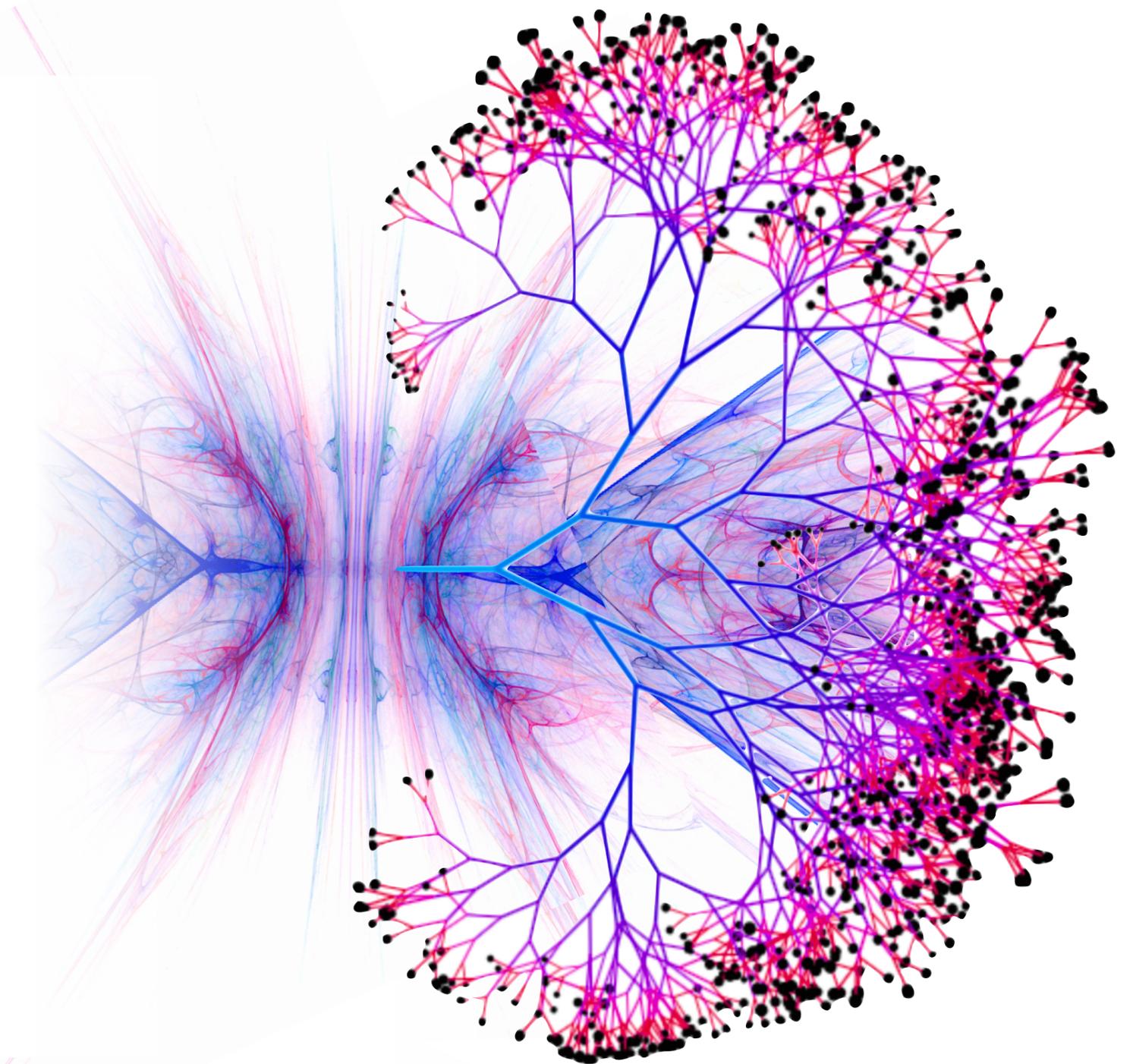
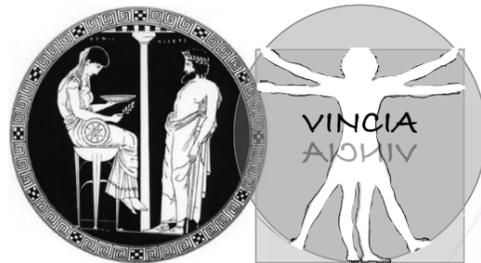


# Hadronization & Underlying Event

QCD and Event Generators  
Lecture 3 of 3

**Peter Skands**  
Monash University  
(Melbourne, Australia)



# From Partons to Pions

Consider a parton emerging from a hard scattering (or decay) process

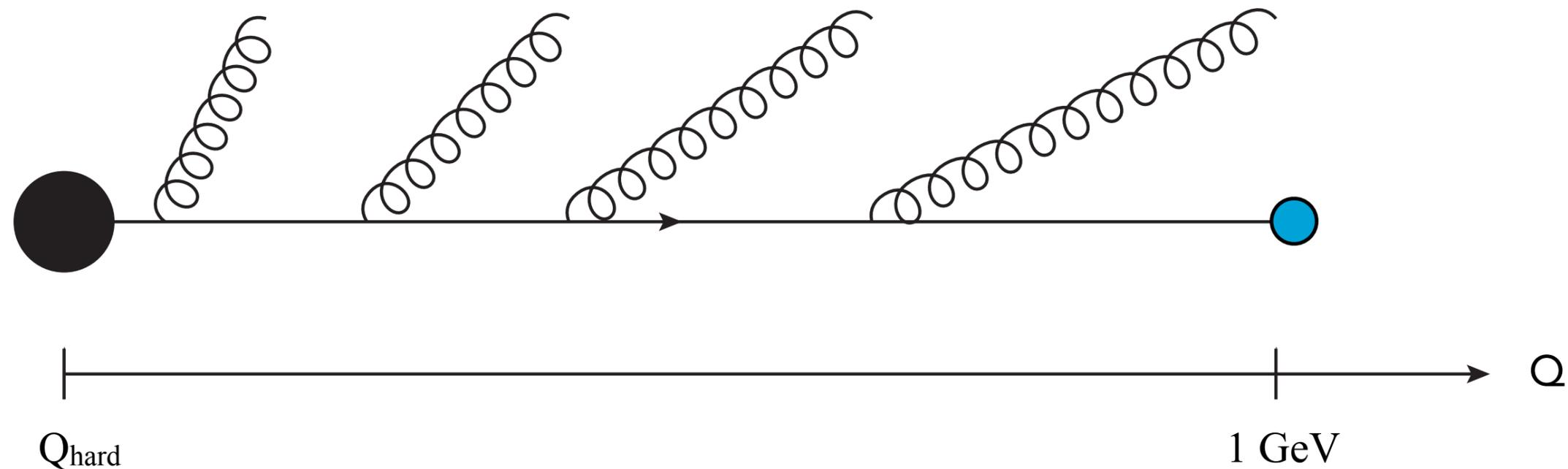
It starts at a high factorization scale

$$Q = Q_F = Q_{\text{hard}}$$

It showers  
(bremsstrahlung)

It ends up  
at a low effective  
factorization scale

$$Q \sim m_p \sim 1 \text{ GeV}$$



**How about I just call it a hadron?**

→ "Local Parton-Hadron Duality"

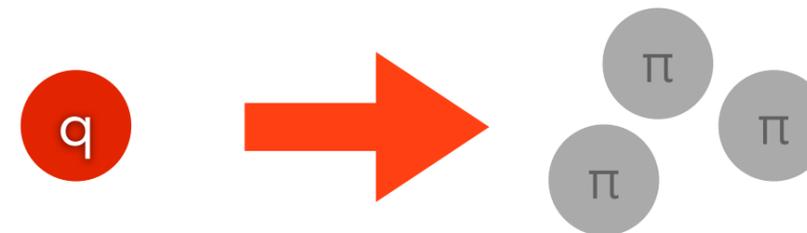
# Parton $\rightarrow$ Hadrons?

## Early models: “Independent Fragmentation”

Local Parton Hadron Duality (**LPHD**) can give useful results for inclusive quantities in collinear fragmentation

Motivates a simple model:

*“Independent Fragmentation”*



But ...

The point of confinement is that partons are **coloured**

Hadronisation = the process of **colour neutralisation**

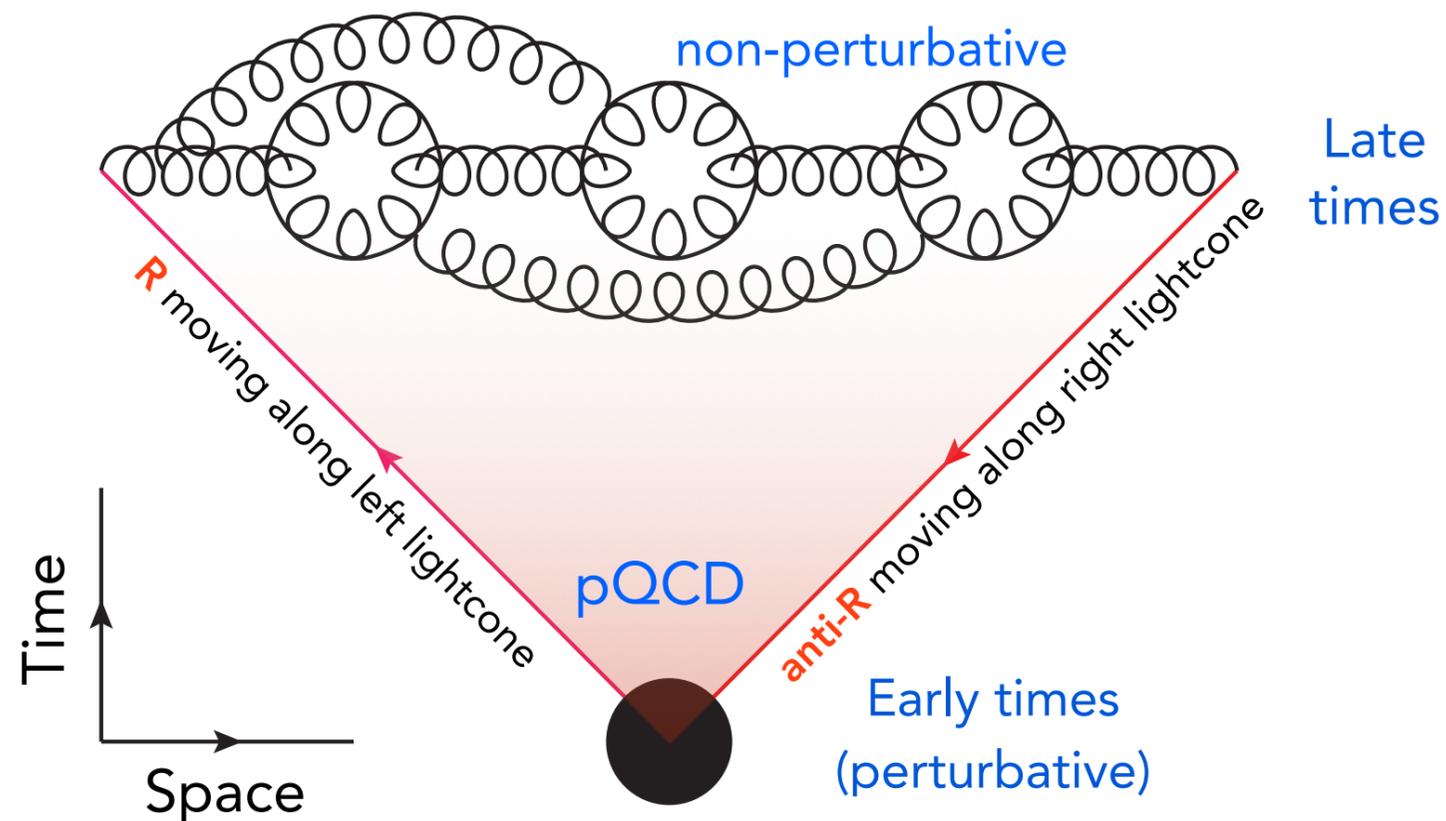
- $\rightarrow$  Unphysical to think about independent fragmentation of a single parton into hadrons
- $\rightarrow$  Too naive to see **LPHD** (inclusive) as a justification for Independent Fragmentation (exclusive)
- $\rightarrow$  More **physics** needed

# Colour Neutralisation

## A **physical** hadronization model

Should involve at least **two** partons, with opposite color charges\*

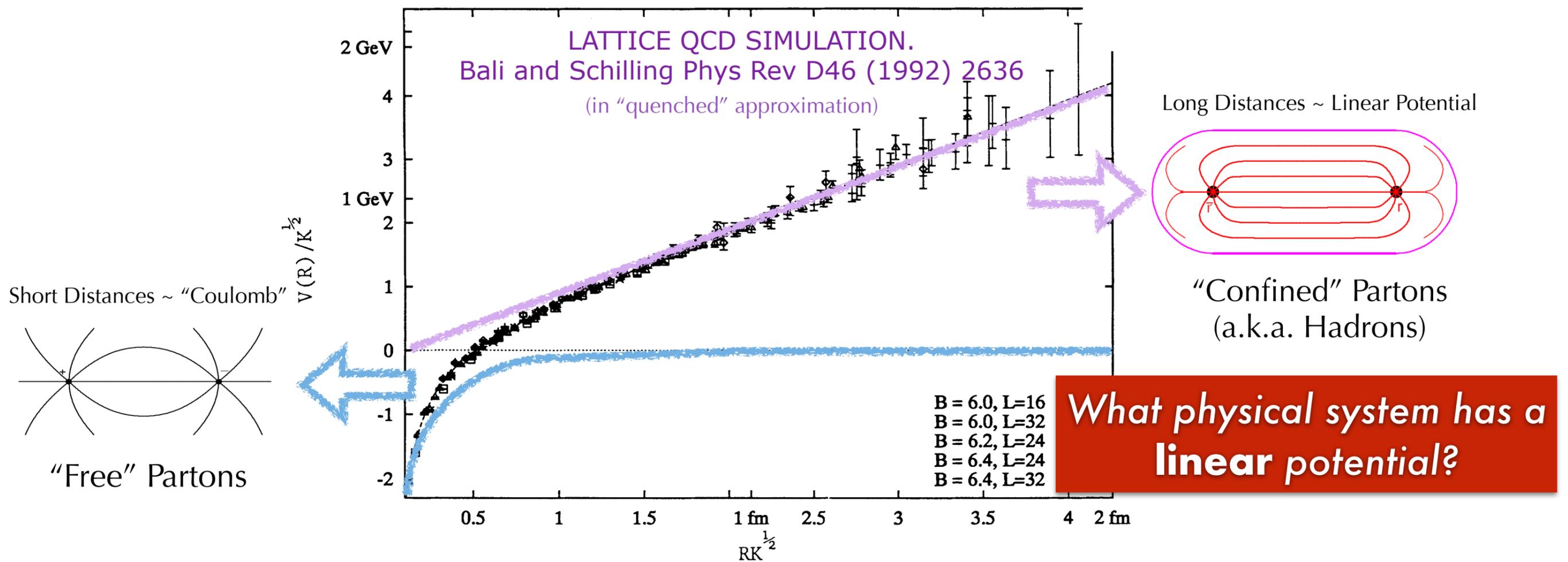
A strong **confining field** emerges between the two when their separation  $\gtrsim 1\text{fm}$



\*) Really, a colour singlet state  $\frac{1}{\sqrt{3}} (|R\bar{R}\rangle + |G\bar{G}\rangle + |B\bar{B}\rangle)$ ; the LC **colour flow rules** discussed in lecture 1 allow us to tell which partons to pair up (at least to LC; see [arXiv:1505.01681](https://arxiv.org/abs/1505.01681))

# Linear Confinement

Using explicit computer simulations of QCD on a 4D “lattice” (lattice QCD), one can compute the potential energy of a colour-singlet  $q\bar{q}$  state, as a function of the distance,  $r$ , between the  $q$  and  $\bar{q}$

