frac{1}{2}$

When  $\langle \bar{\Phi} \rangle \neq 0$ , the vacuum is not gauge invariant.  
Vacuum expectation value (vev)

$-\mu^2 < 0:$

$$\bar{\Phi}^\dagger \bar{\Phi} = \frac{1}{2} (\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2)$$

$$\bar{\Phi}^\dagger \bar{\Phi} = \frac{\mu^2}{2\lambda} = \frac{v^2}{2}, \text{ choose } \phi_3 = v + h(x)$$

$$\bar{\Phi} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ v + h - i\phi_4 \end{pmatrix}$$

$v = 246 \text{ GeV}$

$$\bar{\Phi} = \frac{1}{\sqrt{2}} \exp\left(\frac{i \tilde{s}_y^a \sigma^a}{v}\right) \begin{pmatrix} 0 \\ v + h \end{pmatrix}$$

$$s_{\alpha\beta} = \mp - \frac{1}{2} \int \tilde{s}_y^a(x) \Gamma^a dx$$

$$J_{\mu L} \rightarrow \Psi \rightarrow \exp(i\omega_L t) \frac{1}{2} \not{A}$$

choose  $\lambda_L^a(x) = -2\tilde{\xi}_L^a(x)/\sigma$  : special gauge in which

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix} \quad \begin{matrix} \text{unitarity gauge} \\ \text{unitary gauge} \end{matrix}$$

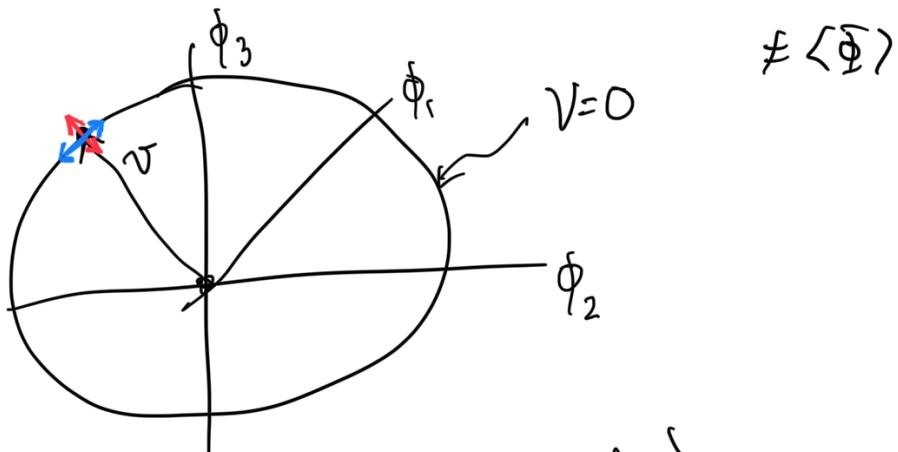

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$$S = \int d^4x \mathcal{L}$$

↑                      ↑  
units:  $\hbar$      $[m]^{-4}$     dimension  $[m]^4$

Vacuum:  $\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$

$U(1)$  gauge xform:  $\langle \Phi \rangle \rightarrow e^{i\lambda(x)} \langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ ve^{i\lambda(x)} \end{pmatrix}$



$$\downarrow \Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} + \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ h(x) \end{pmatrix}$$

$$\mathcal{L} = -g \overline{Q}_L \overline{\Phi} d_R \rightarrow -\frac{g}{\sqrt{2}} \overline{d}_L d_R \cdot v + \dots$$

$\uparrow \pi \Delta \backslash$

$$M_W = \frac{g v}{2}$$

known

known