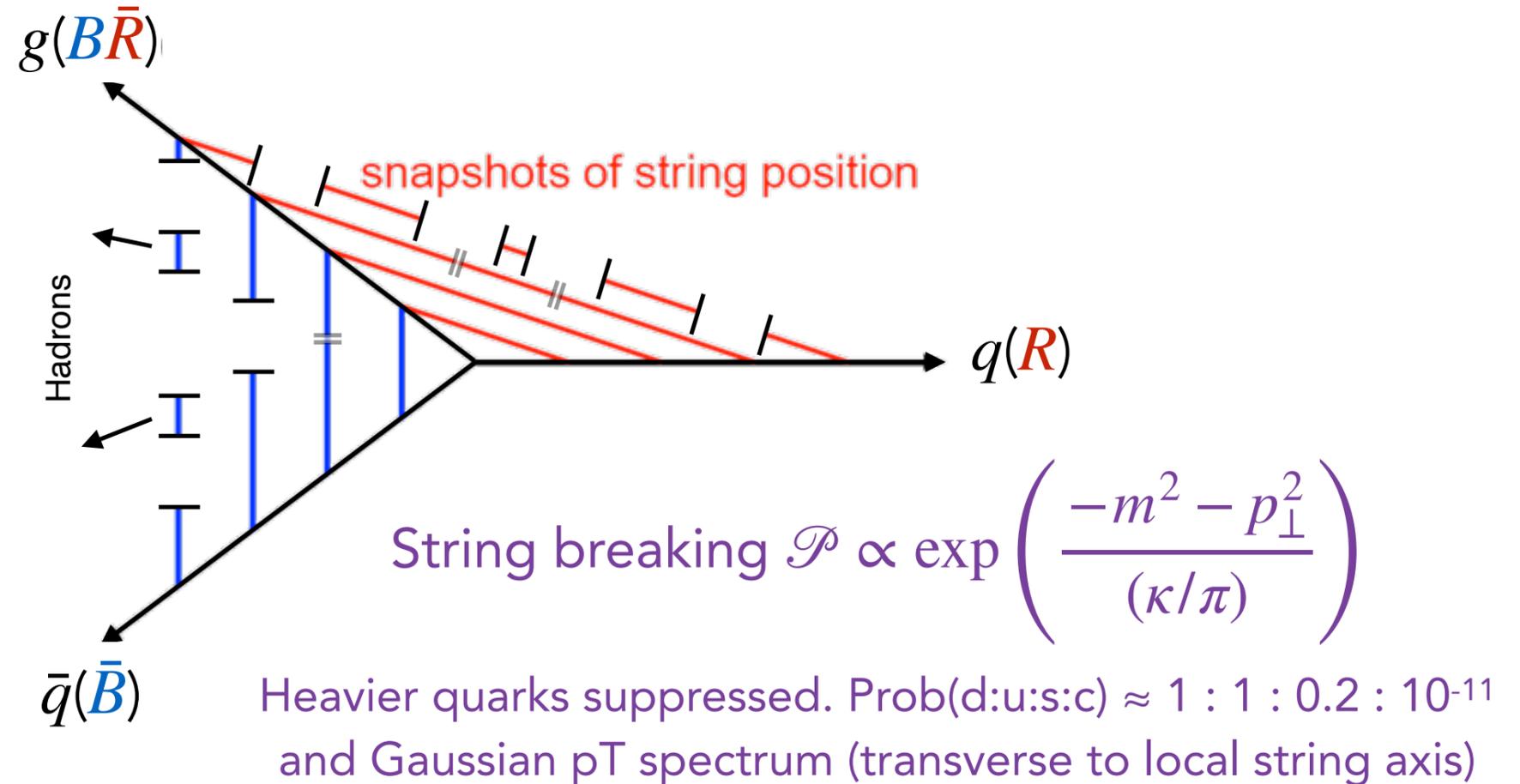


“Cornell Potential” fit:  $V(r) = -\frac{a}{r} + \kappa r$  with  $\kappa \sim 1 \text{ GeV/fm}$  ( $\rightarrow$  could lift a 16-ton truck)

# Motivates a Model

A high-energy quark-gluon-antiquark system is created and starts to fly apart

- **Quarks** → String Endpoints
- **Gluons** → Transverse Excitations (kinks)
- **Physics** then in terms of 1+1-dim **string "worldsheet"** evolving in spacetime
- Probability of **string break** (by quantum tunneling) constant per unit space-time area



Computer algorithms to model this process began to be developed in late 70'ies and early 80'ies

→ **Monte Carlo Event Generators**

Modern MC **hadronization models**: PYTHIA (string), HERWIG (cluster), SHERPA (cluster)

# The (Lund) String Hadronization Model $\leftrightarrow$ PYTHIA (org JETSET)

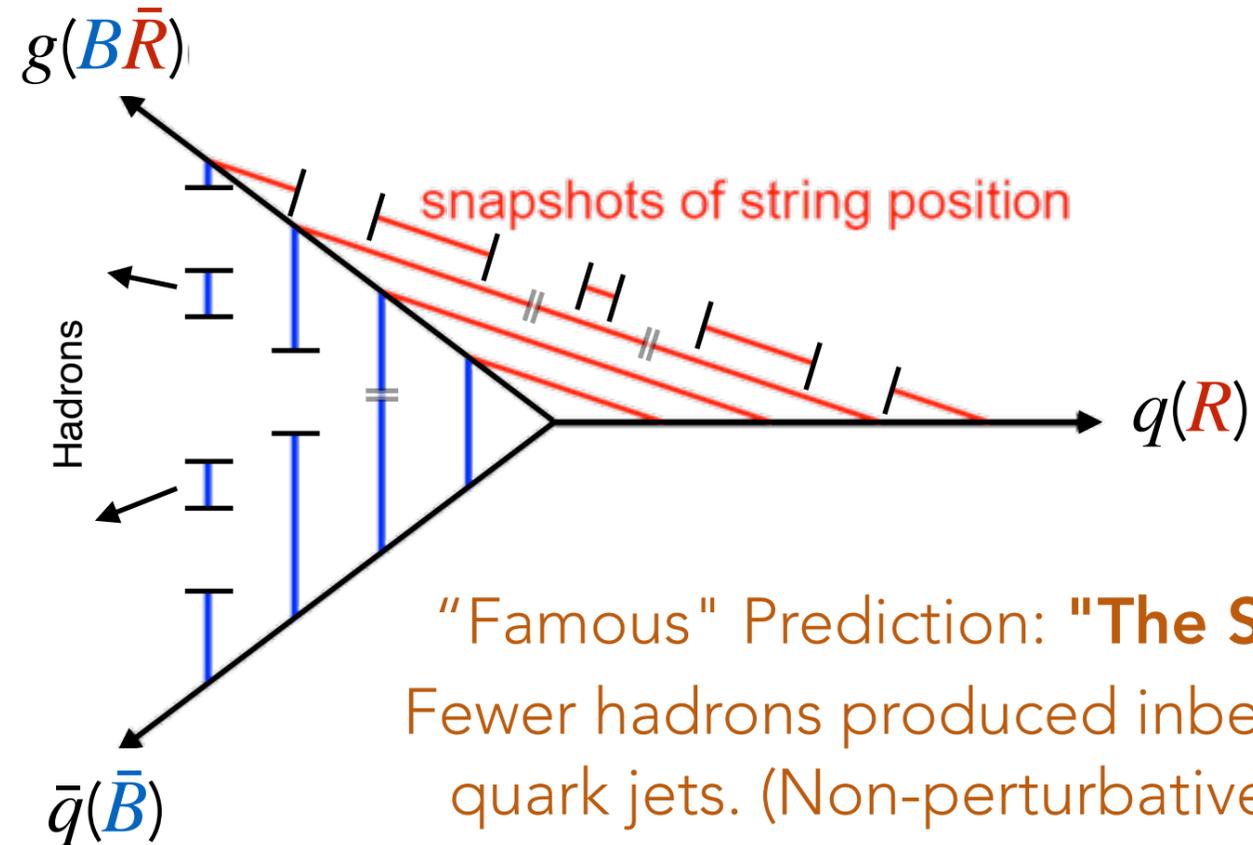
## Simple space-time picture

Highly predictive, few free parameters

Causality and Lorentz invariance  $\implies$  "Lund Symmetric Fragmentation Function" with two free parameters  $a$  and  $b$ :

$$f(z) \propto \frac{(1-z)^a}{z} \exp(-bm_{\perp}^2/z)$$

with  $z \sim E_{\text{hadron}}/E_{\text{quark}}$



"Famous" Prediction: "**The String Effect**"  
Fewer hadrons produced in between the two quark jets. (Non-perturbative coherence.)  
Confirmed by JADE in 1980.

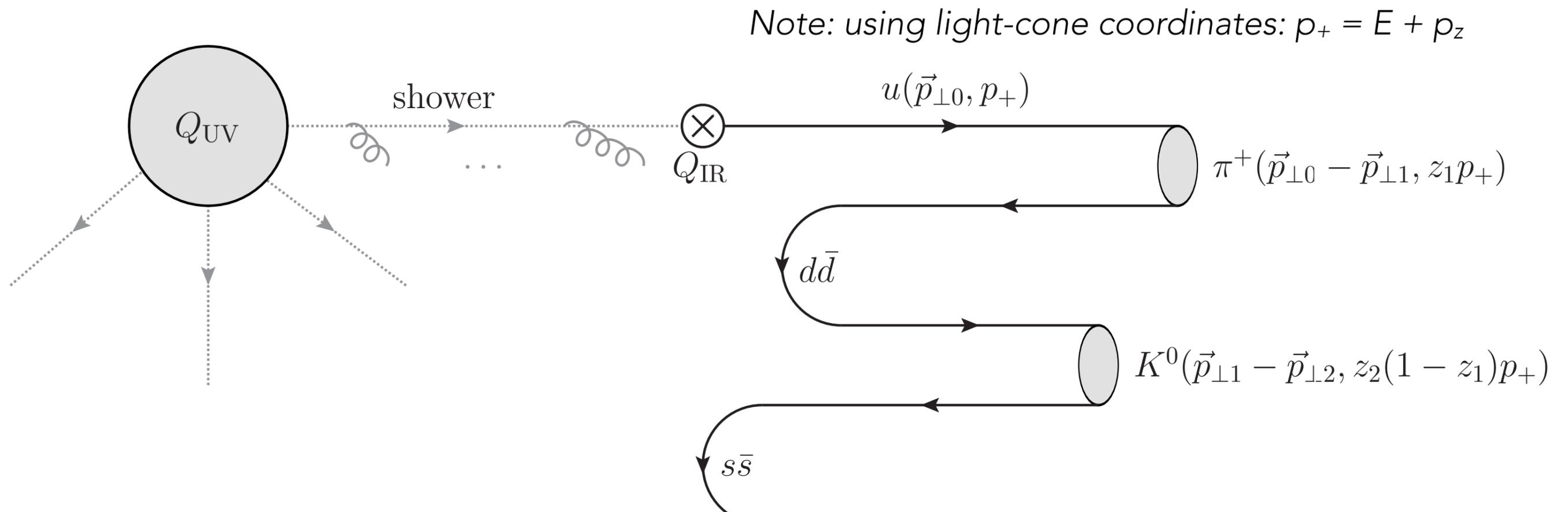
## Details of string breaks more complicated

Many free parameters for **flavour & spin** of produced hadrons  $\rightarrow$  fit to  $e^+e^- \rightarrow$  hadrons

# Iterative String Breaks

String breaks are separated by **spacelike** intervals  $\rightarrow$  **causally disconnected**

$\rightarrow$  We do not have to consider the string breaks in any specific time order  $\rightarrow$  choose the most convenient order for **us**: **starting from the endpoints** ("outside-in")

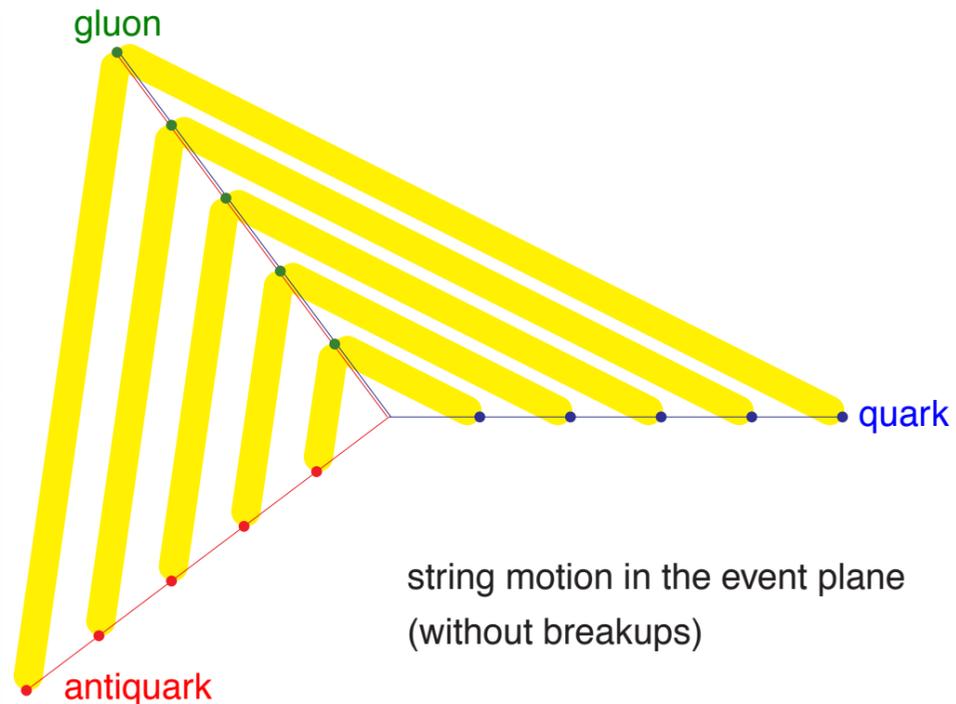


On average, expect energy of  $n^{\text{th}}$  "rank" hadron  $\sim E_n \sim \langle z \rangle^n E_0$

# Quark vs Gluon Jets

Hallmark feature of Lund string model:

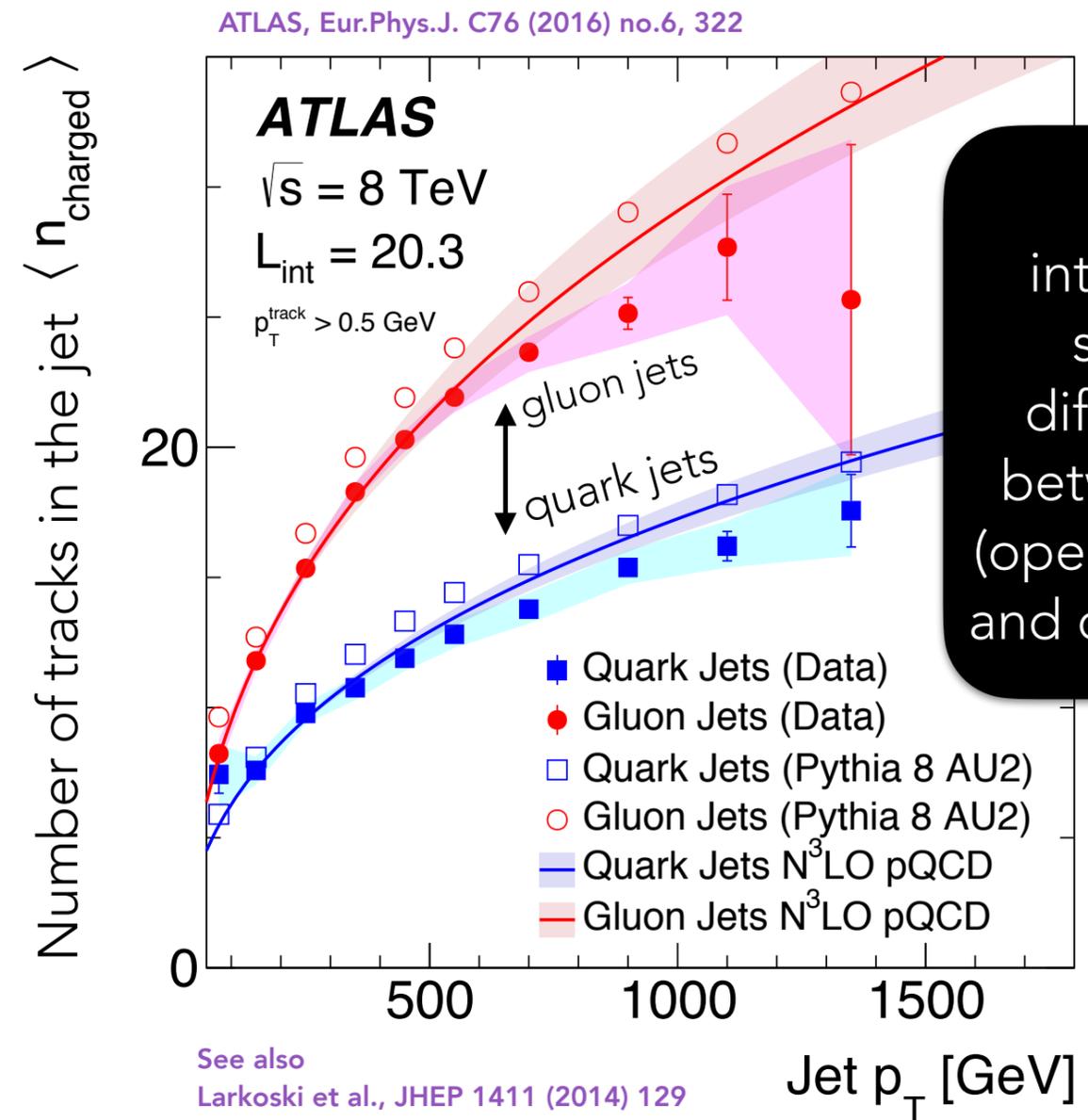
Gluon connected to **two** string pieces



Each quark connected to **one** string piece

→ expect factor  $2 \sim C_A/C_F$  larger particle multiplicity in gluon jets vs quark jets

Can be important for discriminating new-physics signals (decays to quarks vs decays to gluons, vs composition of background and bremsstrahlung combinatorics )



See also  
Larkoski et al., JHEP 1411 (2014) 129  
Thaler et al., Les Houches, arXiv:1605.04692

# The Cluster Model $\leftrightarrow$ HERWIG, SHERPA

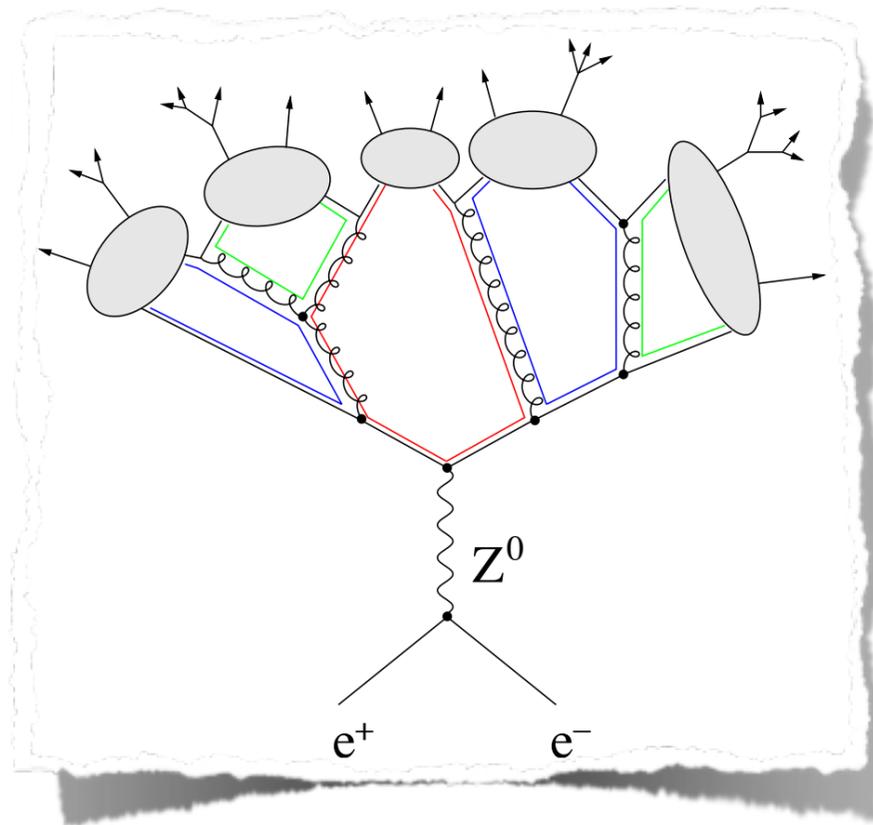
Starting observation: "Preconfinement"

+ Force  $g \rightarrow qq$  splittings at  $Q_0$

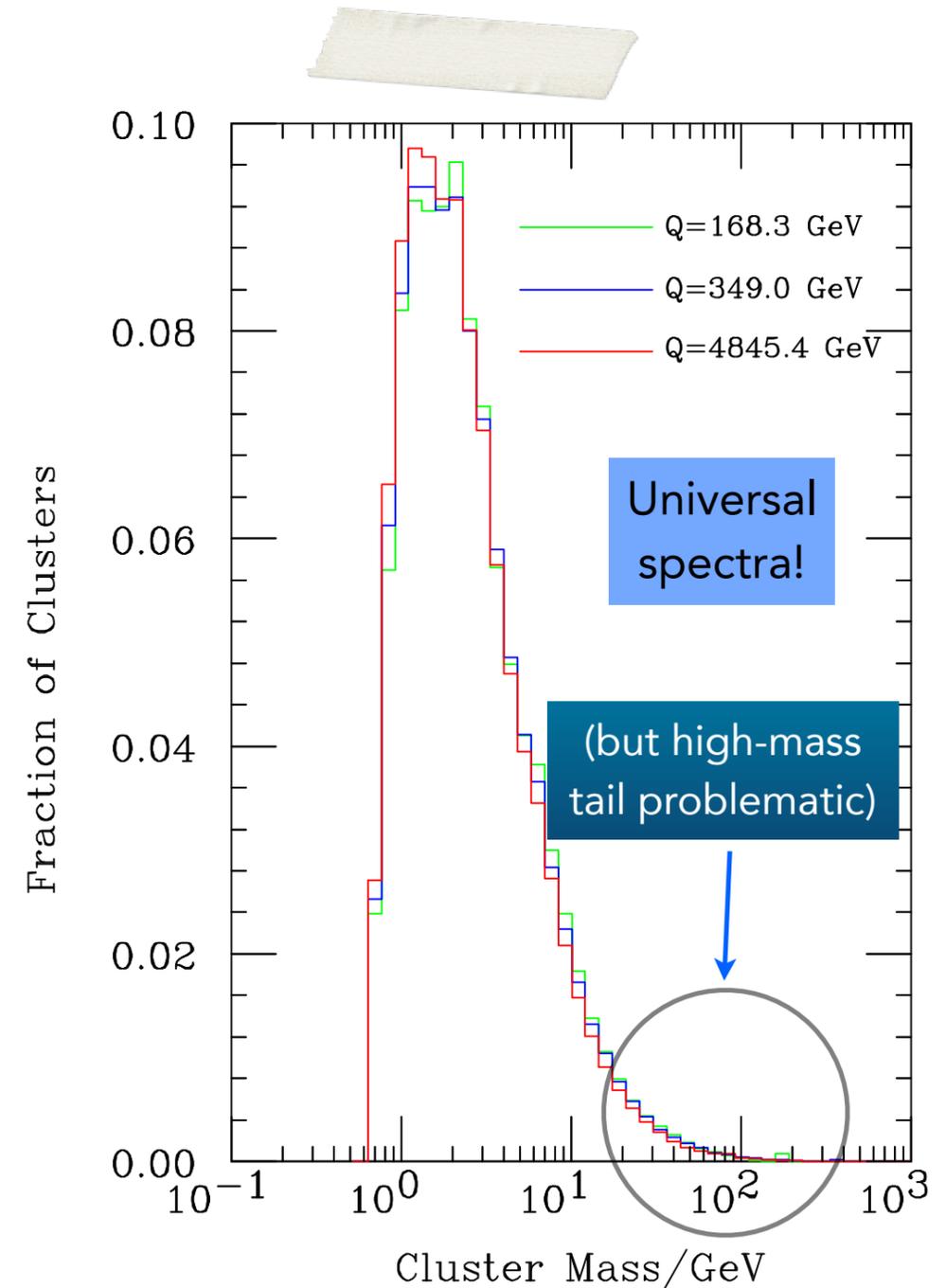
$\rightarrow$  high-mass  $q$ - $q$ bar "clusters"

Isotropic 2-body decays to hadrons

according to PS  $\approx (2s_1+1)(2s_2+1)(p^*/m)$

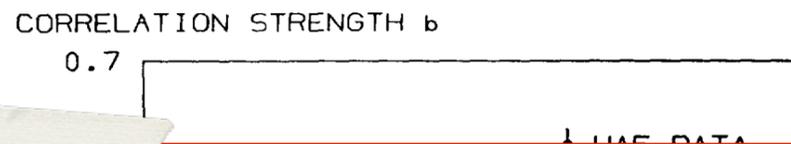
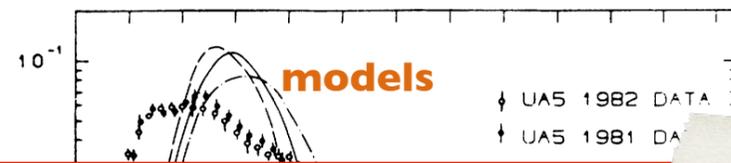


Large clusters  $\rightarrow$  string-like. (In PYTHIA, small strings  $\rightarrow$  cluster-like).



# MC vs Hadron Collisions

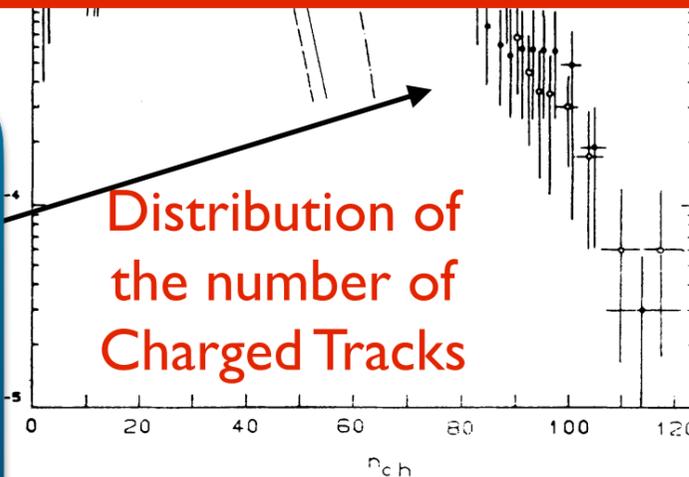
Now that we have a model that includes hard interactions, showers, and string fragmentation, **let's apply it to pp collisions!**



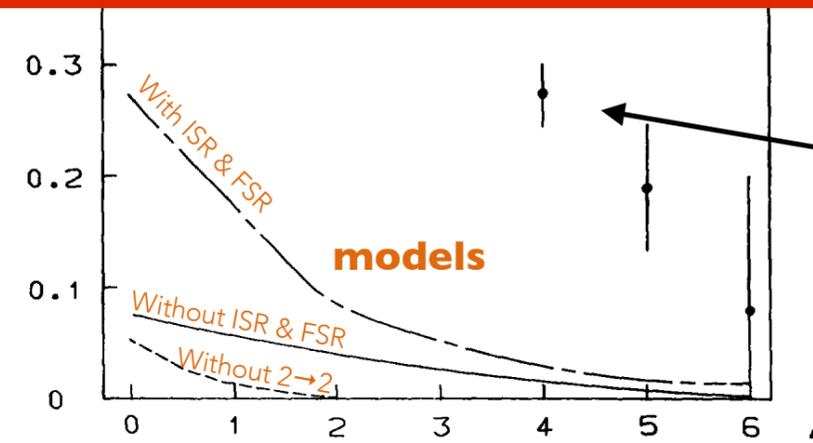
Do not be scared of the failure of physical models (typically points to more interesting physics)

Can get ~ right average but data exhibits **much bigger fluctuations** in multiplicity (here: of charged tracks)

Distribution of the number of Charged Tracks



3. Charged-multiplicity distribution at 540 GeV, UA5 (Ref. 32) vs simple models: dashed low  $p_T$  only, full including hard scatterings, dash-dotted also including initial- and final-state radiation.



Correlation Strength (forward-backward)

some global (quantum) number tells the entire event to fluctuate up or down across many units of rapidity?

Sjöstrand & v. Zijl, Phys.Rev.D36 (1987)2019

# Further evidence of additional physics in hadron-hadron

## 1983: discovery of the "Pedestal Effect"

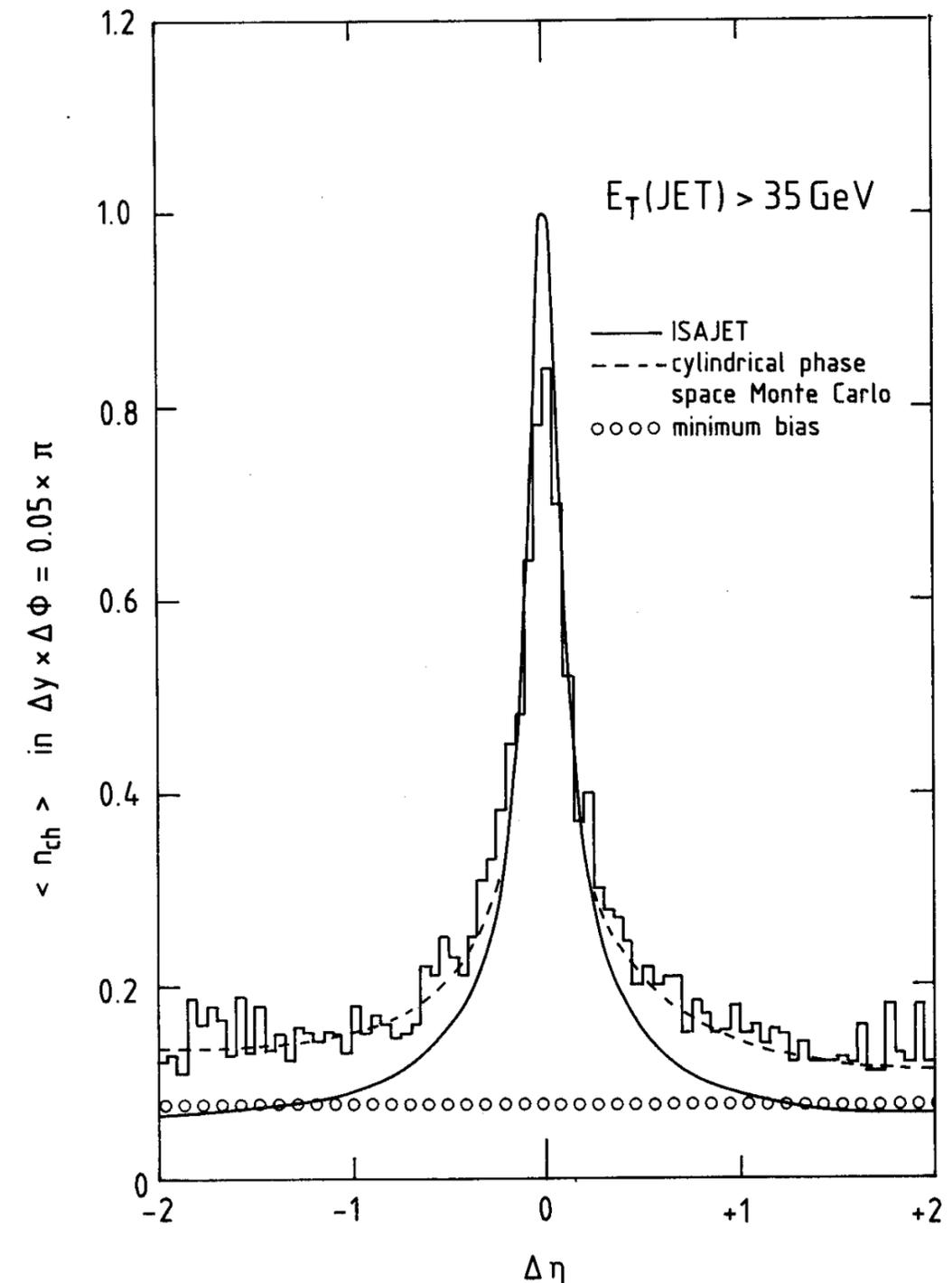
UA1:  $p\bar{p}$  at  $\sqrt{s} = 540$  GeV

Studies of jets with  $E_T$  up to 100 GeV

Phys. Lett. B 132 (1983) 214-222

"Outside the [jet], a constant  $E_T$  plateau is observed, whose height is independent of the jet  $E_T$ . Its value is substantially higher than the one observed for minimum bias events."

In hadron-hadron collisions, **hard jets** sit on "**pedestals**" of increased particle production **extending** far from the jet cores.

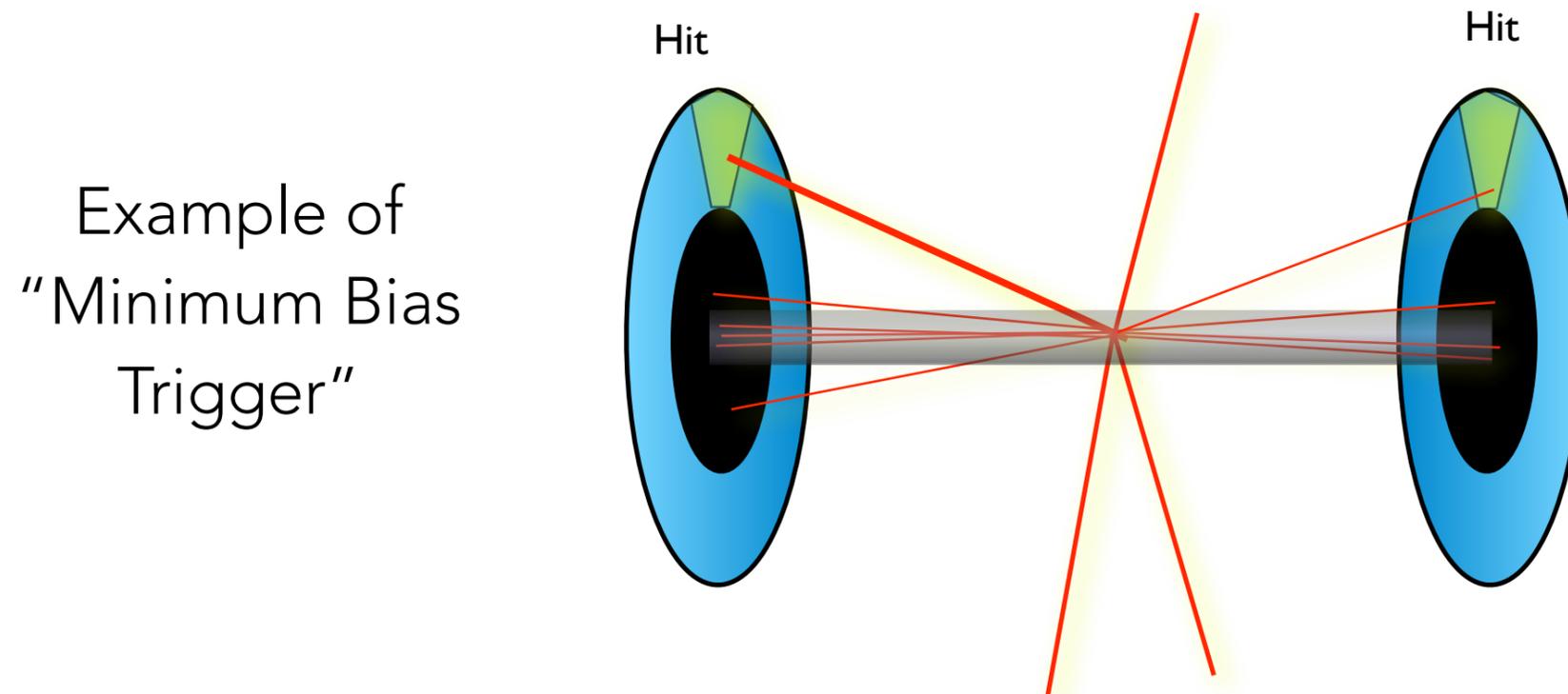


# What's "Minimum-Bias"?

Simple question: what does the **average** LHC collision look like?

First question: how many are there? What is  $\sigma_{\text{tot}}(pp)$  at LHC ?

Around 100mb (of which about half is "inelastic, non-diffractive")



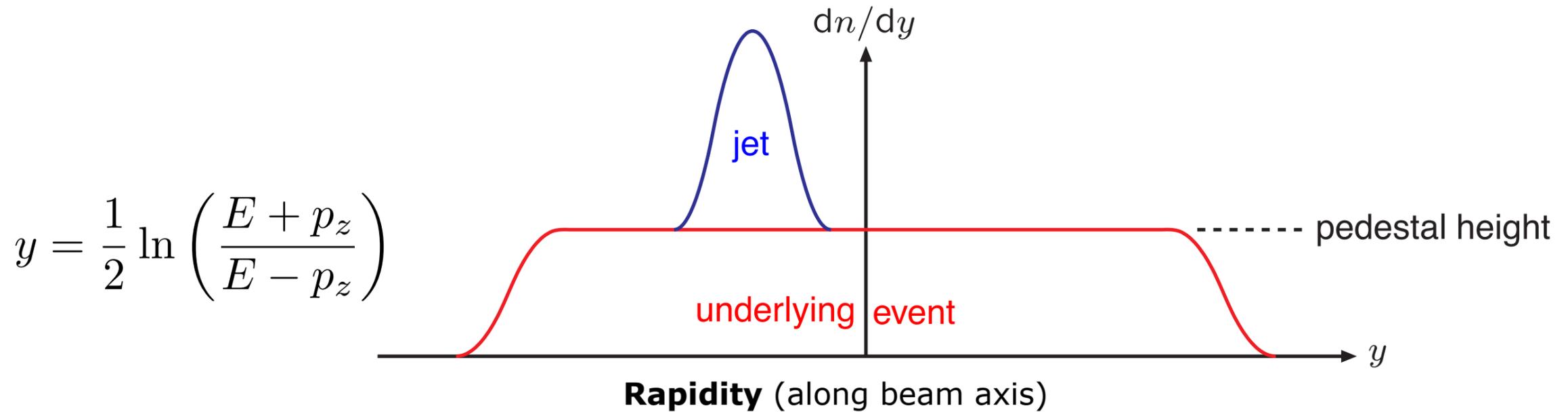
**Minimum Bias = Minimal trigger requirement**

At least one hit in some simple and efficient hit counters (typically at large  $\eta$ )  
(Double-sided trigger requirement suppresses "single diffraction")

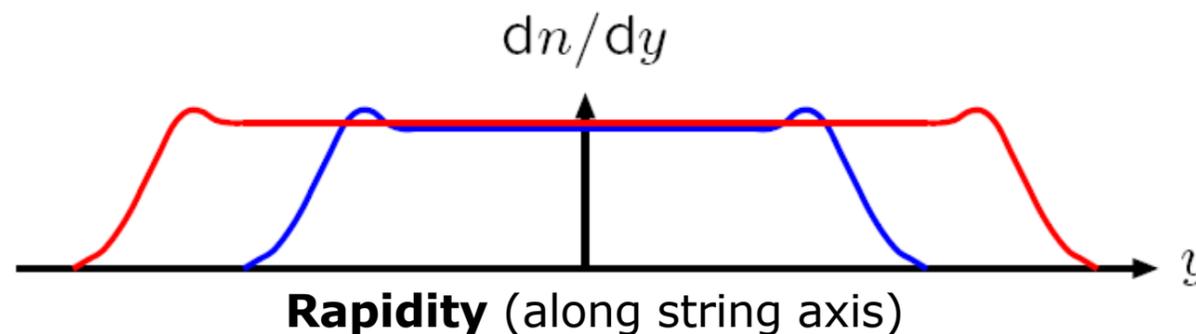
# Dissecting the Pedestal

Illustrations by  
T. Sjöstrand

Today, we call the pedestal **“the Underlying Event”**



A uniform (constant) particle density per rapidity unit is just what a string produces ...



but the **height** of the pedestal was much larger than that of **one** string...

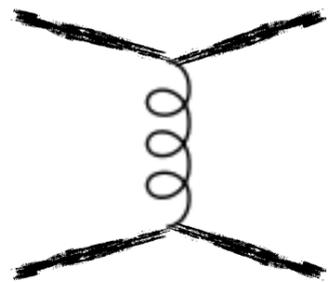
**Multiple Interactions?**

# Parton-Parton vs Proton-Proton Cross Sections

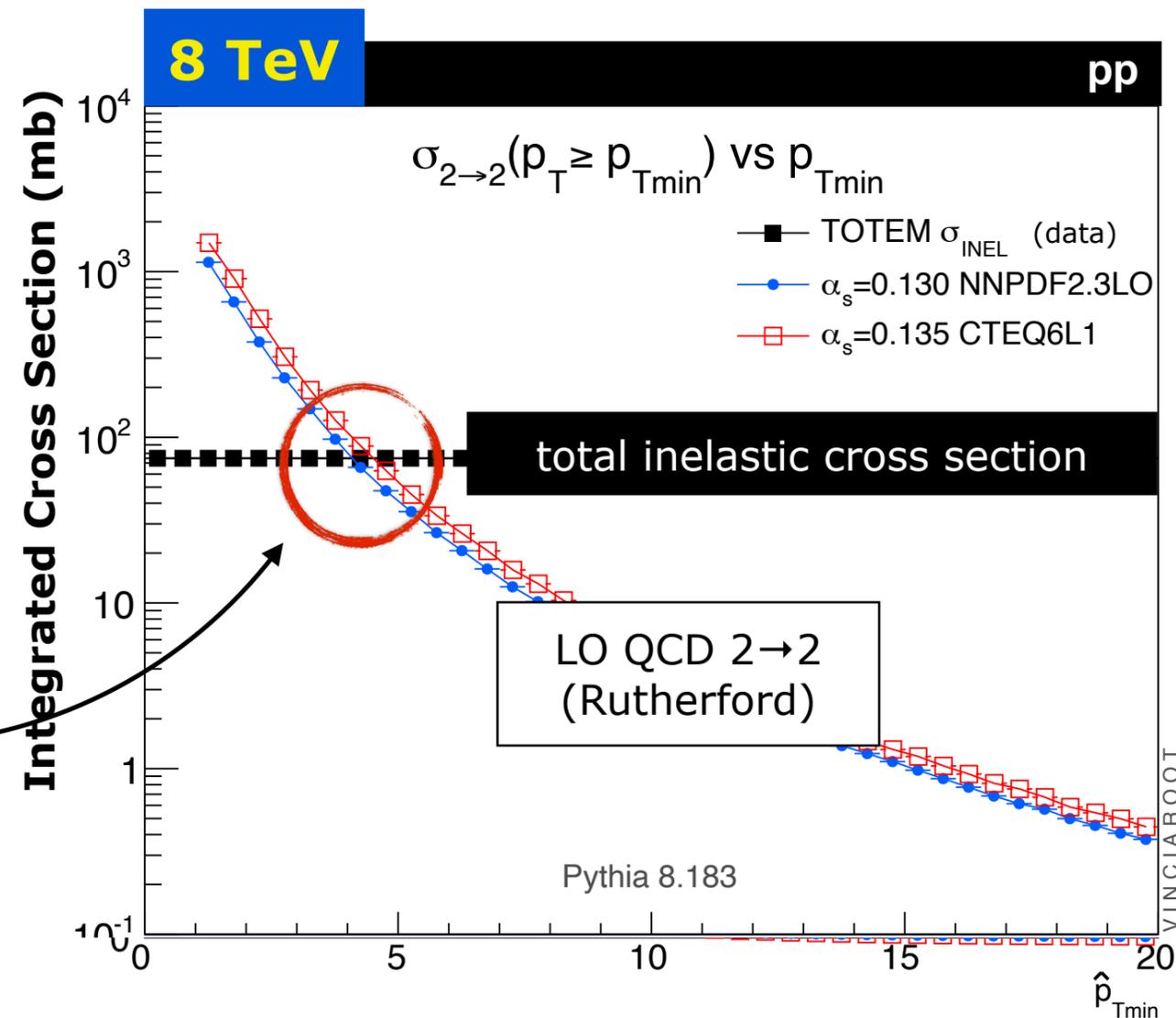
Total **inelastic** pp cross section @ 8 TeV\* ~ 80 mb (measured by TOTEM)

Compare this to perturbative calculation of QCD  $2 \rightarrow 2$  scattering cross section (mainly t-channel gluon exchange; divergent for  $p_T \rightarrow 0$ )

QCD  $2 \rightarrow 2$  cross section dominated by t-channel gluon exchange



**Larger** than total pp cross section for  $\hat{p}_\perp \leq 4$  GeV



Interpret to mean that **every** pp collision has **more than one**  $2 \rightarrow 2$  QCD scattering with  $\hat{p}_\perp \leq 4$  GeV

\*Note: nothing particularly special about 8 TeV; the crossover point would be lower at lower  $E_{CM}$  and higher at higher  $E_{CM}$

# Physics of the Pedestal

Recall **Factorisation**: Subdivide calculation

**Hard scattering**: parton-parton cross section  $d\hat{\sigma}$  independent of non-pert. dynamics

x **PDF factors**  $f(x, Q_F^2)$   
representing: partitioning of proton  
into struck **parton** + unresolved  
**remnant**, at factorisation scale  $Q_F^2$

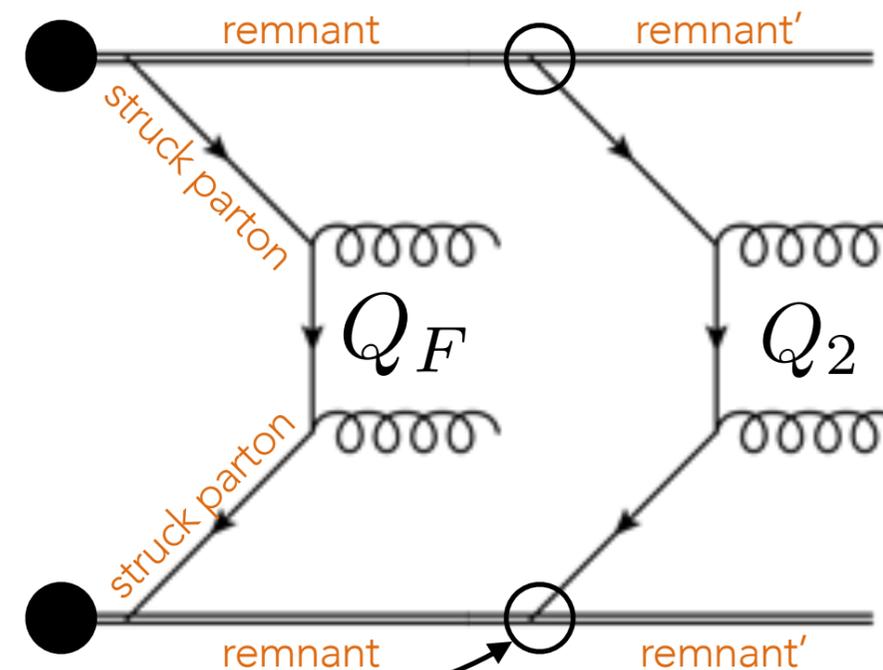
## Multi-Parton Interactions (MPI)

Several QCD  $2 \rightarrow 2$  in **one** pp collision

$\implies$  need **Multi-parton PDFs** (PYTHIA, e.g., Sjöstrand & PS *JHEP* 03 (2004) 053 • [hep-ph/0402078](https://arxiv.org/abs/hep-ph/0402078))

Constructed using **momentum** and **flavour conservation**; goes beyond existing factorisation theorems (though some work on special case Double Parton Scattering)

(More issues such as *colour reconnections, saturation, rescattering, higher twist*, not covered here)



More colour exchanges  
 $\rightarrow$  more strings  
 $\rightarrow$  more hadrons

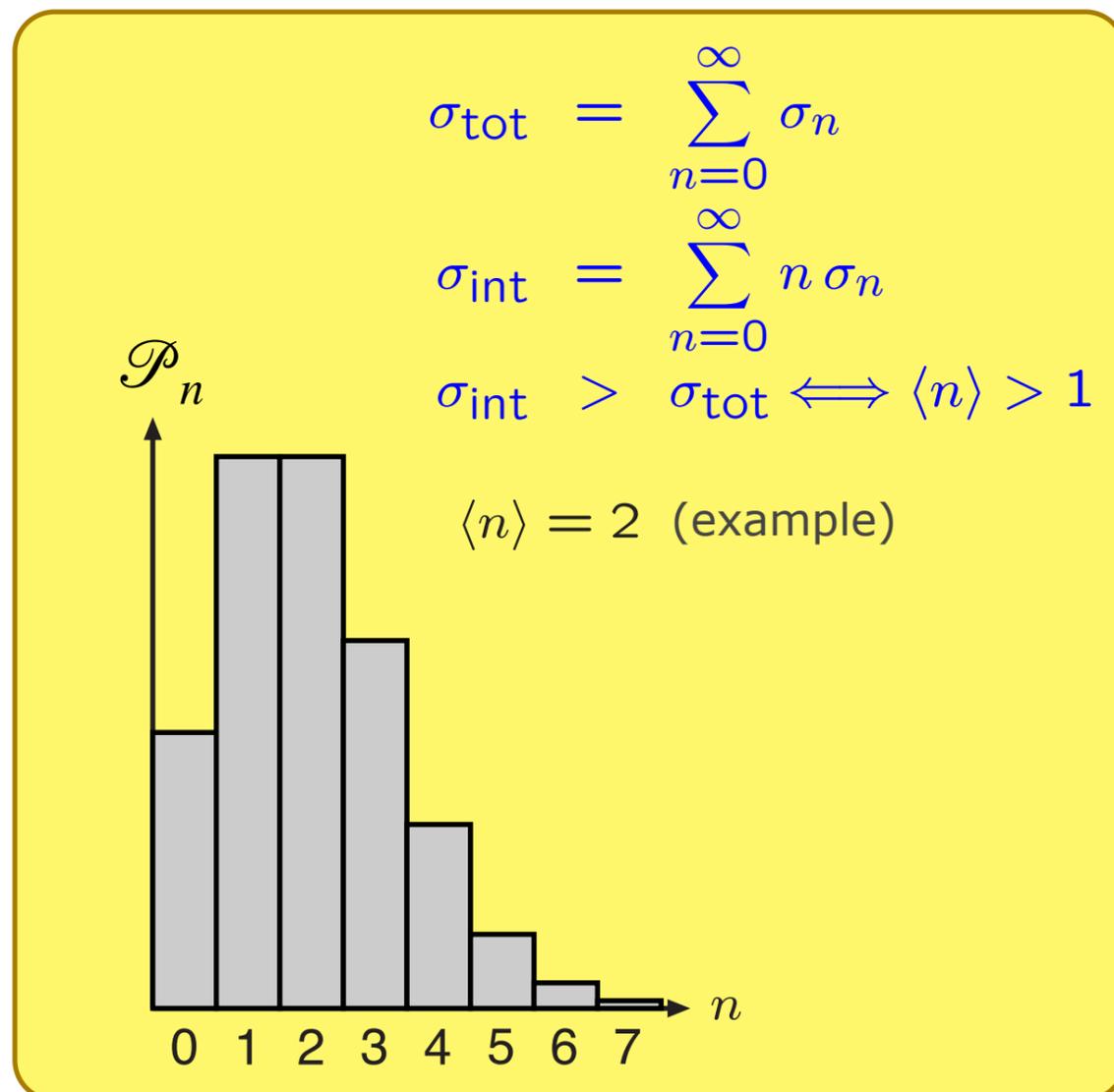
...

+ (mini)-jets  
from tail with  
 $Q_2 \gg 1 \text{ GeV}$

# How many?

Naively 
$$\langle n_{2 \rightarrow 2}(p_{\perp \text{min}}) \rangle = \frac{\sigma_{2 \rightarrow 2}(p_{\perp \text{min}})}{\sigma_{\text{tot}}}$$

If the interactions are assumed  $\sim$  independent (naive factorisation)  $\rightarrow$  Poisson



$$\mathcal{P}_n = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle}$$

## Real Life

Color screening:  $\sigma_{2 \rightarrow 2} \rightarrow 0$  for  $p_{\perp} \rightarrow 0$

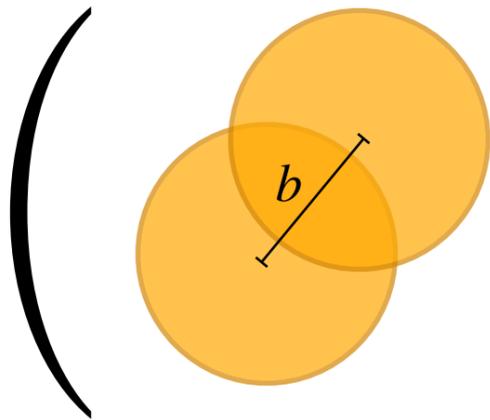
Momentum conservation suppresses high- $n$  tail

Impact-parameter dependence

+ physical correlations

$\rightarrow$  not simple product

# Impact Parameter Dependence



## 1. **Simple Geometry** (in impact-parameter plane)

Simplest idea: smear PDFs across a **uniform disk** of size  $\pi r_p^2$

→ simple geometric overlap factor  $\leq 1$  in dijet cross section

Some collisions have the full overlap, others only partial

⇒ Poisson distribution with different mean  $\langle n_{\text{MPI}} \rangle$  at each  $b$

## 2. More realistic **Proton b-shape** (used by all modern MPI models)

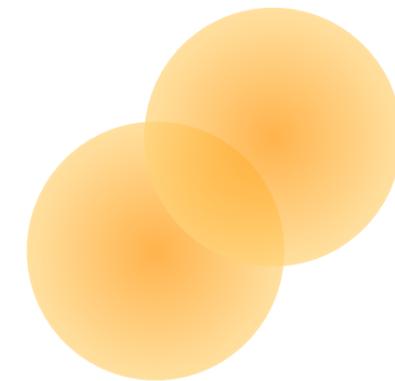
Smear PDFs across a non-uniform disk

E.g., Gaussian(s), or **more**/less peaked (e.g., EM form factor)

Overlap factor = convolution of two such distributions

→ Poisson distribution with different mean  $\langle n \rangle$  at each  $b$

“Lumpy Peaks” → large matter overlap enhancements, higher  $\langle n \rangle$



Note: this is an *effective* description. Not the actual proton mass density. E.g., peak in overlap function ( $\gg 1$ ) can represent unlikely configurations with huge overlap enhancement. Typically use total  $\sigma_{\text{inel}}$  as normalization.

# MC with MPI vs Hadron Collisions

Plots from: Sjöstrand & v. Zijl, Phys.Rev.D36 (1987) 2019

## Fluctuations in $n_{\text{mpi}}$ → Bigger (global) fluctuations

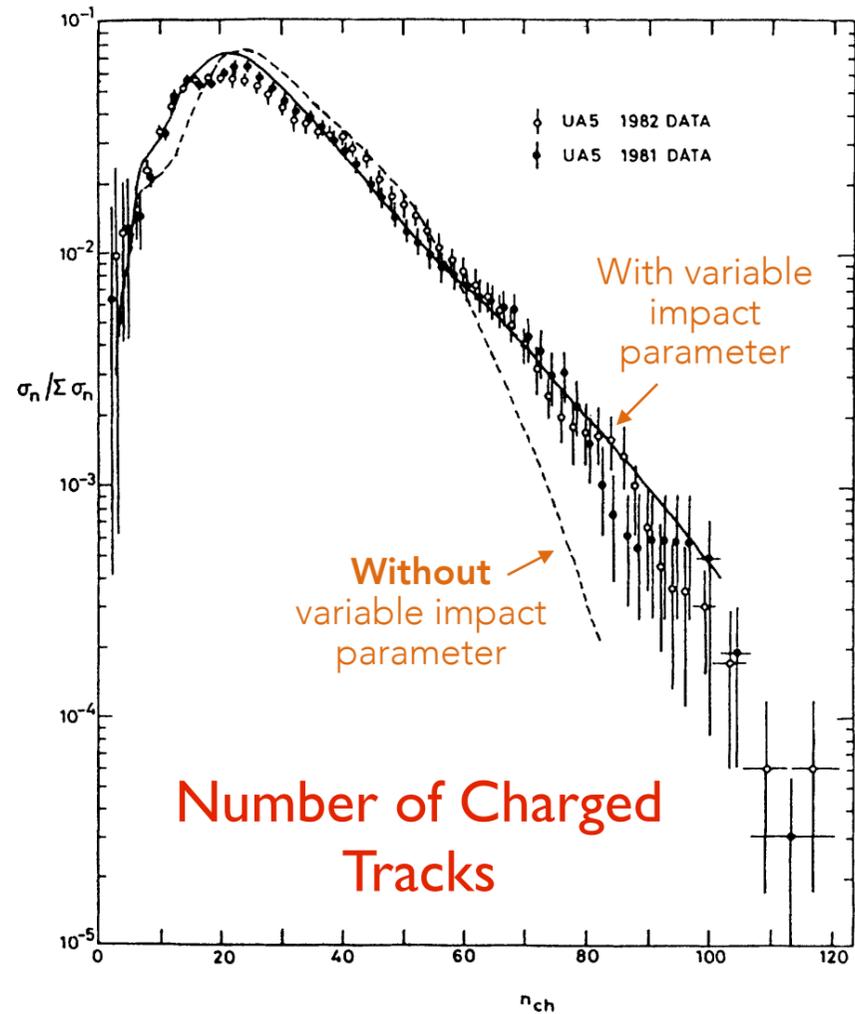
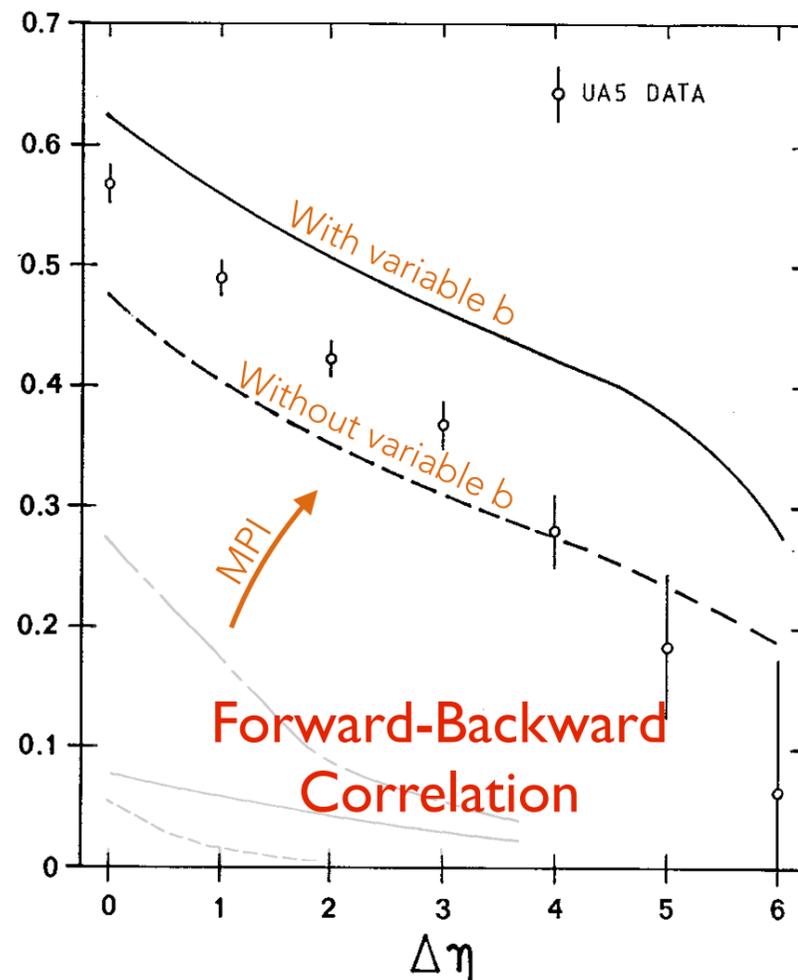
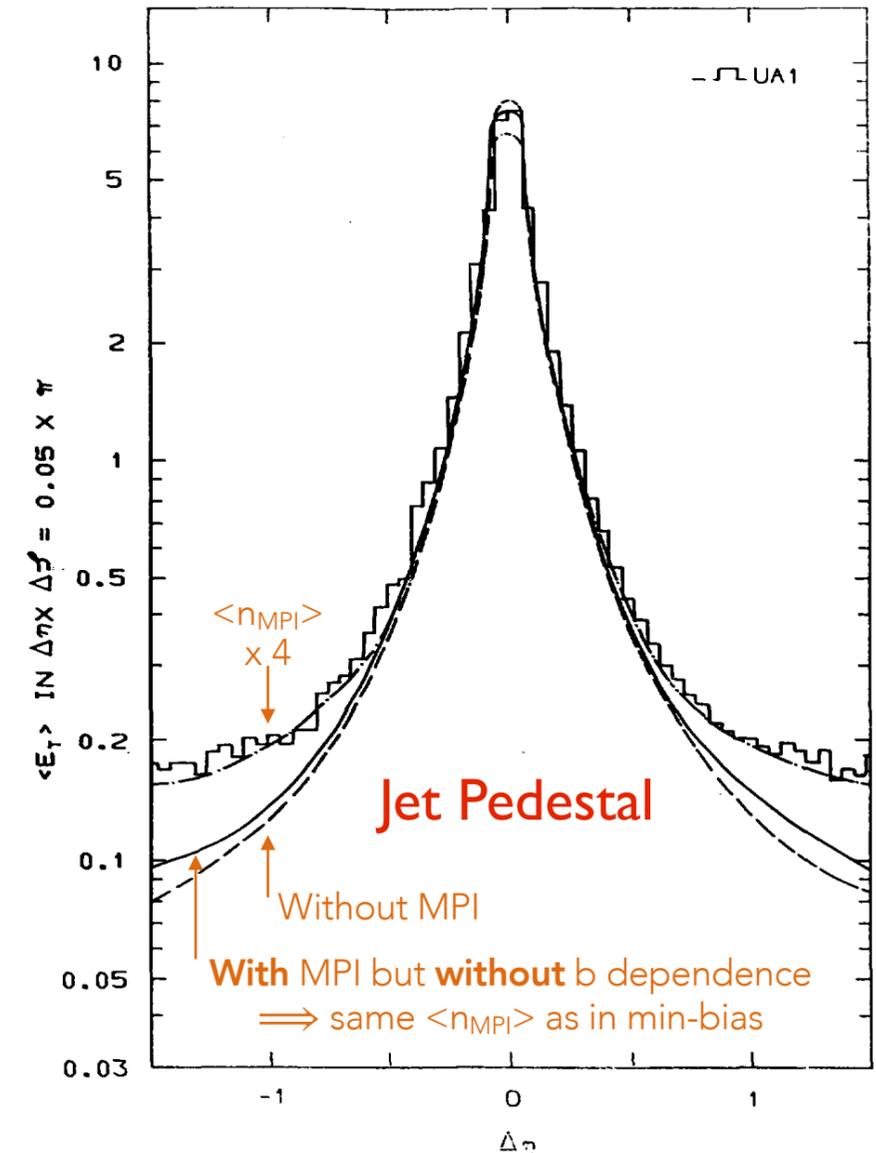


FIG. 12. Charged-multiplicity distribution at 540 GeV, UA5 results (Ref. 32) vs multiple-interaction model with variable impact parameter: solid line, double-Gaussian matter distribution; dashed line, with fix impact parameter [i.e.,  $\tilde{O}_0(b)$ ].



MPI → Long-distance correlations in rapidity

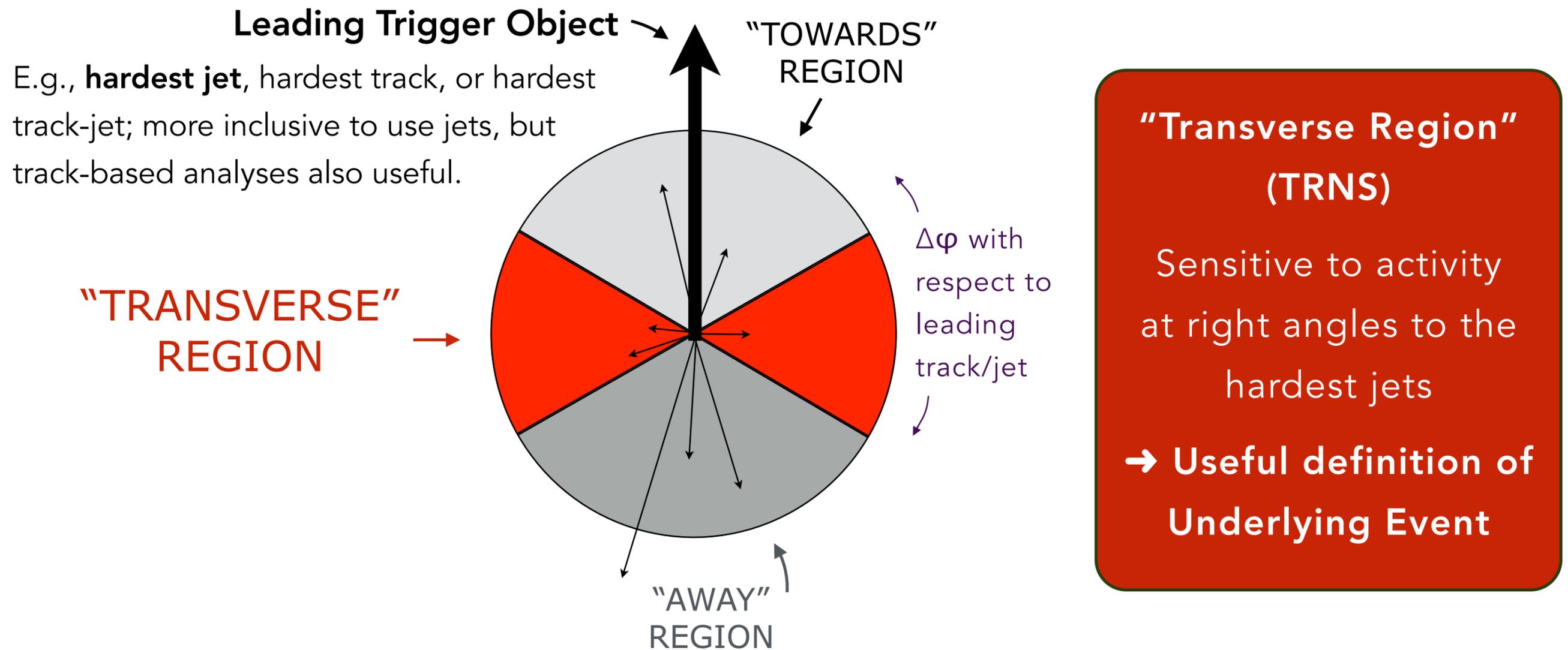


Impact-parameter dependence → UE

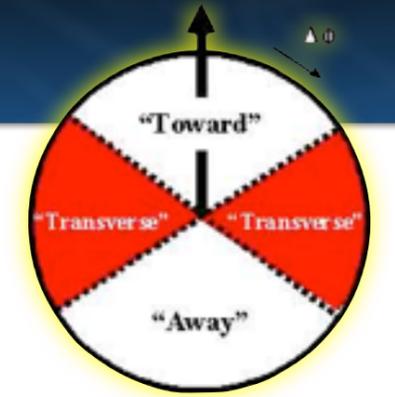
# Characterising The Underlying Event

(The "Rick Field" UE Plots)

There are many UE variables.  
The most important is  $\langle \Sigma p_T \rangle$  in the "Transverse Region"



# Min-Bias VS Underlying Event



Tautology:

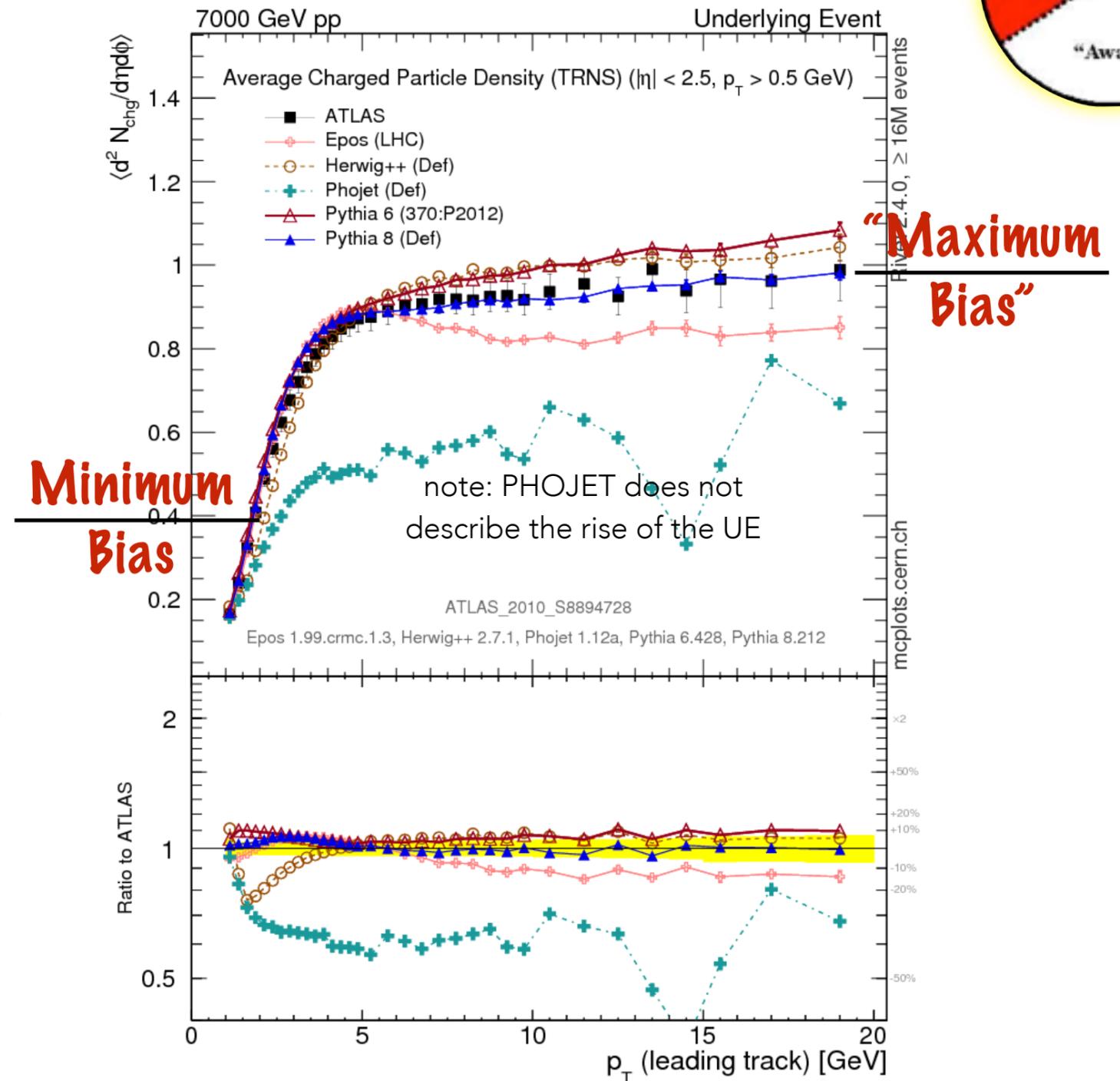
**A jet trigger provides a bias**

(→ subsample of minimum-bias)

Pedestal effect:

Events with a hard jet trigger are accompanied by a higher plateau of ambient activity

**MPI:** interpreted as a biasing effect. Small pp impact parameters → larger matter overlaps → more MPI → higher chances for a hard interaction

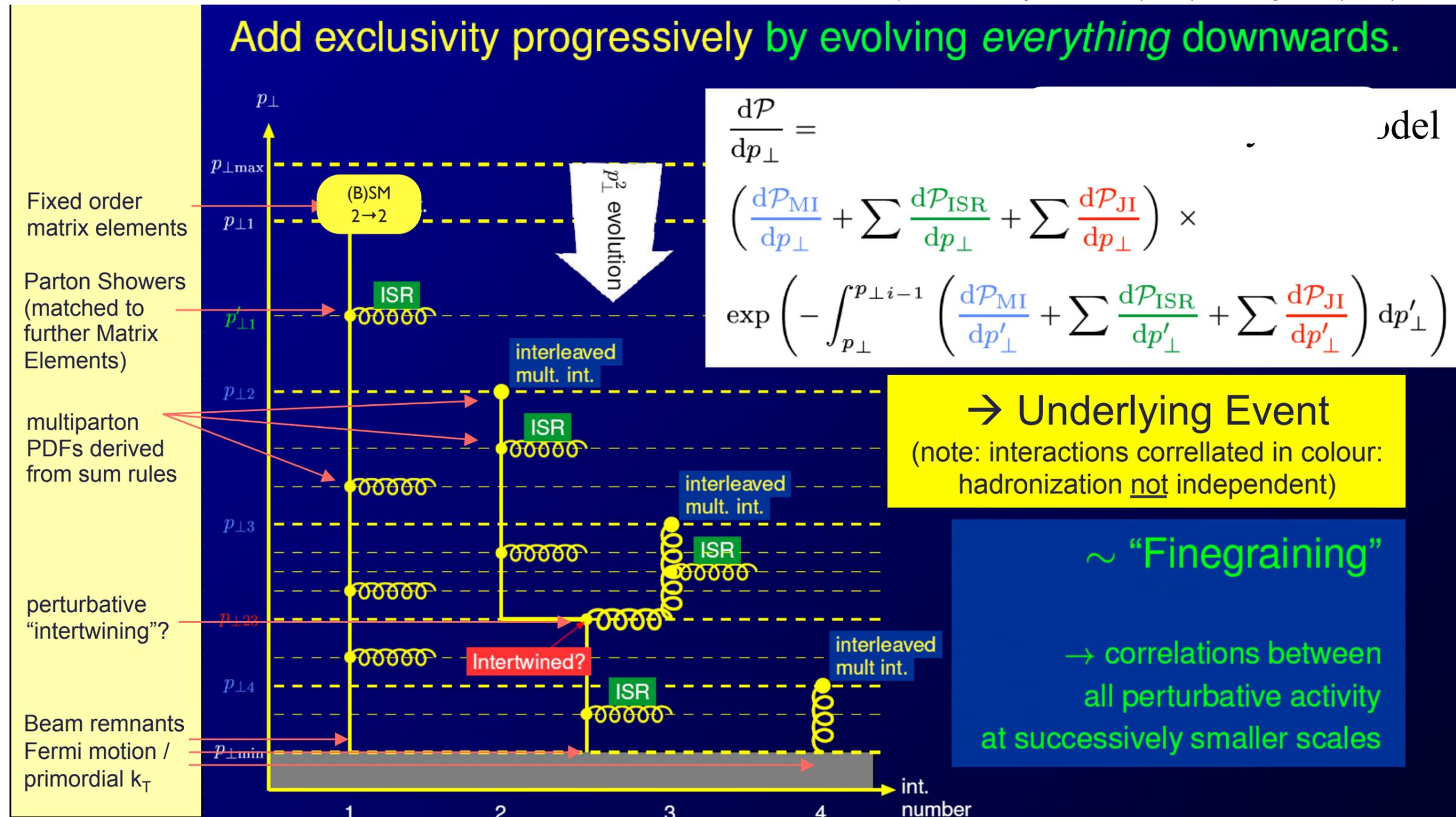


Plot from [mcplots.cern.ch](http://mcplots.cern.ch)

# Interleaved Evolution

## The model in Pythia 8

Sjöstrand, P.S., JHEP 0403 (2004) 053; EPJ C39 (2005) 129

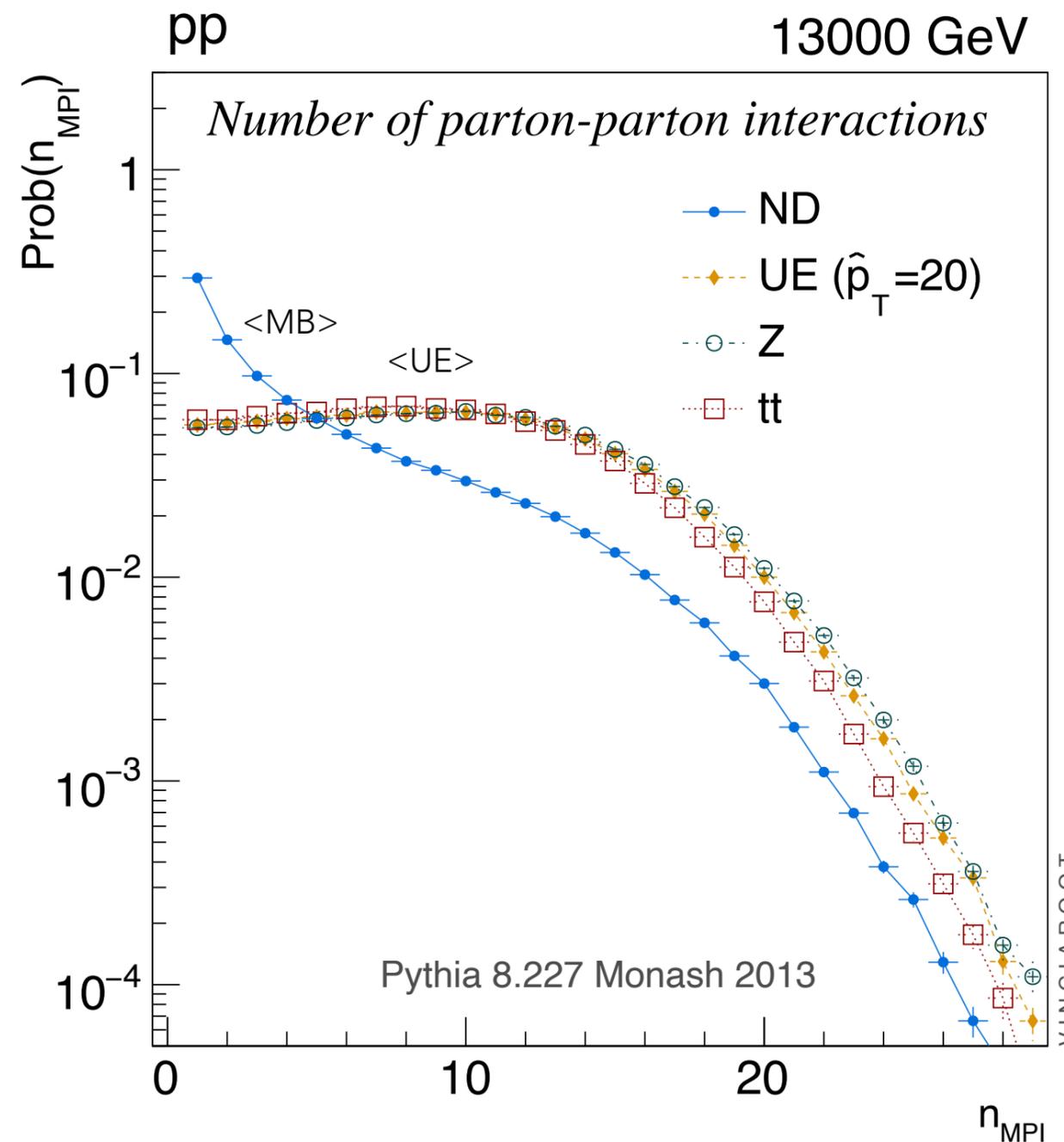


# How many MPI are there? \*

Example for pp collisions at 13 TeV — PYTHIA's default MPI model

Averaged over all pp impact parameters

(Really: averaged over all pp overlap enhancement factors)



\*note: can be arbitrarily soft

# Summary — Divide and Conquer

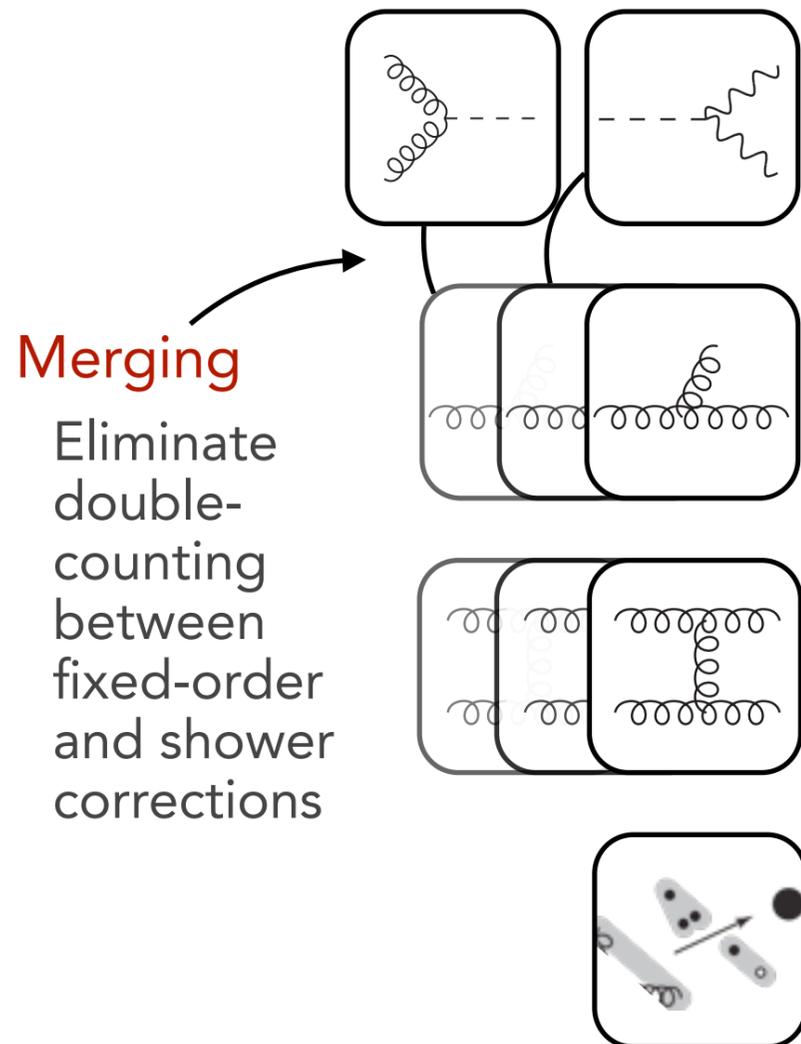
Physics

Separation of time scales ➤ Factorisations

Maths

➔ Can split **big** problem into many (nested) pieces + make random choices (MC)<sup>2</sup> ~ like in nature

$$\mathcal{P}_{\text{event}} = \mathcal{P}_{\text{hard}} \otimes \mathcal{P}_{\text{dec}} \otimes \mathcal{P}_{\text{ISR}} \otimes \mathcal{P}_{\text{FSR}} \otimes \mathcal{P}_{\text{MPI}} \otimes \mathcal{P}_{\text{Had}} \otimes \dots$$



## Hard Process & Decays:

Use process-specific (N)LO matrix elements (e.g.,  $gg \rightarrow H^0 \rightarrow \gamma\gamma$ )  
→ Sets “hard” resolution scale for process:  $Q_{\text{HARD}}$

## ISR & FSR (Initial- & Final-State Radiation):

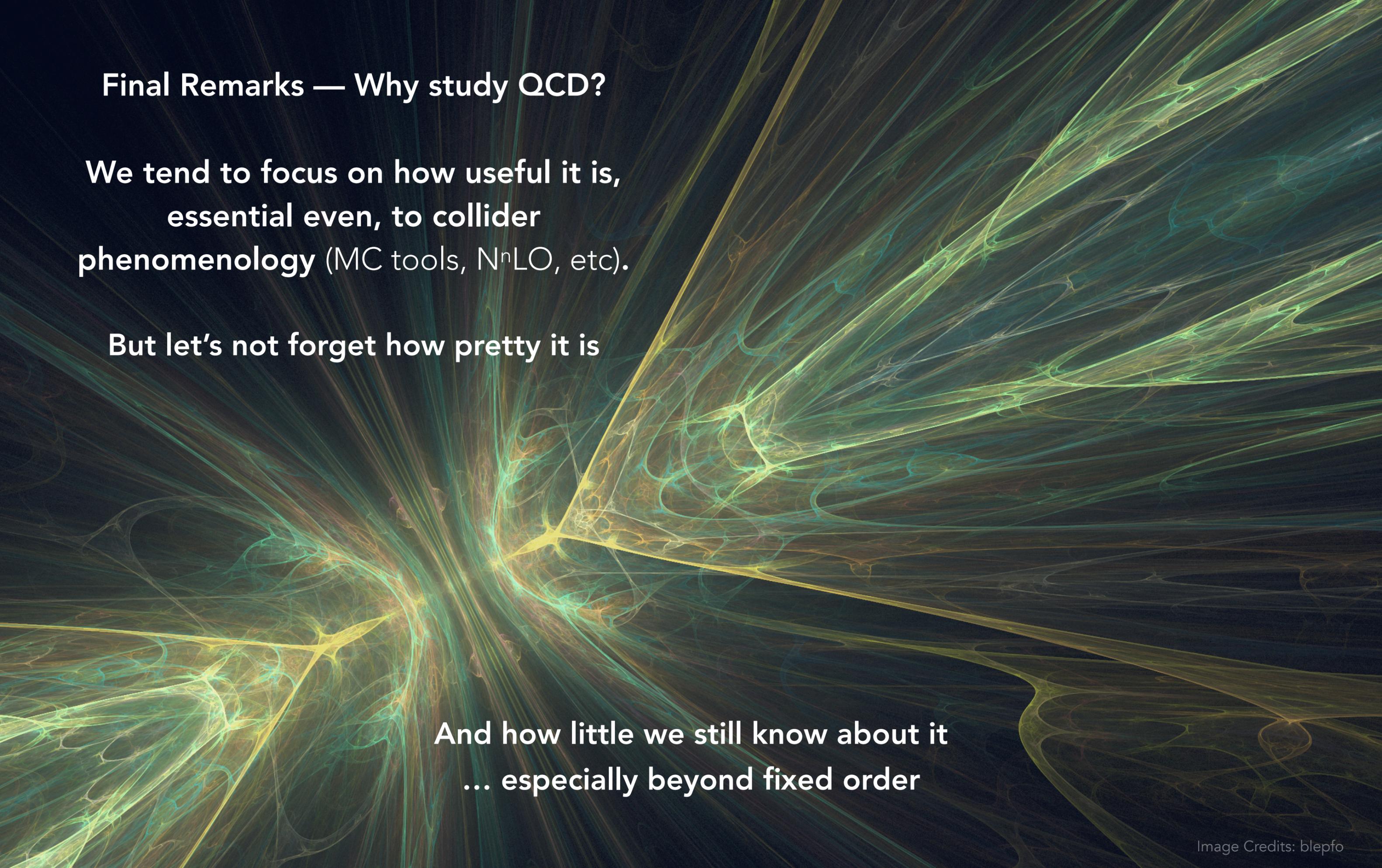
Driven by differential (e.g., DGLAP) evolution equations,  $dP/dQ^2$ , as function of resolution scale; from  $Q_{\text{HARD}}$  to  $Q_{\text{HAD}} \sim 1 \text{ GeV}$

## MPI (Multi-Parton Interactions)

Protons contain lots of partons → can have additional (soft) parton-parton interactions → Additional (soft) “Underlying-Event” activity

## Hadronisation

Non-perturbative modeling of partons → hadrons transition

The background of the slide is a dark blue field filled with intricate, glowing patterns of green and yellow light. These patterns resemble complex, interconnected lines or fibers, possibly representing particle tracks or a network structure. The lines are most concentrated in the center and left side, with some extending towards the right. The overall effect is one of dynamic energy and complexity.

## Final Remarks — Why study QCD?

We tend to focus on how useful it is,  
essential even, to collider  
phenomenology (MC tools, N<sup>n</sup>LO, etc).

But let's not forget how pretty it is

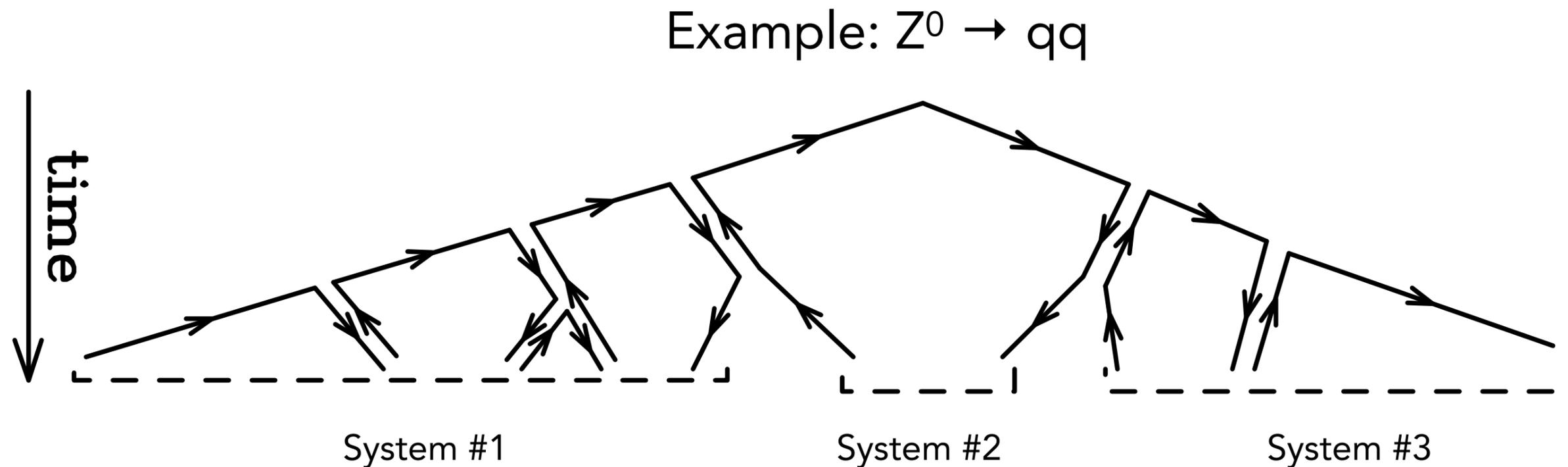
And how little we still know about it  
... especially beyond fixed order

Extra Slides

# RECAP: Colour Flow

## Colour flow in parton showers

(in leading-colour approximation)



**Coherence** of pQCD cascades  $\rightarrow$  not much "overlap" between systems  
 $\rightarrow$  Leading-colour approximation pretty good

(LEP measurements in  $e^+e^- \rightarrow W^+W^- \rightarrow$  hadrons confirm this (at least to order  $10\% \sim 1/N_c^2$ ))

**Note:** (much) more color getting kicked around in hadron collisions. More tomorrow.

# (Note on the Length of Strings)

## In Spacetime:

String tension  $\approx 1$  GeV/fm  $\rightarrow$  a 5-GeV quark can travel 5 fm before all its kinetic energy is transformed to potential energy in the string.

Then it must start moving the other way ( $\rightarrow$  "yo-yo" model of mesons. Note: string breaks  $\rightarrow$  several mesons)

## In Rapidity :

(convenient variable in momentum space)

$$y = \frac{1}{2} \ln \left( \frac{E + p_z}{E - p_z} \right) = \frac{1}{2} \ln \left( \frac{(E + p_z)^2}{E^2 - p_z^2} \right)$$

$$\left( \text{for } m \rightarrow 0 : \frac{1}{2} \ln \left( \frac{1 + \cos \theta}{1 - \cos \theta} \right) = -\ln \tan(\theta/2) = \eta \right)$$

"Pseudorapidity"

Rapidity is useful because it is additive under Lorentz boosts (along the rapidity axis)

$$y' = y + \ln \sqrt{\frac{1 - \beta}{1 + \beta}}$$

$\Rightarrow \Delta y$  **difference** is invariant

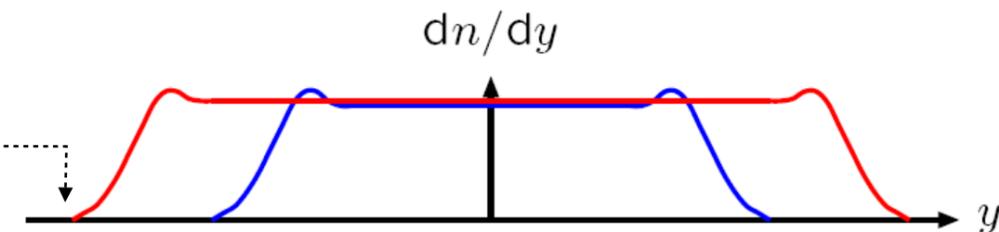
## Particle Production:

If the quark gives all its energy to a single pion traveling along the z axis

$$y_{\max} \sim \ln \left( \frac{2E_q}{m_\pi} \right)$$

**Scaling** in lightcone  $p_\pm = E \pm p_z$

$\Rightarrow$  flat central rapidity plateau (+ some endpoint effects)



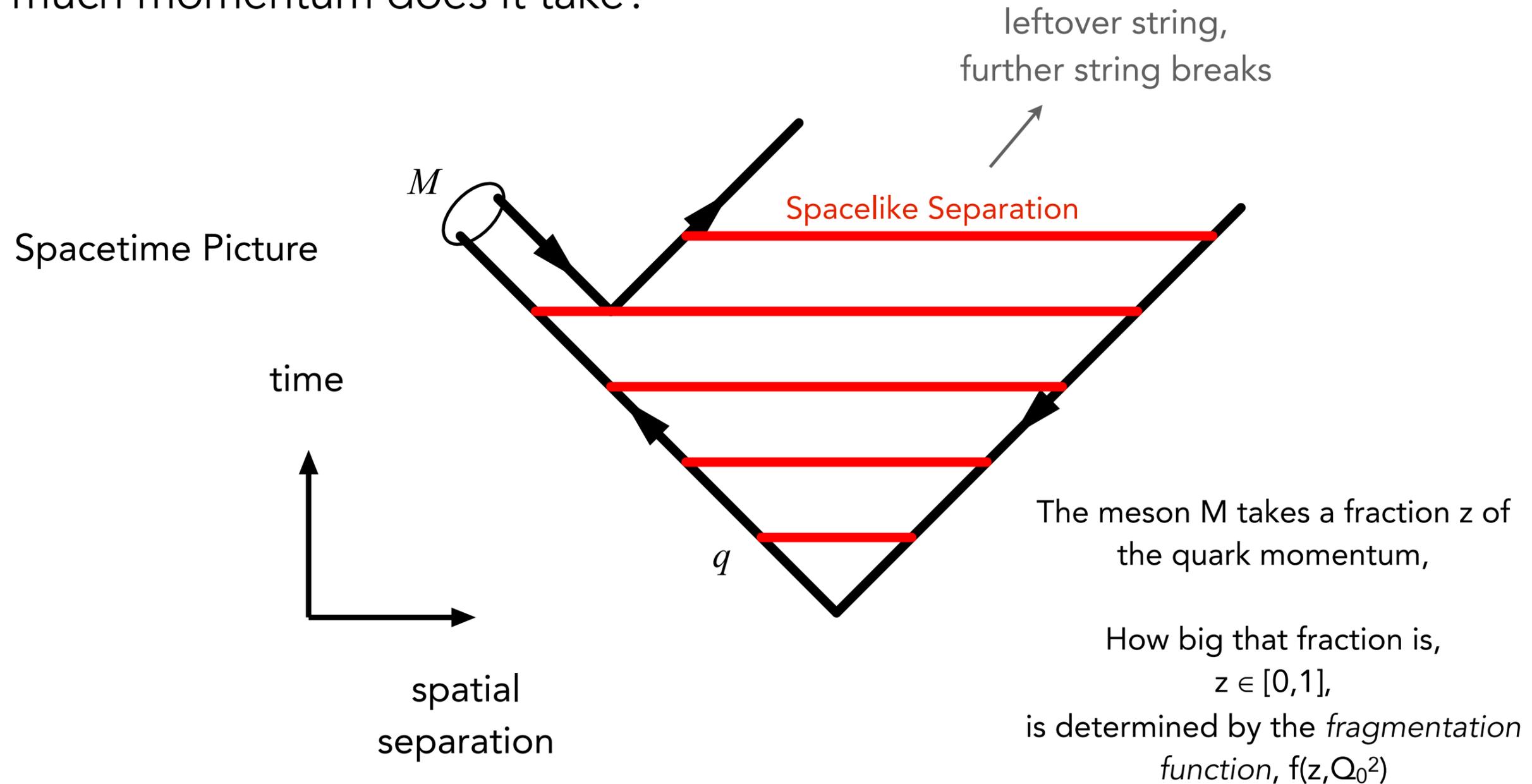
Increasing  $E_q \rightarrow$  logarithmic growth in rapidity range  $\langle n_{\text{ch}} \rangle \approx c_0 + c_1 \ln E_{\text{cm}}, \sim$  Poissonian multiplicity distribution

# Fragmentation Function

Having selected a hadron flavor

(see lecture notes for how selection is made between different spin/excitation states)

How much momentum does it take?

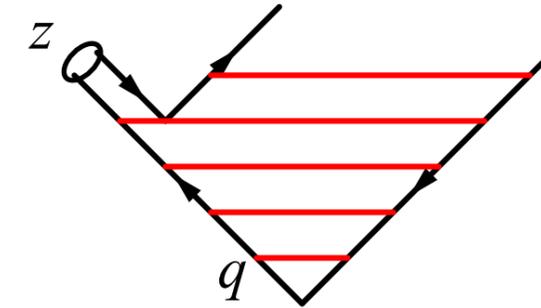


# Left-Right Symmetry

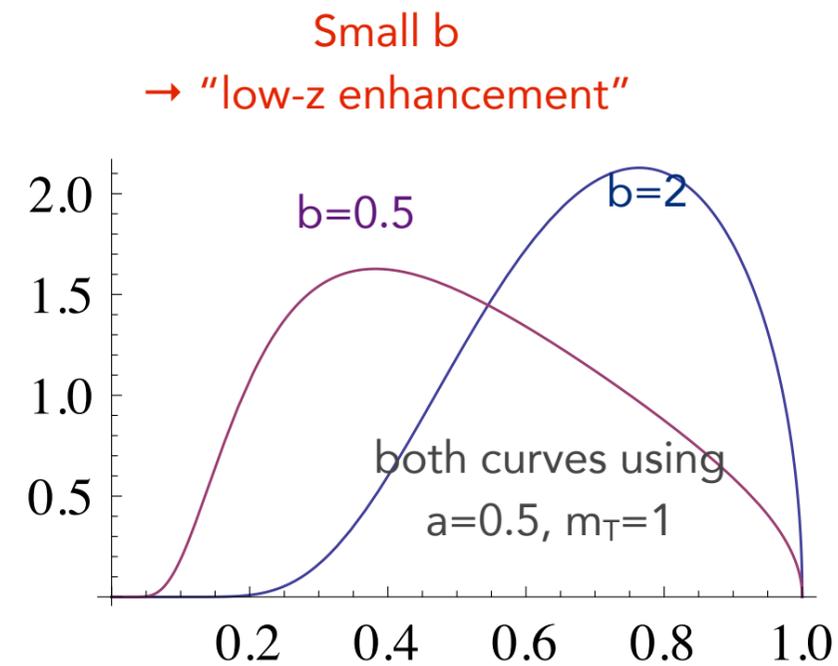
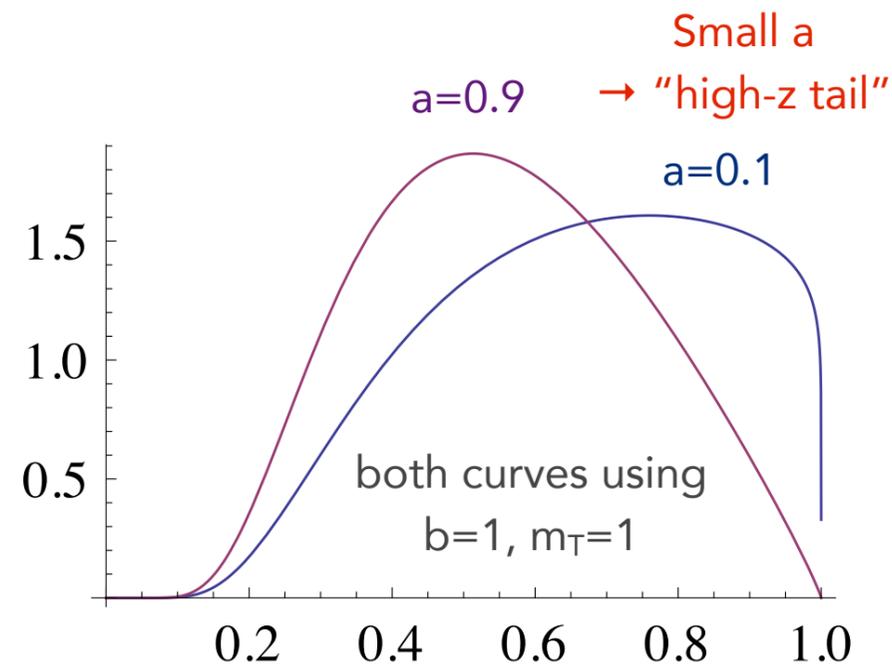
Causality → Left-Right Symmetry

→ Constrains form of fragmentation function!

→ Lund Symmetric Fragmentation Function



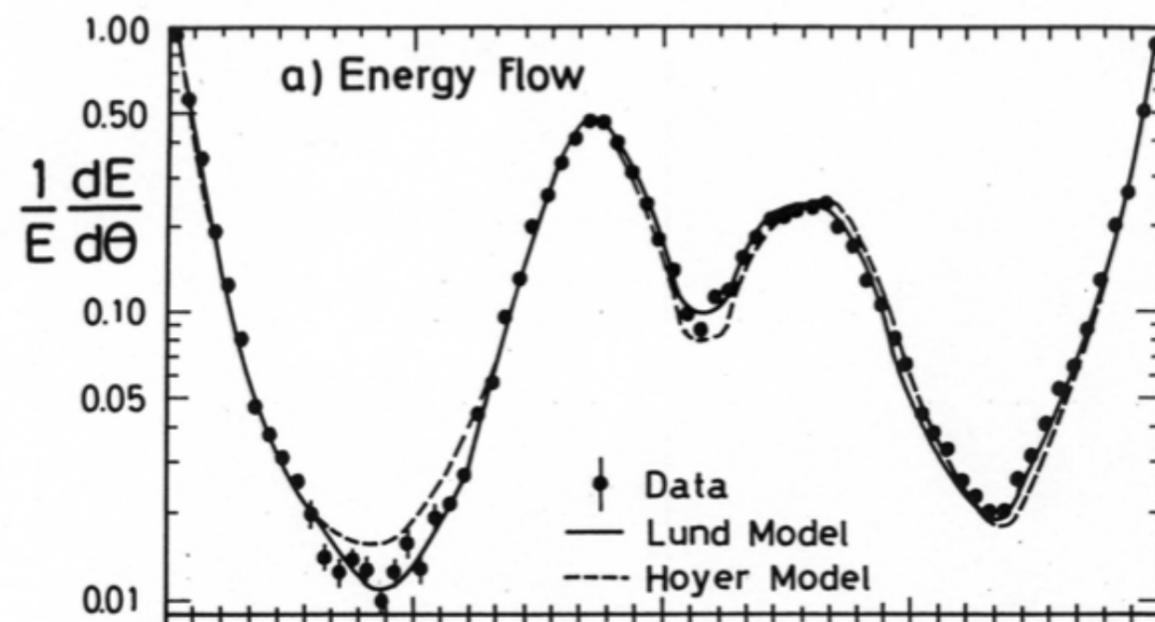
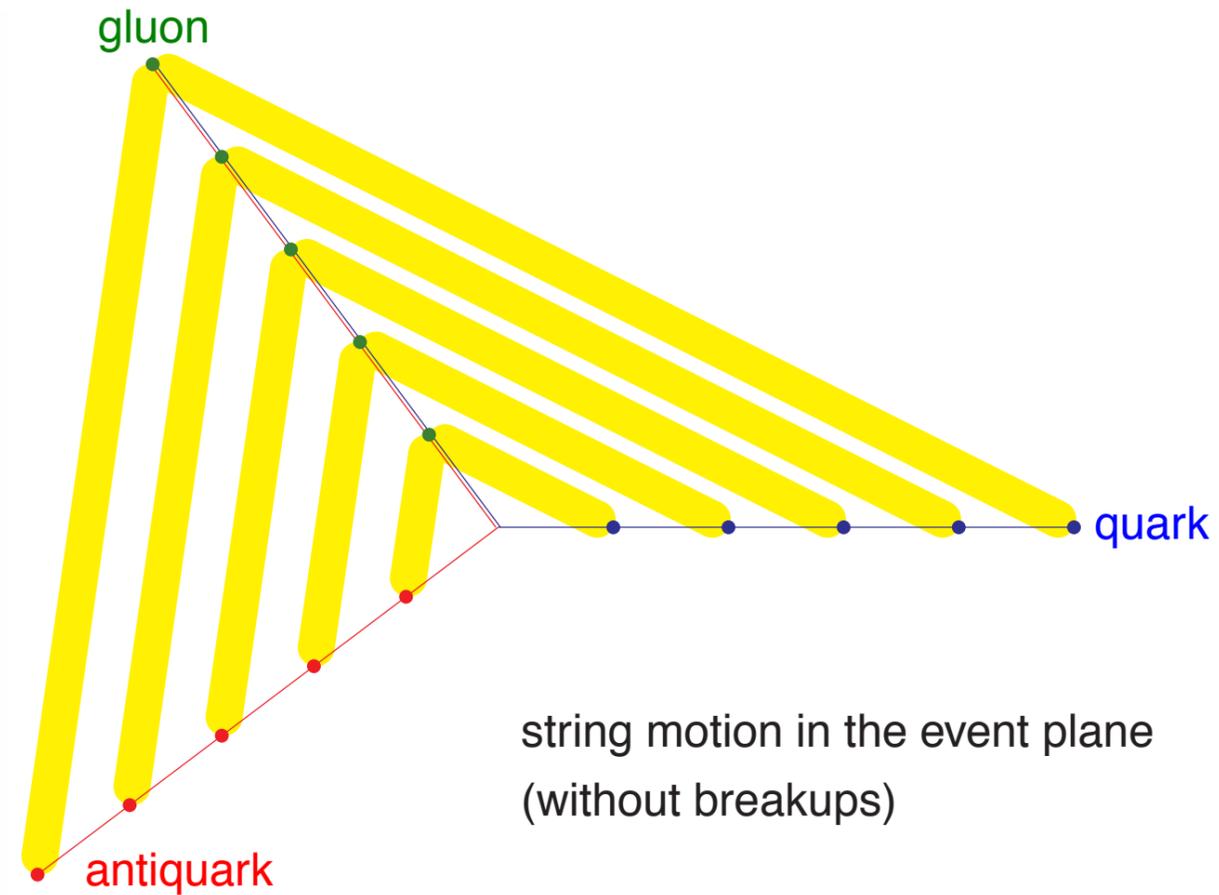
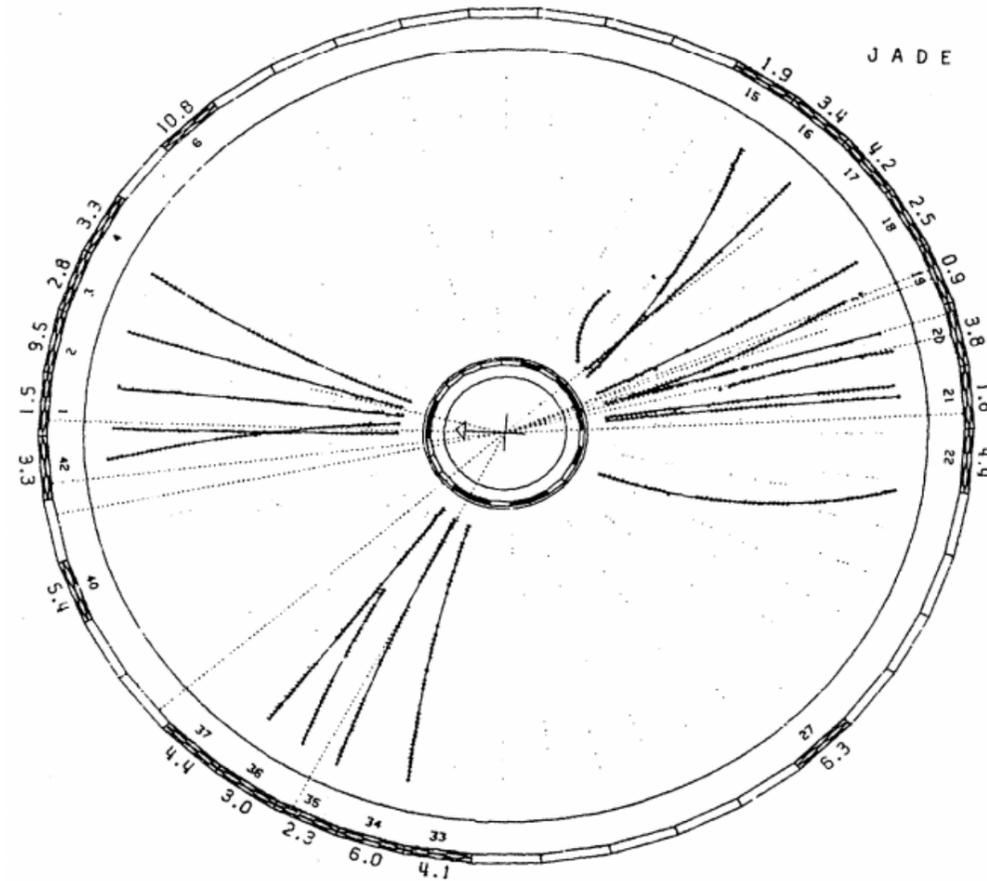
$$f(z) \propto \frac{1}{z} (1-z)^a \exp\left(-\frac{b(m_h^2 + p_{\perp h}^2)}{z}\right)$$



**Note:** In principle,  $a$  can be flavour-dependent. In practice, we only distinguish between baryons and mesons



# 1980: string (colour coherence) effect



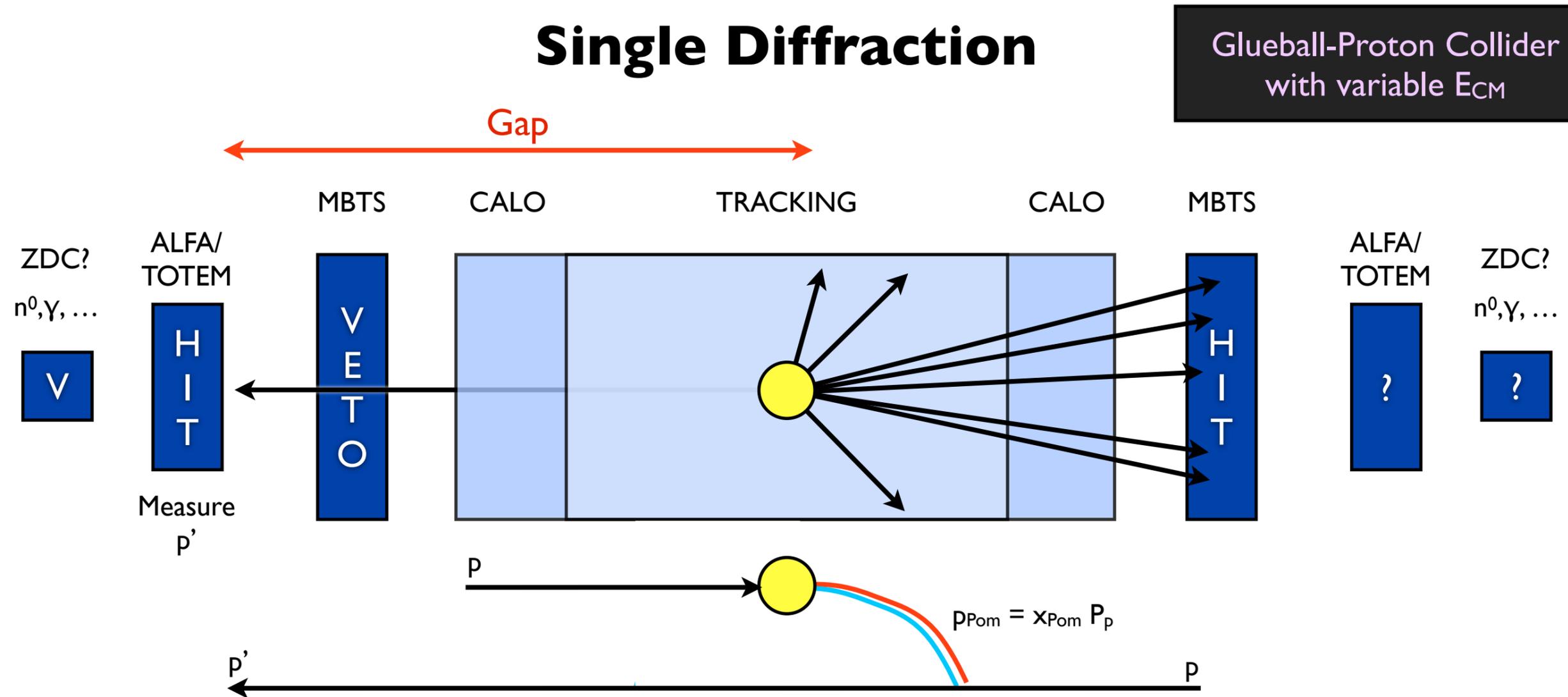
Predicted unique event structure; inside & between jets.

Confirmed first by JADE 1980.

**Generator crucial to sell physics!**

(today: PS, M&M, MPI, ...)

# (Aside: What is diffraction?)



Also:

"Double Diffraction": both protons explode; defined by gap inbetween

"Central Diffraction": two protons + a central (exclusive) system

# 1: A Simple Model

A minimal model incorporating single-parton factorization, perturbative unitarity, and energy-and-momentum conservation

Take literally

$$\sigma_{2 \rightarrow 2}(p_{\perp \min}) = \langle n \rangle(p_{\perp \min}) \sigma_{\text{tot}}$$

Parton-Parton Cross Section Hadron-Hadron Cross Section

## 1. Choose $p_{T\min}$ cutoff

= main tuning parameter

## 2. Interpret $\langle n \rangle(p_{T\min})$ as mean of Poisson distribution

Equivalent to assuming all parton-parton interactions equivalent and independent ~ each take an instantaneous “snapshot” of the proton

## 3. Generate $n$ parton-parton interactions (pQCD 2→2)

Veto if total beam momentum exceeded → overall (E,p) cons

## 4. Add impact-parameter dependence → $\langle n \rangle = \langle n \rangle(b)$

Assume factorization of transverse and longitudinal d.o.f., → PDFs :  $f(x,b) = f(x)g(b)$

$b$  distribution  $\propto$  EM form factor → **JIMMY model (F77 Herwig)**

Constant of proportionality = second main tuning parameter

Ordinary CTEQ, MSTW, NNPDF, ...



← Butterworth, Forshaw, Seymour  
Z.Phys. C72 (1996) 637

## 5. Add separate class of “soft” (zero- $p_T$ ) interactions representing

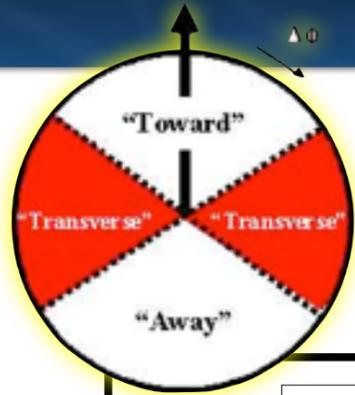
interactions with  $p_T < p_{T\min}$  and require  $\sigma_{\text{soft}} + \sigma_{\text{hard}} = \sigma_{\text{tot}}$

→ **Herwig 7 model**

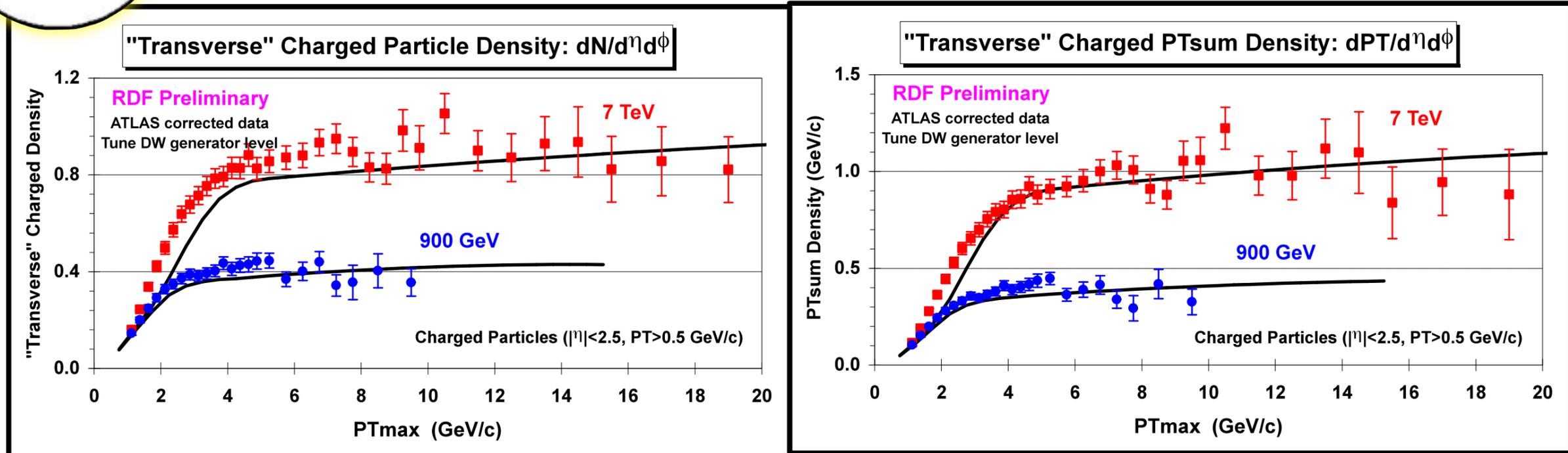
Bähr et al, arXiv:0905.4671

# The Pedestal

(now called the Underlying Event)



LHC from 900 to 7000 GeV - ATLAS



**Track Density (TRANS)**

(Not Infrared Safe)

Large Non-factorizable Corrections

Prediction off by  $\approx 10\%$

**Sum(pT) Density (TRANS)**

(more) Infrared Safe

Large Non-factorizable Corrections

Prediction off by  $< 10\%$

Truth is in the eye of the beholder:

R. Field: "See, I told you!"

Y. Gehrstein: "they have to fudge it again"